Managing Expectations with Exchange Rate Policy*

Giacomo Candian† HEC Montréal

Pierre De Leo‡ University of Maryland

Luca Gemmi§ HEC Lausanne

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Abstract

Should exchange rate policy communication be transparent or intentionally opaque? We develop a macro model in which agents overreact to the information about economic fundamentals contained in private signals as well as in equilibrium prices, such as the exchange rate. In this environment, FX interventions that are publicly announced signal the central bank's view about macro fundamentals, whereas opaque FX interventions influence the informational content of the exchange rate and can thus be used to "manage expectations." If expectations' overreaction is strong enough, it is optimal to intervene opaquely in order to control the informativeness of the exchange rate. Our model rationalizes observed practices in exchange rate policies such as managed floats as well as the widespread opacity around FX interventions.

Keywords: exchange rates, information frictions, foreign exchange interventions, central bank communication.

JEL Classification: D8, F31, F41, E71.

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†Email: giacomo.candian@hec.ca

‡Email: deleop@umd.edu

§Email: luca.gemmi@unil.ch

Central banks around the world frequently engage in foreign exchange (FX) market interventions, buying and selling foreign currency.[1] A notable characteristic of many central banks is their *opaque* communication regarding FX interventions, often not publicly announced or disclosed only after a lag. For example, around two third of central banks do not pre-announce their FX interventions (Patel and Cavallino, 2019). We provide a framework to study the macroeconomic effects of FX interventions and their communication, offering a rationale for the puzzling opacity surrounding observed exchange rate policy.

Our first premise is that exchange rates play an informational role by aggregating agents' knowledge and beliefs about economic fundamentals (Grossman, 1976; Bacchetta and Wincoop, 2006). While often highlighted by policy makers and commentators, the informational role of the exchange rate is largely absent in macroeconomic models. Our first contribution is to formalize this informational role in a small-open economy environment in which equilibrium exchange rate changes lead people to revise their expectations of future fundamentals, and adjust their consumption and capital investment accordingly. This channel is distinct from expenditure switching and wealth effects, and its strength hinges on the *informational content of the exchange rate*, the amount of independent information that the exchange rate contains about fundamentals. A central feature of the model is that FX interventions, and the communication around them, alter the informational content of the exchange rate and can be used to influence markets' expectations about fundamentals.

Our second premise is the growing evidence that expectations of households and financial market participants overreact to incoming news (see, *e.g.*, Bordalo et al., 2022). To speak to this evidence, we extend the diagnostic expectations model of Bordalo et al. (2018) to a setting with endogenous signals. In equilibrium, people overreact to the information contained in the exchange rate as well as in central bank's communication around FX intervention. Our second contribution is to characterize how these biases in beliefs shape the trade-off that central banks face when conducting and communicating FX interventions. In particular, if expectations' overreaction to new information is sufficiently strong, opaque interventions can be optimal so as to limit the informativeness of the exchange rate and, in turn, the inefficiencies stemming from biased beliefs.

Our dynamic small-open economy model exhibits three distinctive features, each of

[1] This practice, traditionally associated with emerging economies, has become more common in advanced economies like Switzerland and Japan in recent decades (Adler et al., 2021).

which finds empirical support. First, we assume that international asset markets are segmented, consistent with evidence from currency demand estimation (see, *e.g.*, Hau et al., 2010, Pandolfi and Williams, 2019, and Aldunate et al., 2022). This implies that financial flows directly influence equilibrium exchange rates and interventions are effective, as in Gabaix and Maggiori (2015), Fanelli and Straub (2021), and Itskhoki and Mukhin (2022). Second, we relax the common, full information assumption. Instead, we assume that each agent has access to private information about future productivity, *i.e.*, the economy's fundamental. As a result, the exchange rate aggregates this private information and is used as a public signal about productivity. Yet, it is a noisy signal because noise-trading shocks blur the relationship between exchange rates and productivity (similar to Bacchetta and Wincoop, 2006). Recent work supports the notion that exchange rates reflect, at least in part, available information about a country's future productivity (Chahrour et al., 2022).[2] Third, individuals overreact to the news from their private signals, as in Bordalo et al. (2018), and mistakenly think of themselves and everyone else as rational agents. This means that agents systematically underestimate the response of other agents (similar to Angeletos and Sastry, 2020) and thus overreact to the news contained in the exchange rate. Expectations' overreaction is well documented in laboratory experiments (Afrouzi et al., 2023) as well as in surveys. Bordalo et al. (2020) document that forecasters overreact to news about macroeconomic and financial variables, while Candian and De Leo (2023) show that this overreaction explains key properties of exchange rate dynamics.

We demonstrate that the macroeconomic consequences of FX interventions depend on the transparency of the communication surrounding them. We consider a central bank that intervenes according to a rule that can respond to productivity and noise-trading shocks. The central bank can choose the strength of the reaction to either shock as well as whether to communicate the size of the intervention. If the size of the FX intervention is publicly announced, it becomes an additional public signal about the beliefs of the central bank. Consequently, the intervention assumes a "signaling role," which expands the information set available to economic agents (akin to Melosi, 2017 and Tang, 2015). In contrast, if the size of the intervention is not publicly announced (*i.e.*, it is kept opaque), the central bank can alter the information content of the exchange rate by appropriate choice of the reaction coefficients in the intervention rule.

[2] Chahrour et al. (2022) show that a large portion of exchange rate variation emanates from anticipated changes in future productivity, with a significant component of expectational noise (see also Engel and West, 2005 and Stavrakeva and Tang, 2020).

This choice affects the extent to which exchange rate fluctuations reflect expectations of future productivity *vis-a-vis* noise trading shocks. For example, a stronger reaction to noise-trading shocks renders the exchange rate a better signal about productivity. Likewise, leaning more strongly against productivity shocks lowers the correlation between productivity and the exchange rate, therefore reducing its informativeness. In this way, the central bank can use systematic FX interventions as a policy tool to control the information content of the exchange rate, and "manage expectations" about economic fundamentals.

Our normative results show that the optimal conduct and communication of FX interventions depend on the strength of the departure from rational expectations. To understand this result let us step back and examine the sources of inefficiency in our small open economy. First, the full-information rational expectation allocation is not efficient because of international asset market segmentation and noise-trading shocks. This friction manifests in an *intermediation wedge* which results in suboptimal external borrowing due to the excess return required by financiers to hold imbalanced currency positions (as in Fanelli and Straub, 2021 and Itskhoki and Mukhin, 2022). Second, dispersed information generally entails that agents underreact to productivity shocks, while the cognitive bias due to diagnostic expectations results in overreaction to new information. Both of these frictions impinge on the *belief wedge* – the gap between the average expectations of future productivity and its actual value. Misaligned expectations of future productivity result in a suboptimal level of external borrowing, and an intertemporal misallocation of consumption and investment.

We show that the optimal exchange rate policy trades off these two sources of inefficiency, and the trade off is shaped by the degree of expectations' overreaction. When agents overreact to new information, the central bank finds it optimal to limit the informativeness of the exchange rates and thus the fluctuations in the belief wedge. To achieve this outcome, the central bank engages in *opaque* FX intervention policy. In particular, it systematically intervenes in the FX market by offsetting only part of the noise-trading shock, but without announcing neither the sign nor the size of the intervention. The exchange rate then remains an imperfect, noisy signal of future productivity. In contrast, in the special case of rational expectations, the central bank can contemporaneously close both intermediation and belief wedges thereby achieving the first-best allocation (a form of "divine coincidence"). To do so, the central bank offsets *all* noise-trading shocks achieving perfect risk sharing while making the exchange rate a perfectly informative public signal. This outcome can be sustained by appropriate opaque or public communication of FX intervention.

The model thus presents a novel interpretation of the widespread practice of "systematic managed floating": central banks regularly respond to changes in total market pressure, with a portion reflected in the exchange rate itself and the rest absorbed through changes in foreign exchange reserves (Frankel, 2019). In our model, when expectations exhibit over-reaction, the optimal opaque intervention policy controls the amount of noise in the exchange rate and achieves a lower *equilibrium* exchange rate volatility. This approach resembles "fear of floating" (Calvo and Reinhart, 2002), yet it is distinct from traditional "exchange rate smoothing" as reducing the volatility of the exchange rate alone does not alter its information content.

Contribution to the literature This paper offers a new perspective on the transmission of exchange rate policy, speaking to central aspects of real-world FX intervention. Notably, the model offers an explanation for two seemingly contradictory empirical observations: policymakers reportedly believe that FX interventions work primarily by affecting market expectations, but their communication about FX interventions is often opaque (Sarno and Taylor, 2001; Patel and Cavallino, 2019). In our model, FX interventions indeed play an important signaling role. However, when expectations overreact to news, central banks may find it advantageous to conduct these interventions opaquely, thereby reducing the informational content of exchange rate fluctuations. The open-economy macro literature on FX intervention has instead focused on distinct, complementary motives of FX interventions, such as alleviating the effects of financial constraints on banks' lending (Chang and Velasco, 2017), counteract the effects of shocks originating in FX markets (Cavallino, 2019; Itskhoki and Mukhin, 2022; Boz et al., 2020), implement a desired exchange rate policy when the interest rate is at the zero lower bound (Amador et al., 2019), mitigate the distributional effects of exchange rate fluctuations due to consumption externalities (Fanelli and Straub, 2021), or address the effects of production externalities (Ottonello et al., 2024).[3,4]

Our paper studies exchange rate policy transparency within a unified general-equilibrium framework that highlights the role of expectations' overreaction, consistent

[3]Recent empirical evidence on FX interventions includes Kearns and Rigobon (2005), Kuersteiner et al. (2018), Fratzscher et al. (2019), Menkhoff et al. (2021), and Adler et al. (2021).

[4]A related literature studies the optimal monetary policy under different currency pricing contracts, *e.g.*, Galí and Monacelli (2005), Benigno and Benigno (2003), Engel (2011), Devereux and Engel (2007), Drenik et al. (2021) and Egorov and Mukhin (2023).

with patterns observed in survey of expectations and laboratory experiments. Instead, the existing literature on exchange rate policy transparency primarily focuses on central bank credibility issues and strategic considerations vis-à-vis market participants (Bhattacharya and Weller, 1997) . Vitale (1999, 2003) study the signaling role of FX interventions in a market micro-structure framework, where the central bank transparency is not about the size of the intervention but about its objective, i.e. the intervention rule. Fernholz (2015) also studies the implications of central bank transparency of FX interventions in a partial equilibrium setting where FX interventions affect fundamentals.[5]

This paper revisits the question of the social value of public information in a novel environment with endogenous signals and departures from rationality.[6] A key lesson from the literature, highlighted in Angeletos and Pavan (2007), is that when economic fluctuations are driven primarily by shocks or other distortions that induce a countercyclical efficiency gap, it is possible that providing markets with information lowers welfare. These conditions may arise, for example, in the presence of markup shocks (Angeletos et al., 2016), distortionary taxes (Fujiwara and Waki, 2020), sticky prices (Fujiwara and Waki, 2022), and even with supply shocks if they are inefficiently shared across countries (Candian, 2021). When prices aggregate information as in Grossman (1976), strategic complementarities that make agents overweight public signals (Amador and Weill, 2010), or correlated expectation errors that are common knowledge (Hassan and Mertens, 2017) are also known to lead to inefficiencies in the usage of public information. The reasons for these inefficiencies in our model is new: correlated errors in expectation formation, *i.e.*, extrapolation of the information contained in the exchange rate, that are not common knowledge.

We also contribute to the literature that studies the effect of macroeconomic policy and their communication, in related but distinct contexts. Bond and Goldstein (2015) investigate how uncertain future government intervention affecting a firm's cash flows impacts the informativeness of prices. Gaballo and Galli (2022) studies the information channel of central bank's asset purchases, but in a closed economy setting where interventions are always publicly observed. Iovino and Sergeyev (2021) study the effects of central bank balance sheet policies in a model where people form expectations

[5]Kimbrough (1983, 1984) show that flexible exchange rate regimes allow agents to learn from the exchange rate, but only consider monetary policy.

[6]To our knowledge we are the first to model extrapolative higher order beliefs formation in the presence of an endogenous public signal.

through an iterative level-k thinking process, also in a closed economy and without information frictions. Angeletos and Sastry (2020) study the separate but complementary question of whether policy communications should anchor expectations about a policy instrument or the targeted outcome, and how it depends on a departure from rational expectations. Brunnermeier et al. (2021) study the incentives of market participants to acquire information when there is uncertainty surrounding the extent of government intervention in financial markets via asset purchases. Their government faces a trade-off between reducing price volatility and enhancing price efficiency. In our environment, there is also a tension between reducing (inefficient) price volatility and improving allocative efficiency but the tension arises because the private sector processes the information contained in prices incorrectly, rather than because the intervention by itself introduces noise.[7]

1 Model

We consider a two-period small-open economy model with a tradable sector and non-tradable sector extended to incorporate three features of interest. First, limited asset market participation gives rise to a finite elasticity of demand for foreign bonds and, therefore, a scope for foreign exchange interventions. Second, the economy is affected by two aggregate shocks that are imperfectly observed by agents in the economy: productivity shocks and "noise" shocks to the demand for foreign bonds. Finally, agents observe the exchange rate and learn from it.

1.1 Model setup

The small-open economy is populated by four types of agents: households, final-good producers, financiers, and a central bank. Households, final-good producers, and financiers are located on a continuum of atomistic islands, $i \in [0, 1]$, as in Lucas (1972). Information is common within islands but heterogeneous across islands. In particular, on each island, households and financiers receive the same private noisy signal on next-period productivity of the small open-economy. Agents observe local output and prices as well as the exchange rate, which serves as a noisy public signal about next-period productivity. Time is discrete and indexed by $t = [0, 1]$. Foreign variables are denoted with a star symbol.

[7] See also Chahrour (2014) and Kohlhas (2020).

1.1.1 Households and goods markets

The preferences of the representative household of island i are described by the following utility function:

$$\frac{{C_0^i}^{1-\sigma}}{1-\sigma} + \beta E_0 \left(\frac{{C_1^i}^{1-\sigma}}{1-\sigma} \right), \tag{1}$$

where C^i denotes consumption and E_0 is an expectation operator, non necessarily rational and conditional on information set in $t = 0$.

Households have an initial endowment of capital, $K_0^i = K_0 > 0$, which fully depreciated between periods and is used in the production of tradable goods:

$$Y_{T,0}^{i,H} = {K_0^i}^{\alpha}, \qquad Y_{T,1}^{i,H} = A_1 {K_1^i}^{\alpha}. \tag{2}$$

Above, A_1 represents stochastic period-1 productivity. In each period, the household also receives an endowment of the non-tradable good: $Y_{N,0} = (1 + \alpha\beta\gamma)Y_{N,1}$.[8] Consumption and period-1 capital are composites of tradable and non-tradable goods:

$$C_0^i + K_1^i = G(Y_{N,0}, Y_{T,0}^i), \qquad C_1^i = G(Y_{N,1}, Y_{T,1}^i) \tag{3}$$

where $G(Y_N, Y_T) = \left[(1-\gamma)^{\frac{1}{\theta}} {Y_N}^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} {Y_T}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$ is homogenous of degree 1. The parameter θ denotes the elasticity of substitution between tradable and non-tradable goods in the production of final goods while γ governs the share of tradable goods in the final composite good. In (3), $Y_{T,t}^i$ represents domestic absorption of the tradable good, which is the sum (difference) of production and imports from (exports to) the rest of the world $Y_{T,t}^i = Y_{T,t}^{i,H} + Y_{T,t}^{i,F}$. We assume that each island trades with the rest of the world but not with other islands to avoid full information revelation by inter-island interactions.

Since the aggregator G is homogenous of degree 1, we have, in equilibrium:

$$P_t^i G(Y_{N,t}, Y_{T,t}^i) = P_{N,t}^i Y_{N,t} + \mathcal{S}_t P_{T,t}^{\star} Y_{T,t}^i, \tag{4}$$

where P_t^i is the island-i price of the composite good, and $P_{N,t}^i$ is the island-i price of the non-tradable good. $\mathcal{S}_t$ is the nominal exchange rate, which is common across islands.

[8] This choice of relative endowment in period 0 and 1 is convenient as it delivers a steady state with $B_1^\star = 0, Q_0 = Q_1 = 1$ and $C_0 = C_1$.

We assume that the foreign-currency price of tradable goods is constant and equal to 1, i.e. $P^{\star}_{T,t} = P^{\star}_{T} = 1$.

The price of the tradable good relative to the non-tradable good is given, in equilibrium, by their marginal rate of transformation:

$$\frac{\mathcal{S}_t}{P^i_{N,t}} = \frac{\partial G(Y_{N,t}, Y^i_{T,t})/\partial Y^i_{T,t}}{\partial G(Y_{N,t}, Y^i_{T,t})/\partial Y_{N,t}} = \left(\frac{\gamma}{1-\gamma}\frac{Y_{N,t}}{Y^i_{T,t}}\right)^{\frac{1}{\theta}}. \tag{5}$$

Combining this expression with (4) yields the equation determining island-i composite price index:

$$P^i_t = \left[(1-\gamma){P^i_{N,t}}^{1-\theta} + \gamma{\mathcal{S}_t}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{6}$$

Combining these last two equations we obtain the demand function for tradable goods:

$$Y^i_{T,t} = \chi\left[\left(\frac{\mathcal{S}_t}{P^i_t}\right)^{-(1-\theta)} - \gamma\right]^{\frac{\theta}{1-\theta}} Y_{N,t} \tag{7}$$

with $\chi = \frac{\gamma}{(1-\gamma)^{\frac{1}{1-\theta}}}$. The household's budget constraints are:

$$\begin{aligned} P^i_0 C^i_0 + P^i_0 K^i_1 + \frac{B^i_1}{R_0} &= P^i_{N,0} Y_{N,0} + \mathcal{S}_0 Y^{i,H}_{T,0} + T^i_0 \\ P^i_1 C^i_1 &= B^i_1 + P^i_{N,1} Y_{N,1} + \mathcal{S}_1 Y^{i,H}_{T,1} + T^i_1 \end{aligned} \tag{8}$$

The date-0 budget constraint assumes no initial debt and states that the household's income from the sale of tradable and non-tradable goods as well as from government nominal transfers, T^i_0, can be used to buy consumption goods, invest in physical capital, or save in a domestic nominal bond, B^i_1, whose interest rate is R_0. The date-1 budget constraint states that all the income of the household is used for consumption.

Implicit in (8) is the assumption of limited asset market participation. Specifically, we assume that the household cannot hold foreign bonds (e.g., Gabaix and Maggiori, 2015, Fanelli and Straub, 2021, Itskhoki and Mukhin, 2021). This assumption captures the idea that it is difficult for many households in emerging markets to access international financial instruments without financial intermediation, especially when borrowing in foreign currency.

Maximizing utility (1) subject to the budget constraints in (8) yields the following optimality conditions:

$$\beta R_0 E_0^i \left[\left(\frac{C_1^i}{C_0^i} \right)^{-\sigma} \left(\frac{P_0^i}{P_1^i} \right) \right] = 1 \tag{9}$$

$$\alpha \beta E_0^i \left[\left(\frac{C_1^i}{C_0^i} \right)^{-\sigma} \frac{\mathcal{S}_1}{P_1^i} A_1 {K_1^i}^{\alpha-1} \right] = 1 \tag{10}$$

Finally, using (3)-(4) in the budget constraints (8), island i households' budget constraints simplifies to:

$$\frac{B_1^i}{R_0} = \mathcal{S}_0 (Y_{T,0}^{H,i} - Y_{T,0}^i) + T_0^i, \qquad -B_1^i = \mathcal{S}_1 (Y_{T,1}^{H,i} - Y_{T,1}^i) + T_1^i. \tag{11}$$

where each island households leave no debt at the end of period 1.

1.1.2 Financial market

Financiers from every island trade home and foreign bonds in the small-open economy-wide financial sector. Along with financiers, the government and a set of noise traders also operate in the financial sector, as we describe next.

Financiers We assume that there are frictions in the financial sector which give rise to a downward-sloping demand for currency from the financiers. In particular, we follow the formulation of Fanelli and Straub (2021). In each island a continuum of risk-neutral financiers trade home and foreign bonds subject to position limits and facing heterogeneous participation costs, as in Alvarez et al. (2009).

In Appendix A.1, we derive the maximization problems of island-i financiers, and show that it results in the following demand for the foreign currency bond:

$$\frac{{D_1^i}^\star}{R_0^\star} = \frac{1}{\hat{\Gamma}} E_0^i \left(R_0^\star - R_0 \frac{\mathcal{S}_0}{\mathcal{S}_1} \right), \tag{12}$$

where the zero-capital portfolio island-i financiers and their carry trade profits are, respectively:

$$\frac{D_1^i}{R_0} + \mathcal{S}_0 \frac{{D_1^i}^\star}{{R_0}^\star} = 0, \qquad\qquad \pi_1^{i,D^\star} \equiv {D_1^i}^\star + \frac{D_1^i}{\mathcal{S}_1} = \cdots = \tilde{R}_1^\star \frac{{D_1^i}^\star}{{R_0}^\star}.$$

Aggregating across islands, the overall demand of financiers for foreign bonds is:

$$\frac{\int D_1^{i\star}\ \mathrm{d}i}{R_0^\star} = \frac{1}{\hat{\Gamma}}\bar{E}_0\left(R_0^\star - R_0\frac{\mathcal{S}_0}{\mathcal{S}_1}\right). \tag{13}$$

where $\bar{E}_0(X_t)$ denotes the average expectation of X_t across islands, i.e. $\bar{E}_0 X_t = \int E_0^i X_t\ \mathrm{d}i$.

Financiers' demand for foreign bonds has a finite (semi-)elasticity to the expected excess return, implying that changes in the net supply of foreign bonds, e.g., induced by FX interventions, indeed affects the equilibrium exchange rate. The critical parameter in equation (13) is the inverse demand elasticity $\hat{\Gamma}$, which governs the strength of frictions in the international financial market. If $\hat{\Gamma}$ is large, e.g., due to tight position limits, intermediation is impeded. In the extreme case where $\hat{\Gamma} \to \infty$ intermediation is absent, and foreign bond demand is nil. By contrast, when $\hat{\Gamma} \to 0$, bond demand adjusts so that expected excess foreign currency returns are nil, and the elasticity is infinite. Henceforth, we assume $\hat{\Gamma} \in (0, \infty)$.

Noise traders Noise traders exogenously demand foreign currency $\frac{N_1^\star}{R_0{}^\star}$. Here $\frac{N_1^\star}{R_0^\star} > 0$ means that noise traders short home-currency bonds to buy foreign-currency bonds. They also hold a zero-capital portfolio in home and foreign bonds denoted $(N_1, N_1^\star)$, which implies:

$$\frac{N_1}{R_0} + \mathcal{S}_0\frac{N_1^\star}{R_0^\star} = 0. \tag{14}$$

Central Bank/Government The economy-wide central bank holds a $(F_1, F_1^\star)$ bond portfolio. The value of the portfolio is $\frac{F_1^i}{R_0} + \mathcal{S}_0\frac{F_1^\star}{R_0^\star}$. We assume that the government finances its operations with transfers:

$$\begin{aligned} \frac{F_1}{R_0} + \mathcal{S}_0\frac{F_1^\star}{R_0^\star} &= -\int T_0^i\ \mathrm{d}i, \\ 0 = F_1 + \mathcal{S}_1 F_1^\star + \tau\mathcal{S}_1\left(\int \pi_1^{i,D\star}\ \mathrm{d}i + \pi_1^{N\star}\right) &- \int T_1^i\ \mathrm{d}i, \end{aligned} \tag{15}$$

where $\pi_1^{i,D\star}$ is the income from financial transactions of financiers on island i, defined above, and $\pi_1^{N\star}$ is the income from financial transactions of noise traders.

Financial market clearing Since home-currency bond are in zero net supply, market clearing implies

$$\int B_1^i \, \mathrm{d}i + N_1 + \int D_1^i \, \mathrm{d}i + F_1 = 0. \tag{16}$$

Combining the market clearing condition with households and government budget constraints, and the portfolios and income from financial transactions of financiers and noise traders, we obtain the aggregate position of the financiers, in foreign currency:

$$\frac{\int D_1^{i\star} \, \mathrm{d}i}{R_0^\star} = \int \left(Y_{T,0}^{i,H} - Y_{T,0}^i \right) \, \mathrm{d}i - \frac{F_1^\star + N_1^\star}{R_0^\star} \tag{17}$$

That is, the aggregate position of financiers equals the portion of households' bond demand – originating from the trade imbalance – that is not met by the foreign currency supplied by the government and noise traders.

1.2 Equilibrium characterization

1.2.1 Economy-wide exchange rate

As standard in the literature, we solve the model using a log-linear approximation around a steady state with $A = 1, N^\star = F^\star = 0$ and $B^i = D^i = 0 \ \forall i$. In Appendix A.2 we report the island-level equilibrium, and in A.3 we derive the solution for the equilibrium aggregate real exchange rate as a function of shocks and expectations thereof:

$$q_0 = \frac{\Gamma \omega_1}{\Gamma \tilde{\theta} \omega_1 + \omega_3} (n_1^\star + f_1^\star) - \frac{\omega_2}{\Gamma \tilde{\theta} \omega_1 + \omega_3} \bar{E}_0 a_1 \tag{18}$$

where $\bar{E}_0$ is the average expectation across all islands and and $\omega_1 > 0, \omega_2 > 0, \omega_3 > 0$, and $\tilde{\theta} > 0$ are convolutions of parameters independent of Γ.

The first term of equation (18) describes how shocks to the demand for foreign bonds by noise traders and central bank induce an exchange rate depreciation, for a given level of expectations about future fundamentals. To see the mechanism at play, note that higher demand for foreign bonds by noise traders and/or central bank requires financiers to take a short position on foreign bonds (eq. (A.8)). To do so, financiers require a compensation in proportion to Γ (A.7): the real exchange rate depreciates today, so that its expected appreciation guarantees financiers an expected profit on their long domestic bond position. For completeness, the exchange rate depreciation also induces an increase in the price of tradable goods, leading to lower import and reduced borrowing. The resulting decline in households' demand for more domestic bonds (eqs. (A.4) and (A.8)) alters the position of financiers and attenuates the equilibrium depreciation of the exchange rate.

The second term of equation (18) describes how an upward revision in expected future fundamental leads to an exchange rate appreciation, keeping the demand for bond of noise traders and central bank constant. First, a higher future technology implies a higher future supply of tradable relatively to non-tradable goods, implying a reduction in the future relative price of tradables, i.e. a future exchange rate appreciation (A.4). By uncovered interest parity, the exchange rate today appreciates (eq. (A.7)). In terms of financial flows, a reduction in the equilibrium price of tradables leads households to increase their import and borrowing, issuing domestic bonds to financiers. Correspondingly, financiers take a short position on foreign bonds and long on domestic bonds, they require an expected exchange rate appreciation (A.7), which attenuates today's equilibrium exchange rate appreciation resulting from the expected improvement in future fundamentals.

1.2.2 Discussion of assumptions

Before we move on, let us discuss some of the assumptions that we made.

First, we have distributed agents along a continuum of islands that do not directly interact with one another other than through a common financial market. Allowing for further interactions among all islands (for example, via inter-island goods trade) would completely reveal average expectations and, therefore, eliminate any marginal informational role of aggregate public signals, may those be aggregate prices such as the exchange rate or quantities such as interventions.

Second, we have assumed that there is only one aggregate price that agents observe, namely the exchange rate, but two economic disturbances, productivity shocks and noise-trading shocks. This assumption ensures that agents cannot fully back out the aggregate state of the economy by simply observing the exchange rate.[9]

These first two assumptions parsimoniously capture the idea that economic agents, for various reasons, do not perfectly observe all the variables that are relevant to their

[9] While the nominal interest rate of the small-open economy bond is also observable, we assume that agents do not use its information to infer the state of fundamentals. However, it is possible to microfound the uninformativeness of the nominal interest rate by having a local bond market and a local shock with an infinite noise, which averages to zero in the aggregate. In this setting, the resulting island-level interest rate would carry no information about aggregates.

decisions but that they use easily accessible information, such as exchange rates, to improve their inference about such variables.

Third, we have assumed that financial intermediaries are owned by the household. This assumption ensures that the profits and losses from carry trade activity do not represent a net benefit or cost to the small-open economy. The implications of FX interventions of "leakages" from carry trade if financial intermediaries were owned by foreigners have already been studied by Fanelli and Straub (2021). Instead, we focus, on the informational role of exchange rates and FX interventions.

Fourth, we have assumed that the small open economy can save in foreign bonds and physical capital. The presence of physical capital plays an important role in our model. The exchange rate, by affecting the relative demand for tradable and non-tradable goods, is a key determinant of the allocation of domestic income between domestic spending and external savings, as can be seen from (7)-(8). The breakdown of domestic spending between current consumption and capital investment depends on the expected marginal product of capital and thus on the expectation of future fundamentals, as embedded in (9)-(10). Absent capital, there is a one-to-one relationship between external saving and current consumption, and that relationship is entirely governed by the current exchange rate. Thus, a policymaker that is interested in affecting the path of consumption has no reason to influence expectations if it can directly affect the exchange rate. The presence of capital ensures that, for a given level of the exchange rate, expectations are a concern for the policymaker because they determine the allocation of domestic spending between current consumption and investment, or (in part) future consumption.

1.3 Laissez-faire information structure

We now consider how expectations are formed and introduce two important assumptions: dispersed information and extrapolative expectations. In particular, we highlight the informational role of the exchange rate and its equilibrium determination. In this section, we discuss the laissez-faire economy, that is the economy without FX interventions, $f_1^\star = 0$. We then introduce FX intervention policy in Section 2. Under laissez-faire, the equilibrium exchange rate is:

$$q_0 = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3} n_1^\star - \frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3} \bar{E}_0 a_1. \tag{19}$$

Dispersed information Households and financiers in each island $i \in [0, 1]$ can observe local fundamentals, prices, and quantities, in addition to a local signal about the future realization of the technology shock a_1:

$$v^i = a_1 + \epsilon^i, \qquad \epsilon^i \sim N(0, \beta_v^{-1}), \tag{20}$$

with $\int_i \epsilon^i \, \mathrm{d}i = 0$ and common prior $a_1 \sim N(0, \beta_a^{-1})$.

While agents in each island cannot observe aggregate prices and quantities, they share the same currency and can therefore observe the aggregate real exchange rate q_0 given in (18). The aggregate exchange rate is an important endogenous signal because it carries information about the aggregate expectation of the common future technology shocks a_1. Last, agents cannot directly observe the amount of noise trading ($n_1^\star$).

Extrapolative expectations Consistent with growing empirical evidence (Bordalo et al., 2020), we consider the possibility that agents do not form beliefs rationally, but have an extrapolation bias that causes them to over-react to new information. First, we follow (Bordalo et al., 2020) in that agents extrapolate their private signal v^i

$$E^i[a_1|v^i] = (1 + \delta)\mathbb{E}^i[a_1|v^i] \tag{21}$$

where $\mathbb{E}$ is the rational expectation operator and the parameter $\delta \geq 0$ governs the degree of extrapolation.[10]

Second, because aggregate prices reflect average beliefs about fundamentals, agents in every island, when extracting information from the exchange rate, inherently need to forecast the forecast of agents in other islands. We thus need to specify how agents form these "higher-order beliefs." We assume that agents are unaware not only of their own extrapolation bias, but also about the bias of all the other agent in the economy. In other words, they think of themselves and every other agents are rational. As a result, they interpret the endogenous public signal, i.e., the exchange rate, as aggregating rational instead of actual beliefs. [11] That is, agents perceive the exchange rate pricing

[10] This setting can be viewed as a special case of the "diagnostic expectations" framework, with i.i.d. shocks and prior beliefs equal to zero (Bordalo et al., 2020). However, while (Bordalo et al., 2020) only consider exogenous private signals, we show how overreaction to exogenous private signals endogenously leads to overreaction to endogenous public signals.

[11] Bastianello and Fontanier (2022) also study mislearning from prices, but starting from a different psychologically-founded bias. They consider agents who fail to understand that other agents learn from prices as well. In our setting, agents understand that other people learn from prices as well, but

equation to differ from the actual pricing equation given in (18), and to be

$$\tilde{q}_0 = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3} n_1^\star - \frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3} \bar{\mathbb{E}}[a_1|v^i, \tilde{q}_0] \tag{22}$$

where $\bar{\mathbb{E}}[a_1|v^i, \tilde{q}_0] \equiv \int^i \mathbb{E}^i[a_1|v^i, \tilde{q}_0] \, \mathrm{d}i$. Hereafter, we refer to $\tilde{q}_0$ as determined by equation (22) as the "perceived exchange rate process." We show in the next section that this higher-order belief formation bias leads endogenously to overreaction to the public signal as well.

Finally, note that the special case of rational expectations corresponds to $\delta = 0$. In this case, not only agents do not extrapolate private information, but they are also correct in assuming that the other agents do not extrapolate either.

1.4 Learning from the exchange rate

The equilibrium exchange rate solves the fixed point problem of clearing the bond market given expectations and determining expectations given market clearing (and the rest of the equilibrium) conditions. To solve for the equilibrium exchange rate in terms of the underlying structural shocks, we adopt the method of undetermined coefficient. However, what determines expectations is not the actual but the perceived exchange rate process. As a result, we conjecture an equilibrium perceived exchange rate equation, derive the resulting average beliefs and then verify that it satisfies condition (22). Then we use the equilibrium perceived exchange rate process to derive the actual beliefs, and the actual equilibrium exchange rate.

1.4.1 Perceived exchange rate process

We conjecture that the equilibrium perceived real exchange rate process depends linearly on the future fundamental a_1 and the noise trader shock $n_1^\star$

$$\tilde{q}_0 = \lambda_a a_1 + \lambda_b n_1^\star, \tag{23}$$

We now define the equilibrium under the laissez-faire information structure.

Definition 1 (Market equilibrium under laissez-faire). *Given shocks realization $\{a_1, n_1^\star\}$ and agents' prior and signals $\{v^i, \tilde{q}_0\}_{i\in[0,1]}$, a symmetric linear market equilibrium is*

fail to internalize their overreaction bias.

defined as

- *an allocation* $(\{c_0^i, c_1^i, k_1^i, y_{T,0}^i, y_{T,1}^i, b_1^{i\star}, d_1^{i\star}\}_{i\in[0,1]})$
- *a vector of prices* $(\{q_0, r_0\}, \{p_0^i, p_1^i\}_{i\in[0,1]})$
- *A perceived real exchange rate as a linear function of the states* $\tilde{q}_0 = \lambda_a a_1 + \lambda_b n_1^\star$.

solving equations (A.2)-(A.8) *and* (22) *with expectations respecting* (21).

The perceived exchange rate process depends on aggregate expectations about the fundamental, but it is itself an information source for agents when forming their beliefs. As a result, the relation between the perceived exchange rate and the two shocks, governed by (λ_a, λ_b), is determined as the solution of a fixed point problem. In particular, one can rewrite (23) as

$$\frac{\tilde{q}_0}{\lambda_a} = a_1 + \frac{\lambda_b}{\lambda_a} n_1^\star. \tag{24}$$

In this formulation, $\frac{\tilde{q}_0}{\lambda_a}$ represents an unbiased signal centered around the fundamental shock a_1 with a error variance of $\beta_q^{-1} \equiv \frac{\lambda_b^2}{\lambda_a^2}\beta_n^{-1}$.

To sum up, agent i has access to three sources of information: (i) the prior distribution of a_1; (ii) the private signal (20); (iii) the perceived exchange rate process (24). Each agent thinks of the other agents as rational conditioning on these three signals, meaning their posterior belief is the average of the signals weighted by their accuracy

$$\mathbb{E}^i[a_1|v^i, \tilde{q}_0] = \frac{\beta_v v^i + \beta_q \frac{\tilde{q}_0}{\lambda_a}}{D}, \tag{25}$$

where $D \equiv \beta^v + \beta^q + \beta^a$ is the posterior belief accuracy. We can average posterior beliefs $\bar{\mathbb{E}}[a_1|v^i, \tilde{q}_0] \equiv \int^i \mathbb{E}^i[a_1|v^i, \tilde{q}_0]\, \mathrm{d}i$ using $\int^i v^i\, \mathrm{d}i = a_1$ and substitute back in the perceived exchange rate process (22) to verify the conjecture (23). The following proposition characterizes the unique equilibrium of the model economy.

Proposition 1. *Let* $\Lambda \equiv \frac{\lambda_a}{\lambda_b}$. *The symmetric linear market equilibrium is unique and the perceived exchange rate process is described by* (23) *with coefficients*

$$\begin{aligned}
\lambda_a &= -\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\frac{\beta_v + \Lambda^2\beta_n}{\beta_a + \beta_v + \Lambda^2\beta_n} \\
\lambda_b &= \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\frac{\beta_v + \Lambda^2\beta_n}{\beta_v}
\end{aligned} \tag{26}$$

where Λ^2 is unique and implicitly defined by

$$\Lambda^2 = \left(\frac{\omega_2}{\Gamma\omega_1}\right)^2 \frac{\beta_v^2}{(\beta_a + \beta_v + \Lambda^2\beta_n)^2} \tag{27}$$

while the explicit solution of Λ is reported in Appendix D.

Proof. See Appendix D. □

1.4.2 Actual exchange rate process

We can now use the equilibrium perceived exchange rate process to derive the actual agent's belief and therefore the actual equilibrium exchange rate. First, agent i forms belief using three sources of information: (i) the prior distribution of a_1; (ii) the private signal (20); (iii) the exchange rate. However, agents do not update rationally but suffer from two biases. First, they extrapolate private information as in (21). Second, they wrongly perceive the exchange rate process as following (23). Agent i's posterior is

$$E_0^i \equiv E^i[a_1|v^i, q_0] = \frac{(1+\delta)\beta_v v^i + \beta_q \frac{q_0}{\lambda_a}}{D} \tag{28}$$

We can average posterior belief $\bar{E}[a_1|v^i, \tilde{q}_0] = \int^i E^i[a_1|v^i, \tilde{q}_0] \, \mathrm{d}i$ and plug in the actual equilibrium exchange rate equation (18). The actual equilibrium exchange rate follows

$$\begin{aligned} q_0 &= \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3} \frac{\beta_v + \Lambda^2\beta_n}{\beta_v} n_1^\star - (1+\delta)\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3} \frac{\beta_v + \Lambda^2\beta_n}{\beta_a + \beta_v + \Lambda^2\beta_n} a_1 \\ &= (1+\delta)\lambda_a a_1 + \lambda_b n_1^\star \end{aligned} \tag{29}$$

Endogenous extrapolation It is useful to compare the perceived and actual exchange rate processes to understand the bias that this misconception introduces in agents' beliefs. Consider the actual public signal in agent's posterior (28)

$$\frac{q_0}{\lambda_a} = (1+\delta)a_1 - \frac{1}{\Lambda} n_1^\star \tag{30}$$

If $\delta = 0$, there is no extrapolation bias and the perceived coincides with the actual exchange rate process. However, $\delta > 0$ leads to a misinterpretation of the endogenous public signal. First, while agents form belief under the impression that they are using an unbiased signal about the fundamental a_1, i.e (24), they are actually using a biased signal, i.e. (30). This is a result of their belief bias in forming higher-order beliefs. Since agents overreact to their own private signal, the aggregate belief loads more on the average private signal, and therefore on the fundamental, by $(1+\delta)$. However, as agents do not internalize this effect, they perceive the exchange rate as loading less on the fundamental a_1 than the actual exchange rate. This leads agents to extrapolate the information about fundamental shocks contained in the exchange rate. Intuitively, they think of movement in the exchange rate as coming from large movements in the fundamental, while they are instead smaller movements amplified by the aggregate extrapolation $(1+\delta)$.

Second, the perceived accuracy of the public signal equals the actual accuracy. In other words, while agents misperceive the mean of the public signal, they correctly perceive its accuracy. The reason is that the higher-order belief bias from $\delta > 0$ does not affect how the exchange rate depends on noise shock $n_1^\star$.[12]

To sum up, since agents are unaware that the other agents in the economy suffer from extrapolation bias, they underestimate the covariance between exchange rate and fundamental shock, and as a result overreact to its information content. If $\delta = 0$, there is no extrapolation bias and perceived coincides with actual exchange rate process.

1.5 The informational role of the exchange rate

What determines the informativeness of the exchange rate? Proposition 1 describes the equilibrium relationship between the exchange rate and the two structural shocks. It also describes how informative the exchange rate is about the fundamental shock, $\beta_q \equiv \Lambda^2 \beta_n$.[13] However, the informational role of the exchange rate not depend on its own accuracy alone, but on its accuracy relative to that of the other signals. The higher its accuracy relative to the other signals, the higher the weight agents assign to the exchange rate when forming beliefs.

Definition 2 (Relative information content of the exchange rate)**.** *Define the relative information content of the exchange rate as its relative accuracy as a signal about the*

[12] This result is due to the particular structure of the diagnostic expectation bias (Bordalo et al., 2020), which affects only the posterior belief mean but not the posterior uncertainty. A belief bias on the perceived accuracy of private signals, e.g. overconfidence, would affect also the perceived accuracy of the endogenous public signal, as in Broer and Kohlhas (2022).

[13] As explained in the previous section, the perceived and actual exchange rate processes have the same loading on the noise shock $n_1^\star$, and therefore the same accuracy as public signal. Therefore, the considerations laid down in this section apply similarly to the rational and extrapolative setting.

fundamental shock a_1 compared to prior and private signal. That is, the Bayesian weight on public signal: $\mathcal{I}_R = \frac{\Lambda^2 \beta_n}{\beta_a + \beta_v + \Lambda^2 \beta_n}$.

To illustrate the difference between absolute and relative accuracy of the exchange rate as signal, let us now examine two limit cases. First, consider a scenario where private signals hold no informative content. In such a case, there is no dispersion of information, as all participants possess identical, incomplete information.

Corollary 1 (Incomplete information economy)**.** *If private signals are fully inaccurate, $\beta^v \to 0$, the exchange rate coefficients equal $\lambda_a = 0$ and $\lambda_b = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}$. The relative information content of the exchange rate is nil, i.e. $\mathcal{I}_R = 0$, and the overall posterior accuracy is nil, i.e. $D = 0$.*

If agents hold no private information, the exchange rate does not have private information to aggregate and therefore it will be uninformative. Both the absolute and relative accuracy of the exchange rate are nil, and the common prior is the only source of information.

Second, consider a scenario where agents receive perfectly informative private signals.

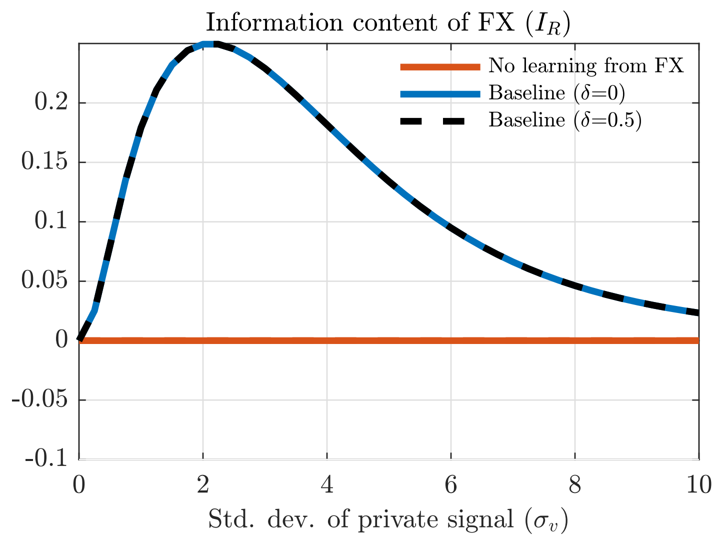
Corollary 2 (Full Information economy)**.** *If private signals are fully accurate, $\beta_v \to \infty$, the exchange rate coefficients equal $\lambda_a = -\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}$ and $\lambda_b = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}$. The relative information content of the exchange rate is nil, i.e. $\mathcal{I}_R = 0$, while the overall posterior accuracy is infinite, i.e. $D \to \infty$.*

Since the private signal is perfectly informative, the exchange rate, even though it is a fully-revealing signal, does not provide any *additional* information to agents. As a result, its absolute informativeness is positive, but its relative informativeness is zero.

Overall, the informational role of the exchange rate is to aggregate individual beliefs. Thus, if the information is commonly shared among agents, the exchange rate does not provide any additional information to them. This situation occurs both when the only information agents have is their common prior (Corollary 1) and when agents are fully informed (Corollary 2)

Away from these two limiting cases, the exchange rate possesses a non-trivial informational role. Figure 1 depicts the informational content of the exchange rate for different degrees of dispersion of private information (and an illustrative calibration of the rest of the parameters). For intermediate values of dispersed information, the

Figure 1: Information content of the exchange rate for different degrees of dispersed information



Notes: This figure reports the equilibrium value of the information content of the exchange rate for different levels of the noise in private signal, σ_v, under laissez faire. The rest of parameters are set as follows: $\beta = 0.99$, $\alpha = 0.3$, $\gamma = 0.3$, $\theta = 1$, $\sigma = 1$. The standard deviation of a_1 is $\sigma_a = 3$, while the standard deviation of $n_1^\star$ is $\sigma_n = 3$. We consider two values for the over-reaction parameter, $\delta = [0; 0.5]$. The "No learning from FX" scenario corresponds to a parametrization of $\sigma_v = \infty$.

informational content of the exchange rate is positive, $\mathcal{I}_R > 0$, as the exchange rate does indeed aggregate individual beliefs.

We now illustrate that the informational role of the exchange rate determines agents' choices and thus macroeconomic outcomes, distinctly from expenditure switching and wealth effects. To do so, it is useful to first rewrite the exchange rate solution as:

$$q_0 = z\frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}n_1^\star - z\left[\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(1+\delta)\frac{\beta^v}{\beta_a + \beta_v + \Lambda^2\beta_n}\right]a_1, \qquad z \equiv 1 + \frac{\Lambda^2\beta_n}{\beta_v} \quad (31)$$

Then, to highlight the informational role of the exchange rate, we examine the model responses to a decline in noise-trader demand for foreign currency ($n_1^\star < 0$, with $a_1 = 0$) as depicted in column 2 of Figure 2.

It is useful to start by considering a version of the model in which agents do not use the exchange rate as a signal about fundamentals (that is, setting $\mathbb{E}_0^i a_1 = \frac{\beta_v v^i}{\beta_v + \beta_a}$, and therefore $\mathcal{I}_R = 0$, $z = 1$ in eq. (31)). The responses under this version of the model are depicted by the red lines in column 2 of Figure 2: a decline in foreign-currency demand causes an appreciation of the exchange rate through portfolio balance, and, in turn, results in higher consumption and investment through a wealth effect. The noise-trading shock only operates through intermediation frictions and its effect on the exchange rate is the same as under full information ($i.e.$, $\frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1+\omega_3}$).

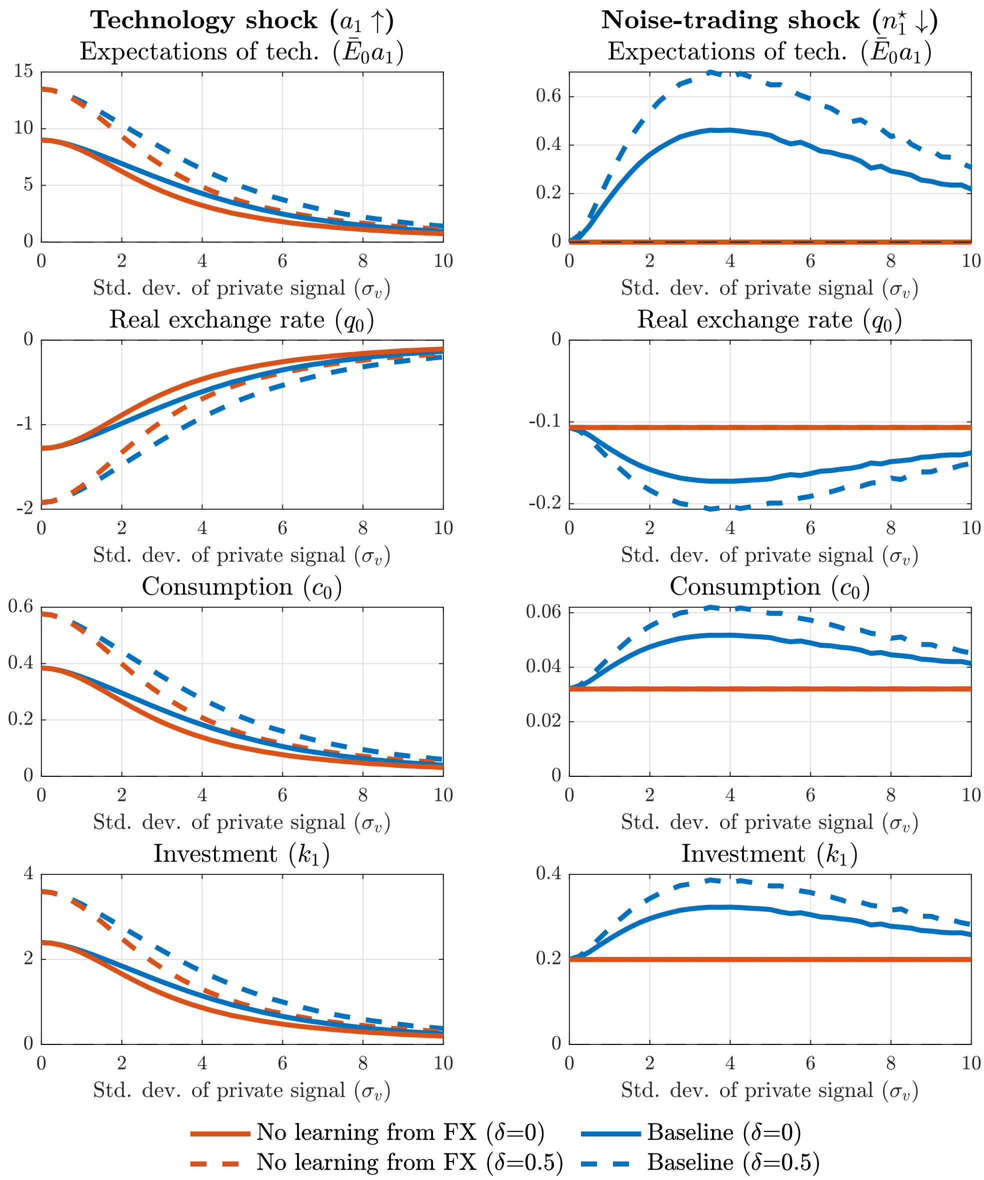
Now consider the baseline case in which agents learn from the exchange rate ($\mathcal{I}_R \geq 0$, and $z \geq 1$ in eq. (31)), as depicted by the blue lines in column 2 of Figure 2: agents in each island observe the exchange rate appreciation, but do not know whether it stems from an expected improvement in future fundamental ($a_1 > 0$) or to a decline in noise-trading demand for foreign currency ($n_1^\star < 0$). Agents thus confound, at least in part, the effect of the noise-trading shock on the exchange rate with the effect of higher future productivity, thereby revising their beliefs about future fundamental upward (according to eq. (25)). As a result, households decide to increase their consumption and investment, relative to the no-learning-from-FX case. This effect is analogous to an exogenous news shock, but it is due to the endogenous response of the exchange rate to the decline in foreign currency demand from noise traders.

This change in beliefs implies a second round of effects on the exchange rate, which appreciates more than in the no-learning-from-FX case. The rational confusion between noise trading and expected technology improvements amplifies the equilibrium effects of noise-trading shocks on the exchange rate, as originally highlighted in Bacchetta and Wincoop (2006), and on macro aggregates, as we emphasize using our fully-specified macro model. In Appendix B we elaborate on the dual role of noise trading, and show that the model reproduces the empirical findings of Chahrour et al. (2022) that a substantial portion of the exchange rate variation can be attributed to correctly anticipated changes in productivity and expectational "noise," which influences expectations of productivity but not the actual realization.

The informational role of the exchange rate is larger when the relative information content of the exchange rate, $\mathcal{I}_R$, is larger, as agents assign a larger weight to the exchange rate in their belief formation. The solid lines in Figure 2 report the equilibrium responses of the model under different levels of noise in the private signal σ_v. As we consider cases closer to the common information economy characterized in Corollary 1 and 2, where σ_v approaches zero or infinity, the exchange rate ceases to be an informative public signal and its information effect becomes zero.

We stress that the information channel of the exchange rate described here does not rely on expectations' overreaction, and is operative even under rational expectations ($\delta = 0$). Yet, expectations over-reaction changes the quantitative role of the information channel, as depicted in Figures 1 and 2 for an illustrative calibration of $\delta = 0.5$.

Figure 2: Equilibrium responses to shocks for different degrees of dispersed information



Notes: This figure reports the equilibrium value of model variables for different levels of the noise in private signal, σ_v, under laissez faire. The rest of parameters are set as follows: $\beta = 0.99$, $\alpha = 0.3$, $\gamma = 0.3$, $\theta = 1$, $\sigma = 1$. The standard deviation of a_1 is $\sigma_a = 3$, while the standard deviation of $n_1^\star$ is $\sigma_n = 3$. We consider two values for the over-reaction parameter, $\delta = [0; 0.5]$. The "No learning from FX" scenario corresponds to a parametrization of $\sigma_v = \infty$.

Because of expectations' overreaction, agents not only confound the noise-trading shock for a fundamental one, but they over-react to this information. As a result, they assign a larger-than-rational informational role to the exchange rate, and thus equilibrium variables exhibit an amplified response relative to the rational expectation case.

We have described the information channel conditional on a noise-trading shock. However, this channel is in place every time agents use the exchange rate as a signal about future fundamentals. In column 1 of Figure 2 we report the equilibrium responses of the model conditional on an increase in future productivity ($a_1 > 0$). The full-information economy response can be seen under $\sigma_v = 0$. In this case, agents perfectly foresee that productivity will increase, and they respond accordingly. They resort to external borrowing to increase current consumption and investment (and thus smooth consumption). When $\mathcal{I}_R > 0$ (and $\delta = 0$), agents' signal about future productivity is imprecise and they use the exchange rate to learn about it. In our illustrative calibration, this results in a dampened macroeconomic response under rational expectations ($\delta = 0$) while it can result in an amplified macroeconomic response under diagnostic expectations ($\delta = 0.5$).

To conclude this section, we provide three recent examples in which the information role of the exchange rate is explicitly discussed by policy makers or commentators. During a recent meeting of African central bank governors, IMF African Department Director Abebe Aemro Selassie highlights that "The exchange rate in most developing countries is the most visible and important price in the economy and so helps to anchor expectations, facilitate planning, as well as investment, and consumption decisions." (Selassie, 2023). Recently, after the Euro depreciated following a increase in the ECB policy rate, *Financial Times* Markets Editor Katie Martin writes that "The euro's latest wobble also forms yet another big signal that investors think Europe's luck has run out." (Martin, 2023). In a recent report to the Bank of International Settlements, economists at the Central Bank of Korea verbally describe how they think depreciation pressure on the Korean Won transmit through financial markets, and highlight that "The depreciation of the won in turn sends negative signals about the Korean economy and can make banks' FX funding more difficult again." (Ryoo et al., 2013).

While anecdotal, these pieces of evidence in favor of the informational role of the exchange rate are fairly common in the media and policy circles.[14] We formalized this channel in a general equilibrium model, and we now turn to study its implications for FX interventions and their communication.

[14] Gholampour and van Wincoop (2019) use Twitter data to compute a measure of investors' private information about the fundamentals driving the Euro-Dollar exchange rate, and use it in a structural estimation of the dispersed information model of Bacchetta and Wincoop (2006). One of their main findings is that Twitter data imply a sizable degree of dispersion in private information.

2 Foreign Exchange Interventions

We now introduce the possibility for the central bank to intervene in the foreign exchange market by purchasing foreign-currency bonds $f_1^\star$. We assume that FX interventions follow:

$$f_1^\star = \kappa_b n_1^\star + \kappa_a \bar{E}_0[a_1], \tag{32}$$

which we assume is known to agents.[15]

We assume that the central bank, as an aggregate agent, is able to observe aggregate quantities and prices, and therefore the average expectation $\bar{E}_0[a_1]$. From average expectation and the exchange rate, it can back out the actual realization a_1, meaning that the central bank has superior information compared to individual agents.[16] However, agents understand that the information source of the central bank is the average belief, and therefore the higher-order belief bias applies to the interpretation of the central bank's FX intervention as well.

FX interventions are intermediated by financiers, analogously to noise-trading demand. In this model, FX interventions are *effective*, i.e. can affect the exchange rate, because they alter the balance-sheet position of financiers. For example, a central bank's purchase of foreign bond $f_1^\star > 0$ requires financiers to take an opposite position (long on domestic bonds and short on foreign ones). As a result, financiers require a compensation, so the real exchange rate depreciates today to allow a premium on financiers position (18). Besides, FX interventions may alter the information available to agents about future fundamentals, as we describe in detail below.

We consider two types of FX intervention communication policy. The first policy is *public* FX intervention, where the central bank communicates the volume of the FX intervention to the public, and thus $f_1^\star$ becomes common knowledge. The second policy is *secret* FX intervention, where the central bank does not reveal the volume of the FX intervention. Nevertheless, the effect of the FX intervention is reflected in the exchange rate, and, by observing equilibrium exchange rates, agents form a forecast of the FX intervention along with the forecast of the other aggregate variables.

[15] In Appendix C, we derive the case in which FX interventions follow an exogenous process.

[16] One could alternatively express the FX intervention rule in terms of the actual fundamental, $f_1^\star = \hat{\kappa}_b n_1^\star + \hat{\kappa}_a a_1 + \varepsilon_1^{f^\star}$, as there is a linear mapping between the two sets of parameters: $\hat{\kappa}_a = \kappa_a(1+\delta)\frac{\beta^v+\Lambda^2\beta^b}{\beta^a+\beta^v+\Lambda^2\beta^b}$ and $\hat{\kappa}_b = \kappa_b + \kappa_a\frac{\Lambda\beta^b}{\beta^a+\beta^v+\Lambda^2\beta^b}$.

2.1 Public rule-based FX intervention

Consider first the case in which agents are able to observe the aggregate volume of the FX intervention, $f_1^\star$. Guess a linear solution for the (perceived) exchange rate process:

$$\tilde{q}_0 = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3} f_1^\star + \lambda_a a_1 + \lambda_b n_1^\star, \tag{33}$$

Define $\hat{\tilde{q}}_0 \equiv \tilde{q}_0 - \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1+\omega_3} f_1^\star$ as the equilibrium exchange rate, after the effect of the FX intervention is "partialed out." Agents use the exchange rate as signal

$$\frac{\hat{\tilde{q}}_0}{\lambda_a} = a_1 + \frac{\lambda_b}{\lambda_a} n_1^\star, \tag{34}$$

with a error variance of $\beta_q^{-1} \equiv \frac{1}{\Lambda^2}\beta_n^{-1}$ with $\Lambda \equiv \frac{\lambda_a^2}{\lambda_b^2}$, the same as in the laissez-faire economy (24). Regardless of the volume of FX intervention $f_1^\star$, as long as it is observed it does not change the information content of the exchange rate.

However, the FX intervention carries independent information about the shocks to which it responds. In particular, the FX intervention becomes a public signal about the average expectations. In fact, rewrite (32) as:

$$\frac{f_1^\star}{\kappa_a} = \bar{E}_0[a_1] + \frac{\kappa_b}{\kappa_a} n_1^\star. \tag{35}$$

Agents can now access two public signals, the exchange rate (22) and the FX intervention (35), which are two independent functions of the same two components, $\bar{E}_0[a_1]$ and $n_1^\star$. As a result, agents are able to perfectly back out the average expectation $\bar{E}_0[a_1]$ and therefore are perfectly informed.[17] If agents are rational, they can perfectly back out the fundamental from average expectations, so that $\bar{\mathbb{E}}_0[a_1] = a_1$. If $\delta > 0$, the average belief exhibits extrapolation bias: $\bar{E}_0[a_1] = (1+\delta)a_1$.

In other words, with a transparent communication strategy by the central bank, the FX intervention has a *signaling* effect that increases agents' information.

Corollary 3 (Public rule-based FX intervention)**.** *Suppose that $f_1^\star$ is observable. The parameters κ_b and κ_a do not directly affect the accuracy of the exchange rate. However, the combined information of the FX intervention and the exchange rate perfectly reveals*

[17] Consider the following linear combination of signals (34) and (22): $\left(\frac{f_1^\star}{\kappa_b} - \tilde{q}_0 \frac{\Gamma\tilde{\theta}\omega_1+\omega_3}{\Gamma\omega_1}\right) / \left(\frac{\kappa_a}{\kappa_b} + \frac{\omega_2}{\Gamma\omega_1}\right)$. This signal would perfectly reveal $\bar{E}_0[a_1]$.

the average expectations $\bar{E}_0[a_1]$, so the economy is in full information. The relative information content of the exchange rate $\mathcal{I}_R = 0$ and the overall agents' posterior accuracy about fundamental $D \to \infty$. The equilibrium perceived exchange rate process is given by (A.22) *with the same* $\lambda_a = -\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1+\omega_3}$ *and* $\lambda_b = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1+\omega_3}$.

As the economy operates under full information, the exchange rate does not carry any *additional* information and thus it does not have any information channel. Since agents are fully informed, the actual exchange rate becomes

$$q_0 = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(1+\kappa_b)n_1^\star - \frac{\omega_2 - \Gamma\omega_1\kappa_a}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(1+\delta)a_1. \tag{36}$$

When the future fundamental increases, $a_1 > 0$, in a laissez-faire economy (such as the one described by (22)) the exchange rate appreciates, $q_0 < 0$. The central bank can amplify this effect by selling foreign bonds, $\kappa_a < 0$, or dampen this effect (and even reverse it) by purchasing foreign bonds, $\kappa_a > 0$. If $\kappa_a = \frac{\omega_2}{\Gamma\omega_1}$, the central bank can completely offset the effect of fundamental on the exchange rate. Similarly, the central bank can amplify noise shocks by purchasing foreign bond when noise traders do, $\kappa_b > 0$, or dampen them (and even reverse them) by taking the opposite position $\kappa_b < 0$. If $\kappa_b = -1$, the central bank completely offsets the noise shocks by taking a symmetrical position.

Discussion Note that the result that public interventions lead to full information is due to our assumption that the central bank is able to observe average beliefs, which are then revealed by the volume of the FX intervention. We make this assumption to simplify the information extraction problem and the exposition. If the central bank had not *superior* but more generally *different* information with respect to agents, then public interventions would still increase agents' information but only partially. Either way, the point is that transparent communication about FX interventions increase agents' information about fundamentals by revealing the central bank's information.

2.2 Secret rule-based FX interventions

Finally, consider the case in which the central bank does *not* reveal the aggregate volume of FX interventions, but still follows the rule described in (32). Substitute it in the exchange rate (18) and get

$$q_0 = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(1+\tilde{\kappa}_b)n_1^\star - \frac{\omega_2 - \Gamma\omega_1\tilde{\kappa}_a}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\bar{E}_0 a_1. \tag{37}$$

The unobserved FX intervention changes the structural relation between the exchange rate and the two shocks, and therefore the information content of the exchange rate. To solve the information problem, we guess a linear solution for the perceived exchange rate process

$$\tilde{q}_0 = \lambda_a a_1 + \lambda_b n_1^\star, \tag{38}$$

which is the same guess as in the laissez-faire economy (23). However, since the exchange rate (37) is different, the equilibrium coefficients λ_a and λ_b are different as well.

Proposition 2. *(Secret rule-based FX Interventions) Suppose the central bank adopts a secret rule-based FX intervention, i.e. $f_1^\star = \tilde{\kappa}_b n_1^\star + \tilde{\kappa}_a \bar{E}_0 a_1$ and $f_1^\star$ is not directly observed. Then*

$$\begin{aligned} \lambda_a &= -\frac{\omega_2 - \Gamma\omega_1\tilde{\kappa}_a}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\frac{\beta_v + \Lambda^2\beta_n}{\beta_a + \beta_v + \Lambda^2\beta_n}, \\ \lambda_b &= \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(1+\tilde{\kappa}_b)\frac{\beta_v + \Lambda^2\beta_n}{\beta_v}, \end{aligned} \tag{39}$$

where Λ^2 is unique and implicitly defined by

$$\Lambda^2 = \left(\frac{\omega_2 - \Gamma\omega_1\tilde{\kappa}_a}{\Gamma\omega_1(1+\tilde{\kappa}_b)}\right)^2 \frac{\beta_v^2}{(\beta_a + \beta_v + \Lambda^2\beta_n)^2}, \tag{40}$$

while the explicit solution of Λ is reported in Appendix D.

Proof. See Appendix D. □

Substituting the actual average belief in (A.21), one gets the actual exchange rate

$$q_0 = (1+\delta)\lambda_a a_1 + \lambda_b n_1^\star. \tag{41}$$

Similarly to the public intervention case, the FX intervention alters the stochastic properties of the exchange rate, i.e. the structural relationship between the exchange rate and the underlying shocks. Differently from the public rule-based case, however, FX interventions are not observed and therefore they alter the information content of the exchange rate.

Corollary 4. *The exchange rate accuracy* $\beta^q \equiv \Lambda^2\beta^n$*, its relative information content* $\mathcal{I}_R$ *and the overall posterior accuracy* D *are proportional to* $(\omega_2 - \Gamma\omega_1\tilde{\kappa}_a)^2$*, the correlation between exchange rate and fundamentals, and inversely proportional to* $(1+\tilde{\kappa}_b)^2$*, the correlation between exchange rate and noise shocks.*

Intuitively, the more the exchange rate is correlated with the fundamental shock (relative to the noise-trading shock), the more information it carries about fundamentals. The central bank can, through FX interventions, increase the equilibrium covariance between exchange rate and fundamental, thereby increasing its information content and, as a result, the the overall amount of information in the economy.

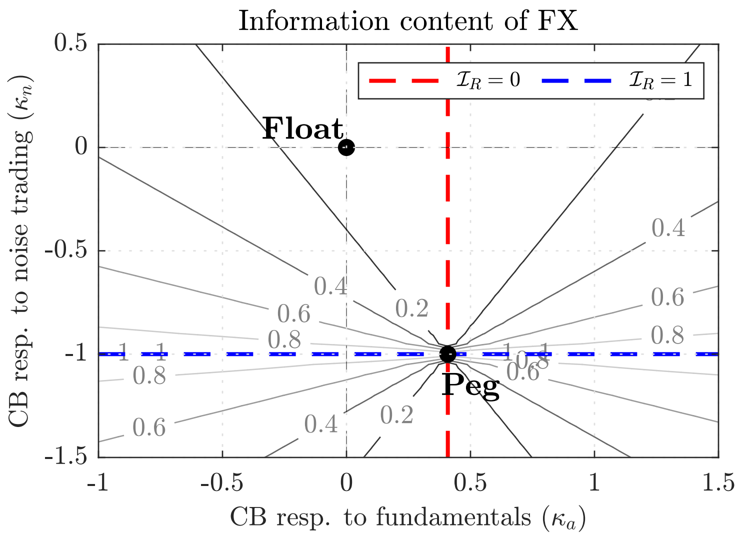
We observe that secret FX policy can *manage* the information content of the exchange rate by appropriate choice of (κ_a, κ_n). Figure 3 reports the equilibrium information content of the exchange rate for different values of the central bank's reaction function (κ_a, κ_n) under an illustrative calibration of the model with $\delta = 0$. It is useful to highlight a number of interesting cases. First, the secret FX policy can attain the full-information equilibrium if it fully offset the noise trading variation in the exchange rate ($\kappa_n = -1$). By doing so, the central banks renders the exchange rate a perfectly informative signal of future fundamentals. Second, the secret FX policy can engineer an equilibrium in which the exchange rate is uninformative, by fully offsetting the fundamental variation in the exchange rate (under $\delta = 0$, this requires $\kappa_a = {}^{\omega_2}/_{\Gamma\omega_1}$). Importantly, a special case of an uninformative equilibrium exchange rate is an *exchange rate peg*, which is obtained when the secret FX policy enforces a constant exchange rate by eliminating all noise and fundamental variation in the exchange rate.

As shown in Figure 3, outside of these two limit cases (fully informative and uninformative exchange rates), there is a spectrum of cases in which the exchange rate is an imperfect signal of future fundamentals, which can be achieved by appropriate choice of (κ_a, κ_b). A special case of imperfectly informative exchange rate is an *exchange rate (free) float*. This regime coincides with the laissez-faire equilibrium, obtained when $\kappa_a = \kappa_b = 0$. Under free floating, exchange rate fluctuations reflect both fundamental and noise, and thus their information content is limited and constrained by the relative amount of noise in exchange rate fluctuations.[18]

[18] Hassan et al. (2022) explore how exchange rate policy influences the riskiness of that country's cur-

Overall, we emphasize that, unlike public FX interventions, secret FX interventions allow a central bank to “manage” the informativeness of the exchange rate. Relative to laissez-faire, secret FX interventions increase or reduce the information content of the exchange rate, depending on how they are conducted. In the next section, we explore whether the central bank may find desirable to intervene publicly or secretly.

Figure 3: Secret FX intervention and the information context of the exchange rate



Notes: The figure reports values of the information content of the exchange rate ($\mathcal{I}_R$) for different values of the central bank's reaction function (κ_a, κ_n) under secret FX intervention policy, for an illustrative calibration of the model under rational expectations (that is, $\delta = 0$).

3 Optimal Foreign Exchange interventions

3.1 Frictionless benchmark

Consider an economy without intermediation frictions, i.e. $\Gamma = 0$, and with full information and rational expectations, i.e. $\bar{E}_0 a_1 = a_1$. The frictionless equilibrium exchange rate is

$$q_0^{FB} = -\frac{\omega_2}{\omega_3} a_1. \tag{42}$$

rency, by altering the stochastic properties of the exchange rate, and derive the implications for optimal exchange rate policy. We also emphasize that FX policy affects the macroeconomic allocation by altering the stochastic properties of the exchange rate, yet through a distinct, complementary channel: the informativeness of the exchange rate.

The difference between the decentralized market equilibrium (18) and the frictionless allocation is

$$q_0 - q_0^{FB} = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3} \underbrace{\left[(n_1^\star + f_1^\star) + \frac{\tilde{\theta}\omega_2}{\omega_3} a_1\right]}_{\text{Intermediation wedge}} - \frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3} \underbrace{\left(\bar{E}_0 a_1 - a_1\right)}_{\text{Belief wedge}}. \tag{43}$$

There are two sources of inefficient fluctuations in the economy's exchange rate. First, the intermediation wedge represents the suboptimal exchange rate variation due to the intermediation frictions in the bond market ($\Gamma > 0$). The same term appears in the literature on models of FX interventions with intermediation frictions (Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2019, 2022). In this literature, FX intervention $f_1^\star$ is typically used to fully offset this term, unless there are other inefficiencies of the decentralized equilibrium. Second, a belief wedge emerges because of frictions in belief formation. It stems from the fact that average beliefs may not coincide with the true value of fundamental technology. The following proposition highlights that a benevolent social planner would need to address both wedges to achieve the frictionless allocation.

Proposition 3. *Assume $\theta \neq \sigma$. Then the frictionless allocation can be achieved only if $\bar{E}_0 a_1 - a_1 = 0$.*

Proof. See Appendix D. □

Proposition 3 means that engineering an intermediation wedge that exactly offsets a non-zero belief wedge is not enough to attain the frictionless allocation of the overall macroeconomic equilibrium. Intuitively, a non-zero belief wedge affects the broad macro allocation, including investment decisions. As explained in Section 1.2.2, a frictionless exchange rate only ensures that the allocation of domestic income between domestic spending and external savings is optimal. Nevertheless, if expectations of future technology are excessively optimistic, then the split of domestic spending between consumption and investment would be sub-optimal. For these reasons, the frictionless allocation can only be attained when both the intermediation wedge and the belief wedge are simultaneously zero.[19]

[19]This argument applies outside of the knife-edge case where the intertemporal elasticity of substitution equals the elasticity of substitution between tradable and non-tradable goods.

3.2 Belief wedge

The departure from the Full Information Rational Expectation Hypothesis introduces a wedge between the frictionless allocation and the decentralized equilibrium which we refer to as "belief wedge", as discussed in Section 3.1. The belief wedge is proportional to the forecast error, which depends on the two frictions on beliefs: dispersed information and extrapolation. We now discuss how these two belief frictions affect the wedge and, in particular, the accuracy of the exchange rate signal $\beta_q \equiv \Lambda^2 \beta_n$ affect the belief wedge. In the next section we show that FX interventions can also alter the accuracy of the exchange rate as a signal.

Rational expectations Consider the case where agents have dispersed information but rational belief, i.e. $\delta = 0$. The individual forecast error unconditional variance equals

$$var(\mathbb{E}^i[a_1|v^i, q_0] - a_1) = \frac{1}{(\beta^v + \beta^q + \beta^a)} \tag{44}$$

It is easy to see that in this case forecast errors, and therefore the belief wedge, are inversely proportional to the accuracy of public signal β_q. Intuitively, higher information lowers forecast errors as long as agents use such information optimally. In other words, under rational expectations, a belief wedge emerges solely because of dispersed information. If dispersed information is resolved with a perfectly informative public signal (as under $\beta_q \to \infty$), then equilibrium average expectations are correct and the belief wedge is zero.

Corollary 5 (Belief wedge with rational expectations)**.** *With rational expectations $\delta = 0$, the unconditional variance of the individual forecast error on fundamental* (44) *is minimized with perfectly informative public signal $\beta_q \to \infty$.*

Extrapolative beliefs Consider a more general case where agents have dispersed information but non-rational beliefs, i.e. $\delta > 0$. The individual forecast error unconditional variance equals

$$var(E_0^i a_1 - a_1) = \frac{[\delta(\beta^v + \beta^q) - \beta^a]^2}{(\beta^v + \beta^q + \beta^a)^2} \frac{1}{\beta^a} + \frac{(1+\delta)^2 \beta^v + \beta^q}{(\beta^v + \beta^q + \beta^a)^2} \tag{45}$$

With extrapolative beliefs, the relation between forecast errors and public signal informativeness is more nuanced. If the exchange rate is perfectly accurate $\beta^q \to \infty$, we are in full information but the consensus error variance is positive, $var(\bar{E}_0^i a_1 - a_1) = \delta^2 \frac{1}{\beta^a}$. Even if the information content of the exchange rate is very precise, agents overreact to this information due to their extrapolation bias. As a result, the belief wedge is positive even in full information.

More generally, for large enough extrapolative bias δ, higher public signal precision has two opposite effects on the belief wedge. First, like in the rational expectation case, higher precision provides more information, lowering the belief wegde. Second, higher precision increases the relative weight on the public signal in posterior belief, and therefore it amplifies the extrapolation bias, increasing the belief wedge. We show the first effect prevails for low values of signal accuracy, while the second effect prevails for larger values of accuracy. As a result, the wedge is minimized for an interior solution of public signal accuracy.

Proposition 4 (Belief wedge with extrapolative expectations)**.** *If $\delta < \bar{\delta}$, then the unconditional variance of the individual forecast error on fundamental* (45) *is minimized with perfectly informative public signal $\beta_q \to \infty$. If $\delta > \bar{\delta}$, then the unconditional variance of the individual forecast error on fundamental* (45) *is minimized with $\beta_q = (\beta_a + \beta_v)\frac{1-2(1+\delta)}{1-2\delta(1+\delta)} > 0$, where $\bar{\delta} \equiv \frac{1+\sqrt{3}}{2}$.*

Proof. See Appendix D. □

To conclude, the notion that more information reduces the volatility of the belief wedge applies to the rational expectation setting, but not to the extrapolative beliefs setting. In that case, higher information might increase forecast errors as agents use information suboptimally. As a result, a perfectly informative public signal is not optimal anymore. In the next section, we study how different communication strategies in FX interventions can alter the information content of the exchange rate.

3.3 FX interventions and macroeconomic wedges

In this section, we turn to how FX interventions affect the macroeconomic equilibrium. In particular, we discuss how public and secret rule-based FX interventions impact the intermediation and the belief wedges described in Section 3.1. We highlight that providing information is welfare-improving in the rational expectation case, yet reducing information may be welfare-improving in the case of extrapolative beliefs. We postpone a detailed characterization of the welfare-maximizing FX policy to Section 3.4.

Rational expectations First, consider a public, rule-based intervention. A public intervention perfectly reveals the information of the central bank, which in this case is full information (Proposition 3). Moreover, with rational expectation and full information, the belief wedge is zero (Proposition 5). As a result, the central bank can use the FX intervention to close the only wedge left, the intermediation wedge:

$$(1+\kappa_b)n_1^\star + \left(\frac{\tilde{\theta}\omega_2}{\omega_3} + \kappa_a\right) a_1 \tag{46}$$

The central bank can eliminate the intermediation wedge, and therefore achieve the frictionless allocation, by setting $\kappa_b = -1$ and $\kappa_a = -\frac{\tilde{\theta}\omega_2}{\omega_3}$.

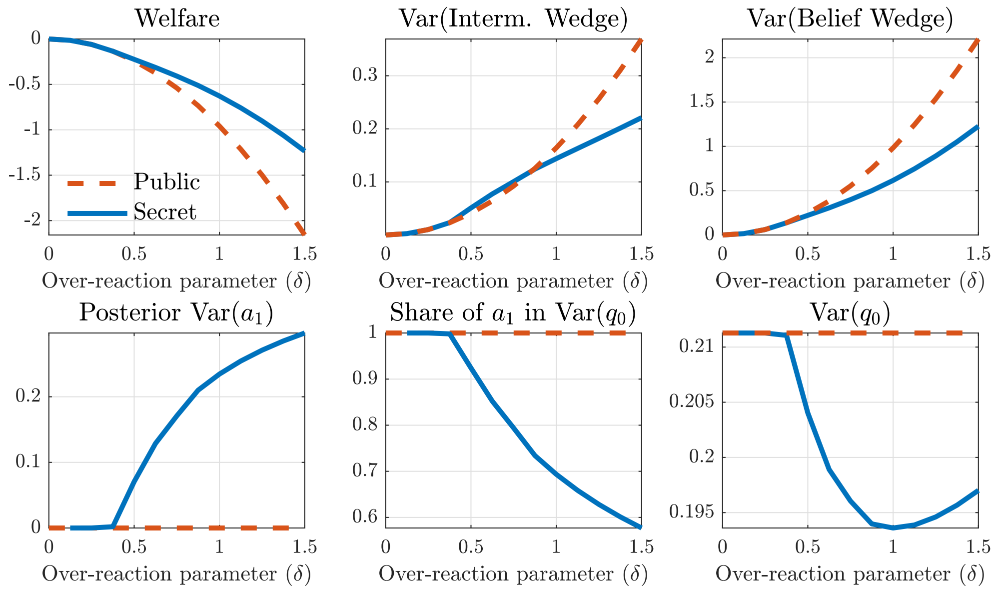
The same outcome can be achieved through a secret intervention, following the same reaction function. In fact, offsetting completely the noise-trading shock ($\kappa_b = -1$) makes the exchange rate signal perfectly informative, and therefore the economy reaches full information endogenously. To sum up, the central bank can achieve the frictionless equilibrium by closing the intermediation gap with either secret or public FX intervention.

Extrapolative expectations With extrapolative expectations it is generally not possible to close the intermediation and belief wedge simultaneously (Proposition 4). Thus a second-best problem arises in which the central bank may want to exploit the intermediation wedge in order to reduce the impact of the belief wedge. A complete characterization of this trade-off requires a welfare analysis, which we undertake next.

3.4 Normative analysis of FX interventions

We evaluate welfare by taking a quadratic approximation of the welfare function around the frictionless allocation (see Appendix E for a derivation). The welfare function is defined as the average expected utility across the islands of the small open economy. The optimal FX intervention policy is described by the values of (κ_a, κ_b) in the central bank's reaction function (eq. (32)) that maximize welfare under either public or secret interventions. Figure 4 reports the value of welfare relative to the frictionless benchmark under the optimal policy for an illustrative calibration and different degrees of over-extrapolation, δ. The figure reports the results for both the optimal secret and public FX intervention policies.

Figure 4: Welfare under optimal public and secret FX intervention policies



Notes: This figure reports values of different variables under optimal FX intervention policy for different levels of the over-reaction, δ, and for both public and secret FXI policy, for an illustrative calibration of the model.

As per Proposition 4, for a low degree of over-extrapolation the belief wedge is minimized with full information. In this calibration, minimizing the belief wedge is (part of the) optimal policy, thus public and secret interventions are designed to reveal the state of the economy, and they achieve the same level of welfare.[20] The state of the economy is revealed by fully offsetting noise-trading shocks so that the equilibrium exchange rate reflects only fundamentals, regardless of whether agents observe the quantity of bonds purchased by the central bank. Conditional on offsetting the noise traders, the central bank chooses κ_a to balance over-borrowing stemming from over-extrapolation and under-borrowing stemming from intermediation frictions in the competitive equilibrium. More specifically, the optimal policy allows part of households' borrowing demand to be reflected in higher borrowing costs (thus exploiting the intermediation wedge) to counteract the over-borrowing due to over-optimism.

Our key finding is that, for a sufficiently high degree of over-extrapolation, the optimal secret FX intervention policy dominates the optimal public FX intervention policy, by leading to lower welfare losses. In this parameterization, the optimal secret

[20]The level of welfare is, however, lower than the one attained in the frictionless equilibrium except when $\delta = 0$.

policy achieves a better outcome by reducing the belief wedge via a reduction of the information content of the exchange rate (as reflected in the non-zero posterior variance of a_1). In fact, interventions do not completely offset noise trader demand for bonds ($\kappa_b \neq 1$) and are less responsive to productivity, overall reducing the correlation between exchange rates and fundamentals. The central bank thus strikes a balance between allowing for some inefficient capital flows driven by noise traders and keeping the information content of the exchange rate low enough to tame over-reaction. Intuitively, the optimal secret intervention achieves a superior welfare outcome because it can affect an additional margin relative to the public intervention – the informativeness of the exchange rate.[21]

In addition, we remark that the secret FXI policy effectively reduces the equilibrium volatility of the exchange rate, relative to the public FXI policy, especially in the region of high extrapolation. Without interventions, agents' over-reaction to the informational content of equilibrium exchange rate causes excessive volatility in macroeconomic allocations, which feeds back into the exchange rate. By letting the exchange rate reflect some non-fundamental volatility, the resulting decline in its information content acts to reduce the amplification due to over-extrapolation and, in turn, the equilibrium volatility of the exchange rate. Such feedback mechanism behind the lower equilibrium exchange rate volatility provides an intuitive rationale for the widespread empirical practices of "systematic managed floating" (Frankel, 2019).

An interesting implication of our analysis is that whether exchange rates reflect fundamental or noise is an equilibrium outcome that depends on the optimal design and communication about FX interventions. In our model, public interventions, if designed optimally, ceteris paribus should imply an exchange rate that is very tightly related to future macroeconomic conditions. On the contrary, optimally secret interventions should result in an exchange rate driven partly by noise trading (as can be seen in the bottom-left panel of Figure 4).

[21] With the rule we consider, the public intervention always fully reveals fundamentals. In a world with more than two shocks, public FXI would not necessarily render the exchange rate fully informative. However, the distinctive feature of secret interventions is that they can make the exchange rate less informative than under no interventions, whereas the public interventions always increase or leave unchanged the information content of the exchange rate relative to no interventions.

4 Conclusions

We studied FX interventions in a macro model in which segmented financial markets and information frictions coexist. Both frictions generate wedges in aggregate consumption relative to its frictionless counterfactual, namely an intermediation wedge and a belief wedge. We formalized a novel informational role of the exchange rate in macroeconomic allocation, as agents use the exchange rate to learn about future fundamentals and make consumption and investment decisions. FX interventions can contemporaneously influence the intermediation wedge, via the standard portfolio balance channel, and the belief wedge, both by altering the information content of the exchange rate and through signaling. We highlighted that their conduct (rule-based vs discretionary) and communication (public vs secret) are important in determining the effects of FX interventions. We then discussed the challenges that a central bank faces when trying to stabilize the economy, and how possible departures from rational expectations shape the central banks' trade-off. A conclusion of our analysis is that managing the information content of the exchange rate can be optimal if individuals overreact to available information, and this is best achieved when FXI communication is opaque.

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Appendix

A Derivations

A.1 Financiers' demand for foreign currency bonds

We follow Fanelli and Straub (2021) and assume that there exists a continuum of risk-neutral financiers, labeled by $j \in [0, \infty)$, in each island i. Financiers also hold a zero-capital portfolio in home and foreign bonds denoted $(d^i_{j,1}, {d^i_{j,1}}^\star)$. Financier's investment decisions are subject to two important restrictions. First, each intermediary is subject to a net open position limit of size $D > 0$. Second, intermediaries face heterogeneous participation costs, as in Alvarez et al. (2009). In particular, each intermediary j active in the foreign bond market at time t is obliged to pay a participation cost of exactly j per unit of foreign currency invested.[22]

Putting these ingredients together, intermediary j on island i chooses ${d^i_{j,1}}^\star$ that solves

$$\max_{\frac{{d^i_{j,1}}^\star}{R^\star_0} \in [-D, D]} \frac{{d^i_{j,1}}^\star}{R^\star_0} E^i_0 \left(\tilde{R}^\star_1 \right) - j \left| \frac{{d^i_{j,1}}^\star}{R^\star_0} \right|,$$

where $\tilde{R}^\star_1 \equiv R^\star_0 - R_0 \frac{\mathcal{S}_0}{\mathcal{S}_1}$ is the return on one foreign-currency unit holding expressed in foreign currency and E^i_0 is the same expectation operator as the island-i household's.

Intermediary j's expected cash flow conditional on investing is $D \left| E^i_0 \left(\tilde{R}^\star_1 \right) \right|$ while participation costs are jD. Thus, investing is optimal for all intermediaries $j \in [0, \bar{j}]$, with the marginal active intermediary $\bar{j}$ given by $\bar{j} = \left| E^i_0 \left({\tilde{R}_1}^\star \right) \right|$. The aggregate investment volume is then

$$\frac{{D^i_1}^\star}{R^\star_0} = \bar{j} D \operatorname{sign} \left\{ E^i_0 \left({\tilde{R}_1}^\star \right) \right\}.$$

Defining $\hat{\Gamma} \equiv D^{-1}$ and substituting out $\bar{j}$, we obtain the total demand for foreign-currency bonds on island i, ${D^i_1}^\star = \int {d^i_{j,1}}^\star \mathrm{d}j$:

$$\frac{{D^i_1}^\star}{R^\star_0} = \frac{1}{\hat{\Gamma}} E^i_0 \left(R^\star_0 - R_0 \frac{\mathcal{S}_0}{\mathcal{S}_1} \right). \tag{A.1}$$

[22] We also assume that participation costs constitute transfers to households in the home island economy. Thus, no extra cost terms enter the household's budget constraint.

which is equation (12) in the text.

A.2 Island-level equilibrium

The log-linearized version of the household's optimality conditions (7), (9), and (10) are:

$$\sigma(E_0^i c_1^i - c_0^i) = r_0 - (E_0^i p_1^i - p_0^i), \tag{A.2}$$

$$(1-\alpha)k_1^i = E_0^i(s_1 - p_1^i) + E_0^i a_1 - r_0 + (E_0^i p_1^i - p_0^i), \tag{A.3}$$

$$s_t - p_t^i = -\frac{1-\gamma}{\theta} y_{T,t}^i, \tag{A.4}$$

Island-i budget in (11) can be combined and loglinearized as:[23]

$$\frac{1+\phi}{\beta} y_{T,0}^i = a_1 + \alpha k_1^i - y_{T,1}^i \tag{A.5}$$

where $\phi = \beta\alpha\gamma$. The final good aggregator in (3) yields:

$$\frac{1}{1+\phi} c_0^i + \frac{\phi}{1+\phi} k_1^i = \gamma y_{T,0}^i \qquad c_1^i = \gamma y_{T,1}^i. \tag{A.6}$$

The log-linear optimality condition of financiers (13):

$$\Gamma \int d_1^{i\star} \, \mathrm{d}i = \bar{E}_0 s_1 - s_0 - (r_0 - r_0^\star) \tag{A.7}$$

where $d_1^{i\star} \equiv \frac{\mathrm{d}D_1^{i\star}}{Y_{T,1}^{ss}}$ and $\Gamma \equiv \hat{\Gamma} \cdot Y_{T,1}^{ss} \cdot \beta^2$.

Finally, bond market clearing, (17) implies:

$$\int d_1^{i\star} \, \mathrm{d}i = -\frac{1+\phi}{\beta} \int y_{T,0}^i \, \mathrm{d}i - n_1^\star - f_1^\star \tag{A.8}$$

We also normalize the average price of the consumption basket, such that $\int p_t^i \, \mathrm{d}i = 0$, for $t = [0, 1]$. This implies that the aggregate real exchange rate equals the nominal exchange rate, that is:

$$q_t = s_t$$

[23] The log-linearized budget constraint is not affected by the size of the tax on financiers and noise traders' carry-trade profits are taxed, nor on how they are distributed across islands, as these represent second-order terms.

A.3 Equilibrium exchange rate of the small open economy

Consider the following set of island-i equilibrium equations:

$$s_0 - p_0^i = -\frac{1-\gamma}{\theta} y_{T,0}^i \tag{A.9}$$

$$s_1 - p_1^i = -\frac{1-\gamma}{\theta} y_{T,1}^i \tag{A.10}$$

$$r_0 - (E_0^i p_1^i - p_0^i) = \sigma\gamma E_0^i y_{T,1}^i - (\sigma\gamma)(1+\phi) y_{T,0}^i + \sigma\phi k_1^i \tag{A.11}$$

$$\frac{(1+\phi)}{\beta} y_{T,0}^i = a_1 + \alpha k_1^i - y_{T,1}^i \tag{A.12}$$

$$k_1^i = \frac{1}{1-\alpha} E_0^i \left(s_1 - p_1^i\right) + \frac{1}{1-\alpha} E_0^i a_1 - \frac{1}{1-\alpha} \left(r_0 - (E_0^i p_1^i - p_0^i)\right) \tag{A.13}$$

where eqs. (A.9) and (A.9) represent island-i's demand for tradables in period 0 and 1, respectively (c.f. (A.4)); eq. (A.11) is obtained by combining the Euler equation and island-i's resource constraint and (cf. (A.2) and (A.6)); eq (A.12) is island-i budget constraint (cf. (A.5)); (A.13) is island-i's demand for capital (c.f. (A.3))

Using eqs (A.9)-(A.13), one can express island-i price level and tradable demand as:

$$p_0^i = s_0 - \frac{\omega_1}{\omega_3} \left(r_0 - (E_0^i s_1 - s_0)\right) + \frac{\omega_2}{\omega_3} E_0^i a_1 \tag{A.14}$$

$$y_{T,0}^i = -\frac{\theta}{1-\gamma} \frac{\omega_1}{\omega_3} \left(r_0 - (E_0^i s_1 - s_0)\right) + \frac{\theta}{1-\gamma} \frac{\omega_2}{\omega_3} E_0^i a_1 \tag{A.15}$$

where $\omega_1 > 0, \omega_2 > 0, \omega_3 > 0$ are all convolution of parameters:

$$\omega_1 \equiv [\theta\sigma\alpha\gamma(1+\beta) + (1-\alpha)\theta + (1-\gamma)\alpha] \qquad \omega_2 \equiv [(1-\gamma) + \sigma\gamma\theta(1+\alpha\beta)]$$

$$\omega_3 \equiv \frac{\theta(1-\alpha)(1+\phi)}{\beta(1-\gamma)} [\sigma\gamma\theta(1+\beta) + (1-\gamma)] + \theta\sigma\alpha\gamma(1+\beta) + \theta(1-\alpha) + (1-\gamma)\alpha$$

Sum (A.16) across islands and use $\int p_t^i \, \mathrm{d}i = 0$, for $t = [0, 1]$, to express the small-open economy (real and nominal) exchange rate as a function of average-expected excess currency returns and average-expected TFP:

$$q_0 = s_0 = \frac{\omega_1}{\omega_3}\big(r_0 - (\bar{E}_0 s_1 - s_0)\big) - \frac{\omega_2}{\omega_3}\bar{E}_0 a_1 \tag{A.16}$$

Consider now the modified UIP condition for the small open economy bond, along with the market clearing condition in the financial market:

$$r_0 = \bar{E}_0 s_1 - s_0 - \Gamma\left(\int d_i^\star \,\mathrm{d}i\right) \qquad \text{w/} \qquad d_1^\star = -n_1^\star - f_1^\star - \frac{(1+\phi)}{\beta}\int y_{T,0}^i \,\mathrm{d}i, \tag{A.17}$$

and note that, by using (A.18):

$$\int y_{T,0}^i \,\mathrm{d}i = -\frac{\theta}{1-\gamma}\frac{\omega_1}{\omega_3}\big(r_0 - (\bar{E}_0 s_1 - s_0)\big) + \frac{\theta}{1-\gamma}\frac{\omega_2}{\omega_3}\bar{E}_0 a_1 \tag{A.18}$$

Using (A.17) and (A.18), one can express the average expectation of aggregate excess home-currency returns as:

$$r_0 - \big(\bar{E}_0 s_1 - s_0\big) = \frac{\Gamma\tilde{\theta}\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\bar{E}_0 a_1 + \frac{\Gamma\omega_3}{\Gamma\tilde{\theta}\omega_1 + \omega_3}n_1^\star + \frac{\Gamma\omega_3}{\Gamma\tilde{\theta}\omega_1 + \omega_3}f_1^\star \tag{A.19}$$

where $\tilde{\theta} \equiv \frac{(1+\alpha\beta\gamma)\theta}{\beta(1-\gamma)} > 0$. Use (A.19) in (A.16) across islands, we obtain the aggregate real exchange rate:

$$q_0 = s_0 = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(n_1^\star + f_1^\star) - \frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\bar{E}_0 a_1 \tag{A.20}$$

which is the equation (18) above.

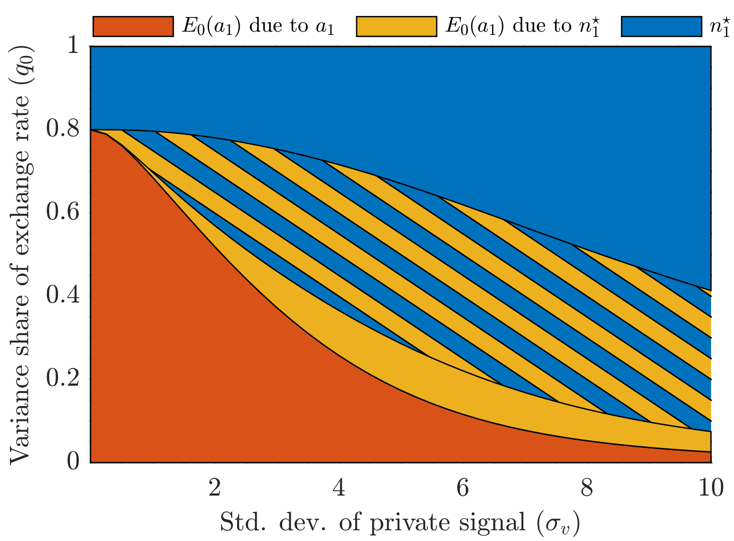
B The dual role of noise-trading shocks

What fraction of exchange rate fluctuations reflects expectations of future fundamentals relative to independent noise trading? To answer this question, Figure A.1 reports the decomposition of the exchange rate into the part that reflects expectations about future fundamentals $E_0(a_1)$, and the part that is due to noise trading $n_1^\star$ (see eq. (18)).

Under common information ($\sigma_v = 0$ or $\sigma_v = \infty$), we observe that there is a clear demarcation between $E_0(a_1)$ and $n_1^\star$. Under full information ($\sigma_v = 0$), agents can perfectly separate fundamental shocks from noise-trading shocks, and thus exchange rate fluctuations can be unequivocally attributed to either $E_0(a_1)$ or $n_1^\star$. Under no information ($\sigma_v = \infty$), all the variation in q_0 inevitably reflects only noise-trading shocks, since agents have no advance information about a_1.

However, when information is dispersed ($\sigma_v \in (0, 1)$) and thus agents learn from the exchange rate ($\mathcal{I}_R > 0$) the distinction between $E_0(a_1)$ and $n_1^\star$ is blurred: the exchange rate is a public signal, and noise-trading demand is the noise in the public signal, blurring the relationship between expectations of fundamentals and their subsequent realization. In fact, noise-trading shocks have a dual role in models where the exchange rate is a public signal (Bacchetta and Wincoop, 2006): they affect the exchange rate by altering the balance sheet position of financiers, and, by affecting the exchange rate, they also influence agents' expectations of a_1. As a result, the portion of fluctuations in $E_0(a_1)$ due to a_1 result in subsequent changes in a_1, while the portion of fluctuations in $E_0(a_1)$ due to $n_1^\star$ are not associated with subsequent changes in a_1. Relatedly, because of this dual role of $n_1^\star$, there is an interaction between the pure noise-trading effect and the effect of noise trading that operates through it being the noise in the public signal ($E_0(a_1)$ due to $n_1^\star$), as depicted in Figure A.1.

Figure A.1: Decomposition of the exchange rate variance for different degrees of dispersed information



Notes: This figure reports the decomposition of the exchange rate variation for different levels of the noise in private signal, σ_v, under laissez faire. The rest of parameters are set as follows: $\beta = 0.99$, $\alpha = 0.3$, $\gamma = 0.3$, $\theta = 1$, $\sigma = 1$. The standard deviation of a_1 is $\sigma_a = 3$, while the standard deviation of $n_1^\star$ is $\sigma_n = 3$. In this figure, the over-reaction parameter, $\delta = 0$.

Our model aligns with the empirical findings of Chahrour et al. (2022) about the source of fluctuations in the USD exchange rate. They reveal that a substantial portion of the exchange rate variation can be attributed to both correctly anticipated changes in productivity and expectational "noise," which influences expectations of productivity but not the actual realization. Our model aligns well with these findings, provided that information is dispersed ($\sigma_v > 0$), and thus agents learn from exchange rates, and thus offers a novel insight into the empirical relationship between exchange rates and macroeconomic fundamentals.

C Exogenous FX intervention

Assume now that the FX interventions follow a random process, i.e. $f_1^\star = \varepsilon_1^{f^\star}$ with $\varepsilon_1^{f^\star} \sim N\left(0, \beta_\varepsilon^{-1}\right)$. While we acknowledge that in practice central banks do not follow a completely random FX intervention rule, this case is useful to build intuition and illustrates how FX interventions affect the information content of the exchange rate.[24] In this case, the equilibrium exchange rate is determined according to:

$$q_0 = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\left(n_1^\star + \varepsilon_1^{f^\star}\right) - \frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\bar{E}_0 a_1. \tag{A.21}$$

Equation (A.21) shows that exogenous FX interventions represent an additional, exogenous shock to the foreign exchange market.

C.1 Public exogenous FX intervention

Let us first consider the case in which agents are able to observe the aggregate volume of the FX intervention, $\varepsilon_1^{f^\star}$. Guess a linear solution for the (perceived) exchange rate process:

$$\tilde{q}_0 = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3} f_1^\star + \lambda_a a_1 + \lambda_b n_1^\star, \tag{A.22}$$

where $f_1^\star = \varepsilon_1^{f^\star}$. Define $\hat{\tilde{q}}_0 \equiv \tilde{q}_0 - \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1+\omega_3} f_1^\star$, as the equilibrium exchange, after the effect of the FX intervention is "partialed out." Agents use the exchange rate as signal

$$\frac{\hat{\tilde{q}}_0}{\lambda_a} = a_1 + \frac{\lambda_b}{\lambda_a} n_1^\star, \tag{A.23}$$

[24] That said, we note that many central banks do not currently conduct FX interventions according to a rule (Patel and Cavallino, 2019).

with a error variance of $\beta_q^{-1} \equiv \frac{1}{\Lambda^2}\beta_n^{-1}$ with $\Lambda \equiv \frac{\lambda_a^2}{\lambda_b^2}$, the same as in the laissez-faire economy (24). Following the same solution method as in section 1.3, one reaches the same equilibrium λ_a and λ_b as in (26).

Corollary 6 (Public exogenous FX intervention)**.** *Suppose the central bank adopts a public exogenous FX intervention rule, i.e. $f_1^\star = \varepsilon_1^{f^\star}$ and $f_1^\star$ is directly observed. A more volatile FX intervention does not affect the relative information content of the exchange rate $\mathcal{I}_R$ nor the overall agents' posterior accuracy about fundamental D. The equilibrium perceived exchange rate process is given by* (A.22) *with the same λ_a and λ_b as in the laissez-faire equilibrium* (26)*.*

Since the intervention is public, agents can partial out the intervention from the exchange rate when they solve their signal extraction problem. It follows that the intervention does not affect the information content of the exchange rate. Moreover, since the intervention is random, it only adds non-fundamental variation to the exchange rate. Substituting the actual average belief in (A.21), one gets the actual exchange rate

$$q_0 = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3} f_1^\star + (1+\delta)\lambda_a a_1 + \lambda_b n_1^\star, \tag{A.24}$$

C.2 Secret exogenous FX intervention

Consider now the case in which the central bank does not reveal the aggregate volume of the FX intervention. Notice that the intervention $\varepsilon_1^{f^\star}$ and the noise shock $n_1^\star$ are both unobservable exogenous shocks to the exchange rate (A.21). Guess a linear solution for the perceived exchange rate process

$$\tilde{q}_0 = \lambda_a a_1 + \lambda_b (n_1^\star + \varepsilon^{f^\star}). \tag{A.25}$$

Agents use the exchange rate as signal

$$\frac{\tilde{q}_0}{\lambda_a} = a_1 + \frac{\lambda_b}{\lambda_a}(n_1^\star + \varepsilon^{f^\star}), \tag{A.26}$$

with a error variance of $\beta_q^{-1} \equiv \frac{1}{\Lambda^2}(\beta_n^{-1} + \beta_\varepsilon^{-1})$ with $\Lambda \equiv \frac{\lambda_a^2}{\lambda_b^2}$. Since the FX intervention is unobserved, it increases non-fundamental volatility to the exchange rate similarly to the liquidity demand from noise traders, and therefore decreases the information content of the exchange rate $\mathcal{I}_R$.[25]

Proposition 5 (Secret exogenous FX intervention)**.** *Suppose the central bank adopts a secret discretionary FX intervention, i.e.* $f_1^\star = \varepsilon_1^{f^\star}$ *and* $f_1^\star$ *is not directly observed. A more volatile FX intervention decreases the relative information content of the exchange rate* $\mathcal{I}_R$ *and agents' posterior accuracy about fundamental* D*. The equilibrium perceived exchange rate process is given by* (A.25) *with* λ_a *and* λ_b *described in Appendix* D.

Proof. See Appendix D. □

Proposition 5 reveals that, when implemented secretly, FX interventions have an information effect. In particular, secret exogenous FX interventions alter agents' expectations of fundamentals by reducing the informativeness of the exchange rate. Substituting the actual average belief in (A.21), one gets the actual exchange rate

$$q_0 = (1+\delta)\lambda_a a_1 + \lambda_b(n_1^\star + \varepsilon^{f^\star}). \tag{A.27}$$

D Proofs

Proof of Proposition 3. Consider the equilibrium exchange rate in case of full information, $\bar{E}_0 a_1 = a_1$ (but with intermediation friction, $\Gamma > 0$)

$$q_0^{FI} = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(n_1{}^\star + f_1{}^\star) - \frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3} a_1 \tag{A.28}$$

Take the difference between (18) and (A.28), and the difference between (A.28) and (42). Sum them and get

$$q_0 - q_0^{FB} = \frac{1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\left(\Gamma\omega_1\left[(n_1{}^\star + f_1{}^\star) + \frac{\tilde{\theta}\omega_2}{\omega_3}a_1\right] - \omega_2(\bar{E}_0 a_1 - a_1)\right) \tag{A.29}$$

[25] In addition to directly increasing exchange rate non-fundamental volatility, higher FX intervention volatility also decreases the load of exchange rate on non-fundamental shock Λ^2. This second effect dampens the initial decrease in exchange rate informativeness $\mathcal{I}_R$, but it cannot reverse it. Intuitively, as the exchange rate becomes less accurate, agents put more weight on their own private signals. As a consequence, the exchange rate can now aggregate more private information and becomes therefore more accurate, attenuating the initial decline in accuracy.

Substitute (A.4), (A.7), and (A.8) in (A.3) to get

$$k_1 = \frac{1}{1-\alpha}q_0 + \frac{1}{1-\alpha}E_0a_1 - \frac{\tilde{\Gamma}}{1-\alpha}\left(-\frac{1}{\beta}\frac{\theta(1+\phi)}{1-\gamma}q_0 + (n_1^\star + f_1^\star)\right) \tag{A.30}$$

Consider the frictionless investment allocation, i.e. with $\Gamma = 0$ and $\bar{E}_0 a_1 = a_1$

$$k_1^{FB} = \frac{1}{1-\alpha}q_0^{FB} + \frac{1}{1-\alpha}a_1 \tag{A.31}$$

Take the difference and get

$$k_1 - k_1^{FB} = \frac{1}{1-\alpha}(q_0 - q_0^{FB}) + \frac{1}{1-\alpha}(E_0a_1 - a_1) - \frac{\tilde{\Gamma}}{1-\alpha}\left(-\frac{1}{\beta}\frac{\theta(1+\phi)}{1-\gamma}q_0 + (n_1^\star + f_1^\star)\right) \tag{A.32}$$

Using (18)

$$\begin{aligned}
k_1 - k_1^{FB} =& \frac{1}{1-\alpha}\left(\bar{E}_0a_1 - a_1\right) + \frac{1}{1-\alpha}(q_0 - q_0^{FB}) + \\
&+ \frac{\Gamma}{1-\alpha}\left(\tilde{\theta}\frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_4} - 1\right)(n_1^\star + f_1^\star) - \frac{\Gamma}{1-\alpha}\tilde{\theta}\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\bar{E}_0a_1 \\
k_1 - k_1^{FB} =& \frac{1}{1-\alpha}\left(1 - \Gamma\,\tilde{\theta}\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\right)\left(\bar{E}_0a_1 - a_1\right) + \frac{1}{1-\alpha}(q_0 - q_0^{FB}) + \\
&\frac{\Gamma}{1-\alpha}\left(\tilde{\theta}\frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_4} - 1\right)(n_1^\star + f_1^\star) - \frac{\Gamma}{1-\alpha}\tilde{\theta}\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}a_1 \\
k_1 - k_1^{FB} =& \frac{1}{1-\alpha}(q_0 - q_0^{FB}) + \\
&- \frac{1}{(1-\alpha)}\frac{1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\left(\Gamma\omega_3\left[(n_1^\star + f_1^\star) + \frac{\tilde{\theta}\omega_2}{\omega_3}a_1\right] - \left[\theta\Gamma(\omega_1 - \omega_2) + \omega_3\right]\left(\bar{E}_0a_1 - a_1\right)\right)
\end{aligned} \tag{A.33}$$

First, consider the case $\theta \neq \sigma$, which implies $\omega_1 \neq \omega_2$. Suppose the belief wedge $(\bar{E}_0a_1 - a_1) \neq 0$. Then the exchange rate is optimal $q_0 = q_0^{FB}$ only if

$$\frac{\Gamma\omega_2}{\beta(1-\gamma)\omega_4}\left[\beta(1-\gamma)\omega_4(n_1^\star + f_1^\star) + \theta\omega_3 a_1\right] = \omega_3(\bar{E}_0a_1 - a_1).$$

However, in that case investment is not at optimum, $k_1 \neq k_1^{FB}$. Therefore, both capital and investment can't be simultaneously at optimum if $(\bar{E}_0a_1 - a_1) \neq 0$.

Second, consider the case $\theta = \sigma$, which implies $\omega_1 = \omega_2$. In this case, one can write the wedge in capital accumulation solely as a function of the wedge in exchange rate.

$$k_1 - k_1^{FB} = \frac{1}{1-\alpha}\left(\frac{\omega_2 - \omega_3}{\omega_2}\right)(q_0 - q_0^{FB}) \tag{A.34}$$

In this case, if the exchange rate equals the frictionless value, so does the capital accumulation. As a result, it is sufficient to have $\Gamma\left[(n_1^\star + f_1^\star) + \frac{\tilde{\theta}\omega_2}{\omega_3}a_1\right] - \left(\bar{E}_0 a_1 - a_1\right) = 0$ to obtain the frictionless allocation, even if $\bar{E}_0 a_1 - a_1 \neq 0$

□

Proof of Proposition 1. Plug (25) in the solution for the perceived exchange rate process (22):

$$q_0 = \left[1 + \frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\frac{\beta^q}{D\lambda_a}\right]^{-1}\left\{\frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}n_1^\star - \left[\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\frac{\beta^v}{D}\right]a_1\right\} \tag{A.35}$$

To find the undetermined coefficients, set (A.35) equal to the guess (23). You get

$$\begin{aligned} -\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(1+\delta)\frac{\beta^v}{D} &= \lambda_a\left[1 + \frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\frac{\beta^q}{D\lambda_a}\right] \\ \lambda_a &= -\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\frac{\beta^v + \beta^q}{D} \end{aligned} \tag{A.36}$$

and

$$\begin{aligned} \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3} &= \lambda_b\left[1 + \frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\frac{\beta^q}{D\lambda_a}\right] \\ \lambda_b &= \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\frac{\beta_v + \beta_q}{\beta_v} \end{aligned} \tag{A.37}$$

Take the ratio

$$\frac{\lambda_a}{\lambda_b} = -\frac{\omega_2}{\Gamma\omega_1}\frac{\beta_v}{D} \tag{A.38}$$

Define $\Lambda \equiv \frac{\lambda_a}{\lambda_b}$. Then:

$$\begin{aligned} \Lambda &= -\frac{\omega_2}{\Gamma\omega_1}\frac{\beta_v}{\beta_v + \beta_a + \Lambda^2\beta_n} \\ \Lambda^3 + \left(\frac{\beta_v}{\beta_n} + \frac{\beta_a}{\beta_n}\right)\Lambda + \frac{\omega_2}{\Gamma\omega_1}\frac{\beta_v}{\beta_n} &= 0 \end{aligned} \tag{A.39}$$

Define $\rho_1 \equiv \frac{(\beta_v+\beta_a)}{\beta_n}$ and $\rho_2 \equiv \frac{\omega_2}{\Gamma\omega_1}\frac{\beta_v}{\beta_n}$. Thus, rewrite (A.52) as:

$$\Lambda^3 + \rho_1\Lambda + \rho_2 = 0 \tag{A.40}$$

Cubics of this form are said to be "depressed." Cardano's formula states the following. If

1. the cubic equation is of the form in (A.53)

2. ρ_1 and ρ_2 are real numbers

3. $\frac{\rho_2^2}{4} + \frac{\rho_1^3}{27} > 0$ (which is satisfied in our context for any real value of $\frac{\omega_3}{\Gamma\omega_2}$)

Then, equation (A.53) has:

(i) the real root:

$$\sqrt[3]{-\frac{\rho_2}{2} + \sqrt{\frac{\rho_2^2}{4} + \frac{\rho_1^3}{27}}} + \sqrt[3]{-\frac{\rho_2}{2} - \sqrt{\frac{\rho_2^2}{4} + \frac{\rho_1^3}{27}}} \tag{A.41}$$

(ii) and two other roots that are non-real complex conjugate numbers.

□

Proof of Proposition 4. Take the limit of (45),

$$\begin{aligned} \lim_{\beta_q\to\infty} var(E_0^i a_1 - a_1) &= \delta^2 \frac{1}{\beta^a} \\ \lim_{\beta_q\to 0} var(E_0^i a_1 - a_1) &= \frac{(\delta\beta^v - \beta^a)^2 \frac{1}{\beta^a} + (1+\delta)^2\beta_v}{(\beta^v+\beta^a)^2} \end{aligned} \tag{A.42}$$

As a result, $\lim_{\beta_q\to\infty} var(E_0^i a_1 - a_1) < \lim_{\beta_q\to 0} var(E_0^i a_1 - a_1)$ if $\delta < 1$.

Moreover,

$$\frac{\partial var(E_0^i a_1 - a_1)}{\partial \beta_q} = \frac{1}{(\beta^v+\beta^q+\beta^a)^3}\left\{(\beta^v+\beta^a)[1-2(1+\delta)] - \beta^q[1-2\delta(1+\delta)]\right\} \tag{A.43}$$

we can distinguish two cases. If $[1-2\delta(1+\delta)] > 0 \Leftrightarrow \delta < \frac{-1+\sqrt{3}}{2}$, then $\frac{\partial var(E_0^i a_1 - a_1)}{\partial \beta_q} < 0$, so the belief wegde decline in public signal accuracy. If $[1-2\delta(1+\delta)] < 0 \Leftrightarrow \delta > \frac{-1+\sqrt{3}}{2}$, then $\frac{\partial var(E_0^i a_1 - a_1)}{\partial \beta_q} < 0$ as long as $0 < \beta_q < (\beta_a+\beta_v)\frac{1-2(1+\delta)}{1-2\delta(1+\delta)}$ and $\frac{\partial var(E_0^i a_1 - a_1)}{\partial \beta_q} > 0$ as long as $(\beta_a+\beta_v)\frac{1-2(1+\delta)}{1-2\delta(1+\delta)} < \beta_q < \infty$.

As a result, if $\delta < \frac{-1+\sqrt{3}}{2}$ the global minimum is reached at $\beta \to \infty$. if $\delta > \frac{-1+\sqrt{3}}{2}$, the global minimum is reached at $\beta_q = (\beta_a + \beta_v)\frac{1-2(1+\delta)}{1-2\delta(1+\delta)}$. $\square$

Proof of Proposition 2. Plug (25) in the solution for the exchange rate (37):

$$q_0 = \left[1 + \frac{\omega_2 - \Gamma\omega_1\tilde{\kappa}_a}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(1+\delta)\frac{\beta^q}{D\lambda_a}\right]^{-1}\left\{\frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(1+\tilde{\kappa}_b)n_1^\star - \left[\frac{\omega_2 - \Gamma\omega_1\tilde{\kappa}_a}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(1+\delta)\frac{\beta^v}{D}\right]a_1\right\} \tag{A.44}$$

To find the undetermined coefficients, set (A.44) equal to the guess (23). You get

$$\lambda_a = -\frac{\omega_2 - \Gamma\omega_1\tilde{\kappa}_a}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(1+\delta)\frac{\beta^v + \beta^q}{D} \tag{A.45}$$

and

$$\lambda_b = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(1+\tilde{\kappa}_b)\frac{\beta_v + \beta_q}{\beta_v} \tag{A.46}$$

Take the ratio

$$\frac{\lambda_a}{\lambda_b} = -\frac{\omega_2 - \Gamma\omega_1\tilde{\kappa}_a}{\Gamma\omega_1(1+\tilde{\kappa}_b)}(1+\delta)\frac{\beta_v}{D} \tag{A.47}$$

Define $\Lambda \equiv \frac{\lambda_a}{\lambda_b}$. Then:

$$\begin{aligned} \Lambda &= -\frac{\omega_2}{\Gamma\omega_1}(1+\delta)\frac{\beta_v}{\beta_v + \beta_a + \Lambda^2\beta_n} \\ \Lambda^3 + \left(\frac{\beta_v}{\beta_n} + \frac{\beta_a}{\beta_n}\right)\Lambda + \frac{\omega_2 - \Gamma\omega_1\tilde{\kappa}_a}{\Gamma\omega_1(1+\tilde{\kappa}_b)}(1+\delta)\frac{\beta_v}{\beta_n} &= 0 \end{aligned} \tag{A.48}$$

Define $\rho_1 \equiv \frac{(\beta_v+\beta_a)}{\beta_n}$ and $\rho_2 \equiv \frac{\omega_2-\Gamma\omega_1\tilde{\kappa}_a}{\Gamma\omega_1(1+\tilde{\kappa}_b)}(1+\delta)\frac{\beta_v}{\beta_n}$. Thus, rewrite (A.52) as:

$$\Lambda^3 + \rho_1\Lambda + \rho_2 = 0 \tag{A.49}$$

Applying the Cardano's formula as in Proposition 1, one gets the following unique solution:

$$\sqrt[3]{-\frac{\rho_2}{2} + \sqrt{\frac{\rho_2^2}{4} + \frac{\rho_1^3}{27}}} + \sqrt[3]{-\frac{\rho_2}{2} - \sqrt{\frac{\rho_2^2}{4} + \frac{\rho_1^3}{27}}} \tag{A.50}$$

$\square$

Proof of Proposition 5. Following the proof for Proposition 1, Plug (25) in the solution for the exchange rate (22) and get the equilibrium λs

$$\begin{aligned}\lambda_a =& -\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1+\omega_3}\frac{\beta_v+\beta_q}{\beta_v+\beta_a+\beta_q}\\ \lambda_b =& \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1+\omega_3}\frac{\beta_v+\beta_q}{\beta_v}\end{aligned} \tag{A.51}$$

Define $\Lambda \equiv \frac{\lambda_a}{\lambda_b}$. Then, since $\beta_q \equiv \Lambda^2(\beta_n^{-1}+\beta_\varepsilon^{-1})^{-1}$:

$$\begin{aligned}\Lambda =& -\frac{\omega_2}{\Gamma\omega_1}\frac{\beta_v}{\beta_v+\beta_a+\Lambda^2(\beta_n^{-1}+\beta_\varepsilon^{-1})^{-1}}\\ \Lambda^3 + &\left(\frac{\beta_v+\beta_a}{(\beta_n^{-1}+\beta_\varepsilon^{-1})^{-1}}\right)\Lambda + \frac{\omega_2}{\Gamma\omega_1}\frac{\beta_v}{(\beta_n^{-1}+\beta_\varepsilon^{-1})^{-1}} = 0\end{aligned} \tag{A.52}$$

Define $\rho_1 \equiv \frac{(\beta_v+\beta_a)}{(\beta_n^{-1}+\beta_\varepsilon^{-1})^{-1}}$ and $\rho_2 \equiv \frac{\omega_2}{\Gamma\omega_1}\frac{\beta_v}{(\beta_n^{-1}+\beta_\varepsilon^{-1})^{-1}}$. Thus, rewrite (A.52) as:

$$\Lambda^3 + \rho_1\Lambda + \rho_2 = 0 \tag{A.53}$$

Applying the Cardano's formula as in Proposition 1, one gets the following unique solution:

$$\sqrt[3]{-\frac{\rho_2}{2}+\sqrt{\frac{\rho_2^2}{4}+\frac{\rho_1^3}{27}}}+\sqrt[3]{-\frac{\rho_2}{2}-\sqrt{\frac{\rho_2^2}{4}+\frac{\rho_1^3}{27}}} \tag{A.54}$$

□

E Welfare Approximation

We evaluate welfare using an utilitarian criterion in which every island of the small open economy receives the same Pareto weight. Welfare is therefore defined as:

$$W = \int \mathbb{E}W_i di = \int \mathbb{E}\left[\frac{C_0^{i\,1-\sigma}}{1-\sigma} + \beta\left(\frac{C_1^{i\,1-\sigma}}{1-\sigma}\right)\right] di. \tag{A.55}$$

We consider a second-order approximation of the above welfare function around the steady state of the frictionless economy, meaning with no intermediation frictions $\Gamma = 0$ and with perfect information $E_0^i a_1 = a_1$:

$$W_i = C^{1-\sigma}\left\{[\hat{c}_0 + \frac{1}{2}(1-\sigma)(\hat{c}_0^i)^2] + \beta[\hat{c}_1 + \frac{1}{2}(1-\sigma)(\hat{c}_1^i)^2]\right\} + t.i.p. + \mathcal{O}(||\xi||^3), \tag{A.56}$$

where hatted variables are expressed in log-deviations from steady state, $t.i.p.$ stands for terms independent of policy, and $\mathcal{O}(||\xi||^3)$ denotes terms that are of third or higher order. Utility is maximized when consumption takes on its efficient values

$$W^{\max} \approx [\bar{c}_0 + \frac{1}{2}(1-\sigma)\bar{c}_0^2] + \beta[\bar{c}_1 + \frac{1}{2}(1-\sigma)\bar{c}_1^2] \tag{A.57}$$

where barred variables are log-deviations from the steady state in the efficient allocation (which is the same for every island). In general, this maximum may not be attainable. We can write $\hat{x}_t = \bar{x}_t + \tilde{x}_t$ so that $\tilde{x}_t = \hat{x}_t - \bar{x}_t$ represents gaps from the efficient allocation. We then have:

$$W^i - W^{max} \approx \tilde{c}_0 + \beta\tilde{c}_1 + \frac{1}{2}(1-\sigma)\left(\hat{c}_0^2 - \bar{c}_0^2\right) + \beta\frac{1}{2}(1-\sigma)\left(\hat{c}_1^2 - \bar{c}_1^2\right) \tag{A.58}$$

To eliminate the linear terms in (A.58), we characterize C_0 and C_1 by taking a second-order approximation of the equilibrium market clearing conditions in (3). To lighten notation we drop the i subscript in the following derivations. Starting from C_1, we obtain:

$$\hat{c}_1 + \frac{1}{2}\hat{c}_1^2 = \gamma\hat{y}_{T,1} + \frac{1}{2}\frac{\gamma}{\theta}(\gamma+\theta-1)\hat{y}_{T,1}^2$$

Now square the first-order approximation:

$$c_1^2 = \gamma^2 y_{T,1}^2$$

to get rid of $\hat{c}_1^2$ above and obtain:

$$\hat{c}_1 = \gamma\hat{y}_{T,1} + \frac{1}{2}\gamma(1-\gamma)\frac{\theta-1}{\theta}\hat{y}_{T,1}^2 \tag{A.59}$$

Similarly for c_0, the second-order approximation of (3) yields:

$$\hat{c}_0 + \frac{1}{2}\hat{c}_0^2 + \phi(\hat{k}_1 + \frac{1}{2}\hat{k}_1^2) = (1+\phi)\gamma\hat{y}_{T,0} + \frac{1}{2}(1+\phi)\frac{\gamma}{\theta}(\gamma+\theta-1)\hat{y}_{T,0}^2 \tag{A.60}$$

Once again, use the square of the first-order approximation:

$$\hat{c}_0^2 = \phi^2\hat{k}_1^2 + (1+\phi)^2\gamma^2\hat{y}_{T,0}^2 - 2\gamma(1+\phi)\phi\hat{k}_1\hat{y}_{T,0} \tag{A.61}$$

to get rid of c_0^2 above and obtain, after some manipulations,

$$\hat{c}_0 = \gamma(1+\phi)\hat{y}_{T,0} - \phi k_1 - \frac{1}{2}\phi(1+\phi)\hat{k}_1^2 + \frac{1}{2}\xi\hat{y}_{T,0}^2 + \gamma(1+\phi)\phi\hat{k}_1\hat{y}_{T,0}, \tag{A.62}$$

where we defined $\xi = (1+\phi)\gamma\left[(1-\gamma)\frac{\theta-1}{\theta} - \gamma\phi\right]$. Now we consolidate the budget constraints, imposing $\tau = 1$:

$$Y_{T,1} = \frac{1}{\beta}K_0^\alpha - \frac{1}{\beta}Y_{T,0} + A_1 K_1^\alpha \tag{A.63}$$

to obtain a second-order approximation for $Y_{T,1}$

$$\hat{y}_{T,1} + \frac{1}{2}\hat{y}_{T,1}^2 = -\frac{1}{\beta}(1+\phi)\left(\hat{y}_{T,0} + \frac{1}{2}\hat{y}_{T,0}^2\right) + \left(\hat{a}_1 + \alpha\hat{k}_1 + \frac{1}{2}\left(\alpha^2\hat{k}_1^2 + 2\alpha\hat{a}_1\hat{k}_1 + \hat{a}_1^2\right)\right)$$

and substitute it into (A.59) and simplify to obtain:

$$\begin{aligned}\hat{c}_1 =& -\gamma\frac{1}{\beta}(1+\phi)\hat{y}_{T,0} + \gamma\hat{a}_1 + \gamma\alpha\hat{k}_1 + \frac{\gamma}{2}\left\{\alpha^2\hat{k}_1^2 + 2\alpha\hat{a}_1\hat{k}_1 + \hat{a}_1^2 - \frac{(1+\phi)}{\beta}\hat{y}_{T,0}^2\right\}\\ &+ \frac{1}{2}\gamma\left[(1-\gamma)\frac{\theta-1}{\theta} - 1\right]\hat{y}_{T,1}^2\end{aligned}$$

Multiply the above expression by β to obtain:

$$\begin{aligned}\beta\hat{c}_1 =& -\gamma(1+\phi)\hat{y}_{T,0} + \gamma\beta\hat{a}_1 + \gamma\alpha\beta\hat{k}_1 + \frac{\beta\gamma}{2}\left\{\alpha^2\hat{k}_1^2 + 2\alpha\hat{a}_1\hat{k}_1 + \hat{a}_1^2 - \frac{(1+\phi)}{\beta}\hat{y}_{T,0}^2\right\}\\ &+ \frac{1}{2}\beta\gamma\left[(1-\gamma)\frac{\theta-1}{\theta} - 1\right]\hat{y}_{T,1}^2\end{aligned} \tag{A.64}$$

Then use (A.62) and (A.64) to evaluate $\tilde{c}_0 + \beta\tilde{c}_1 = (\hat{c}_0 - \bar{c}_0) + \beta(\hat{c}_1 - \bar{c}_1)$:

$$\begin{aligned}\tilde{c}_0 + \beta\tilde{c}_1 =& -\frac{\phi}{2}(1+\phi-\alpha)(\hat{k}_1^2 - \bar{k}_1^2) + \frac{1}{2}(\xi - \beta\gamma\frac{(1+\phi)}{\beta})(\hat{y}_{T,0}^2 - \bar{y}_{T,0}^2) + \gamma(1+\phi)\phi(\hat{k}_1\hat{y}_{T,0} - \bar{k}_1\bar{y}_{T,0}) +\\ &+ \phi\hat{a}_1(\hat{k}_1 - \bar{k}_1) + \frac{1}{2}\beta\gamma\left[(1-\gamma)\frac{\theta-1}{\theta} - 1\right](\hat{y}_{T,1}^2 - \bar{y}_{T,1}^2)\end{aligned} \tag{A.65}$$

Finally, substitute (A.65) in (A.58) to eliminate linear terms from the welfare expression:

$$
\begin{aligned}
W - W^{max} \approx & -\frac{\phi}{2}(1+\phi-\alpha)(\hat{k}_1^2 - \bar{k}_1^2) + \frac{1}{2}(\xi - \beta\gamma\frac{(1+\phi)}{\beta})(\hat{y}_{T,0}^2 - \bar{y}_{T,0}^2) \\
& + \gamma(1+\phi)\phi(\hat{k}_1\hat{y}_{T,0} - \bar{k}_1\bar{y}_{T,0}) + \phi\hat{a}_1(\hat{k}_1 - \bar{k}_1) + \frac{1}{2}\beta\gamma\left[(1-\gamma)\frac{\theta-1}{\theta} - 1\right](\hat{y}_{T,1}^2 - \bar{y}_{T,1}^2) \\
& + \frac{1}{2}(1-\sigma)\left(\hat{c}_0^2 - \bar{c}_0^2\right) + \beta\frac{1}{2}(1-\sigma)\left(\hat{c}_1^2 - \bar{c}_1^2\right) \qquad \text{(A.66)}
\end{aligned}
$$

Equation (A.66) is the expression we use in all our welfare-related calculations and results.