

# Rational Overoptimism and Limited Liability \*

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## Abstract

Is excessive risk-taking in credit cycles driven by convex incentives or biased beliefs? I provide a framework where one follows from the other: procyclical overoptimism results rationally from risk-taking incentives. First, I show that payoff structure with limited liability lowers firms and banks incentive to pay attention to the aggregate conditions that generate risk. Second, inattentive agents systematically underestimate the endogenous accumulation of risk during booms. Since they are overoptimistic about future revenues, they over-lend and borrow, further increasing downside risk. Credit cycles driven by this new “uninformed” risk taking are consistent with the existing evidence: low risk premia predict higher default rates, higher probability of crises and negative banks excess returns. My model imply that regulating incentives can reduce overoptimistic beliefs and therefore mitigate boom-and-bust cycles.

**Keywords:** Overoptimism, expectations, information, limited liability, credit cycles

**JEL classification:** D83, D84, E32, G01

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# 1 Introduction

Recent empirical works has revived the longstanding hypothesis that boom-and-bust credit cycles are driven by overoptimistic beliefs (Minsky, 1977; Kindleberger, 1978). In particular, empirical evidence documents that high credit growth and low risk premia significantly predict financial crises (Schularick and Taylor, 2012; Jordà et al., 2013; Krishnamurthy and Muir, 2017; Greenwood et al., 2020). A recent literature ascribes this evidence to overoptimistic beliefs, supported by two additional facts. First, credit booms also predict low and even negative excess returns on bank stocks (Baron and Xiong, 2017). Second, forecasts are systematically too optimistic when credit spreads are low (Bordalo et al., 2018b; Gulen et al., 2019). Behavioral models of extrapolative beliefs have been particularly successful in explaining such systematic bias in belief formation and excess risk taking (Maxted, 2019; Bordalo et al., 2021; Krishnamurthy and Li, 2021). As a result, they shifted away the focus from the role of risk taking incentives, which has been studied in particular with respect to the excess risk taking that led to the recent financial crisis (DeYoung et al., 2013; Boyallian and Ruiz-Verdú, 2018; Armstrong et al., 2022). In this paper I show how biased beliefs might be in fact a consequence of risk taking incentives.

I develop a theory where overoptimism originates from agents' rational choice of dismissing information about the endogenous accumulation of aggregate risk. Furthermore, I show that this inattention may be driven by risk taking incentives. I present these two contributions sequentially. First, I show how the overoptimism driving credit-boom-and-busts originates from inattention to aggregate risk factors. I present a model where aggregate productivity shock leads to an increase in borrowing and production of firms who face the same downward sloping demand for their combined output. Because higher aggregate production implies lower selling price, inattention to competitors' investment decisions cause firms and banks to form overoptimistic expectations about their own revenues. Inattentive firms over-borrow and over-invest, causing an excess supply in the good market which further amplifies the decline in price. As firms' revenues are lower than expected, their default risk increases. My model implies that even fully rational agents can be systematically overoptimistic in credit booms (and overpessimistic in busts). Moreover, because inattentive banks underestimate borrower's probability of default, they misprice risk and register negative excess returns after credit booms, consistently with the existing evidence.

Second, I show that inattention to risk factors can be driven by to risk taking incentives in information choice. Since agents form beliefs rationally, I can use the model to study the incentives that leads them to ignore or pay attention to risk factors. I introduce limited liability in their payoff and allow them to pay a information cost to observe aggregate economic conditions. As convex payoff structures insure agents from risk, they have a lower marginal benefit of information, resulting in lower attention choice. Uninformed firms underestimate the increase in competition and decline in revenues after booms and are overoptimistic about their company's revenues. As a result, limited liability doesn't just lead to excessive risk taking for given beliefs, but also inattention to risk and overoptimistic beliefs in booming periods. This result helps connect the two narratives of excess risk taking before the financial crisis of 2008-2009: the initial criticisms toward managers' moral hazard incentives (e.g. [Blinder 2009](#)) and the following behavioral overoptimism view (e.g., [Gennaioli and Shleifer 2018](#)). I show that overoptimism is in fact a consequence of risk taking incentives, and therefore regulating incentives can attenuate biases in belief formation.

**Model** I embed limited liability and information choice in a macroeconomic model with endogenous default. The model features a continuum of bank-firm pairs, which I refer as islands (as in [Lucas \(1972\)](#)). Firms demand loans from banks in order to finance investment, while banks get funding at a constant risk free rate on international markets. I assume limited liability on firm and bank payoff structures, meaning that the agent managing the company is partially insured from losses (one can think of either manager's compensation convexity, e.g. option and bonuses versus stock holdings, or shareholders payoff, e.g. loan government guarantees or public bailout policies).

I introduce two important elements in an otherwise standard setting. First, strategic substitutability between islands. I assume each firm produces intermediate goods to be sold to an aggregate final good producer with downward sloping demand. Firm's productivity depends on local and aggregate shocks but, because of the competition in the intermediate goods market, firms benefit more from local than aggregate shocks. Local shocks improve firm's fundamentals and reduce its default probability, resulting in higher equilibrium debt and lower spreads. On the other hand, aggregate shocks also increase production of competitors and therefore lower firm's expected revenue and increase its default probability relative to a local shock with the same magnitude. While the first effect

is standard in the literature that abstracts from competition between islands, the second effect is novel and implies a strategic interaction between islands.

Second, I introduce incomplete information. Following the Lucas island framework, I assume agents face an information friction in observing aggregate economic condition. In particular, I allow bank and firm managers on each island to pay an information cost to observe aggregate shocks and therefore investment decisions of competitors.<sup>1</sup> This assumption is in line with [Coibion et al. \(2018\)](#)'s survey evidence of large dispersion in managers' beliefs not only about future economic condition, but also current ones.

I compare the model's implications in two limit cases, full information and dispersed information. First, I show that the full information model is not able to qualitatively match the existing evidence on risk premia in a credit boom. If companies observe aggregate shocks, they internalize the negative effect on revenues coming from the increase in competition, and therefore they reduce their investment and borrowing. The economy is always safer during a credit boom, and risk premia are lower. Even if the negative price externality has a dampening effect on the credit boom, the model is qualitative similar to a standard model without this additional channel ([Strebulaev and Whited, 2011](#)). Because the economy is safer after a boom, the model does not match the existing evidence.

The model with dispersed information is instead able to match the existing evidence on credit cycles. If agents do not observe aggregates, they confound aggregate and local shocks. This is consistent with the empirical evidence that firms' expectations about aggregate economic conditions respond to industry-specific shocks even though these shocks have no aggregate effects ([Andrade et al., 2022](#)). Similarly, after aggregate shocks agents incorrectly attribute the higher productivity at least partly to a local shock and underestimate the increase in production of competitors. As a result, they over-borrow and over-invest, further overheating the economy. Even if perceived risk and risk premia decline, default rate increases. The model is consistent with the existing empirical evidence. First, credit growth predicts higher average probability of default ([Krishnamurthy and Muir, 2017](#)). Second, low risk premia also predict higher average probability of default ([Krishnamurthy and Muir, 2017](#)). Third, bank's excess return during the boom and bust is negative on average ([Baron and Xiong, 2017](#)).

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<sup>1</sup> I follow the rational inattention literature ([Sims, 2003, 2006](#)) in interpreting the information cost as a cognitive cost agents pay in order to processing information which could be freely accessible.

Next, I allow agents to pay an attention cost to observe aggregate conditions and show that limited liability in payoff discourages them to collect this information. I show that limited liability makes companies less exposed to risk and therefore lowers their marginal benefit of information, discouraging them to collect information. As a result, they will be inattentive to the endogenous increase in risk during credit booms. Importantly, I show that the procyclicality of risk is a consequences of this information channel of risk taking incentives, and not the standard “informed” risk taking. In order to isolate the information channel of limited liability, I shut down information choice and allow agents to observe aggregates. I show that in my baseline calibration limited liability increases unconditional risk taking, but conditional on aggregate shocks the economy is safer. On the other hand, with endogenous information limited liability discourages attention allocation, which results in excessive “uninformed” risk taking in booms and procyclical risk.

Finally, I embed the model in a infinite-periods framework to study its implication for credit cycles and relate it to the existing evidence. I show that the model with a realistic calibration is able to reproduce two important sets of moments in the data. First, my model matches the systematic decrease in spreads and increase in credit growth before financial crises. Second, it reproduces the predictive power of decline in spreads and increase in credit in forecasting financial crises.

As beliefs depend on incentives, my model implies that policy makers can reduce overoptimism in credit booms by regulating incentives to collect information. Informed agents reduce borrowing and investment in credit booms, mitigating economic fluctuations. Information provision through public announcement or direct communication could improve risk assessment, but public information could still be costly for agents to process (Sims, 2003, 2006). Instead, reducing risk taking incentives by acting on payoffs, for example by regulating managers’ compensation, would not only solve their “informed” excess risk taking, but also encourage them to pay attention to aggregate risk factors.

**Contribution to the literature** This paper contributes to several strands of the literature. First, the growing body of research about credit cycles. In addition to the already mentioned empirical work (Schularick and Taylor, 2012; Jordà et al., 2013; Krishnamurthy and Muir, 2017; Baron and Xiong, 2017; López-Salido et al., 2017; Mian et al., 2017; Greenwood et al., 2020), this paper relates to the theoretical research on financial crises, which can be divided in two categories. The first emphasizes the role of behavioral bias in belief

formation and credit market sentiments (Bordalo et al., 2018b; Greenwood et al., 2019; Maxted, 2019; Farhi and Werning, 2020). The most related is Bordalo et al. (2021), which embeds extrapolative expectations in a firm dynamic model with lending and default. In their model, beliefs overreact to good news, leading to overoptimism in credit booms. In my model overoptimism originates instead from rational underreaction to bad news. As a result, forecast errors exhibits predictability even in a fully rational setting.

A second line of research emphasizes the role of financial frictions in intermediation as sources of fragility (Gertler and Kiyotaki, 2010; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2019; Jeanne and Korinek, 2019; Bianchi and Mendoza, 2020). This class of models use full information and strategic complementarity in leverage choices to rationalize the overaccumulation of debt during booms, as individuals do not internalize the externality effects of their decision on the whole economy. In other words, investors ride the bubble as long as others ride it. Differently from them, in my model financial fragility originates from strategic substitutability and incomplete information. If investors knew about the increase in aggregate risk, they would reduce leverage and therefore reduce risk. In other word, they would like to exit the bubble before it burst. The lack of information is what lead them to accumulate risk, resulting in a unexpected boom-and-bust.

A third line of research emphasize moral hazard incentives in investment and lending as a source of procyclical risk taking. Coimbra and Rey (2017) show how limited liability due to government guarantees on banks produces a risk-shifting behavior when interest rates decreases from a certain threshold. Similarly, in Martinez-Miera and Repullo (2017) a decline in interest rates produces risk taking incentives due to the limited liability of borrowing banks against lending investors. In both papers, lenders are informed about borrowers risk and they do not accept systematic negative returns on booms. Differently from them, I show that limited liability also produces inattention to risk and overoptimistic beliefs in boom, which leads to systematic losses by lenders, even after an improvement in fundamentals.

My paper also relates to the literature on strategic games with incomplete information (Woodford, 2001; Coibion and Gorodnichenko, 2012; Maćkowiak and Wiederholt, 2015; Benhima, 2019). While dispersed information and strategic substitutability lead to amplification of partial equilibrium effects as in Angeletos and Lian (2017), I study its implication

for pricing of risk in credit booms. Similarly to [Kohlhas and Walther \(2020\)](#), agents here pay asymmetric attention to local and aggregate quantities, which leads to “extrapolative beliefs” even in a rational setting. Differently from them, the determinant of the attention allocation is not the difference in shock volatility, but risk taking incentives.

Finally, this paper contributes to the literature on compensation incentives. In addition to the large body of research on CEO compensation (see [Edmans et al. 2017](#) for a review), I mostly relate to the works studying the impact of compensation on information. [Mackowiak and Wiederholt \(2012\)](#) show that limited liability reduces optimal information choice, while [Lindbeck and Weibull \(2017\)](#) study optimal contracts between principal and manager in rational inattention setting. Differently from them, this paper abstracts from optimal contracts, and embeds the link between risk taking incentives and information in a macro-financial model with endogenous risk. On the empirical side, [Cole et al. \(2014\)](#) provides experimental evidence on the impact of compensation on loan officers’ screening effort.

**Structure of the paper** The remaining sections of the paper are organized as follows. In section 2 I provide some motivational evidence on the importance of information friction in expectations surveys. In section 3 I propose a rational model of endogenous credit booms-and-bust driven by information frictions. Following the literature on endogenous information models, I present the result of the model backwards. In section 4 I provide analytical and numerical solutions of the second stage equilibrium, i.e. the lending and investment choice problem for a given information structure. In section 5 I present the solution of the first stage equilibrium, i.e. the information choice problem. In section 6 I extend the baseline model to a dynamic setting and relate it to existing evidence on credit cycles. Section 7 discusses policy implications and section 8 concludes.

## 2 Motivational Evidence on Beliefs in Booms

While existing theories of overoptimism preserve full information and depart from rational expectation, in this section I provide suggestive evidence which points towards the importance of information frictions in business cycles. In particular, I document that aggregate beliefs under-react to changes in macroeconomic quantities in booms and busts, consistent



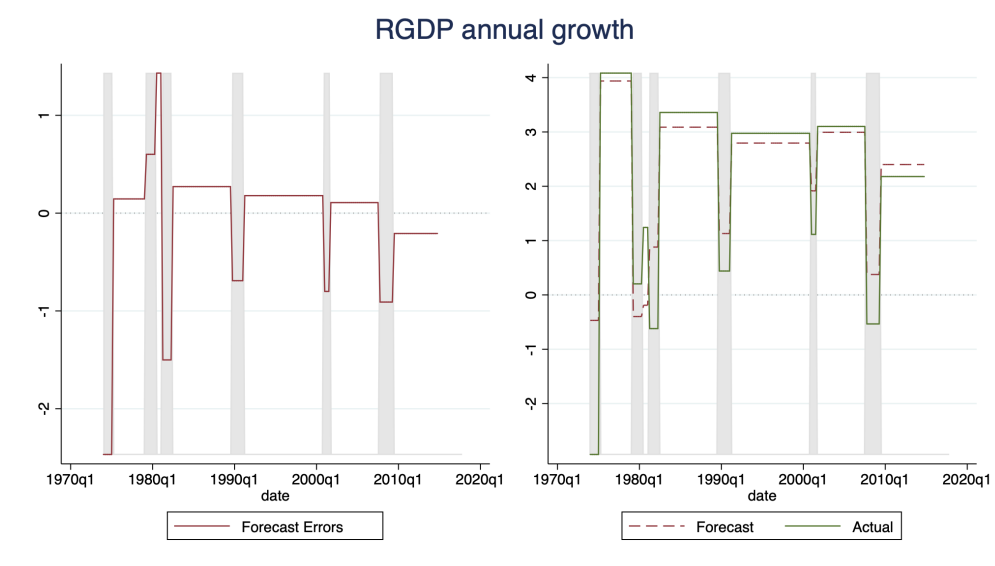


Figure 1: Forecast errors on Real GDP growth

*Notes:* Left panel: the red line plots the forecast errors on annualized real GDP growth averaged between shaded area. Forecast errors are defined as  $f_{e_t} = x_t - f_t(x_t)$ , where  $x_t$  is the average annualized growth of real GDP in the current and the next three quarters, and  $f_t(x_t)$  the average (consensus) forecast in quarter  $t$  about annualized growth of real GDP in the current and the next three quarters. The shaded area indicates the NBER recession dates. Right panel: the dashed red line plots the average forecast on annualized real GDP growth  $f_t(x_t)$ , while the solid green line the actual real GDP growth  $x_t$ . All expectation data are from the Survey of Professional Forecasters, collected by the Federal reserve's Bank of Philadelphia

with models of dispersed information.<sup>2</sup>

First, I look at business cycle frequency fluctuations of forecast errors on real GDP growth by comparing the average errors in booms and recessions. Forecast errors are defined as  $f_{e_t} = x_t - f_t(x_t)$ , where  $x_t$  is the average annualized growth of real GDP in the current and the next three quarters, and  $f_t(x_t)$  the average (consensus) forecast in quarter  $t$  about annualized growth of real GDP at the same horizon. Forecast data are from the Survey of Professional Forecasters, and a positive forecast errors imply that the consensus forecast underestimate the actual GDP growth. Figure 1 shows that forecasters underestimate real output during booms and overestimate them during NBER recessions. This evidence suggests that at the aggregate level expectations display underreaction to changes in macroeconomic quantities.

In addition to the business cycle frequency, I provide suggestive evidence for belief under-reaction in the most recent credit boom-and-bust episode. Financial crises are less

<sup>2</sup> A leading behavioral theory of overoptimism is belief extrapolation, and in particular diagnostic expectations, which causes agents to over-react to recent news (Gennaioli and Shleifer, 2010; Bordalo et al., 2018b, 2021).



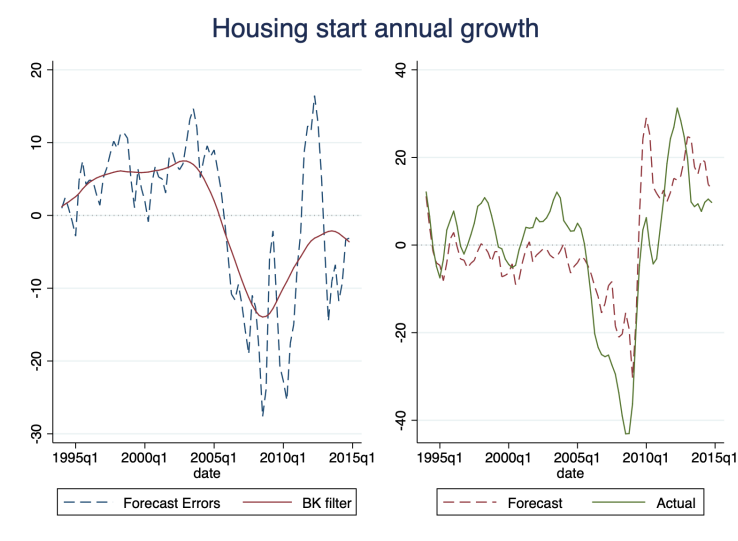


Figure 2: Forecast errors on Housing Start

*Notes:* The blue line plots the forecast errors on annualized housing start growth from the Survey of Professional Forecasters, collected by the Federal reserve's Bank of Philadelphia. Forecast errors are defined as  $fe_t = x_t - f_t(x_t)$ , where  $x_t$  is the average annualized growth of housing starts in the current and the next three quarters, and  $f_t(x_t)$  the average (consensus) forecast in quarter  $t$  about annualized growth of housing starts in the current and the next three quarters. The red line plot the Baxter-King filtered trend, where I filtered out periods lower than 32.

frequent than business cycle recession, and given the limited time span of expectations data the only meaningful credit boom-and-bust I can consider is the recent financial crisis of 2007-2008. Figure 2 plots annualized growth forecasts and realizations of housing starts, averaged across the current and the next three quarters. The pattern is similar to the previous figure and it suggests that forecasters underestimated housing starts growth during the boom. In the next section I show how underestimation of an increase in supply leads to overestimation of the equilibrium market price, which might shed some light on the apparent overoptimism that boosted the housing bubble in the years preceding the crisis.

In addition to the evidence reported here, a growing literature employs surveys of professional forecasters to document the importance of information frictions against the full information hypothesis (Coibion and Gorodnichenko, 2012, 2015; Gemmi and Valchev, 2022).<sup>3</sup> The evidence of aggregate stickiness in belief updating supports model of dis-

<sup>3</sup> Bordalo et al. (2018a) provides evidence supporting behavioral overreaction in survey individual-level forecasts on financial and macroeconomic variables. However, they still find dispersed information and belief stickiness at the consensus level. Moreover, Gemmi and Valchev (2022) provide further evidence on survey individual forecast which are inconsistent with the diagnostic expectation framework.

persed information, where agents have access to different information and are always in disagreement about the fundamentals. Moreover, the professional forecaster's expectations data I use here are likely to underestimate the amount of information friction of firms. In line with this, [Coibion et al. \(2018\)](#) study firm's level expectation and find higher information frictions: managers' expectations display much more disagreement than professional forecasters, and this disagreement applies to both future and current economic condition. Moreover, they find that their belief updating is consistent with the Bayesian framework and their attention allocation to aggregates depends on incentives.

In summary, the evidence on aggregate expectations are consistent with information frictions that hinder the diffusion of information or the incorporation of new information in agent's beliefs ([Sims, 2003](#); [Woodford, 2001](#)). In the following section I present a model consistent with the data, where overoptimism originates from incomplete information about aggregate quantities.

### 3 Model of inattentive credit booms

The economy is populated by a continuum of islands  $j \in [0, 1]$  and each island is populated by a firm-bank pair.<sup>4</sup> Banks in each island collect funds at the risk free rate in international markets and lend to the firm at a premium above the funding rate to cover for repayment risk. Firms borrow from banks in order to finance investment and production of intermediate goods, which they sell to a unique aggregate final good producer. If revenues are higher than outstanding debt, the firm repays the bank and keep the net profit, and otherwise it defaults.

**Timeline** The model is divided in three stages. First, before receiving any information each bank-firm pair decides whether they want to observe aggregate shocks in the next stage. Second, they observe information and bargain on loans and loan rates. Finally, shocks realize and firms repay or default. Rather than a description of business cycles, the model is intended to describe the phases of a financial bubble, with the second stage representing the building up of the bubble and the third stage its burst.

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<sup>4</sup> The island assumption reflects the importance of banking relationship and the cost faced by borrowers in switching lender ([Chodorow-Reich, 2014](#)). I assume that the sorting of lenders and borrowers across island happens before markets open and information is observed, when there is no heterogeneity in firms and banks characteristics.

**Final good producer** The economy features a representative final good producer, acquiring a bundle of intermediate goods  $M = \left[ \int^j M_j^\xi dj \right]^{\frac{1}{\xi}}$  with elasticity of substitution  $\frac{1}{1-\xi}$ , in order to produce final good with production function  $Y = M^\nu$ . Therefore, the demand function for intermediate goods  $M_j$  in stage 3 equals:

$$p_j = \nu M^{\nu-\xi} M_j^{\xi-1} \quad (1)$$

The demand for intermediate good  $M_j$  could increase or decrease in aggregate production  $M$  depending on the degree of decreasing return to scale in final good production and the elasticity of substitution between goods. If  $\nu < \xi$ , we have a negative production externality: higher aggregate supply of intermediates  $M$  lead to lower price  $p_j$  and therefore lower revenues for intermediate producer  $j$ . Conversely, if  $\nu > \xi$ , we have a positive production externality: higher aggregate supply of intermediates  $M$  lead to higher price  $p_j$  and therefore higher revenues for intermediate producer  $j$ .

**Firms** In the second stage, firms in island  $j$  borrows  $b_j$  from the bank in order to purchase capital inputs and cover the capital adjustment cost. For simplicity, I assume firms start with zero net worth and therefore borrowing equal  $b_j = k_j + \phi \frac{k_j^2}{2}$ . In the third stage, firms combine labor  $l_j$ , pre-installed capital  $k_j$  and productivity  $A_j$  with production function  $M_j = A_j^\zeta k_j^{\tilde{\alpha}} l_j^{1-\tilde{\alpha}}$ , where  $\tilde{\alpha} \in (0, 1)$  represents the capital share. Firms hire labor in the third stage after observing the shocks realization and pay workers before repaying their debt to the bank. Define the operating profit of the firm as  $\pi_j = p_j M_j - w l_j$ . We can maximize labor out of the problem and substitute for the demand function (1) to obtain net operating profit as function of only capital, technology and aggregate supply of intermediates

$$\pi(A_j, k_j, M) = \Lambda(M) A_j k_j^\alpha \quad (2)$$

where  $\alpha = \frac{\tilde{\alpha}\xi}{1-(1-\tilde{\alpha})\xi}$ ,  $\Lambda(M) = \nu^{\frac{1}{1-(1-\alpha)\xi}} M^{\frac{\nu-\xi}{1-(1-\alpha)\xi}}$  with

$$M = \left\{ \left[ \frac{w}{(1-\alpha)\xi\nu} \right]^{\frac{(1-\alpha)}{(1-\alpha)\xi-1}} \left[ \int^N A_j k_j^\alpha dj \right]^{\frac{1}{\xi}} \right\}^{\frac{1-(1-\alpha)\xi}{1-(1-\alpha)\nu}} \quad (3)$$

Here I have normalized the parameter  $\zeta$  so that the profit function is linear in technology and the real wage  $w$  so that the constant multiplying  $\Lambda(M)$  in the profit function equals 1.

Firms payoff in stage 3 are as follows:

$$d_{firm,j} = \begin{cases} (1 - \tau)[\pi(A_j, k_j, M) - (1 + r_j)b_j] & \text{if } \pi(A_j, k_j, M) \geq (1 + r_j)b_j \\ -c_d k_j, & \text{if } \pi(A_j, k_j, M) < (1 + r_j)b_j \end{cases} \quad (4)$$

If profits are larger than the outstanding debt  $(1 + r_j)b_j$ , the firm repays the bank and keep the difference as dividends, minus a tax rate  $\tau$ . If the profits are not enough to repay the outstanding debt, the firm pays a default cost  $c_d$  proportional to installed capital, which can be thought as a liquidation or reorganization cost following the bankruptcy procedure.<sup>5</sup>

**Banks** Banks in each island  $j$  are deep-pocketed and risk-neutral. In the second stage they borrow at risk free rate  $r^f$  in the international market to finance the risky loan to firms  $b_j$  at loan rate  $r_j$ . They maximize their expected profits in the third stage, which equal

$$d_{bank,j} = \begin{cases} [(1 + r_j) - (1 + r^f)]b_j & \text{if } \pi(A_j, k_j, M) \geq (1 + r_j)b_j \\ -(1 + r_j)b_j & \text{if } \pi(A_j, k_j, M) < (1 + r_j)b_j \end{cases} \quad (5)$$

where risk free rate  $r^f$  is exogenous and equilibrium loan rate  $r_j$  is determined in stage 2. Firm's revenues are lost when the firm defaults, therefore default represents a net loss for the economy.<sup>6</sup>

**Exogenous shocks** The logarithm of local technology  $A_j$  in each island  $j$  is the sum of two independent components: an i.i.d. local island component  $\epsilon_j$  and an aggregate component  $\theta$ :

$$\ln(A_j) = \epsilon_j + \theta \quad (6)$$

Agents in each island have common prior  $\epsilon_j \sim N(0, \sigma_\epsilon^2)$  and  $\theta \sim N(0, \sigma_\theta^2)$ . The local shock average out in the aggregate,  $\int^j \epsilon_j dj = 0$  Both shocks realize in stage 3 and determine aggregate and local production.

<sup>5</sup> I consider here a form of “reorganization” bankruptcy, as in Chapter 11 of US bankruptcy code, under which the firm is allow to keep operating after a period of reorganization. This procedure implies some cost such as reputation costs, asset fire sales, loss of customer or supplier relationships, legal and accounting fees, and costs of changing management, which I assume depend on the size of the firm (Branch, 2002; Bris et al., 2006).

<sup>6</sup> While I assume a zero recovery rate for simplicity, a positive recovery rate would not change qualitatively the implications of the model.

### 3.1 Limited Liability

I assume a limited liability friction on firms and banks' payoff that insures them from downside risk. In particular, I assume they are insured against a fraction  $\psi$  of their losses: higher  $\psi$  implies a more convex payoff structure and therefore higher risk taking incentives. The payoff structures of bank and firm becomes then

$$w_{firm,j} = \begin{cases} (1 - \tau)[\pi(A_j, k_j, M) - (1 + r_j)b_j] & \text{if } \pi(A_j, k_j, M) \geq (1 + r_j)b_j \\ -(1 - \psi)c_d k_j, & \text{if } \pi(A_j, k_j, M) < (1 + r_j)b_j \end{cases} \quad (7)$$

$$w_{bank,j} = \begin{cases} [(1 + r_j) - (1 + r^f)]b_j & \text{if } \pi(A_j, k_j, M) \geq (1 + r_j)b_j \\ -(1 - \psi)(1 + r_j)b_j & \text{if } \pi(A_j, k_j, M) < (1 + r_j)b_j \end{cases} \quad (8)$$

I consider a general limited liability constraint specification which embeds different real world cases. First, it can represent the compensation convexity of the managers, as the relation between bonuses and option holdings versus shares. Option compensation in particular is one of the most studied source of moral hazard incentives (Edmans et al., 2017).<sup>7</sup> Moreover, in the aftermath of the financial crisis of 2008-2009 compensation policies have been suggested as likely culprits for the excessive risk-taking that led to the crisis (e.g. Bebchuk et al. (2010)). Second, one can interpret the limited liability of the firm as resulting from the borrower-lender moral hazard, such as a lower default cost or public bailout policies. Third, one can interpret the limited liability of the bank as the share of funds coming from insured deposit versus equity (Dell'Ariccia et al., 2014) or as the share of the loan value covered by government guarantees, a major part of the COVID-19 support packages offered by European governments to companies (OECD, 2020).

<sup>7</sup> Stock option compensation in US companies has increased considerably during the 1980s, and especially in the 1990s, becoming the largest component of executive pay. Options increased from only 19% of manager's pay in 1992 to 49% by 2000, and start declining from mid-2000 and in 2014 they represent 16% of the pay (Edmans et al., 2017).

### 3.2 Stage 2: lending and borrowing

I describe the two stages backwards, starting from the second stage. Before shocks realize and production takes place, banks and firms in each island decide loan quantity  $b_j$  and rate  $r_j$  based on their expectation about profits in stage 3. They share the island's surplus by Nash bargaining with the firm holding all the bargaining power, which implies a zero expected profit condition on the lender in line with the literature (e.g. [Strebulaev and Whited \(2011\)](#)).

**Information structure** The bank and firm on island  $j$  share the same information.<sup>8</sup> Before deciding on their borrowing and lending, they receive up to two signals. First, they observe a free noisy signal about local productivity:

$$z_j = \ln(A_j) + \eta_j \quad (9)$$

with  $\eta_j \sim N(0, \sigma_\eta^2)$  and local technology described in [6](#).

Second, they may or may not perfectly observe aggregate productivity. Following the Lucas island setting, I assume managers in each islands do not freely observe aggregate quantities and prices. However, in stage 1 bank and firm managers in island  $j$  can decide whether to pay a information cost to perfectly observe the aggregate shock  $\theta$ . Let  $\Omega_j$  be the (common) stage-2 information set of managers in island  $j$ : if they pay the cost in stage 1,  $\Omega_j = \{z_j, \theta\}$ , otherwise  $\Omega_j = \{z_j\}$ . One can think of this as a cognitive cost of not only collecting, but also processing information and compute the optimal individual best response ([Sims, 2003, 2006](#)).

**Lending and borrowing decision** The bank's expected excess return equals

$$\begin{aligned} E[w_{bank,j}|\Omega_j] = & b_j[1 - p(default_j|\Omega_j)](1 + r_j)b_j \\ & - [(1 - \psi) + \psi[1 - p(default_j|\Omega_j)]](1 + r^f)b_j \end{aligned} \quad (10)$$

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<sup>8</sup> Any private information between agents in the same island would be perfectly revealed by local prices.

the expected firm payoff conditioning on second stage information set is

$$E[w_{firm,j}|\Omega_j] = (1 - \tau) \int_0^\infty \int_{\ln\left(\frac{b_j}{\Lambda(M)k(b_j)^\alpha}\right)}^\infty \Lambda(M) A_j k_j^\alpha f(\ln(A_j), M|\Omega_j) dA_j dM \\ - [1 - p(\text{default}_j|\Omega_j)] (1 + r_j(b_j)) b_j - [p(\text{default}_j|\Omega_j)] (1 - \psi) c_d k_j \quad (11)$$

with the posterior default risk being defined by

$$p(\text{default}_j|\Omega_j) = \int_0^\infty \int_{-\infty}^{\ln\left(\frac{b_j}{\Lambda(M)k(b_j)^\alpha}\right)} f(\ln(A_j), M|\Omega_j) dA_j dM \quad (12)$$

where  $\Omega_j$  is the information set of island  $j$ ,  $f(\ln(A_j), M|\Omega_j)$  is the joint posterior density function of  $\ln(A_j)$  and  $M_j$ , and capital purchased is a monotonic function of borrowing  $k(b_j) = \phi^{-1}(\sqrt{1 + 2b_j\phi} - 1)$ . Finally,  $\Lambda(M) = \nu^{\frac{1}{1-(1-\alpha)\xi}} M^{\frac{\nu-\xi}{1-(1-\alpha)\xi}}$  is the revenue shifter due to the effect of aggregate production on intermediate good price, where  $M$  is defined in 3.

**Definition 1 (Stage 2 equilibrium)** *Given local shock realization  $\{\epsilon, \eta\}_{j \in [0,1]}$ , aggregate shock realization  $\{\theta\}$  and agents information set  $\Omega_{j \in [0,1]}$ , the market equilibrium in stage 2 is defined as a set of local loan prices  $r_{j \in [0,1]}$  and local loan quantities  $b_{j \in [0,1]}$  such that*

- The bank  $j$ 's expected profits 10 equal zero
- Firm  $j$  internalizes loan supply  $r_j(b_j)$  and maximizes expected profits 11

Appendix B describes in the detail the bargaining process behind the stage-2 equilibrium. The loan rate is implicitly determined by the zero expected profit condition on the bank

$$\frac{1 + r_j}{1 + r^f} = \frac{(1 - \psi) + \psi[1 - p(\text{default}_j|\Omega_j)]}{[1 - p(\text{default}_j|\Omega_j)]} \quad (13)$$

The loan rate is proportional to the perceived probability of default, implying that the risk premium on the loan is only proportional to the perceived risk with no time-varying price of risk.<sup>9</sup>

<sup>9</sup> This result derives from the assumption that the firm retains all the bargaining power, which implies a zero expected profits condition for the bank. A non-zero bargaining power on the bank will not change the mechanism of the model, but it will change the determination of the risk premium, which could decline for higher quantity of risk if the price of risk decline as well. See appendix for an alternative calibration of the model where the bank has a non-zero bargaining power.



The firm internalizes the bank's supply of loan  $r_j(b_j)$  and decides the optimal borrowing  $b_j$  to maximize expected payoff

$$k_j = \operatorname{argmax} E[w_{firm,j}(r_j(b_j), M_j, \ln(A_j)) | \Omega_j] \quad (14)$$

**Strategic motives** The impact of aggregate production of intermediate  $M$  on firm  $j$ 's revenue is described by the term

$$\Lambda(M) = \nu^{\frac{1}{1-(1-\alpha)\xi}} M^{\frac{\nu-\xi}{1-(1-\alpha)\xi}} \quad (15)$$

Depending on the sign of  $\nu - \xi$  we can have strategic substitutability or complementarity between islands in lending and borrowing decisions. First, suppose that  $\nu < \xi$ : the shifter  $\Lambda(M)$  is decreasing in aggregate production of intermediates  $M$  and we have *strategic substitutability*. For a given level of local production  $M_j$ , a higher aggregate production  $M$  implies a lower price  $p_j$  and lower revenues for firm  $j$ . As a result, island  $j$ 's optimal borrowing  $b_j$  and loan rate  $r_j$  is decreasing in aggregate production  $M$ . Second, suppose that  $\nu > \xi$ : the shifter  $\Lambda(M)$  is increasing in aggregate production of intermediates  $M$  and we have *strategic complementarity*. For a given level of local production  $M_j$ , a higher aggregate production  $M$  implies a higher price  $p_j$  and higher revenues for firm  $j$ . As a result, island  $j$ 's optimal borrowing  $b_j$  and loan rate  $r_j$  is increasing in aggregate production  $M$ . I formalize this relation in the next section.

### 3.3 Stage 1: Information choice

Before observing any signal, each island decides whether to pay an information cost  $c$  to perfectly observe aggregate shock  $\theta$  stage 2, which is informative about aggregate production  $M$ . Similarly to the lending decision in sage 2, I assume banks and firms share information and decide cooperatively through Nash bargaining with the firm holding all

the bargaining power.<sup>10</sup> As a result, island  $j$  information problem is

$$\max_{n_j \in \{0,1\}} E[E[w_{firm,j}(b_j, r_j) | \Omega_j(n_j)] - n_j c] \quad (16)$$

where  $n = 1$  if they decide to pay the cost  $c$  and  $n = 0$  otherwise. The first expectation term is conditional on the information set in stage 1, which consists of only priors, while the second expectation operator is conditioning on stage-2 information set  $\Omega_j$ . If they pay the cost, they will be able to observe aggregates in the next stage:  $\Omega_j(1) = \{z_j, \theta\}$ . If they do not pay the cost, they will be not able to observe aggregates:  $\Omega_j(0) = \{z_j\}$ . In other words, island  $j$  decides to pay the attention cost if

$$E[w_{firm,j}^*(\theta \in \Omega_j, \lambda) - c] \geq E[w_{firm,j}^*(\theta \notin \Omega_j, \lambda)] \quad (17)$$

where  $w_{firm}^*$  is the firm's payoff given stage-2 equilibrium  $r_j$  from equation 13 and  $b_j$  from equation 14. The equilibrium price and quantities, and therefore the payoff, are functions of stage-2 information set  $\Omega_j$ , which is the object of this stage choice problem. Expectation in stage 1 are instead conditional only on common priors, as agents have no access to any signal at this stage.

As argued in the previous section, optimal local prices and quantities in stage 2 depends on aggregate decision through the price externality  $\Lambda(M)$ . As a result, the optimal individual information choice will depend on the information choice of the other islands in the economy  $\lambda \in [0, 1]$ . In particular,  $\lambda = 1$  if all islands decide to pay the cost to observe aggregate shocks and  $\lambda = 0$  if none decides so. In equilibrium,  $\lambda^*$  is such that all islands are indifferent between paying the cost or not.

**Definition 2 (Stage 1 equilibrium)** *Given prior beliefs about local shock realization  $\{\epsilon, \eta\}_{j \in [0,1]}$  and aggregate shock realization  $\{\theta\}$ , the market equilibrium in stage 1 is defined by a share  $\lambda \in [0, 1]$  of islands such that all islands  $j \in [0, 1]$  are indifferent between paying and not paying the information cost, i.e. equation 17 holds with equality  $\forall j \in [0, 1]$ .*

<sup>10</sup> Any private information between agents in the same island would be perfectly revealed by local prices. Therefore any individual decision on whether to observe private information would need to account for this information spillover, introducing strategic considerations between agents in the same island. To avoid this, I use a Nash bargaining setting where the decision is taken cooperatively with the same bargaining power as in stage-2 bargaining. As a result, the firm manager gets the surplus and pay the information cost. I allow for a different split of surplus and cost in the appendix.

While I model information choice on the extensive margin, i.e. observing or not aggregates, the result are qualitative similar to modeling information choice on the intensive margin, i.e. deciding the accuracy of a signal about aggregates given a convex cost as in the rational inattention literature (for a review, [Mackowiak et al. \(2018\)](#)).

## 4 Credit booms and Inattention

### 4.1 Analytical results

In order to provide intuition for the model mechanism, I consider a first order approximation of the second stage model around the risky steady state ([Coeurdacier et al., 2011](#)).<sup>11</sup> At the steady state, all islands observes the same signal  $z_j = 0$  and the aggregate shock  $\theta = 0$ , but there is still uncertainty about the local shock realization  $\epsilon_j$ . This risk is priced in the steady state spread  $r_j > r^f$ , meaning there is a positive steady state risk premium. In this section I assume for simplicity no adjustment cost  $\phi = 0$ , no limited liability  $\psi = 0$  and no default cost  $c_d = 0$ . Because of these assumptions, the equilibrium perceived default risk and risk premium are constant (while actual default risk might not be), but the remaining qualitative implications of the model are unaffected. I relax all these assumptions in section 4.2 where I solve the full model numerically.

**Proposition 1 (Linearized model)** *Consider the first order approximation of the second-stage equilibrium defined by equations (13) and (14) assuming  $\phi = 0$ ,  $\psi = 0$  and  $c_d = 0$ . Let  $\hat{x}$  indicate the log-deviation of any variable  $x$  from its steady state value and with  $\tilde{x}$  the level deviation from steady state.*

- *Equilibrium local investment equals*

$$\hat{k}_j = \frac{1}{1 - \alpha} (E[\ln A_j | \Omega_j] - \gamma E[\hat{M} | \Omega_j]) \quad (18)$$

where  $\hat{M} = \mu(\theta + \alpha \hat{K})$ , with  $\mu > 0$  and  $\hat{K} = \int^j \hat{k}_j dj$ . Let  $\Omega_j$  denote the information set of island  $j$  (bank and firm) and  $\gamma \equiv \frac{\nu - \xi}{1 - (1 - \alpha)\xi}$  the elasticity of the operating profit  $\pi_j(A_j, k_j, M)$  with respect to aggregate production  $M$ . if  $\nu < \xi$ , then  $\gamma < 0$  and the

<sup>11</sup> While the economy at the proximity of the steady state is not suitable to study large and rare financial crises like the one considered in this paper, the basic model mechanism does not rely on non-linearity and there preserve its main idea in the linearized version

economy exhibits strategic substitutability in firms investment decisions. If  $\nu > \xi$ , then  $\gamma > 0$  and the economy exhibits strategic complementarity in firms investment decisions.

- The loan rate is proportional to perceived default risk

$$\hat{r}_j \propto -\hat{p}(def_j|\Omega_j) \quad (19)$$

where  $\hat{p}(def_j|\Omega_j)$  is the perceived default risk of island  $j$  conditioning on information set  $\Omega_j$ .

- Equilibrium perceived default risk is constant

$$\hat{p}(def_j|\Omega_j) = 0 \quad (20)$$

- Equilibrium aggregate bank's profits in state  $\theta$  equal

$$E[\tilde{\pi}_{bank}|z_j, \theta] \propto - \int^j [\hat{p}(def_j|z_j, \theta) - E[\hat{p}(def_j|\Omega_j)|\theta]]dj \quad (21)$$

where  $\hat{p}(def_j|z_j, \theta)$  is the default risk conditional on signal  $z_j$  and aggregate shock  $\theta$ , which I define as actual default risk.

See the appendix for the derivations.

The proposition highlights some interesting results. First, the equilibrium loan rate  $\hat{r}_j$  is negatively related to the perceived default probability. This result follows directly from the price equation (13) and it implies that changes in risk premia only reflects changes in perceived quantity of risk. Second, perceived default risk is constant in equilibrium (or zero in log-deviation from the steady state). This is a knife-edge result that depends on the simplifying assumptions introduced in this section, which I relax in the numerical solution. Third, as the loan pricing condition implies no expected profits for the bank, aggregate bank profits in state  $\theta$  depend on whether agents correctly perceived risk, i.e. the loan is correctly priced conditioning on  $\theta$ .

**PE vs GE** A positive aggregate shock  $\theta$  has two effects on equilibrium investment: a partial equilibrium effect and a general equilibrium effect.<sup>12</sup>

$$\frac{\partial \hat{k}_j}{\partial \theta} = \frac{1}{1 - \alpha} \left( \underbrace{\frac{\partial E[\ln A_j | \Omega_j]}{\partial \theta}}_{\text{PE effect}} - \gamma \underbrace{\frac{\partial E[\hat{M} | \Omega_j]}{\partial \theta}}_{\text{GE effect}} \right) \quad (22)$$

First, local productivity  $A_j$  in each island increases. Because firm's fundamental is higher, island  $j$ 's posterior probability of default decreases, boosting borrowing and investment  $\hat{k}_j$ . This is the standard positive channel of productivity shocks in the existing literature and it does not depend on the interaction between islands (PE). Second, higher aggregate supply of intermediates can imply lower or higher demand for intermediate good  $j$  depending on the degree of decreasing return to scale ( $\nu$ ) with respect to the elasticity of substitution between intermediates ( $\theta$ ). Since I assume  $\nu < \xi$ ,  $\gamma > 0$  and the higher competition in the intermediate good market implies a lower demand and revenues for firm  $j$ . As a result, optimal investment  $\hat{k}_j$  is lower.

**Assumption 1 (Strategic substitutability)** Assume that  $\nu < \xi$ : firms exhibit strategic substitutability in investment decisions.

In section 4.2 I show that this assumption holds under fairly mild conditions, such as a similar or higher markup in intermediate compared to final good sectors, which is supported by empirical evidence. Because of this assumption, higher aggregate investment  $\hat{K}$  implies a lower selling price  $p_j$  and revenue for island  $j$  and as a result a lower optimal investment  $\hat{k}_j$ .

While  $\lambda$  depends endogenously on the stage-1 information choice, I consider here two limit cases to illustrate the mechanism of the model. First, I assume all islands decide to pay attention to aggregates in the first stage ( $\lambda = 1$ , i.e. full information). Second, I assume no island decide to pay attention to aggregates in the first stage ( $\lambda = 0$ , i.e. dispersed information).

<sup>12</sup> Here I use partial equilibrium effect to refer to an effect related only to the island  $j$ 's problem, and the term general equilibrium effect to indicate an effect related to the interaction between islands (Angeletos and Lian, 2017).

#### 4.1.1 Full information $\lambda = 1$

Consider the full information case, meaning all islands decide to observe aggregate shock  $\theta$  in the first stage in addition to the free signal  $z_j$  defined by equation (9).

**Proposition 2 (Full information)** *If  $\Omega_j = \{z_j, \theta\}$ , the solution to the linear game in proposition 1 is*

$$\hat{K}^{fi} = \frac{1 - \gamma\mu}{1 - \alpha + \gamma\mu\alpha} \theta \quad (23)$$

See the appendix for the proof.

After an aggregate shock, the improvement in local technology increases equilibrium aggregate debt and investment, but its effect is dampened by the endogenous decrease in intermediate good prices, which lower firms' optimal investment. The stronger the elasticity of intermediate price with respect to the increase in aggregate supply of intermediate  $0 < \gamma < 1$ , the stronger is the dampening force of the GE effect.

**Corollary 1 (Actual default rate in FI)** *If  $\Omega_j = \{z_j, \theta\}$ , actual default risk coincides with perceived default risk, which is constant by proposition 1.*

$$\hat{p}(def_j|z_j, \theta) = \hat{p}(def_j|\Omega_j) = 0 \quad (24)$$

*As a result the default rate, which equals the average actual default risk across firms, is also constant.*

Notice that the negative endogenous GE effect on expected firm's revenue can not be larger than the positive PE effect in full information, which implies that the actual default risk can not be larger either. If that was the case, then the lower expected revenues would lead the firms to decrease debt and investment (proposition 1), resulting in lower aggregate supply, higher price and a positive endogenous GE effect. In other words, if the default risk was higher, the agents in the economy would optimally limit leverage and reduce it. This is a consequence of the strategic substitutability game between firms.<sup>13</sup> As a result, the full information economy is not riskier in credit boom, which is at odds with the existing empirical evidence (Schularick and Taylor, 2012; Krishnamurthy and Muir, 2017).

<sup>13</sup> A large body of research focuses instead on strategic complementarity to rationalize the procyclical leverage in full information (Gertler and Kiyotaki, 2010; Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2019; Bianchi and Mendoza, 2020).

**Corollary 2 (Bank's profit in FI)** *If  $\Omega_j = \{z_j, \theta\}$ , bank's profit are zero conditioning on  $z_j$  and  $\theta$ .*

$$E[\tilde{\pi}_{bank}|z_j, \theta] = 0 \quad (25)$$

Because perceived risk coincides with actual risk, default risk is correctly priced conditioning on aggregate economic conditions. In other words, because banks observe  $\theta$ , they do not make systematic errors conditioning on it. The zero expected profit condition implies that banks make zero excess return on average for each  $\theta$ . While this model implies zero expected profits for banks, a different bargaining power could imply positive profits. However bank shareholders would not accept predictable losses, which is at odds with the evidence of systematic negative excess returns in bank stocks after large credit booms (Baron and Xiong, 2017).

#### 4.1.2 Dispersed information $\lambda = 0$

Consider the dispersed information case, meaning no island decides to observe aggregate shock  $\theta$  in the first stage, so they only observe the free signal  $z_j$  defined by equation 9.

**Proposition 3 (Dispersed information)** *If  $\Omega_j = \{z_j\}$ , the solution to the linear game in proposition 1 is*

$$K^{di} = \frac{(m - \gamma\mu\delta)}{1 - \alpha + \gamma\mu\alpha\delta}\theta \quad (26)$$

where  $m = \frac{\sigma_e^2 + \sigma_\theta^2}{\sigma_e^2 + \sigma_\theta^2 + \sigma_\eta^2}$  and  $\delta = \frac{\sigma_e^2}{\sigma_e^2 + \sigma_\theta^2 + \sigma_\eta^2}$  are the Bayesian weights on signal  $z_j$  in the posterior means of  $\ln(A_j)$  and  $\theta$  respectively, with  $0 < \delta < m < 1$ .

See the appendix for the proof.

Agents do not observe aggregates but only the local signal, which is informative about local technology. Since local technology is the sum of local and aggregate shocks, they can not distinguish between the two without additional information. Agents are rational and after observing the signal they form Bayesian posterior beliefs that assign a positive probability to both shocks. This is consistent with the empirical evidence that firms' expectations about aggregate economic conditions respond to industry-specific shocks even though these shocks have no aggregate effects (Andrade et al., 2022).



**Corollary 3 (Boom amplification)** *The difference in aggregate investment in dispersed information 26 and full information 23 depends positively on  $\theta$ , and therefore the information friction leads to an amplification of credit booms if*

$$(m - \gamma\mu\delta)(1 - \alpha + \gamma\mu\alpha) > (1 - \gamma\mu)(1 - \alpha + \gamma\mu\alpha\delta) \quad (27)$$

Assuming the condition in 3 holds, after a positive aggregate shock agents observe a signal about higher local technology and they partially confound it for a local shock. As a result, they underestimate the endogenous increase in aggregate production and the implied decrease in selling price. Incomplete information therefore dampens the negative general equilibrium effect on investment and leads to an amplification of individual borrowing and investment.<sup>14</sup>

The intuition above is accurate only if condition 27 holds. Generally, aggregate shock  $\theta$  affects both the local fundamental (PE effect) and the aggregate production (GE effect). As a result, not observing  $\theta$  leads to an underestimation of both, with opposite effects on optimal investment. Whether investment in dispersed information is larger than in full information depends on how much observing aggregates increases (i) posterior belief on local productivity (PE) and (ii) posterior belief on aggregate production of intermediates (GE). First, suppose the signal  $z_j$  is infinitely noisy,  $\sigma_\eta \rightarrow \infty$ , then  $m = \delta = 0$  and condition 27 doesn't hold. The intuition is as follows. Without signals on local productivity, the aggregate shock is the only source of information. If agents do not observe it either, investment equals the steady state level in every state. If agents are instead able to observe it, higher aggregate shock  $\theta$  increases their posterior on both local technology (PE) and aggregate investment (GE), but the only equilibrium is one in which the first prevails on the second and optimal local investment increases.<sup>15</sup> Second, suppose the signal  $z_j$  is noiseless,  $\sigma_\eta \rightarrow 0$ , then  $m = 1, \delta < 1$  and condition 27 holds. In this case agents observe perfectly local productivity regardless of their information on aggregate shock. However, observing aggregates is informative on the investment decisions of the other firms in the economy, and therefore on the negative endogenous GE effect. In the dispersed informa-

<sup>14</sup> Angeletos and Lian (2017) shows a similar result in a more stylized endowment economy.

<sup>15</sup> To see it, suppose that the negative GE force from higher aggregate investment was stronger than the positive PE effect from higher local technology an optimal local investment decreased in  $\theta$ . Aggregate investment would then be inversely related to  $\theta$ , and the GE force would be positive for the island and not negative, leading to a contradiction.

tion setting, after an aggregate shock agents underestimate the increase in competition and over-invest with respect to the economy with informed agents.

Now consider the case of an individual island, both bank and firm, forming expectation on local firm's operating profits. Define the forecast errors as the difference between realized and expected revenues,  $fe \equiv \hat{\pi}(A_j, k_j, M) - E[\hat{\pi}(A_j, k_j, M)|\Omega_j]$ .

**Corollary 4 (Rationally extrapolative beliefs and underreaction)** *If  $\Omega_j = \{z_j\}$ , the average forecast errors on firm's revenues in state  $\theta$  is proportional to*

$$E[\hat{\pi}_j|z_j, \theta] - E[E[\hat{\pi}_j|z_j]|\theta] = \alpha - [(m - \gamma\mu\delta)(1 - \alpha + \gamma\mu\alpha) - (1 - \gamma\mu)(1 - \alpha + \gamma\mu\alpha\delta)]\theta \quad (28)$$

while the forecast error on aggregate output is

$$E[\hat{Y}|z_j, \theta] - E[E[\hat{Y}|z_j]|\theta] = (1 - \gamma\mu) \left( \frac{1 - \alpha + \alpha m}{1 - \alpha + \gamma\mu\alpha\delta} \right) \theta \quad (29)$$

If condition (27) holds, then

- $\theta > 0$ : agents underestimate aggregate output and overestimate individual revenues (overoptimism in booms).
- $\theta < 0$ : agents overestimate aggregate output and underestimate individual revenues (overpessimism in busts).

Firm's revenues depend positively on the PE effect and negatively on the GE effect. Because agents do not observe aggregates, they rationally confound an aggregate shock for a local shock and underestimate the negative GE effect. The information incompleteness produces extrapolative-like beliefs, as agents are systematically overoptimistic after positive aggregate shocks and overpessimistic after negative ones. Differently from behavioral models where extrapolation originates from overreaction to positive news (Bordalo et al., 2018b, 2019), here it is due to rational underreaction to the endogenous negative general equilibrium effect. As a result, booms are associated with both overoptimism about local revenues and underestimation of aggregate quantities, consistently with the evidence in section 2. Importantly, even if agents are rational and correct on average conditioning on their information set, they are consistently mistaken conditioning on unobserved aggregate states.

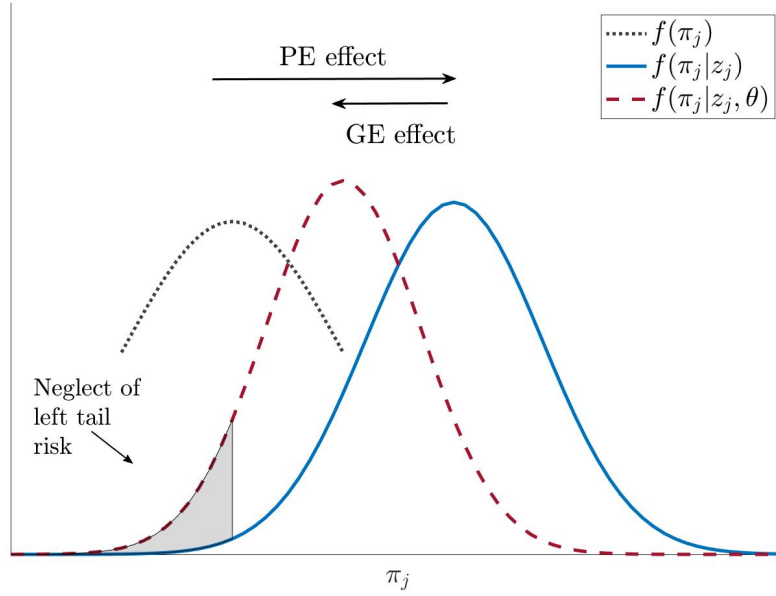


Figure 3: Rationally extrapolative beliefs in booms

*Notes:* The figure illustrates the posterior belief on firm's operating profits after a positive aggregate shock under three different information sets. The black dotted line represents the posterior of an agent not observing any new information. The blue solid line represents the posterior of an agent observing only local signal  $z_j$ . The red dashed line represents the posterior of an agent observing both local signal  $z_j$  and aggregate shock  $\theta$ . Not observing aggregate shock  $\theta$  leads to overestimating equilibrium price  $p_j$  and therefore individual revenues  $\pi_j$ .

Figure 3 illustrates this mechanism. The dotted line represents the prior belief about firm's revenues before receiving any information. A positive aggregate technology shock increases firm's fundamentals and implies on average a good signal  $z_j$  that shifts the posterior beliefs on revenues to the blue solid line (positive PE effect). However, because of the endogenous increase in intermediate good supply, price of good  $j$  will be lower and the actual posterior revenues of an informed agent would shift back to the middle dashed line (negative GE effect). However, if agent do not observe aggregates, they underestimate this last shift and by consequence left tail risk, illustrated in the figure as the shaded area between their posterior and the actual posterior distribution of revenues.<sup>16</sup>

**Corollary 5 (Actual default rate in DI)** *If  $\Omega_j = \{z_j\}$ , the equilibrium default rate is pro-*

<sup>16</sup> Notice that, because more information also implies lower posterior uncertainty, the difference between informed and non-informed posterior is not only lower posterior mean but also lower posterior variance.

portional to

$$\hat{p}(def|\theta) \propto [(m - \gamma\mu\delta)(1 - \alpha + \gamma\mu\alpha) - (1 - \gamma\mu)(1 - \alpha + \gamma\mu\alpha\delta)]\theta \quad (30)$$

where  $\hat{p}(def|\theta) = \int^j \hat{p}(def_j|z_j, \theta) dj$ . If condition (27) holds, default rate increases in aggregate shock  $\theta$

See the appendix for the proof.

As dispersed information amplifies booms, the larger supply of intermediates lowers further prices and firms' revenues. Agents confound aggregate for local shocks and increase leverage too much with respect to their future revenues, which leads to higher default rate. As a result, credit booms are period in which default risk is larger, consistently with the evidence that low risk premium and high credit growth predict higher financial fragility (Krishnamurthy and Muir, 2017).

**Corollary 6 (Bank's profit in DI)** *If  $\Omega_j = \{z_j\}$ , the equilibrium average bank profits are proportional to*

$$E[\tilde{\pi}_{bank}|z_j, \theta] \propto [(m - \gamma\mu\delta)(1 - \alpha + \gamma\mu\alpha) - (1 - \gamma\mu)(1 - \alpha + \gamma\mu\alpha\delta)]\theta \quad (31)$$

*If condition (27) holds, average bank profits are negative after a credit boom.*

Because in equilibrium the risk premium is such that banks get zero expected profit on average, when banks underestimate default risk they misprice loans and get negative profits. This result is consistent with the evidence that credit booms predict negative returns on bank stocks in Baron and Xiong (2017).

**Information choice** In the first stage, firms and banks in each island decide whether they want to observe aggregates based on their expected profits in the final stage. In general, a share  $\lambda \in [0, 1]$  of islands decides to acquire the information. While figure 3 illustrates individual beliefs for a given aggregate production  $M$ , this quantity is endogenous to the aggregate amount of information in the economy. If all managers in each island are informed,  $\lambda = 1$ , proposition 3 states that the increase in aggregate supply in boom is lower, and the decrease in price as well. In figure 3, this would mean a shorter distance between informed and uninformed posterior, as the neglected GE effect is lower. On the other hand,

if managers and firms in each island are uninformed,  $\lambda = 0$ , the credit boom is amplified, and the decline in price is larger. In figure 3, this would mean a larger distance between informed and uninformed posterior, as the neglected GE effect is higher. Therefore, the benefit of information for the individual island depends negatively on the average level of information in the economy. In particular, there is strategic substitutability in information choice, as higher aggregate information implies lower individual benefit of information. In section 5 I illustrate numerically how risk taking incentives also affect benefit of information and equilibrium  $\lambda$ .

## 4.2 Numerical illustrations

I provide a numerical illustration of the non-linear model. The contribution of studying numerical solutions of the model is twofold. First, I relax some parametric assumptions needed to keep the analytical model tractable. Second, non-linear global solution are more suitable than approximation around the steady state to analyze the nature of large and rare credit booms, as the ones considered in this paper.

**Calibration** Table 1 reports the model's calibration. First, I set  $\xi = 0.833$  to match a markup of 20%, which is inside the set of values estimated in the macro literature (for a review, see Basu (2019)). Together with a capital share  $\tilde{\alpha} = 0.33$ , it implies  $\alpha = \frac{\tilde{\alpha}\xi}{1-(1-\tilde{\alpha})\xi} = 0.624$ . The return to scale of final good producer  $\nu$  can be expressed similarly as a function of the final good sector markup and the intermediate good share in production. Assuming the latter equal 0.5 (approximately the average value for the US economy over a long period of time) and a markup of 50% gives  $\nu = 0.5$ . The larger markup in the retail and wholesale sectors with respect to other sectors is in line with the evidence in De Loecker et al. (2020). However, my modeling assumption of  $\nu < \xi$  would be satisfied by any final good sector markup larger than 13%.<sup>17</sup>

Since TFP in my model is i.i.d., I set the aggregate volatility equal to the unconditional volatility implied by a standard autoregressive process with quarterly shock volatility 0.02 and autoregressive coefficient 0.995, which gives  $\sigma_\theta = 0.2$ . I set the idiosyncratic TFP volatility  $\sigma_e = 3\sigma_\theta$ , where the ratio 3 is somewhere between the macro structural estimates

<sup>17</sup> Assume the final good sector face a demand given by  $P = Y^{\tilde{\xi}-1}$  and have a production function  $Y = M^{\tilde{\nu}} X^{1-\tilde{\nu}}$ , where  $X$  is some other variable input. After maximizing  $X$  out, the profit function would be proportional to  $\pi \propto M^{\frac{\tilde{\nu}\tilde{\xi}}{1-(1-\tilde{\nu})\tilde{\xi}}} \equiv M^\nu$ . Given an intermediate share of  $\tilde{\nu} = 0.5$  and  $\xi = 0.833$ , the condition  $\nu < \xi$  implies a final good sector markup  $\frac{1}{\xi} > 1.13$ .

Table 1: Calibration

Parameter	Interpretation	Value
$\alpha$	Return to scale intermediate good sector	0.624
$\nu$	Return to scale final good sector	0.5
$r^f$	Risk free rate	0.1
$\phi$	Investment adj cost coefficient	1
$\sigma_\theta$	Volatility aggregate shock	0.2
$\sigma_e$	Volatility local shock	0.6
$\sigma_\eta$	Volatility signal noise	0.64
$\psi$	Limited liability	0
$c_d$	Default cost	0.5
$\tau$	Corporate tax	0.20
$c$	Information cost	0.0017

(e.g.  $\approx 15$ , [Maćkowiak and Wiederholt \(2015\)](#)) and the micro empirical estimates (e.g.  $\approx 1.1$ , [Castro et al. \(2015\)](#)). Moreover, I set the private noise  $\sigma_\eta = \sigma_a$ , where  $\sigma_a$  is the total volatility of TFP. Because the model aims to capture low frequency credit boom&busts as in the macro-finance empirical literature, I set the risk free rate to the 5-year implied return from a one-year T-bill of 2%, which gives  $r^f = 0.1$ . The corporate tax rate is set to 20% ([CBO, 2017](#)).

In this section I abstract from limited liability and set  $\psi = 0$ . In section 5 I increase risk taking incentivess and study its implications on lending and information choice. Finally, I calibrate the cost of information  $c$  such that with no limited liability it is optimal for all islands to be collect information ( $\lambda = 1$ ), which corresponds to around 3% of firm's dividends in the full information economy.

**Full information  $\lambda = 1$**  Consider the full information case, in which all islands decide to observe aggregate shock  $\theta$  in the first stage. The blue dashed lines in figure 4 reports the response of aggregate credit  $B = \int^j b_j dj$  (proportional to aggregate investment), average risk premium  $R - r^f$ , default rate and average bank profits in this economy as functions of standard deviations of the aggregate shock  $\theta$ . The figure confirms the analytical results in the previous section, as large values of aggregate shock  $\theta$  are associated with a credit boom. Differently from the linear model in the previous section, I allow for a non-zero investment

adjustment cost. As a consequence, the probability of default is not constant but declines after boom and, because agents know the default risk is lower, the risk premium declines as well. Risk is correctly priced and banks make zero average profits conditional on each aggregate state. The model's implications are qualitatively similar to a benchmark model that abstracts from strategic interactions between firms, but with the price externality dampening the boom.

The model is not consistent with the existing evidence. First, [Schularick and Taylor \(2012\)](#) show that booms are periods where financial risks accumulates, which in my model would imply a larger default rates after a credit boom. Second, [Krishnamurthy and Muir \(2017\)](#) document that low risk premium predict financial crisis, but in full information, because risk is correctly priced conditioning on aggregates, risk premia are positively correlated with default risk. Finally, [Baron and Xiong \(2017\)](#) document that average excess return on bank stocks is negative after a boom, while informed bank in the model would not accept to make average negative returns.<sup>18</sup>

**Dispersed information**  $\lambda = 0$  Consider the dispersed information case, where no island decides to observe aggregate shock  $\theta$  in the first stage. The red solid lines in figure 4 reports the response of aggregate credit  $B = \int^j b_j dj$ , average risk premium  $R - r^f$ , default rate and average bank profits in this economy as functions of standard deviations of the aggregate shock  $\theta$ . The figure confirms the analytical results in the previous section. Because agents underestimate the size of the negative GE effect, the credit boom is amplified, as depicted by the solid red line in the upper left panel. The excess supply of intermediate goods lowers intermediate price and revenues, but firms are inattentive to aggregates and take on too much debt. Default risk peaks after credit booms, consistent with the evidence on credit boom-and-busts ([Schularick and Taylor, 2012](#)). Banks are also inattentive to aggregates and confound the aggregate shock for a local shock. As a result, the risk premium on lending is lower in credit booms when the default risk is larger. The model's results are consistent with existing evidence that high credit and low risk premia predict subsequent financial downturn ([Krishnamurthy and Muir, 2017](#)).

<sup>18</sup> While it would be possible to set up a model where firms had higher risk tolerance and were willing to take on more risk during credit booms, bond pricing equation (13) implies that the risk premium would increase as a consequence, inconsistently with the evidence in [Krishnamurthy and Muir \(2017\)](#). If the bankers had higher risk tolerance in booms as well, risk premia could be lower in periods of high risk (e.g. [Krishnamurthy and Li 2021](#)), but it would still not be possible to have rational bankers accepting negative excess returns on average, as documented in [Baron and Xiong \(2017\)](#).



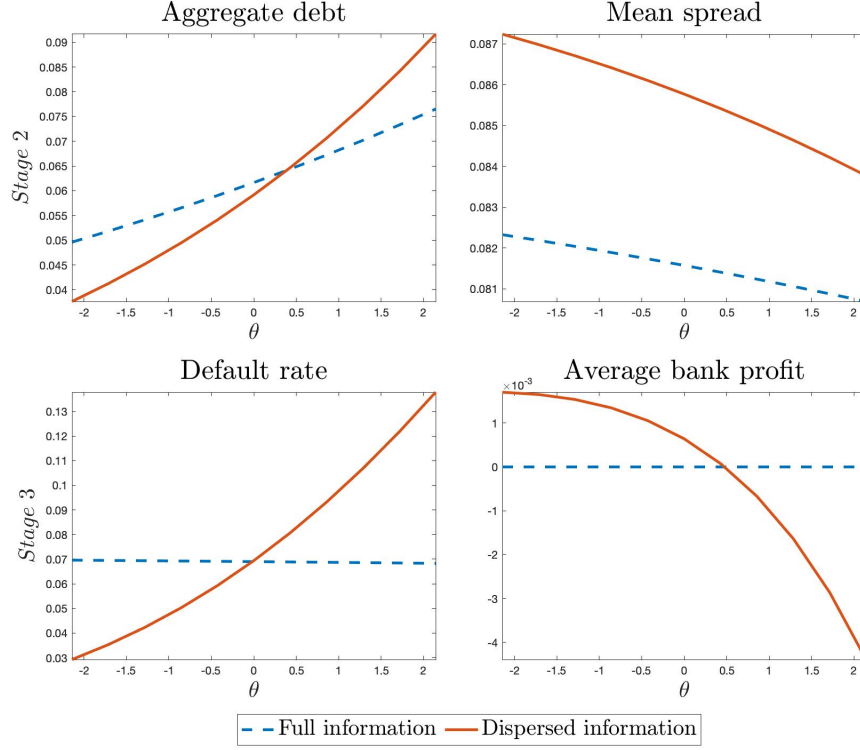


Figure 4

Notes: The figure illustrates the equilibrium of stage-2 investment and borrowing choice in the full information ( $\theta \in \Omega_j$ ) and dispersed information economy ( $\theta \notin \Omega_j$ ). The aggregate shock  $\theta$  in the x-axis is expressed in standard deviations.

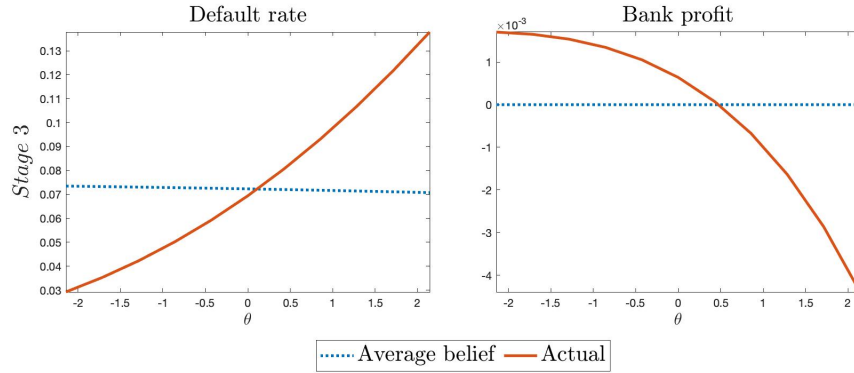


Figure 5

Notes: The figure illustrates the actual and the average expectation of bank excess return and default rate in the dispersed information economy ( $\theta \notin \Omega_j$ ). The aggregate shock  $\theta$  in the x-axis is expressed in standard deviations.

The decline in risk premium is not due to a change in risk tolerance, but to the underestimation of the endogenous increase in default risk. Figure 5 clarifies this point by plotting actual bank's profits (solid red) and mean bank's expected profits (dotted blue)

in the left panel and actual average default rate (solid red) and mean expected default rate (dotted blue) on the right. Banks do not internalize the increase in default risk and expect zero average excess return. However, because of the increase in default risk, excess returns during credit booms are negative on average. Under the assumption that bank's stock price is correlated with operating profits, the results is in line with the evidence of average negative returns on bank's stock during booms in [Baron and Xiong \(2017\)](#).

The equilibrium share of informed islands  $\lambda$  is endogenous and depends on the optimal attention decisions in stage 1. While it would be possible to rationalize a high level of information friction with a high enough information cost  $c$ , such high cost might not be realistic. In the next section I show how limited liability lead to lower optimal attention choices and therefore explain a high level of information dispersion in the model even when information costs are low.

## 5 Inattention and limited liability

While the previous section illustrates how information frictions explain the observed frothiness and overoptimism in credit booms, I now turn to the determinant of such information friction. I show that limited liability cause agents to optimally pay less attention to aggregates even for low information costs, causing them to be overoptimistic in booms and overpessimistic in busts. I connect the moral hazard narratives of the excessive risk taking before the financial crisis of 2008-2009 (e.g. [Blinder 2009](#)) with the behavioral overoptimism view (e.g., [Gennaioli and Shleifer 2018](#)) by showing that overoptimism is in fact a consequence of risk taking incentives.

**Stage 2: Risk taking in lending** An increase in limited liability has a standard risk-taking effect on stage-2 borrowing and lending decisions. First, consider firms' decision in equation 14. For a given interest rate schedule  $r_j(b_j)$ , the firm faces a trade-off in their debt issuance  $b_j$  between higher expected profits in the no-default states and higher default probability. Higher payoff convexity  $\psi$  lowers firms' losses in case of default, encouraging them to take on more risk. Second, consider banks' decision in equation 13. Higher payoff convexity  $\psi$  implies lower losses in case of default and therefore lower elasticity of credit spread  $\frac{1+r^i}{1+r^f}$  with respect to default risk. This is the standard effect of risk taking incentives for a given information structure, i.e. "informed" risk taking.

In order to isolate the effect of limited liability on borrowing decisions, I initially shut

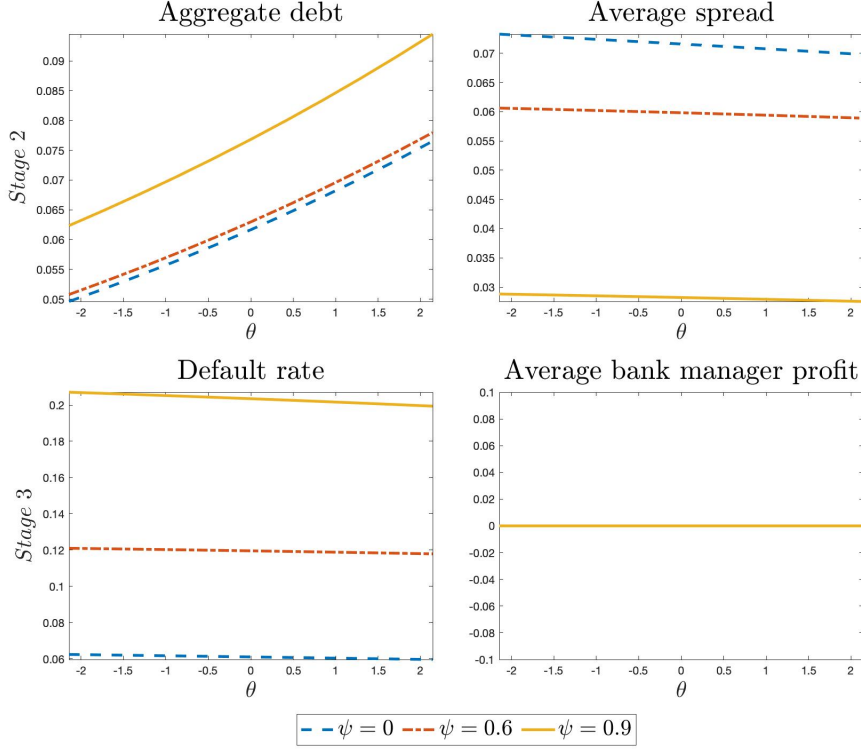


Figure 6: Full information and limited liability

*Notes:* The figure illustrates the stage-2 investment and borrowing choice in full information economy ( $\theta \in \Omega_j$ ) for different values of limited liability parameter  $\psi$ . The aggregate shock  $\theta$  in the x-axis is expressed in standard deviations.

down the information choice in stage 1. Figure 6 reports the equilibrium debt, average spread, default rate and bank's profits in an economy in full information for different values of limited liability  $\psi$ . Higher payoff convexity lead to higher risk taking and lower price of risk, resulting in higher unconditional default rate. However, similarly to the full information model in the previous section, in the baseline calibration credit booms are period where the economy is safer and default rate decreases, which is not consistent with the empirical evidence (Schularick and Taylor, 2012; Krishnamurthy and Muir, 2017). Therefore the full information model with only moral hazard incentives in stage-2 borrowing decisions is not able to match qualitatively the empirical evidence on credit cycles.

**Stage 1: Risk taking in information** In the first stage, banks and firms in each island decide whether to pay or not the information cost to observe aggregate shocks in stage 2. Both agents benefit from information, as neglecting aggregate shocks leads to higher default risk and losses. I set the attention cost such that, with no limited liability  $\psi = 0$ , it

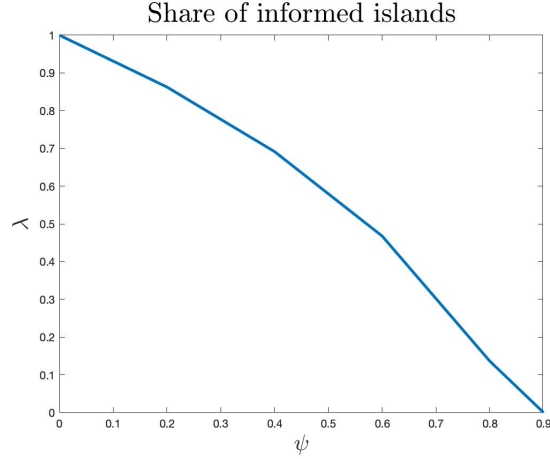


Figure 7: Limited liability and information choice

Notes: The figure illustrates the result of stage-1 information choice under different calibration for payoff convexity  $\psi$ . It shows that higher payoff convexity is associated with lower information choice.

is optimal for all islands to pay the cost and be fully informed in next stage,  $\lambda = 1$ . Figure 7 shows that the equilibrium share of informed island  $\lambda$  declines in limited liability  $\psi$ .<sup>19</sup> Intuitively, the larger is agent's payoff convexity, the lower is their exposure to losses and therefore the lower is their marginal benefit of information.<sup>20</sup>

Figure 8 reports the equilibrium debt, average spread, default rate and bank's profits for different values of limited liability  $\psi$ , which endogenously lead to different value of attention  $\lambda$ . The higher is limited liability, the lower is the optimal attention choice, which leads to higher default rate and lower bank's profits in booms as discussed in the previous section. As a consequence, credit booms are period where default risk is larger but risk premium lower, consistently with the empirical evidence on credit cycles. The comparison between figure 6 and figure 8 reveals that risk taking in information choice is able to explain the existing evidence on credit cycles, while "informed" risk taking in investment decision alone is not.

<sup>19</sup> This result relies on the contemporaneous increase both firm and bank limited liability. First, higher firms' payoff convexity leads to higher risk taking and lower optimal information for a given credit spreads, but lower information also results in higher uncertainty and higher average credit spreads. Because firms want to take on more risk, depending on the calibration they might prefer to collect more information just to decrease price of risk. However, if banks' payoff convexity increases as well, then price of risk declines and the island collectively is better off with lower information.

<sup>20</sup> The intuition behind this result is similar to Mackowiak and Wiederholt (2012), who show that limited liability reduces optimal information choice in a general setting, while Lindbeck and Weibull (2017) study optimal contracts between principal and manager in rational inattention setting.

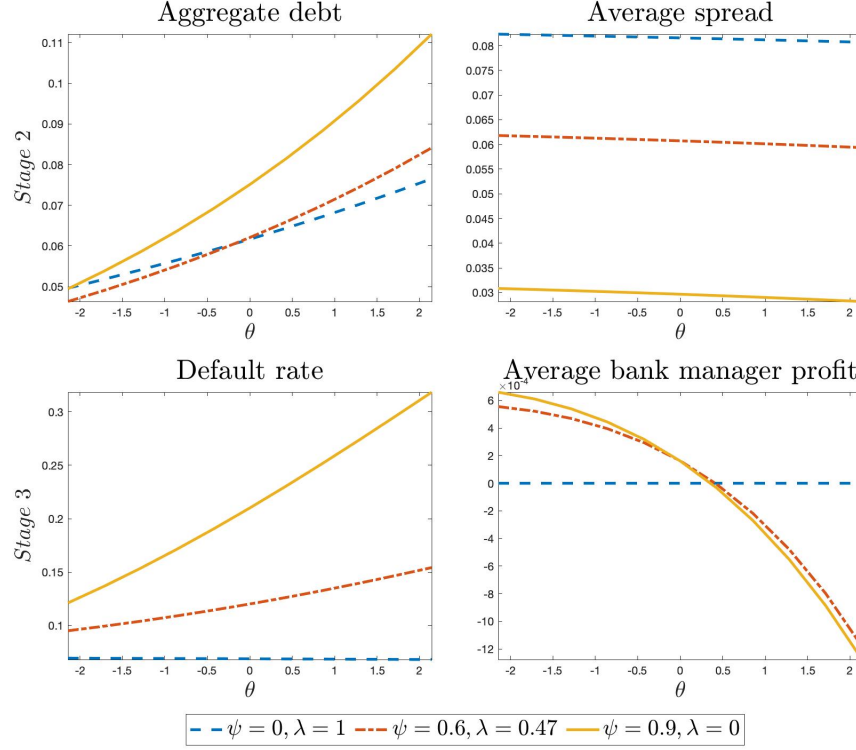


Figure 8: Information Choice and limited liability

Notes: The figure illustrates the equilibrium of the model (both stage 1 and stage 2) for different values of the limited liability parameter  $\psi$ . The aggregate shock  $\theta$  in the x-axis is expressed in standard deviations.

## 6 Dynamic extension

I extend the model to an infinite-periods setting to compare its predictions to the existing evidence on credit cycles. First, I review the existing evidence on the paths of spreads and credit before financial crises, then I compare the performance of my model against the data. While a full quantitative estimation of the model is beyond the scope of this paper, I show that the model with a standard calibration is nonetheless able to produce realistic boom-and-busts dynamics.

I focus on financial crises, defined by the literature “as events during which a country’s banking sector experiences bank runs, sharp increases in default rates accompanied by large losses of capital that result in public intervention, bankruptcy, or forced merger of financial institutions” (Jordà et al., 2013). I compare my model against two sets of evidence from Krishnamurthy and Li (2021): first, the pre-crisis path of spreads and credit; second, the predictive power of spreads and credit growth in forecasting financial crises.

**Pre-crisis period** Conditioning on a crisis at time  $t$ , consider the path of spreads and credit in the 5-years preceding the crisis. First, credit spreads are  $0.34\sigma$ s below their country mean, where the mean is defined to exclude the crisis and the 5 years after the crisis. Second, credit/GDP is 5% above the country mean.

**Predicting crises** The most important evidence for the scope of this paper is the ability of spreads and credit growth to predict crises. First, [Krishnamurthy and Muir \(2017\)](#) find that conditioning on an episode where credit spreads are below their median value 5 years in a row, the probability of a financial crisis increase by 1.76%. Second, [Schularick and Taylor \(2012\)](#) shows that a one standard deviation increase in credit growth over the preceding 5 years implies an increased in probability of a crisis of 2.8% over the next year.

**Dynamic model** In order to related my model to the existing evidence, I embed my three-stage game in a infinite period setting. I consider an overlapping generation of bank and firm managers living for two periods. In each period a new generation of managers is born and decide information (stage 1) and lending and borrowing (stage 2). In the following period, shocks described in equation (6) realize, production take place and firms repay or default (stage 3). In this period, the old generation of managers receive their payoffs and die, while a new generation of managers is born and repeats the cycle.

I assume that in case of default, firms can not re-enter in the economy immediately as it takes one period for the firm to re-build its productive capacity. This simple friction can be interpreted as time needed for new firms to collect the funding to cover some fixed cost of production or to organize the production process. Define the number of defaulted firms  $N_{def,t}$  as the default rate times the number of firms in the economy  $N_t$ . Then the number of firms operating in period  $t$  is given by

$$N_t = N_{t-1} - N_{def,t} + N_{def,t-1} \quad (32)$$

As illustrated in the previous section, in presence of limited liability credit booms are followed by a larger default rate, which implies a lower number of productive firms in the economy in the following period. As a result, booms are followed by a burst as in the existing evidence.<sup>21</sup>

<sup>21</sup> While in the framework considered here booms translates into busts through a credit demand channel, one could think of a framework where the mechanism works through a credit supply channel instead. As showed in the previous section, banks balance sheets are also impaired after booms as they suffer losses on their

Table 2: Model and Data Moments

	Data	Model	
		$\psi = 0$	$\psi = .8$
<i>Pre-crisis period (5 years)</i>			
Credit spreads ( $\sigma$ below mean)	0.34	0.00	0.06
Credit/GDP (% above mean)	5	0	7
<i>Predicting crises (5 years)</i>			
Credit spreads (% increase in probability)	1.76	0.00	2.02
Credit/GDP (% increase in probability)	2.8	0.00	4.8

In order to relate to the existing evidence on credit cycles, I calibrate one period in the model to represent a 5-years time span in the data. I follow [Krishnamurthy and Li \(2021\)](#) and target an annual unconditional frequency of financial crisis of 4%, which is the mean value of the different frequencies estimated in the literature. As a result, I define a financial crisis as an event in which the output drops below the 20% percentile. I solve for the model equilibrium stage-1 information and stage-2 aggregate quantities and prices for each node in 15x9 grid of aggregate shock  $\theta_t$  and number of firms  $N_t$ , then I simulate 100,000 periods by drawing from the distribution of  $\theta$  and interpolating from the grid. I simulate the theoretical moments for both the baseline model without payoff convexity  $\psi = 0$  and with payoff convexity  $\psi > 0$ .

Table 2 reports the empirical moments and the ones generated by the model in the two different calibrations. First, the baseline model without limited liability is not able to produce systematic movement in spreads or credit before crises, or to predict financial crisis with movements in spreads or credit. In this model, crises happens only when the economy is hit by negative technological shock, with no boom-and-bust dynamics. On the other hand, the model with limited liability is qualitatively consistent with the evidence. First, crises are systematically preceded by credit boom with an increase in credit and a decline in spreads. Similarly, increase in credit and decline in spreads have predictive power on the probability of a crises in the future. Inattentive managers neglect default risk and over-invest, over-heating the economy which will end up in a recession in the

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loans.



following period.

## 7 Discussion and policy implications

My model implies that inattentive agents over-accumulate debt and investment during booms, which leads to higher default risk and economic fragility. While this results is similar to a large strand of the macroeconomic literature on financial frictions, the underlying mechanism is different and it highlights novel macro-prudential policy implications.

A large class of models in the macro-financial literature rationalizes the over-accumulation of debt during booms with strategic complementarity in leverage choices with full information: it is individually optimal to increase leverage when other agents do it, as individuals do not internalize the impact of their decision on the aggregate economy (Gertler and Kiyotaki, 2010; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2019; Bianchi and Mendoza, 2020). However, it is socially suboptimal, as it leads to high levels of leverage and financial fragility. In this framework, a Pigouvian tax on investment corrects this externality by mitigating the increase in leverage (Jeanne and Korinek, 2019).

In my model, the socially suboptimal high borrowing and investment during booms results from the combination of strategic substitutability and imperfect information. As aggregate investment increases, informed firms and banks would decrease their own lending and investment, making the economy safer. However, because they are not informed about the increase in aggregate investment, they contribute in making the economy riskier by increasing their own lending and investment. Information provision would then mitigate the overoptimism and therefore the boom-and-busts cycles.

The model implies two alternative policies to mitigate the credit boom-and-bust. First, an increase in public information about accumulation of risk. Second, a change in individual incentives to privately collect information.

**Public communication** The procyclical aggregate fragility in the model stems from a lack of coordination due to information dispersion. Policy makers can attenuate this problem by providing free public information about the accumulation of risk. A recent and developing literature studies central banks' financial stability communication and their impact on financial stability (Born et al., 2014; Harris et al., 2019; Londono et al., 2021). While the literature on financial stability communication is still developing, this paper

highlights how information about the aggregate economic fragility can attenuates credit boom-and-busts. However, even if the central bank can provide free public information, a standard interpretation in the rational inattention literature implies that agents might still have to pay a cognitive cost to process this information (Sims, 2003; Mackowiak et al., 2018).

**Risk taking incentives** My model highlights how risk taking incentives discourage agents from correctly assess risk, leading to procyclical overoptimistic beliefs. Policy makers can then encourage information collection by affecting agents' incentives, and in particular lowering risk-taking incentives. An example is the regulation of managers' compensation structure by limiting stock options compensation. An example of this policy is the Tax Cuts and Jobs Act (TCJA), that in 2017 reduced the scope of tax deductability for performance-based compensation as stock options (Durrant et al., 2020). On the empirical side, Cole et al. (2014) provides experimental evidence on the impact of compensation incentives, and in particular limited liability, on loan officers' effort to asses risk of borrower of a commercial bank.

## 8 Conclusions

I presented a theoretical framework where overoptimism originates from risk taking incentives in information choice. While existing models explain overoptimism during credit booms with behavioral extrapolation to good news, I propose a rational framework where overoptimism originates instead from inattention to negative news. In particular, large credit booms are associated with an increase in aggregate supply and decrease in price, and therefore inattention to aggregates leads to overestimation of own revenues. As a result, agents over-borrow and over-invest, overheating further the economy. Periods of low risk premium predict higher default rate and systematic bank losses, in line with existing evidence. Moreover, I show that such information friction can result from limited liability in payoffs, as convex payoff structures discourage managers to collect information. Because beliefs depend on incentives, my model suggests that compensation regulation has important implication in terms of macro-prudential policy.

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## A Manager compensation

Suppose we interpret the limited liability constraint as coming from a manager's convex compensation structure. The manager's compensation structure is as follows:

$$w = \begin{cases} (1 - \psi)d_j + \psi(d_j - \tilde{P}) & \text{if } d_j \geq \tilde{P} \\ (1 - \psi)d_j & \text{if } d_j < \tilde{P} \end{cases} \quad (33)$$

where  $d_j$  is the company's payoff, bank or firm, and  $\tilde{P}$  is the profit level corresponding to the exercise price of manager's options. The larger the amount of options in manager's compensation scheme  $\psi$ , the lower is his exposure to company's losses and therefore higher his insurance against company's losses.<sup>22</sup>

I assume for simplicity  $\tilde{P} = 0$ , meaning that manager's options are in the money when the profits of the firm are positive, i.e. in the non-default state. Therefore the payoff structure is equivalent as in the main text.

## B Stage-2 equilibrium

The stage-2 equilibrium can be equivalently expressed in terms of firm's issuance of bond  $\tilde{b}_j$  and bond price  $q_j$  instead of loan rate  $r_j$  and loan quantity  $b_j$ , where  $q_j = \frac{1}{1+r_j}$ , and  $\tilde{b}_j = \frac{b_j}{q_j}$ .

**Information** Agents observe the signal  $z = \epsilon_j + \theta + \eta_j$ , with  $\epsilon_j \sim N(0, \sigma_\epsilon^2)$  and  $\eta_j \sim N(0, \sigma_\eta^2)$ , and may observe  $\theta \sim N(0, \sigma_\theta^2)$ . Therefore information set of agent  $j$  is  $\Omega_j = \{z_j, \theta\}$  or  $\Omega_j = \{z_j\}$  depending on their choice in the first stage.

Define  $\tilde{z} = z - \theta$ . Posteriors are  $e|\tilde{z} \sim N(E[e|\tilde{z}], Var[e|\tilde{z}])$  with  $E[e|\tilde{z}] = \tilde{m}\tilde{z}$  with  $\tilde{m} = \frac{\sigma_e^2}{\sigma_e^2 + \sigma_\eta^2}$  and  $Var[e|\tilde{z}] = \frac{\sigma_e^2 \sigma_\eta^2}{\sigma_e^2 + \sigma_\eta^2}$ , and  $\theta|z \sim N(E[\theta|z], Var[\theta|z])$  with  $E[\theta|z] = \delta z$  with  $\delta = \frac{\sigma_\theta^2}{\sigma_e^2 + \sigma_\eta^2 + \sigma_\theta^2}$  and  $Var[\theta|z] = \frac{\sigma_\theta^2(\sigma_e^2 + \sigma_\eta^2)}{\sigma_e^2 + \sigma_\eta^2 + \sigma_\theta^2}$ .

<sup>22</sup> A more general compensation structure would consist of  $\beta_m$  shares of company's equity, of which  $\psi$  are options.

$$w_j = \begin{cases} \beta_m(1 - \psi)d_j + \beta_m\psi(d_j - \tilde{P}) & \text{if } d_j \geq \tilde{P} \\ \beta_m(1 - \psi)d_j & \text{if } d_j < \tilde{P} \end{cases} \quad (34)$$

The net profits for the shareholder are  $(1 - \beta_m)d_j$  if profit are positive and  $\beta_m\psi d_j$  otherwise. In particular,  $\beta_m < 1$  in order to ensure a positive expected leftover profit for the shareholders. However, setting  $\beta_m = 1$  does not affect qualitatively the results. Moreover, an additional fixed compensation  $\bar{w}$  would not affect the manager's incentives and therefore his decisions.

**Bargaining process** Define  $C(\theta) = \ln \left( \frac{k + \frac{1}{2}\phi k^2}{q\Lambda(M)k^\alpha} \right) - \theta$ . The expected payoff of firm manager conditioning on stage-2 information set  $\Omega_j$  is

$$\begin{aligned} E[w_{firm,j}|\Omega_j] = & - \left[ 1 - \int_{-\infty}^{\infty} \int_{-\infty}^{C(\theta)} \phi(\epsilon_j|\theta, z_j) d\epsilon_j \phi(\theta|\Omega_j) d\theta \right] \tilde{b}_j \\ & - \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{C(\theta)} \phi(\epsilon_j|\theta, z_j) d\epsilon_j \phi(\theta|\Omega_j) d\theta \right] \psi c_d k_j(q_j, \tilde{b}_j) \\ & + k_j(q_j, \tilde{b}_j)^\alpha \int_{-\infty}^{\infty} \int_{C(\theta)}^{\infty} \Lambda(\theta) e^{\epsilon_j} \phi(\epsilon_j|\theta, z_j) d\epsilon_j e^\theta \phi(\theta|\Omega_j) d\theta \end{aligned}$$

while the expected payoff of the bank manager conditioning on stage-2 information set  $\Omega_j$  is

$$\begin{aligned} E[w_{bank}|\Omega_j] = & b_j \left( \left[ 1 - \int_{-\infty}^{\infty} \int_{-\infty}^{C(\theta)} \phi(\epsilon_j|\theta, z_j) d\epsilon_j \phi(\theta|\Omega_j) d\theta \right] \right) \\ & - b_j \left( 1 - \psi \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{C(\theta)} \phi(\epsilon_j|\theta, z_j) d\epsilon_j \phi(\theta|\Omega_j) d\theta \right] \right) \frac{q_j}{q} \end{aligned} \quad (35)$$

where  $\phi(\epsilon_j|\theta, z_j) = \phi \left( \frac{C - E[\epsilon_j|\theta, z_j]}{\sqrt{Var[\epsilon_j|\theta, z_j]}} \right)$  is the posterior distribution of  $\epsilon_j$  conditioning on  $\theta$  and  $z_j$ , and  $\phi(\theta|\Omega_j) = \phi \left( \frac{\theta - E[\theta|\Omega_j]}{\sqrt{Var[\theta|\Omega_j]}} \right)$  is the posterior distribution of  $\theta$  conditioning on information set  $\Omega_j$ , which may or not include  $\theta$ .

Bank and firm manager decide collectively bond issued  $\tilde{b}_j$  and price  $q_j$  through Nash Bargaining

$$\max_{q_j, \tilde{b}_j} (E[w_{firm,j}|\Omega_j])^\beta (E[w_{bank,j}|\Omega_j])^{1-\beta} \quad (36)$$

Since I assume  $\beta \rightarrow 1$ , the problem becomes

$$\begin{aligned} & \max_{q_j, b_j} E[w_{firm,j}|\Omega_j] \\ & s.t. \quad E[w_{bank,j}|\Omega_j] \geq 0 \end{aligned} \quad (37)$$

Note that maximizing in terms of  $k_j$  is equivalent to maximizing in terms of  $\tilde{b}_j$ . The result-

ing first order conditions are given by

$$E[w_{bank,j}|\Omega_j] = 0$$

$$\frac{\frac{\partial E[w_{firm,j}|\Omega_j]}{\partial \tilde{b}_j}}{\frac{\partial E[w_{bank,j}|\Omega_j]}{\partial \tilde{b}_j}} = \frac{\frac{\partial E[w_{firm,j}|\Omega_j]}{\partial q_j}}{\frac{\partial E[w_{bank,j}|\Omega_j]}{\partial q_j}} \quad (38)$$

where each term is defined as follow. Define  $pdef_j = \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{C(\theta)} \phi(\epsilon_j|\theta, z_j) d\epsilon_j \phi(\theta|\Omega_j) d\theta \right]$ . Then,

$$\begin{aligned} \frac{\partial E[w_{firm,j}|\Omega_j]}{\partial \tilde{b}_j} = & -[1 - pdef_j] - [pdef_j] \psi_{cd} \frac{\partial k_j}{\partial \tilde{b}_j} - \left[ \int_{-\infty}^{\infty} \phi(C|\theta, z_j) \frac{\partial C}{\partial \tilde{b}_j} d\epsilon_j \phi(\theta|\Omega_j) d\theta \right] \psi_{cd} k_j \\ & + \alpha k_j^{\alpha-1} \frac{\partial k_j}{\partial \tilde{b}_j} \int_{-\infty}^{\infty} \int_{C(\theta)}^{\infty} \Lambda(\theta) e^{\epsilon_j} \phi(\epsilon_j|\theta, z_j) d\epsilon_j e^{\theta} \phi(\theta|\Omega_j) d\theta \end{aligned} \quad (39)$$

where  $\frac{\partial k_j}{\partial \tilde{b}_j} = \frac{q_j}{\sqrt{1+2\phi\tilde{b}_j q_j}}$ , and  $\frac{\partial C}{\partial \tilde{b}_j} = \frac{1}{\tilde{b}_j} - \alpha \frac{1}{k_j} \frac{\partial k_j}{\partial \tilde{b}_j}$ .

$$\begin{aligned} \frac{\partial E[w_{firm,j}|\Omega_j]}{\partial \tilde{q}_j} = & -[pdef_j] \psi_{cd} \frac{\partial k_j}{\partial q_j} - \left[ \int_{-\infty}^{\infty} \phi(C|\theta, z_j) \frac{\partial C}{\partial q_j} d\epsilon_j \phi(\theta|\Omega_j) d\theta \right] \psi_{cd} k_j \\ & + \alpha k_j^{\alpha-1} \frac{\partial k_j}{\partial q_j} \int_{-\infty}^{\infty} \int_{C(\theta)}^{\infty} \Lambda(\theta) e^{\epsilon_j} \phi(\epsilon_j|\theta, z_j) d\epsilon_j e^{\theta} \phi(\theta|\Omega_j) d\theta \end{aligned} \quad (40)$$

where  $\frac{\partial k_j}{\partial q_j} = \frac{\tilde{b}_j}{\sqrt{1+2\phi\tilde{b}_j q_j}}$ , and  $\frac{\partial C}{\partial q_j} = -\alpha \frac{1}{k_j} \frac{\partial k_j}{\partial q_j}$ .

$$\frac{\partial E[w_{bank,j}|\Omega_j]}{\partial \tilde{b}_j} = \left[ (1 - pdef_j) - (1 - \psi pdef_j) \frac{q_j}{q^f} \right] + \tilde{b}_j \left[ -\frac{\partial pdef_j}{\partial \tilde{b}_j} + \psi \frac{q_j}{q^f} \frac{\partial pdef_j}{\partial \tilde{b}_j} \right] \quad (41)$$

where

$$\frac{\partial pdef_j}{\partial \tilde{b}_j} = \left[ \int_{-\infty}^{\infty} \phi(C|\theta, z_j) \frac{\partial C}{\partial \tilde{b}_j} d\epsilon_j \phi(\theta|\Omega_j) d\theta \right] \quad (42)$$

Finally,

$$\frac{\partial E[w_{bank,j}|\Omega_j]}{\partial q_j} = +\tilde{b}_j \left[ -\frac{\partial pdef_j}{\partial q_j} + \psi \frac{q_j}{q^f} \frac{\partial pdef_j}{\partial q_j} - (1 - \psi pdef_j) \frac{1}{q^f} \right] \quad (43)$$

where

$$\frac{\partial p d e f_j}{\partial q_j} = \left[ \int_{-\infty}^{\infty} \phi(C|\theta, z_j) \frac{\partial C}{\partial q_j} d\epsilon_j \phi(\theta|\Omega_j) d\theta \right] \quad (44)$$

## C Proofs

**Proposition 1.** Assume no limited liability and no investment adjustment cost  $c_d = \psi = \phi = 0$ . To simplify the exposition, I drop the subscript  $j$ . Use the definition of  $q = \frac{1}{1+r}$  and  $q\tilde{b} = k$ . As a result,  $C = \left( \frac{k^{1-\alpha}}{q\Lambda(\theta)} \right) - \theta$ .

**Foc 1** Consider the first first order condition in 38.

$$q = q^f \left[ 1 - \int_{-\infty}^{\infty} \Phi_e(C(\theta)|z, \theta) \phi_{\theta}(\theta|\Omega) d\theta \right] \quad (45)$$

In steady state

$$q^* = q^f \left[ 1 - \Phi \left( \frac{C^*}{\sqrt{Var[\epsilon|\theta, z]}} \right) \right] \quad (46)$$

where  $x^*$  is the steady state value of variable  $x$ . Differentiating

$$dq = -q^f \Phi \left( \frac{C^*}{\sqrt{Var[\epsilon|\theta, z]}} \right) \int_{-\infty}^{\infty} [dC - dE[e|z, \theta]] \phi_{\theta}(\theta|z) d\theta \quad (47)$$

where  $dC = (1 - \alpha)\hat{k} - \hat{q} - (\eta_{\Lambda(M), \theta} - 1)d\theta$ , where  $\eta_{\Lambda(M), \theta} \equiv -\frac{1}{\Lambda(M)}\Lambda'(M)M'(\theta)$ , and  $dE[\epsilon|z, \theta] = \frac{\partial E[\epsilon|\tilde{z}]}{\partial \theta}d\theta + \frac{\partial E[\epsilon|\tilde{z}]}{\partial z}dz$ . Therefore

$$dq = -q^f \Phi \left( \frac{C^*}{\sqrt{Var[\epsilon|\theta, z]}} \right) \int_{-\infty}^{\infty} \left[ (1 - \alpha)\hat{k} - \hat{q} - \eta_{\Lambda(M), \theta}d\theta - \frac{\partial E[\epsilon|\tilde{z}]}{\partial \theta}d\theta - \frac{\partial E[\epsilon|\tilde{z}]}{\partial z}dz \right] \phi_{\theta}(\theta|z) d\theta \quad (48)$$

Denote  $a \equiv \ln(A)$  and notice that

$$\begin{aligned} E[a|z, \theta] &= \tilde{m}(z - \theta) + \theta \\ &= \frac{\partial E[\epsilon|\tilde{z}]}{\partial z}z + \frac{\partial E[\epsilon|\tilde{z}]}{\partial \theta}\theta + \theta \end{aligned}$$

Moreover,  $\hat{M} \equiv \frac{dM}{M} = \frac{M'(\theta)d\theta}{M}$  and therefore

$$\begin{aligned}\eta_{\Lambda(M),\theta}d\theta &= -\frac{M}{\Lambda(M)}\Lambda'(M)\frac{M'(\theta)d\theta}{M} \\ &= \eta_{\Lambda,M}\hat{M}\end{aligned}\tag{49}$$

where  $\eta_{\Lambda,M} \equiv \frac{\nu-\xi}{1-(1-\alpha)\xi}$ . Substitute back and divided by steady state value

$$\hat{q} = \tilde{L}_1 \left\{ -(1-\alpha)\hat{k} + E[a|z] + \eta_{\Lambda,M}E[\hat{M}|z] \right\}\tag{50}$$

where  $\tilde{L}_1 = \frac{\phi\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)}{\left[1-\Phi\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)-\phi\left(\frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}}\right)\right]}$ .

**Foc 2** Differentiate the second first order condition in 38

$$\frac{d\frac{\partial E[w_{firm,j}|\Omega_j]}{\partial \tilde{b}_j}}{\frac{\partial E[w_{firm,j}|\Omega_j]}{\partial \tilde{b}_j}} - \frac{d\frac{\partial E[w_{bank,j}|\Omega_j]}{\partial \tilde{b}_j}}{\frac{\partial E[w_{bank,j}|\Omega_j]}{\partial \tilde{b}_j}} = \frac{d\frac{\partial E[w_{firm,j}|\Omega_j]}{\partial q_j}}{\frac{\partial E[w_{firm,j}|\Omega_j]}{\partial q_j}} - \frac{d\frac{\partial E[w_{bank,j}|\Omega_j]}{\partial q_j}}{\frac{\partial E[w_{bank,j}|\Omega_j]}{\partial q_j}}\tag{51}$$

and let's see each term individually.

- From equation 39, the derivative of expected firm manager payoff with respect to bond  $\tilde{b}$  is given by

$$\begin{aligned}\frac{\partial E[w_{firm}|\Omega]}{\partial \tilde{b}} &= -\left[1 - \int_{-\infty}^{\infty} \Phi_e(C(\theta)|z, \theta)\phi_{\theta}(\theta|\Omega)d\theta\right] \\ &\quad + \alpha k_j^{\alpha-1}q \int_{-\infty}^{\infty} \Lambda(\theta)e^{\frac{Var[\epsilon|\theta,z]}{2} + E[\epsilon|\theta,z]}\Phi_{\epsilon}\left(\frac{Var[\epsilon|\theta,z] + E[\epsilon|\theta,z] - C(\theta)}{\sqrt{Var[\epsilon|\theta,z]}}\right)e^{\theta}\phi(\theta|\Omega_j)d\theta\end{aligned}$$

Differentiating,

$$\begin{aligned}
d \frac{\partial E[w_{firm}|\Omega]}{\partial \tilde{b}} &= \phi_e \left( \frac{C^*}{\sqrt{Var[\epsilon|\theta, z]}} \right) \left\{ \hat{q} - (1 - \alpha) \hat{k} + E[a|z] + \eta_{\Lambda, M} E[\hat{M}|z] \right\} \\
&\quad + \alpha k_j^{\alpha-1} q \Lambda e^{\frac{Var[\epsilon|\theta, z]}{2}} \Phi_\epsilon(\cdot) \left\{ \hat{q} - (1 - \alpha) \hat{k} + E[a|z] + \eta_{\Lambda, M} E[\hat{M}|z] \right\} \\
&\quad - \alpha k_j^{\alpha-1} q \Lambda e^{\frac{Var[\epsilon|\theta, z]}{2}} \phi_\epsilon(\cdot) \left\{ -\hat{q} + (1 - \alpha) \hat{k} - E[a|z] - \eta_{\Lambda, M} E[\hat{M}|z] \right\} \\
&= \left\{ \alpha k_j^{\alpha-1} q \Lambda e^{\frac{Var[\epsilon|\theta, z]}{2}} [\Phi_\epsilon(\cdot) + \phi_\epsilon(\cdot)] + \phi_e \left( \frac{C^*}{\sqrt{Var[\epsilon|\theta, z]}} \right) \right\} \times \\
&\quad \times \left\{ \hat{q} - (1 - \alpha) \hat{k} + E[a|z] + \eta_{\Lambda, M} E[\hat{M}|z] \right\}
\end{aligned} \tag{52}$$

As a result,

$$\begin{aligned}
\frac{d \frac{\partial E[w_{firm, j}|\Omega_j]}{\partial \tilde{b}_j}}{\frac{\partial E[w_{firm, j}|\Omega_j]}{\partial \tilde{b}_j}} &= \frac{\left\{ \alpha k_j^{\alpha-1} q \Lambda e^{\frac{Var[\epsilon|\theta, z]}{2}} [\Phi_\epsilon(\cdot) + \phi_\epsilon(\cdot)] + \phi_e \left( \frac{C^*}{\sqrt{Var[\epsilon|\theta, z]}} \right) \right\}}{\frac{\partial E[w_{firm, j}|\Omega_j]}{\partial \tilde{b}_j}} \times \\
&\quad \times \left\{ \hat{q} - (1 - \alpha) \hat{k} + E[a|z] + \eta_{\Lambda, M} E[\hat{M}|z] \right\} \\
&= L_1 \left\{ \hat{q} - (1 - \alpha) \hat{k} + E[a|z] + \eta_{\Lambda, M} E[\hat{M}|z] \right\}
\end{aligned} \tag{53}$$

$$\text{where } L_1 \equiv \frac{\left\{ \alpha k_j^{\alpha-1} q \Lambda e^{\frac{Var[\epsilon|\theta, z]}{2}} [\Phi_\epsilon(\cdot) + \phi_\epsilon(\cdot)] + \phi_e \left( \frac{C^*}{\sqrt{Var[\epsilon|\theta, z]}} \right) \right\}}{\frac{\partial E[w_{firm, j}|\Omega_j]}{\partial \tilde{b}_j}}.$$

- From equation 40, the derivative of expected firm manager payoff with respect to bond price  $q$  is given by

$$\frac{\partial E[d_{firm}|\Omega]}{\partial q} = \alpha k_j^{\alpha-1} \frac{k}{q} \int_{-\infty}^{\infty} \Lambda(\theta) e^{\frac{Var[\epsilon|\theta, z]}{2} + E[\epsilon|\theta, z]} \Phi_\epsilon \left( \frac{Var[\epsilon|\theta, z] + E[\epsilon|\theta, z] - C(\theta)}{\sqrt{Var[\epsilon|\theta, z]}} \right) e^\theta \phi(\theta|\Omega_j) d\theta \tag{54}$$

Differentiating,

$$\begin{aligned} d \frac{\partial E[d_{firm}|\Omega]}{\partial q} = & \alpha k_j^{\alpha-1} \frac{k}{q} \Lambda e^{\frac{Var[\epsilon|\theta,z]}{2}} [\Phi_\epsilon(\cdot) + \phi_\epsilon(\cdot)] \left\{ \hat{q} - (1-\alpha)\hat{k} + E[a|z] + \eta_{\Lambda,M} E[\hat{M}|z] \right\} \\ & + \alpha k_j^{\alpha-1} \frac{k}{q} \Lambda e^{\frac{Var[\epsilon|\theta,z]}{2}} \Phi_\epsilon(\cdot) (\hat{k} - 2\hat{q}) \end{aligned} \quad (55)$$

therefore

$$\begin{aligned} \frac{d \frac{\partial E[w_{firm,j}|\Omega]}{\partial q}}{\frac{\partial E[w_{firm,j}|\Omega]}{\partial q}} &= \frac{\alpha k_j^{\alpha-1} \frac{k}{q} \Lambda e^{\frac{Var[\epsilon|\theta,z]}{2}} [\Phi_\epsilon(\cdot) + \phi_\epsilon(\cdot)]}{\frac{\partial E[w_{firm,j}|\Omega]}{\partial q}} \times \\ &\times \left\{ \hat{q} - (1-\alpha)\hat{k} + E[a|z] + \eta_{\Lambda,M} E[\hat{M}|z] \right\} + (\hat{k} - 2\hat{q}) \\ &= L_2 \left\{ \hat{q} - (1-\alpha)\hat{k} + E[a|z] + \eta_{\Lambda,M} E[\hat{M}|z] \right\} + (\hat{k} - 2\hat{q}) \end{aligned} \quad (56)$$

where  $L_2 \equiv \frac{\alpha k_j^{\alpha-1} \frac{k}{q} \Lambda e^{\frac{Var[\epsilon|\theta,z]}{2}} [\Phi_\epsilon(\cdot) + \phi_\epsilon(\cdot)]}{\frac{\partial E[w_{firm,j}|\Omega]}{\partial q}}$ .

- From equation 41, the derivative of the expected bank manager payoff with respect to bond  $\tilde{b}_j$  is given by

$$\begin{aligned} \frac{\partial E[d_{bank}|\Omega^i]}{\partial \tilde{b}} &= \left[ \left( 1 - \int_{-\infty}^{\infty} \Phi_e(C(\theta)|z, \theta) \phi_\theta(\theta|\Omega) d\theta \right) - \frac{q}{q^f} \right] \\ &\quad - (1-\alpha) \int_{-\infty}^{\infty} \phi_e(C(\theta)|z, \theta) \phi_\theta(\theta|\Omega) d\theta \end{aligned}$$

Differentiating,

$$\begin{aligned} d \frac{\partial E[d_{bank}|\Omega]}{\partial \tilde{b}} &= -\phi_e \left( \frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}} \right) \left\{ \hat{q} - (1-\alpha)\hat{k} + E[a|z] + \eta_{\Lambda,M} E[\hat{M}|z] \right\} \\ &\quad - \frac{q}{q^f} \hat{q} - (1-\alpha) \phi_e \left( \frac{C^*}{\sqrt{Var[\epsilon|\theta,z]}} \right) \frac{C^*}{Var[\epsilon|\theta,z]} \left\{ \hat{q} - (1-\alpha)\hat{k} + E[a|z] + \eta_{\Lambda,M} E[\hat{M}|z] \right\} \end{aligned} \quad (57)$$

therefore

$$\begin{aligned}
\frac{d \frac{\partial E[d_{bank}|\Omega]}{\partial \hat{b}}}{\frac{\partial E[d_{bank}|\Omega]}{\partial b}} &= \frac{\phi_e \left( \frac{C^*}{\sqrt{Var[\epsilon|\theta, z]}} \right) (1 + (1 - \alpha) \frac{C}{Var[\epsilon|\theta, z]})}{\frac{\partial E[d_{bank}|\Omega]}{\partial b}} \left\{ \hat{q} - (1 - \alpha) \hat{k} + E[a|z] + \eta_{\Lambda, M} E[\hat{M}|z] \right\} \\
&\quad - \frac{\frac{q}{q^f}}{\frac{\partial E[d_{bank}|\Omega]}{\partial \hat{b}}} \hat{q} \\
&= L_3 \left\{ \hat{q} - (1 - \alpha) \hat{k} + E[a|z] + \eta_{\Lambda, M} E[\hat{M}|z] \right\} - L_4 \hat{q}
\end{aligned} \tag{58}$$

where  $L_3 \equiv \frac{\phi_e \left( \frac{C^*}{\sqrt{Var[\epsilon|\theta, z]}} \right) (1 + (1 - \alpha) \frac{C}{Var[\epsilon|\theta, z]})}{\frac{\partial E[d_{bank}|\Omega]}{\partial b}} \left\{ \hat{q} - (1 - \alpha) \hat{k} + E[a|z] + \eta_{\Lambda, M} E[\hat{M}|z] \right\}$  and  $L_4 \equiv \frac{\frac{q}{q^f}}{\frac{\partial E[d_{bank}|\Omega]}{\partial b}}.$

- From equation 43, the derivative of the expected bank manager payoff with respect to bond price  $q_j$  is given by

$$\frac{\partial E[d_{bank}|\Omega]}{\partial q} = \frac{k}{q} \left[ \alpha \int_{-\infty}^{\infty} \phi_e(C(\theta)|z, \theta) \phi_{\theta}(\theta|\Omega) d\theta \frac{1}{q} - \frac{1}{q^f} \right]$$

differentiating,

$$\begin{aligned}
d \frac{\partial E[d_{bank}|\Omega]}{\partial q} &= \frac{k}{q} (\hat{k} - \hat{q}) \left[ \alpha \phi_e \left( \frac{C^*}{\sqrt{Var[\epsilon|\theta, z]}} \right) \frac{1}{q} - \frac{1}{q^f} \right] + \\
&\quad + \frac{k}{q} \alpha \phi_e \left( \frac{C^*}{\sqrt{Var[\epsilon|\theta, z]}} \right) \frac{C^*}{Var[\epsilon|\theta, z]} \frac{1}{q} \left\{ \hat{q} - (1 - \alpha) \hat{k} + E[a|z] + \eta_{\Lambda, M} E[\hat{M}|z] \right\} \\
&\quad - \frac{k}{q} \alpha \phi_e \left( \frac{C^*}{\sqrt{Var[\epsilon|\theta, z]}} \right) \frac{1}{q} \hat{q}
\end{aligned}$$



Therefore

$$\begin{aligned}
\frac{d \frac{\partial E[d_{bank}|\Omega]}{\partial q}}{\frac{\partial E[d_{bank}|\Omega]}{\partial q}} &= (\hat{k} - \hat{q}) - \frac{\frac{k}{q} \alpha \phi_e \left( \frac{C^*}{\sqrt{\text{Var}[\epsilon|\theta, z]}} \right)}{\frac{\partial E[d_{bank}|\Omega]}{\partial q}} \hat{q} \\
&\quad - \frac{\frac{k}{q} \alpha \phi_e \left( \frac{C^*}{\sqrt{\text{Var}[\epsilon|\theta, z]}} \right) \frac{C^*}{\text{Var}[\epsilon|\theta, z]}}{\frac{\partial E[d_{bank}|\Omega]}{\partial q}} \left\{ \hat{q} - (1 - \alpha) \hat{k} + E[a|z] + \eta_{\Lambda, M} E[\hat{M}|z] \right\} \\
&= (\hat{k} - \hat{q}) - L_5 \left\{ \hat{q} - (1 - \alpha) \hat{k} + E[a|z] + \eta_{\Lambda, M} E[\hat{M}|z] \right\} - L_6 \hat{q}
\end{aligned} \tag{59}$$

$$\text{where } L_5 = \frac{\frac{k}{q} \alpha \phi_e \left( \frac{C^*}{\sqrt{\text{Var}[\epsilon|\theta, z]}} \right) \frac{C^*}{\text{Var}[\epsilon|\theta, z]}}{\frac{\partial E[d_{bank}|\Omega]}{\partial q}} \text{ and } L_6 = \frac{\frac{k}{q} \alpha \phi_e \left( \frac{C^*}{\sqrt{\text{Var}[\epsilon|\theta, z]}} \right)}{\frac{\partial E[d_{bank}|\Omega]}{\partial q}}.$$

Finally, substitute equations 53, 56, 58, and 59 in equation 51 and get

$$\begin{aligned}
(L_1 - L_2 - L_3 - L_5 + L_4 + 1 - L_6) \hat{q} &= -(L_1 - L_2 - L_3 - L_5) \left\{ -(1 - \alpha) \hat{k} + E[a|z] + \eta_{\Lambda, M} E[\hat{M}|z] \right\} \\
\hat{q} &= \tilde{L}_2 \left\{ -(1 - \alpha) \hat{k} + E[a|z] + \eta_{\Lambda, M} E[\hat{M}|z] \right\}
\end{aligned} \tag{60}$$

$$\text{where } \tilde{L}_2 \equiv \frac{-(L_1 - L_2 - L_3 - L_5)}{(L_1 - L_2 - L_3 - L_5 + L_4 + 1 - L_6)}.$$

**Equilibrium** Substitute equation 50 in 60

$$\begin{aligned}
\tilde{L}_1 \left\{ -(1 - \alpha) \hat{k} + E[a|z] + \eta_{\Lambda, M} E[\hat{M}|z] \right\} &= \tilde{L}_2 \left\{ -(1 - \alpha) \hat{k} + E[a|z] + \eta_{\Lambda, M} E[\hat{M}|z] \right\} \\
(\tilde{L}_1 - \tilde{L}_2) \left\{ -(1 - \alpha) \hat{k} + E[a|z] + \eta_{\Lambda, M} E[\hat{M}|z] \right\} &= 0
\end{aligned} \tag{61}$$

therefore, the stage-2 equilibrium  $k$  and  $q$  are given by

$$\begin{aligned}
\hat{k} &= \frac{1}{1 - \alpha} (E[a|z] - \gamma E[\hat{M}|z]) \\
\hat{q} &= 0
\end{aligned} \tag{62}$$

Where  $\gamma \equiv -\eta_{\Lambda(M), M} = -\frac{\nu - \xi}{1 - (1 - \alpha)\xi}$ . If  $\nu < \xi$ , then  $\gamma > 0$ . Therefore  $\hat{r}_j \propto \hat{q} = 0$ .

Since  $M = \left\{ \left[ \frac{w}{(1 - \alpha)\xi^\nu} \right]^{\frac{(1 - \alpha)}{(1 - \alpha)\xi - 1}} \left[ \int^N A_j k_j^\alpha dj \right]^{\frac{1}{\xi}} \right\}^{\frac{1 - (1 - \alpha)\xi}{1 - (1 - \alpha)\nu}}$ , log deviation of  $M$  around the

stochastic steady state equals

$$\hat{M} = \mu(\alpha\hat{K} + \theta)$$

where  $\mu \equiv \frac{1}{\xi} \frac{1-(1-\alpha)\xi}{1-(1-\alpha)\nu} > 0$  and  $\hat{K} = \int^j k_j dj$ . One can write

$$\begin{aligned}\hat{k} &= \frac{1}{1-\alpha}(E[a|z] - \gamma\mu E[\theta + \alpha\hat{K}|z]) \\ \hat{q} &= 0\end{aligned}$$

Moreover, from 45

$$\hat{q}_j = -\zeta\hat{p}(def_j|\Omega_j) = 0 \quad (63)$$

where  $\zeta \equiv \frac{p^*(def|0)}{1-p^*(def|0)}$ .

The expected level deviation of bank  $j$  profits from steady state conditioning on state  $\theta$  equals

$$\begin{aligned}E[w_{bank,j}|z_j, \theta] &= -p^*(def|0)\hat{p}(def_j|z_j, \theta) - \frac{q^*}{q_j}\hat{q}_j \\ &= -p^*(def|0)[\hat{p}(def_j|z_j, \theta) - E[\hat{p}(def_j|\Omega_j)|\theta]]\end{aligned} \quad (64)$$

which is zero for each  $\theta$  if  $\theta \in \Omega_j$ . ■

**Proposition 2.** Consider the global game when  $\theta$  is observed

$$\hat{k} = \frac{1}{1-\alpha}E[a_j|z] - \frac{1}{1-\alpha}\gamma\mu(\theta + \alpha\hat{K}) \quad (65)$$

where  $E[a_j|z_j, \theta] = \tilde{m}(z_j - \theta) + \theta$ , where  $z_j = a_j + \eta_j$  and  $\tilde{m} = \frac{\sigma_e^2}{\sigma_e^2 + \sigma_\eta^2}$ . Aggregating across islands

$$\begin{aligned}K &= \frac{1}{1-\alpha}(1-\gamma\mu)\theta - \frac{\alpha}{1-\alpha}\gamma\mu K \\ K &= \frac{(1-\gamma\mu)}{1-\alpha+\alpha\gamma\mu}\theta\end{aligned} \quad (66)$$

■

**Proposition 3.** Consider the global game when  $\theta$  is not observed

$$\hat{k} = \frac{1}{1-\alpha}E[a_j|z_j] - \frac{1}{1-\alpha}\gamma\mu(E[\hat{\theta}|z_j] + \alpha E[\hat{K}|z_j]) \quad (67)$$

where  $E[a_j|z_j] = mz_j$ , where  $m = \frac{\sigma_e^2 + \sigma_\theta^2}{\sigma_e^2 + \sigma_\theta^2 + \sigma_\eta^2}$ , and  $E[\theta|z_j] = \delta z_j$  where  $\delta = \frac{\sigma_e^2}{\sigma_e^2 + \sigma_\theta^2 + \sigma_\eta^2}$ . Following **Morris and Shin (2002)**, I guess the linear solution  $k_j = \chi z_j$

$$\begin{aligned} k_j &= \frac{1}{1-\alpha}(m - \gamma\mu[1 + \alpha\chi]\delta)z_j \\ \chi &= \frac{1}{1-\alpha}(m - \gamma\mu[1 + \alpha\chi]\delta) \\ \chi &= \frac{(m - \gamma\mu\delta)}{1 - \alpha + \gamma\mu\alpha\delta} \\ K &= \frac{(m - \gamma\mu\delta)}{1 - \alpha + \gamma\mu\alpha\delta}\theta \end{aligned} \tag{68}$$

■

**Corollary 4.** The loglinearized individual revenues  $\hat{\pi}_j$  if  $\theta \notin \Omega_j$  equals

$$\begin{aligned} \hat{\pi}_j &= -\gamma\hat{M} + a_j + \alpha k_j \\ &= -\gamma\mu \left( \theta + \alpha \frac{(m - \gamma\mu\delta)}{1 - \alpha + \gamma\mu\alpha\delta} \theta \right) + a_j + \alpha k_j \end{aligned} \tag{69}$$

Since  $E[a_j|z_j] = mz_j$  and  $E[\theta|z_j] = \delta z_j$ ,

$$\begin{aligned} E[\hat{\pi}_j|z_j, \theta] - E[E[\hat{\pi}_j|z_j]|\theta] &= E[a_j|z_j, \theta] - E[E[a_j|z_j]|\theta] - \gamma(\hat{M} - E[\hat{M}|z_j]) \\ &= \left[ (1 - m) - \gamma\mu(1 - \delta) \left( 1 + \alpha \frac{(m - \gamma\mu\delta)}{1 - \alpha + \gamma\mu\alpha\delta} \right) \right] \theta \end{aligned} \tag{70}$$

It implies that average forecast errors are a positive function of  $\theta$  if

$$\begin{aligned} (1 - m) - \gamma\mu(1 - \delta) \left( 1 + \alpha \frac{(m - \gamma\mu\delta)}{1 - \alpha + \gamma\mu\alpha\delta} \right) &> 0 \\ (m - \gamma\mu\delta)(1 - \alpha + \alpha\gamma\mu) &> (1 - \gamma\mu)(1 - \alpha + \gamma\mu\alpha\delta) \end{aligned} \tag{71}$$

■

**Corollary 5.** Consider actual probability of default of firm  $j$  in dispersed information conditioning on aggregate shock  $\theta$ :  $p(def_j|z_j, \theta) \equiv \Phi_{e|\bar{z}}(C(\theta))$ . The first order approximation around the risky steady state is

$$\hat{p}(def_j|z_j, \theta) = \frac{\phi_{e|0}(C^*)}{\Phi_{e0}(C^*)} \left[ (1 - \alpha) \hat{k}_j - \hat{q}_j + \gamma\hat{M} - E[a_j|z_j, \theta] \right] \tag{72}$$

Aggregating across islands

$$\begin{aligned}
\hat{p}(def|z_j, \theta) &= \xi \left[ (1 - \alpha) \hat{K} - \hat{Q} + \gamma \hat{M} - \theta \right] \\
\hat{p}(def|z_j, \theta) &= \xi \left[ (1 - \alpha + \alpha\gamma\mu) \hat{K} - (1 - \gamma\mu)\theta \right] \\
\hat{p}(def|z_j, \theta) &= \xi \left[ (1 - \alpha + \alpha\gamma\mu) \frac{(m - \gamma\mu\delta)}{1 - \alpha + \gamma\mu\alpha\delta} - (1 - \gamma\mu) \right] \theta
\end{aligned} \tag{73}$$

Then it implies that  $\frac{\partial \hat{p}(def|\theta)}{\partial \theta} > 0$  if

$$(m - \gamma\mu\delta)(1 - \alpha + \alpha\gamma\mu) > (1 - \gamma\mu)(1 - \alpha + \gamma\mu\alpha\delta) \tag{74}$$

■

**Corollary 6.** Consider the logdeviation of perceived probability of default from steady state, meaning conditioning on info set  $\Omega_j = \{z_j\}$ .

$$\hat{p}(def_j|z_j) = \frac{\phi_{e|0}(C^*)}{\Phi_{e_0}(C^*)} \left[ (1 - \alpha) \hat{k}_j - \hat{q}_j + \gamma E[\hat{M}|z_j] - E[a_j|z_j] \right] \tag{75}$$

Consider the logdeviation of actual probability of default from steady state, meaning conditioning on info set  $\Omega_j = \{z_j, \theta\}$ .

$$\hat{p}(def_j|z_j, \theta) = \frac{\phi_{e|0}(C^*)}{\Phi_{e_0}(C^*)} \left[ (1 - \alpha) \hat{k}_j - \hat{q}_j + \gamma \hat{M} - E[a_j|z_j, \theta] \right] \tag{76}$$

The average bank profits equal the difference between the two

$$\begin{aligned}
E[\tilde{\pi}_{bank,j}|z_j, \theta] &\propto -[\hat{p}(def_j|z_j, \theta) - E[\hat{p}(def_j|z_j)|\theta]] \\
&\propto -[E[a_j|z_j, \theta] - E[E[a_j|z_j]|\theta] - \gamma(M - E[M|z_j])]
\end{aligned} \tag{77}$$

from the proof of corollary 4, it follow that average bank profits are a negative function of  $\theta$  if

$$(m - \gamma\mu\delta)(1 - \alpha + \alpha\gamma\mu) > (1 - \gamma\mu)(1 - \alpha + \gamma\mu\alpha\delta) \tag{78}$$

■

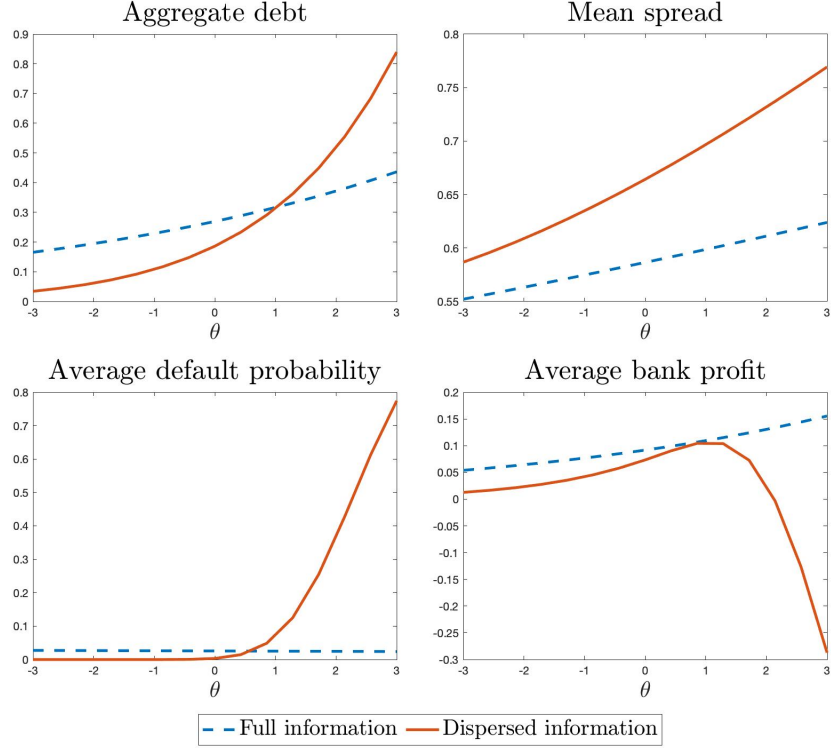


Figure 9

## D Equal bargaining power

In the baseline model I assume firm and bank managers decide loan quantity and prices in second stage and information in the first stage through Nash bargaining, with the firms retaining all bargaining power. This yields the standard implication that the price of the loan reflects only quantity of risk, with no changes in price of risk. I relax this assumption here by setting the same bargaining power on bank and firm.

**Second stage** The second-stage optimal  $k_j^*$  and  $q_j^*$  maximize

$$\max_{q_j, b_j} (E[w_{firm}|\Omega_j])^\beta (E[w_{bank}|\Omega_j])^{1-\beta} \quad (79)$$

Figure 9, and 10 illustrate the equilibrium where  $\beta = 0.5$ . Differently from the baseline model, risk premium increases in booms even if risk declines, as the bank extract more profit from the firm. As a result, bank's profits increase in moderate booms, but decline for very large booms as the losses for mispricing of risk becomes larger than the rent extraction from the firm.

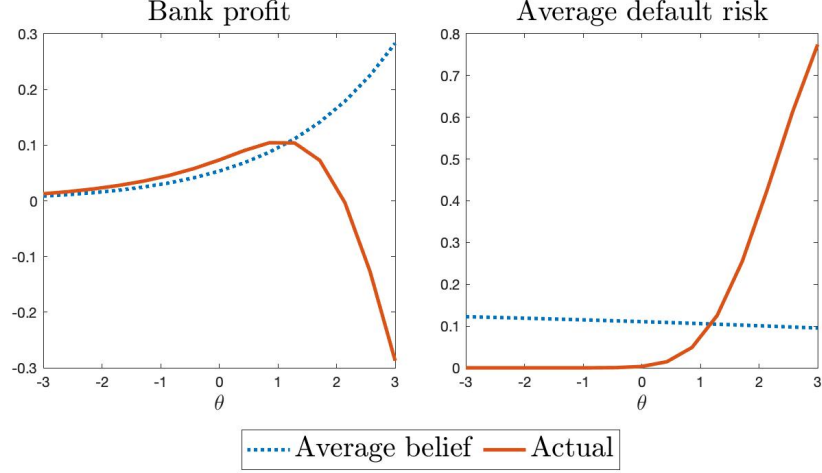


Figure 10

**First stage** Next , consider the same convex payoff structure 33 on the bank manager instead

$$w_{bank} = \begin{cases} b_j(1 - q_j) - b^f(1 - q^f) & \text{if } \Lambda(M)A_jk_j^\alpha \geq b_j \quad (repay) \\ (1 - \alpha_b)[b_j(-q_j) - b^f(1 - q^f)] & \text{if } \Lambda(M)A_jk_j^\alpha < b_j \quad (default) \end{cases} \quad (80)$$

where  $\alpha_b$  is the option holding of the bank manager. In the first stage the island decide to pay the information cost if

$$\begin{aligned} & (E[\pi_{firm}^*(\theta \in \Omega_j, \lambda)] - \beta c)^\beta (E[w_{bank}^*(\theta \in \Omega_j, \lambda)] - (1 - \beta)c)^{1-\beta} \\ & \geq (E[\pi_{firm}^*(\theta \notin \Omega_j, \lambda)])^\beta (E[w_{bank}^*(\theta \notin \Omega_j, \lambda)])^{1-\beta} \end{aligned} \quad (81)$$

where I assume that bank and firm split the information cost  $c$  according to their bargaining power  $\beta$  as well. Figure 11 reports the equilibrium information  $\lambda$  for different values of bank payoff convexity (assuming no convexity on firm payoff). Higher limited liability on bank manager also reduces optimal information choice.

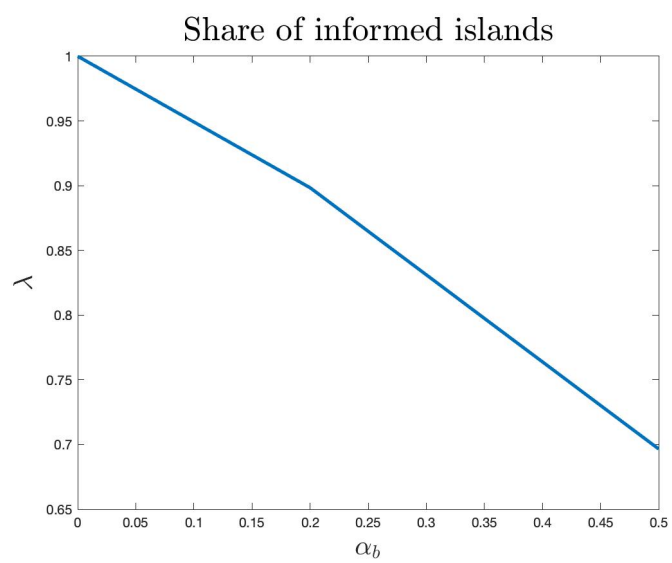


Figure 11