Managing Expectations with Exchange Rate Policy*

Giacomo Candian† HEC Montréal

Pierre De Leo‡ University of Maryland

Luca Gemmi§ University of Bologna

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Abstract

We study a small open economy in which the exchange rate aggregates private information about macro fundamentals, yet imperfectly because of non-fundamental financial flows. Foreign exchange interventions influence economic conditions via traditional portfolio balance and novel informational effects. Publicly announced interventions reveal the central bank's knowledge about macro fundamentals, while secret interventions alter the informational content of the exchange rate. When the private sector overreacts to public information, the optimal policy is to intervene secretly and tolerate some non-fundamental fluctuations in the exchange rate in order to limit its informativeness. Our results speak to practical questions about exchange rate policy design.

Keywords: exchange rates, informational frictions, foreign exchange interventions, central bank communication.

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†Email: giacomo.candian@hec.ca

‡Email: deleop@umd.edu

§Email: luca.gemmi@unibo.it

Despite a historical consensus favoring freely floating exchange rates, central banks worldwide frequently intervene in foreign exchange (FX) markets by buying and selling foreign currencies—a practice traditionally associated with emerging economies and increasingly common in advanced economies (Adler et al., 2021). A remarkable feature of these interventions is the wide variation in central banks' communication practices. While some central banks transparently announce the timing and size of their interventions, about two-thirds either do not disclose these activities or reveal them only after a delay, effectively conducting *secret* interventions (Patel and Cavallino, 2019). Despite these facts, leading macroeconomic models remain silent on the informational dimensions of FX interventions.

This paper develops a micro-founded small open economy model in which agents learn from both exchange rates and FX interventions. Within this framework, we show that interventions have markedly different informational effects depending on whether they are conducted publicly or secretly. We explain how the central bank can leverage communication as an additional tool for formulating optimal exchange rate policy, and show that the optimal communication strategy depends on how agents form beliefs. Our central insight is that when the private sector overreacts to public information, secret interventions can be optimal.

Our dynamic small open economy exhibits three distinctive features within an otherwise standard framework that includes non-tradable and tradable goods, physical capital investment, and stochastic productivity. First, international asset markets are segmented, implying that financial flows directly affect exchange rates as in Gabaix and Maggiori (2015), Fanelli and Straub (2021), and Itskhoki and Mukhin (2022). Second, we relax the full-information assumption. Instead, each agent has access to private information about future productivity—the economy's fundamental. The exchange rate serves to aggregate this dispersed private information and acts as a public signal about productivity. However, this signal is noisy, as noise-trading shocks blur the relationship between exchange rates and productivity, similar to Bacchetta and Wincoop (2006). Third, individuals are diagnostic—overreacting to the information from their private signals, as in Bordalo et al. (2018)—and mistakenly perceive themselves and everyone else as rational. This means that agents systematically underestimate the response of others and endogenously overreact to the news contained in the exchange rate. All three features are supported by empirical evidence in the literature.[1]

[1] The implications of international asset market segmentation are consistent with the evidence from currency demand estimation (*e.g.*, Hau et al., 2010; Pandolfi and Williams, 2019). The notion that

The paper provides three main contributions. First, we formalize the informational role of the exchange rate, an aspect often emphasized by policymakers and commentators yet largely overlooked in international macroeconomics. In our model, agents use the exchange rate as a public signal to form their expectations of future productivity, influencing their external borrowing, consumption, and capital investment. This channel is distinct from traditional mechanisms of exchange rate transmission, such as expenditure switching, and its strength hinges on the *informational content of the exchange rate*—the amount of independent information that the exchange rate contains about future productivity. Because the informational content of the exchange rate is endogenous, it also depends on the conduct and communication of FX interventions.

Second, we show that FX interventions have radically different macroeconomic effects depending on whether they are revealed to the public or conducted secretly. We consider a central bank that observes aggregate financial flows, consistent with the supervisory role many monetary authorities hold over financial institutions. By observing financial flows and the exchange rate, the central bank can infer productivity. Moreover, we assume that the central bank buys and sells foreign bonds according to a publicly known rule that responds to both noise-trading and productivity shocks. The central bank can choose the strength of the reaction to either shock as well as whether to communicate the size of the intervention. If the size of the intervention is publicly announced, it becomes an additional public signal about the beliefs of the central bank. Consequently, the intervention assumes a "signaling role," which expands the information set of agents, akin to the closed-economy models of Melosi (2017) and Tang (2015).[2] In contrast, if the intervention is not publicly announced, *i.e.*, it is conducted secretly, the central bank alters the informational content of the exchange rate through the intervention rule. For example, a rule that leans against noise-trading shocks makes the exchange rate a better signal about productivity; on the contrary, a rule that leans against productivity shocks reduces the exchange rate informativeness. Secret FX interventions, in this sense, can be used to "manage expectations" about

exchange rates partly reflect expectations about a country's future productivity or macroeconomic conditions is consistent with the analysis of Engel and West (2005) and the evidence in Chahrour et al. (2024) and Stavrakeva and Tang (2023). Expectations' overreaction is well documented in laboratory experiments (Afrouzi et al., 2023) and surveys. Bordalo et al. (2020) document that forecasters overreact to news about macro and financial variables, while Candian and De Leo (2023) show that overreaction explains key properties of exchange rate dynamics.

[2] Our notion of signaling differs from contexts in which FX interventions signal future changes in monetary policy (Mussa, 1981), or new target levels for the exchange rate (Bhattacharya and Weller, 1997; Vitale, 1999).

future productivity. However, we stress that the information effects of public and secret FX interventions exist irrespective of whether the central bank is actively trying to influence private sector's expectations.

Third, our normative results show that suboptimal use of public information by the private sector, which in our model arises due to expectations' overreaction to news, can justify the optimal choice of conducting secret FX interventions. To understand this result, it is useful to highlight that our small open economy features two sources of inefficiency relative to the first-best allocation with frictionless financial markets and full-information rational expectations. First, asset market segmentation gives rise to expected excess currency returns that result in suboptimal external borrowing, as in Fanelli and Straub (2021) and Itskhoki and Mukhin (2022). Second, dispersed information entails that agents underreact to productivity shocks, while the cognitive bias due to diagnostic expectations results in overreaction to new information. Both of these frictions drive a wedge between the expectations of future productivity and its actual value. These expectations misalignments are detrimental as they result in suboptimal consumption and investment for a given level of external borrowing.[3]

Publicly announced interventions cannot manipulate the informational content of the exchange rate and therefore their optimal design is aimed exclusively at reducing suboptimal external borrowing. To do so, the central bank completely offsets inefficient financial flows due to noise-trading shocks and ensures an efficient level of external borrowing in response to productivity shock by appropriately buying or selling foreign currency. While this policy guarantees the efficient amount of external borrowing, it nevertheless cannot mitigate beliefs overreaction, implying that the ensuing consumption and investment allocations are generally distorted.

In contrast, *secret* interventions can exploit a novel trade-off between inefficient external borrowing and expectations' overreaction by altering the informativeness of the exchange rate. Intuitively, when the central bank does not announce the size of its interventions, a policy that dampens the effects of noise-trading shocks makes the exchange rate more informative about productivity and causes agents to place more weight on a public signal to which they overreact. On the other hand, mitigating this overreaction requires tolerating some inefficient noise-driven exchange rate fluctuations exactly to reduce their informational content.

[3]As in Gabaix (2020) and most of the behavioral literature, we assume that agents operate under non-rational beliefs but experience utility as rational agents, and that the central bank is aware of this feature.

We show that when overreaction due to diagnostic expectations is sufficiently strong, the central bank optimally chooses to exploit this trade-off through secret interventions. Under these circumstances, agents effectively place an excessively high weight on the exchange rate; thus, lowering its informational content *reduces*, rather than increases, the volatility of expectational mistakes. In such scenarios, it is indeed optimal to limit the informativeness of the exchange rate. To achieve this, the central bank allows some degree of non-fundamental variation in the exchange rate arising from noise trading, even though it results in suboptimal external borrowing.

Our model reconciles two seemingly conflicting aspects of central banking practices. On the one hand, surveys of central bankers point to a consensus that FX interventions work primarily by affecting market expectations through a signaling channel. On the other hand, most central banks conduct secret interventions in the FX market. Together, these observations compose a *secrecy puzzle* (Sarno and Taylor, 2001) as the signaling channel is generally thought to work through transparent policy announcements. Our model rationalizes these puzzling observations. Publicly announced interventions indeed have signaling effects. Yet, because market participants tend to overreact, it can be desirable in some contexts to conceal interventions, and thus its ensuing signaling role, to control the amount of information available to the market. The model thus offers a new interpretation of the widespread practice of "systematic managed floating," whereby central banks regularly respond to changes in market pressure, with a portion reflected in the exchange rate and the rest absorbed through changes in FX reserves (Frankel, 2019).

Relation to the literature This paper contributes to several strands of the literature in international macroeconomics and information economics.

First, a growing body of work in open-economy models has highlighted motives for FX interventions, including alleviating intermediaries' financial constraints (Chang and Velasco, 2017), counteracting the effects of shocks originating in FX markets (Cavallino, 2019; Itskhoki and Mukhin, 2022; Basu et al., 2023), implementing a desired exchange rate policy when the interest rate is at the zero lower bound (Amador et al., 2019), mitigating the distributional effects of exchange rate fluctuations due to consumption externalities (Fanelli and Straub, 2021), or addressing the effects of production externalities (Ottonello et al., 2024). This paper contributes to this literature by studying the informational effects of FX interventions. Information and communication aspects are important in real-world FX policies, as indicated directly by central bankers in surveys and suggested by the observed opacity surrounding many interventions (Patel and Cavallino, 2019).[4] Our contribution is twofold: first, we describe a novel informational channel of FX policies, which applies irrespective of the rationale behind the intervention; second, we show how optimal exchange rate policy can leverage this informational channel to address inefficiencies stemming from frictions in belief formation.[5]

The paper contributes also to the literature on the social value of public information. When information is exogenous, more precise public communication can be detrimental to welfare in the presence of a socially harmful desire to coordinate (Morris and Shin, 2002) or when economic fluctuations are driven primarily by shocks or other distortions that make the full-information equilibrium inefficient (Angeletos and Pavan, 2007). These conditions may arise, for example, in the presence of markup shocks (Angeletos et al., 2016), distortionary taxes (Fujiwara and Waki, 2020), nominal rigidities (Fujiwara and Waki, 2022), and even with supply shocks if they are inefficiently shared across countries (Candian, 2021). When information is endogenous as in learning from prices (Grossman, 1976), more public information can be harmful because of strategic complementarities that make agents overweight public signals (Amador and Weill, 2010), or correlated expectation errors that are common knowledge (Hassan and Mertens, 2017). Our paper formalizes a new setting in which public information can be socially harmful, namely in the presence of extrapolation of endogenous public signals. It also highlights how opaque balance sheet policies can limit price informativeness.

Finally, we contribute to the literature that studies the effect of macroeconomic policy and their communication, in related but distinct contexts. Bond and Goldstein (2015) investigate how uncertain future government intervention affecting a firm's cash flows impacts the informativeness of prices. Gaballo and Galli (2022) studies the informational channel of central bank's asset purchases, but in a closed economy setting where interventions are always publicly observed. Iovino and Sergeyev (2021) study the effects of central bank's balance-sheet policies in a model where people form expectations through an iterative level-k thinking process, also in a closed economy and without informational frictions. Angeletos and Sastry (2020) delve into the separate but complementary question of whether policy communications should anchor expec-

[4]Recent empirical evidence on FX interventions includes Kearns and Rigobon (2005), Kuersteiner et al. (2018), Fratzscher et al. (2019), Menkhoff et al. (2021), Adler et al. (2021), and Ferreira et al. (2024).

[5]Earlier theoretical research has employed partial equilibrium models to explore other aspects of FX interventions such as central bank transparency and credibility about their exchange rate target (Stein, 1989; Bhattacharya and Weller, 1997; Vitale, 1999, 2003).

tations about a policy instrument or the targeted outcome, and how it depends on a departure from rational expectations. Brunnermeier et al. (2021) study the incentives of market participants to acquire information when there is uncertainty surrounding the extent of government intervention in financial markets via asset purchases. Their government faces a trade-off between reducing price volatility and enhancing price efficiency. In our environment, there is also a tension between reducing (inefficient) price volatility and improving allocative efficiency but the tension arises because the private sector processes the information contained in prices incorrectly, rather than because the intervention by itself introduces noise.[6]

1 Belief formation in an open economy model

We consider a two-period small open economy model with a tradable sector and non-tradable sector, and extend it to incorporate two features of interest. First, limited asset market participation gives rise to a finite elasticity of demand for foreign bonds and, therefore, a scope for foreign exchange interventions. Second, the economy is affected by two aggregate shocks that are imperfectly observed by agents in the economy: productivity shocks and "noise-trading" shocks to the demand for foreign bonds.

1.1 Model setup

The small open economy is populated by private agents and a central bank. Private agents, that include households, firms, and financiers, are located on a continuum of atomistic islands, $i \in [0, 1]$, as in Lucas (1972). Information is common within islands but heterogeneous across islands. In particular, on each island, households and financiers receive the same private noisy signal on next-period productivity of the small open economy. Agents observe local output and prices as well as the exchange rate, which serves as a noisy public signal about next-period productivity. Time is discrete and indexed by $t = [0, 1]$. Foreign variables are denoted with a star symbol.

[6] See also Kimbrough (1983, 1984), Chahrour (2014), Paciello and Wiederholt (2014), Angeletos et al. (2020), Kohlhas (2020), Dávila and Walther (2023), and Morelli and Moretti (2023).

1.1.1 Households and goods markets

The preferences of the representative household of island i are described by the following utility function:

$$E_0^i \left\{ \frac{{C_0^i}^{1-\sigma}}{1-\sigma} + \beta \frac{{C_1^i}^{1-\sigma}}{1-\sigma} \right\}, \tag{1}$$

where C^i denotes consumption and E_0^i is an expectation operator, non necessarily rational and conditional on information set in $t = 0$.

Households have an initial endowment of capital, $K_0^i = K_0 > 0$ and must invest final goods to accumulate capital for the next period, K_1^i. The capital stock fully depreciates between periods and is used in the production of tradable goods:

$$Y_{T,0}^{i,H} = {K_0^i}^{\alpha}, \qquad Y_{T,1}^{i,H} = A_1 {K_1^i}^{\alpha}. \tag{2}$$

Above, A_1 represents stochastic period-1 productivity that is common across islands. In each period, the household also receives an endowment of the non-tradable good: $Y_{N,0} = (1+\alpha\beta\gamma)Y_{N,1}$.[7] Final goods that are used for consumption and period-1 capital are composites of tradable and non-tradable goods:

$$C_0^i + K_1^i = G(Y_{N,0}, Y_{T,0}^i), \qquad C_1^i = G(Y_{N,1}, Y_{T,1}^i), \tag{3}$$

where $G(Y_N, Y_T) = \left[(1-\gamma)^{\frac{1}{\theta}} {Y_N}^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} {Y_T}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$ is homogenous of degree 1. The parameter θ denotes the elasticity of substitution between tradable and non-tradable goods in the production of final goods while γ governs the share of tradable goods in the final composite good. In equation (3), $Y_{T,t}^i$ represents domestic absorption of the tradable good, which is the sum of production and imports from the rest of the world $Y_{T,t}^i = Y_{T,t}^{i,H} + Y_{T,t}^{i,F}$. We assume that each island trades with the rest of the world but not with other islands to avoid information revelation by inter-island interactions.

Since the aggregator G is homogenous of degree 1, we have, in equilibrium:

$$P_t^i G(Y_{N,t}, Y_{T,t}^i) = P_{N,t}^i Y_{N,t} + \mathcal{S}_t P_{T,t}^{\star} Y_{T,t}^i, \tag{4}$$

where P_t^i is the island-i price of the composite good, and $P_{N,t}^i$ is the island-i price of the non-tradable good. $\mathcal{S}_t$ is the nominal exchange rate, which is common across islands.

[7] The endowment in period 0 relative to period 1 implies a steady state with $B_1^{\star} = 0, Q_0 = Q_1 = 1$ and $C_0 = C_1$.

We assume that the foreign-currency price of tradable goods is constant and equal to 1, *i.e.*, $P^{\star}_{T,t} = P^{\star}_{T} = 1$.

The price of the tradable good relative to the non-tradable good is given, in equilibrium, by their marginal rate of transformation:

$$\frac{\mathcal{S}_t}{P^i_{N,t}} = \frac{\partial G(Y_{N,t}, Y^i_{T,t})/\partial Y^i_{T,t}}{\partial G(Y_{N,t}, Y^i_{T,t})/\partial Y_{N,t}} = \left(\frac{\gamma}{1-\gamma}\frac{Y_{N,t}}{Y^i_{T,t}}\right)^{\frac{1}{\theta}}. \tag{5}$$

Combining this expression with eq. (4) yields the equation determining island-i composite price index:

$$P^i_t = \left[(1-\gamma){P^i_{N,t}}^{1-\theta} + \gamma {\mathcal{S}_t}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{6}$$

Combining these last two equations we obtain the demand function for tradable goods:

$$Y^i_{T,t} = \chi \left[\left(\frac{\mathcal{S}_t}{P^i_t}\right)^{-(1-\theta)} - \gamma\right]^{\frac{\theta}{1-\theta}} Y_{N,t}, \tag{7}$$

with $\chi = \gamma(1-\gamma)^{-\frac{1}{1-\theta}}$. The household's budget constraints are:

$$\begin{aligned} P^i_0 C^i_0 + P^i_0 K^i_1 + \frac{B^i_1}{R_0} &= P^i_{N,0} Y_{N,0} + \mathcal{S}_0 Y^{i,H}_{T,0} + T^i_0, \\ P^i_1 C^i_1 &= B^i_1 + P^i_{N,1} Y_{N,1} + \mathcal{S}_1 Y^{i,H}_{T,1} + T^i_1. \end{aligned} \tag{8}$$

The date-0 budget constraint assumes no initial debt and states that the household's income from the sale of tradable and non-tradable goods as well as from government nominal transfers, T^i_0, can be used to buy consumption goods, invest in physical capital, or save in a domestic nominal bond, B^i_1, whose interest rate is R_0. The date-1 budget constraint states that all the income of the household is used for consumption.

Implicit in equation (8) is the assumption that the household cannot hold foreign bonds (*e.g.*, Gabaix and Maggiori, 2015, Fanelli and Straub, 2021, Itskhoki and Mukhin, 2021). Limited asset market participation captures the idea that it is difficult for many households in emerging markets to access international financial instruments without financial intermediation, especially when borrowing in foreign currency.

Maximizing utility (1) subject to the budget constraints (8) yields the following optimality conditions:

$$\beta R_0 E_0^i \left[\left(\frac{C_1^i}{C_0^i} \right)^{-\sigma} \left(\frac{P_0^i}{P_1^i} \right) \right] = 1; \tag{9}$$

$$\alpha\beta E_0^i \left[\left(\frac{C_1^i}{C_0^i} \right)^{-\sigma} \frac{\mathcal{S}_1}{P_1^i} A_1 {K_1^i}^{\alpha-1} \right] = 1. \tag{10}$$

Finally, using equations (3) and (4) in the budget constraints (8), island i households' budget constraints simplifies to:

$$\frac{B_1^i}{R_0} = \mathcal{S}_0 (Y_{T,0}^{H,i} - Y_{T,0}^i) + T_0^i; \qquad -B_1^i = \mathcal{S}_1 (Y_{T,1}^{H,i} - Y_{T,1}^i) + T_1^i. \tag{11}$$

where each island households leave no debt at the end of period 1.

1.1.2 Financial market

Financiers from every island trade home and foreign bonds in the small open economy-wide financial sector. Along with financiers, the government and a set of noise traders also operate in the financial sector, as we describe next.

Financiers We assume that there are frictions in the financial sector which result in a downward-sloping demand for currency from financiers. In particular, we follow the formulation of Fanelli and Straub (2021). In each island a continuum of risk-neutral financiers owned by the island household trade home and foreign bonds subject to position limits and heterogeneous participation costs, as in Alvarez et al. (2009).[8]

In Appendix A.1, we derive the maximization problems of island-i financiers, and show that it results in the following demand for the foreign currency bond:

$$\frac{{D_1^i}^\star}{R_0^\star} = \frac{1}{\hat{\Gamma}} E_0^i \left(R_0^\star - R_0 \frac{\mathcal{S}_0}{\mathcal{S}_1} \right), \tag{12}$$

where the zero-capital portfolio island-i financiers and their carry trade profits are, respectively:

$$\frac{D_1^i}{R_0} + \mathcal{S}_0 \frac{{D_1^i}^\star}{{R_0}^\star} = 0, \qquad\qquad \pi_1^{i,D\star} \equiv {D_1^i}^\star + \frac{D_1^i}{\mathcal{S}_1} = \cdots = \tilde{R}_1^\star \frac{{D_1^i}^\star}{{R_0}^\star}.$$

[8]The assumption that financial intermediaries are owned by the household ensures that profits and losses from carry trade activity do not represent a net benefit or cost to the small open economy. Fanelli and Straub (2021) already studied the implications for FX interventions of such "leakages." Our novel focus is instead on the informational role of exchange rates and FX interventions.

Aggregating across islands, the overall demand of financiers for foreign bonds is:

$$\frac{\int D_1^{i\star}\,\mathrm{d}i}{R_0^\star} = \frac{1}{\hat{\Gamma}}\bar{E}_0\left(R_0^\star - R_0\frac{\mathcal{S}_0}{\mathcal{S}_1}\right). \tag{13}$$

where $\bar{E}_0(X_t)$ denotes the average expectation of X_t across islands, *i.e.*, $\bar{E}_0 X_t = \int E_0^i X_t\,\mathrm{d}i$.

Financiers' demand for foreign bonds has a finite (semi-)elasticity to the expected excess return, implying that changes in the net supply of foreign bonds, *e.g.*, induced by noise trading or FX interventions, affect the equilibrium exchange rate. The critical parameter in equation (13) is the inverse demand elasticity $\hat{\Gamma}$, which governs the strength of frictions in the international financial market. If $\hat{\Gamma}$ is large, *e.g.*, due to tight position limits, intermediation is impeded. In the extreme case where $\hat{\Gamma} \to \infty$ intermediation is absent, and the country is in financial autarky. By contrast, when $\hat{\Gamma} \to 0$, financiers are not subject to position limits and expected excess currency returns are driven to zero. Henceforth, we assume $\hat{\Gamma} \in (0, \infty)$.

Noise traders Noise traders exogenously demand foreign currency $\frac{N_1^\star}{R_0{}^\star}$. Here $\frac{N_1^\star}{R_0^\star} > 0$ means that noise traders short home-currency bonds to buy foreign-currency bonds. They also hold a zero-capital portfolio in home and foreign bonds denoted $(N_1, N_1^\star)$, which implies:

$$\frac{N_1}{R_0} + \mathcal{S}_0\frac{N_1^\star}{R_0^\star} = 0. \tag{14}$$

Central bank/Government The economy-wide central bank holds a $(F_1, F_1^\star)$ bond portfolio. The value of the portfolio is $\frac{F_1^i}{R_0} + \mathcal{S}_0\frac{F_1^\star}{R_0^\star}$. We assume that the government finances its operations with transfers:

$$\begin{aligned} \frac{F_1}{R_0} + \mathcal{S}_0\frac{F_1^\star}{R_0^\star} &= -\int T_0^i\,\mathrm{d}i, \\ 0 = F_1 + \mathcal{S}_1 F_1^\star + \tau\mathcal{S}_1\left(\int \pi_1^{i,D\star}\,\mathrm{d}i + \pi_1^{N\star}\right) &- \int T_1^i\,\mathrm{d}i, \end{aligned} \tag{15}$$

where $\pi_1^{i,D\star}$ is the income from financial transactions of financiers on island i, defined above, and $\pi_1^{N\star}$ is the income from financial transactions of noise traders. In addition, the central bank ensures stable aggregate price levels, *i.e.*, $P_0 = P_1 = 1$. This implies a real exchange rate of the small open economy $\mathcal{Q}_t = \frac{\mathcal{S}_t P_t}{P_{T,t}^\star} = \mathcal{S}_t$ in every period.

Financial market clearing Since home-currency bond are in zero net supply, market clearing implies

$$\int B_1^i \, \mathrm{d}i + N_1 + \int D_1^i \, \mathrm{d}i + F_1 = 0. \tag{16}$$

Combining the market clearing condition with households and government budget constraints, and the portfolios and income from financial transactions of financiers and noise traders, we obtain the aggregate position of the financiers, in foreign currency:

$$\frac{\int D_1^{i\star} \, \mathrm{d}i}{R_0^\star} = \int \left(Y_{T,0}^{i,H} - Y_{T,0}^i \right) \, \mathrm{d}i - \frac{F_1^\star + N_1^\star}{R_0^\star} \tag{17}$$

That is, the aggregate position of financiers equals the portion of households' bond demand—originating from the trade imbalance—that is not met by the foreign currency supplied by the government and noise traders.

1.2 Exchange rate determination

We solve the model using a log-linear approximation around a steady state with $A_1 = 1, N^\star = F^\star = 0$ and $B^i = D^i = 0, \forall i$. Lower-case variables denote log-deviation from steady state of corresponding upper-case variables. In Appendix A.2 we report the island-level equilibrium, and in Appendix A.3 we derive the equilibrium aggregate real exchange rate as a function of shocks and expectations thereof:

$$q_0 = \frac{\Gamma \omega_1}{\Gamma \tilde{\theta} \omega_1 + \omega_3} (n_1^\star + f_1^\star) - \frac{\omega_2}{\Gamma \tilde{\theta} \omega_1 + \omega_3} \bar{E}_0 a_1 \tag{18}$$

where $\bar{E}_0$ is the average expectation across all islands, $\Gamma \equiv \hat{\Gamma} \beta^2 (\alpha \beta)^{\frac{\alpha}{1-\alpha}}$, and $\omega_1 > 0, \omega_2 > 0, \omega_3 > 0$, and $\tilde{\theta} > 0$ are convolutions of parameters independent of Γ defined in Appendix A.3.

Equation (18) says that, *ceteris paribus*, the exchange rate appreciates following a fall in foreign bond demand (by noise traders or by the central bank) or an improvement in expectations of productivity. Lower foreign bond demand requires financiers to take a long position on foreign bonds, commanding an expected profit, in proportion to Γ. As a result, the exchange rate appreciates today, so that its expected depreciation earns financiers an expected return on their long foreign-bond position.[9]

[9] For completeness, the exchange rate appreciation also lowers the price of tradable goods, leading to higher imports and borrowing. The ensuing households' demand for more domestic bonds alters the position of financiers and attenuates the equilibrium appreciation of the exchange rate.

Higher expected future productivity in the tradable sector implies a reduction in the future relative price of tradables, *i.e.* a future exchange rate appreciation. By uncovered interest parity (UIP), the exchange rate today appreciates. The equilibrium appreciation, however, is attenuated relative to UIP due to the effects of higher expected future productivity on financial flows: households increase imports and borrowing today, prompting financiers to absorb more domestic bonds. As a result, domestic bonds must offer a positive excess return, exerting downward pressure on the equilibrium exchange rate.

1.3 Information structure

We now consider how expectations are formed and introduce two important assumptions: dispersed information and extrapolative expectations.

Priors, private and public signals Households and financiers in each island $i \in [0,1]$ have a common prior about next-period productivity $a_1 \sim N(0, \beta_a^{-1})$. Moreover, these agents observe local prices and quantities, in addition to a local signal about the future realization of productivity a_1:

$$v^i = a_1 + \epsilon^i, \qquad \epsilon^i \sim N(0, \beta_v^{-1}), \tag{19}$$

with $\int_i \epsilon^i \, \mathrm{d}i = 0$.

We make the following assumptions regarding the observability of aggregates. First, while agents in each island cannot observe aggregate prices and quantities, they share the same currency and can therefore observe the aggregate exchange rate q_0. The exchange rate is an important endogenous signal because it carries information about the average expectation of the common future productivity shock a_1 as per eq. (18). Second, agents in each island cannot directly observe the amount of aggregate noise trading $n_1^\star$. Third, agents may also be able to observe a second endogenous public signal namely the quantity of FX interventions, $f_1^\star$, depending on the communication policy adopted by the central bank. Nevertheless, irrespective of such communication policy we can guess and verify that the public information available to every island i can be summarized by a sufficient statistic

$$z | a_1 \sim \mathcal{N}(\mu_z, \beta_z^{-1}).$$

The form of this sufficient statistic will be detailed in each intervention scenario analyzed below.

Extrapolative expectations Consistent with growing empirical evidence, we consider the possibility that agents do not form beliefs rationally but instead overreact to news (Bordalo et al., 2020; Broer and Kohlhas, 2022; Afrouzi et al., 2023). To this end, we build on the Diagnostic Expectations (DE) model, first proposed by Gennaioli and Shleifer (2010) and widely adopted in the literature (for a review, see Bordalo et al., 2022). The foundational psychological principle in this model is limited and selective memory retrieval: in their subjective probability assessments, individuals overweight event outcomes that are easily recalled because they are 'representative' (Kahneman and Tversky, 1972). In the DE framework, events associated with new information are more representative, which leads to overreaction to news. Appendix F provides additional details on the psychological foundation of DE. We make two assumptions to formalize diagnosticity in our context.

DE1 (Extrapolation of private signal). Agents are diagnostic with respect to the news contained in their private signals v^i: the probability assessment of a_1 after observing v^i is

$$h^{\delta}(a_1|v^i) = h(a_1|v^i)\left[\frac{h(a_1|v^i)}{h(a_1|v^i=0)}\right]^{\delta}\frac{1}{Z}, \tag{20}$$

where Z is a normalizing constant, $h(a_1|v^i)$ is the true Bayesian posterior distribution of a_1 after observing the signal v^i, and $h(a_1|v^i = 0)$ is the posterior if the signal realization were to equal the prior mean, which in this case is zero. The term $\frac{h(a_1|v^i)}{h(a_1|v^i=0)}$ reflects the representativeness of a_1, which is higher if the observed signal raises the probability of that state relative to the case where the news equals the prior. The parameter $\delta \geq 0$ captures the impact of representativeness on judgments (if $\delta = 0$, memory is frictionless). The DE framework in equation (20) combined with the assumption of normality leads to a convenient representation: the DE bias does not affect the subjective posterior variance, which equals the rational one, while the diagnostic posterior mean equals

$$E^i[a_1|v^i] = (1+\delta)\mathbb{E}^i[a_1|v^i], \tag{21}$$

where $\mathbb{E}$ is the rational expectation operator (Bordalo et al., 2019, 2020). After a positive (negative) news, the subjective assessment overweights positive (negative) realizations of a_1, leading to overreaction.

Combining the information of the private signal v^i with the prior and and the public signal z, we can write individual agents' posterior beliefs as:

$$E_0^i a_1 \equiv E[a_1|v^i, z] = \frac{(1+\delta)\beta_v v^i + \beta_z z}{\beta_v + \beta_z + \beta_a} \tag{22}$$

Above, $\beta_v + \beta_z + \beta_a$ is the overall posterior accuracy, given by the sum of the precisions of the prior and of the private and public signals. The term $\frac{\beta_v}{\beta_v+\beta_z+\beta_a}$ is the rational weight on the private signal v^i, which is multiplied by $(1+\delta)$ because of assumption DE1. Finally, $\frac{\beta_z}{\beta_v+\beta_z+\beta_a}$ is the weight on the public signal, z.

While the existing literature applies diagnosticity only to exogenous information, we extend DE to endogenous signals such as exchange rates. Because the exchange rate reflects average beliefs about future productivity, this extension requires specifying how agents form these "higher-order beliefs."

DE2 (Misperception about others). We assume that agents are unaware not only of their own diagnosticity bias, but also of the bias of all the other agents in the economy. In other words, they perceive themselves and every other agent as rational:

$$E_0^i[E_0^j[a_1]] = E_0^i[\mathbb{E}_0^j[a_1]]; \qquad \forall j \neq i. \tag{23}$$

As a result of this misperception, they interpret endogenous public signals as aggregating rational beliefs instead of actual beliefs. We elaborate on the implications of this misperception below.[10]

We note that the special case of rational expectations corresponds to $\delta = 0$. In this case, agents not only refrain from extrapolating their private information ($E^i[a_1|v^i] = \mathbb{E}^i[a_1|v^i]$), but also correctly assume that the other agents do not extrapolate either.

1.4 Discussion of assumptions

Before we move on, let us discuss some of the assumptions that we made.

First, we ruled out inter-island interactions other than through a common financial market. Allowing for further interactions among islands (for example, via inter-island goods trade) would completely reveal average expectations and, therefore, eliminate

[10] Bastianello and Fontanier (2022) also study mislearning from prices, but starting from a different psychologically-founded bias. They consider agents who fail to understand that other agents learn from prices as well. In our setting, agents understand that other people learn from prices as well, but fail to internalize that they overreact to private information.

any marginal informational role of public signals such as exchange rate or interventions. Second, we assumed that there is only one aggregate price that agents observe, namely the exchange rate, but two economic disturbances, productivity shocks and noise-trading shocks. This assumption ensures that agents cannot fully back out the aggregate state of the economy by simply observing the exchange rate. These first two assumptions parsimoniously capture the idea that economic agents, for various reasons, do not perfectly observe all the variables that are relevant to their decisions but that they use easily accessible information, such as exchange rates, to improve their inference about such variables.

Third, we assumed that the small open economy can save in physical capital and foreign bonds. Capital plays an important role in our model. The exchange rate, by affecting the relative demand for tradable and non-tradable goods, is a key determinant of the allocation of domestic income between domestic spending (consumption and capital investment) and external savings. Absent capital, there is a one-to-one relationship between external saving and current consumption, and that relationship is entirely governed by the exchange rate. Thus, a policymaker interested in affecting the path of consumption has no reason to influence expectations if it can directly affect the exchange rate. The presence of capital ensures that, for a given level of the exchange rate, expectations are a distinct concern for the policymaker because they determine the repartition of domestic spending between consumption and investment.

Finally, we have postulated a departure from rationality in the form of diagnostic expectations extended to a setting with endogenous public signals. Diagnostic expectations are now widely adopted in the literature. An essential aspect of the diagnostic expectation in equation (20) is the 'kernel of truth' property, *i.e.*, the idea that distortions in beliefs stem from incorrect interpretation of true information. This property has been shown to provide a common explanation to biases in probability assessments documented in laboratory experiments, including base rate neglect, the conjunction fallacy, and the disjunction fallacy (Gennaioli and Shleifer, 2010), as well as to real-world behaviors, such as stereotyping (Bordalo et al., 2016). Furthermore, it has been used to rationalize the formation of expectations in financial markets (Bordalo et al., 2018, 2019) and surveys (Bordalo et al., 2020; Candian and De Leo, 2023).

Regarding higher-order beliefs, our assumption DE2 implies that each agent systematically underestimates the response of others, similar to Angeletos and Sastry (2020). This form of higher order belief bias leads to endogenous overreaction to public signals, a result we elaborate upon below. While we find the endogenous aspect conceptually appealing, in Appendix H we show that our substantive results obtain also under the following alternative formulation of beliefs: (i) a setting without higher-order bias but in which agents are diagnostic with respect to both private and public signals; (ii) a Partial Equilibrium Thinking model as in Bastianello and Fontanier (2022), where agents use private signals optimally, but fail to realize other agents also learn from prices.

2 Learning and mislearning from the exchange rate

A central feature of our model is that agents learn about the economy's fundamentals from exchange rate fluctuations. In turn, learning affects their consumption and investment decisions, which feed back into the exchange rate via the forces that clear the foreign exchange market described in Section 1.2. To examine this general equilibrium feedback loop in the cleanest way and to show that this mechanism exists independently of exchange rate policies, in this section we abstract from interventions by setting $f_1^\star = 0$. We refer to this economy as the *laissez-faire* economy and use a superscript l where useful.

2.1 Perceived and actual exchange rate process

In the laissez-faire economy the only source of public information is the exchange rate.

Public signals including exchange rates are perceived by private agents to follow a different process than the actual one because agents fail to realize that others extrapolate their private information, a direct consequence of DE2. Despite this difference, in the log-linearized economy perceived and actual processes for aggregate variables are both linear functions of aggregate shocks a_1 and $n_1^\star$. To correctly account for this misperception, throughout we will denote the solution coefficients of perceived and actual aggregate variables with and without a $\tilde{\cdot}$ notation, respectively.

Our solution approach starts with a guess for the *perceived* exchange rate process, which, together with the prior and the private signal, determines markets' expectations. Given those expectations we can go back to the perceived version of market clearing equation (18) and verify that the perceived exchange rate process is consistent with our guess. Finally, we can use these equilibrium beliefs into the actual market clearing equation (18) to solve for the *actual* exchange rate process.

Accordingly, we begin with a guess for the *perceived* exchange rate process:

$$q_0 = \tilde{\lambda}_a^l a_1 + \tilde{\lambda}_n^l n_1^\star. \tag{24}$$

Given this perceived process, individuals form posterior beliefs in (22) using the signal:

$$z^l | a_1 \equiv \left. \frac{q_0}{\tilde{\lambda}_a^l} \right| a_1 \overset{p}{\sim} \mathcal{N}(a_1, ({\tilde{\lambda}_n^l}/{\tilde{\lambda}_a^l})^2 \beta_n^{-1}) \tag{25}$$

where $\overset{p}{\sim}$ introduces the distribution that agents perceive the signal to follow. In this formulation, z^l represents a signal that is perceived to be centered around future productivity a_1 with a error variance of $\beta_z^{-1} \equiv ({\tilde{\lambda}_n^l}/{\tilde{\lambda}_a^l})^2 \beta_n^{-1}$.

Next, because agents are unaware of others' extrapolative bias (DE2), they perceive the market clearing condition (18) to be

$$q_0 = \frac{\Gamma \omega_1}{\Gamma \tilde{\theta} \omega_1 + \omega_3} n_1^\star - \frac{\omega_2}{\Gamma \tilde{\theta} \omega_1 + \omega_3} \bar{\mathbb{E}}[a_1 | v^i, z^l], \tag{26}$$

where $\bar{\mathbb{E}}[a_1|v^i, z^l] \equiv \int^i \mathbb{E}^i[a_1|v^i, z^l] \, \mathrm{d}i = \frac{\beta_v a_1 + \beta_z z^l}{\beta_v + \beta_z + \beta_a}$ are average rational (instead of diagnostic) expectations. Matching coefficients in equations (24) and (26) gives the solution $(\tilde{\lambda}_a, \tilde{\lambda}_n)$ to the *perceived* exchange rate process.

Next, guess the following solution for the actual exchange rate process: $q_0 = \lambda_a^l a_1 + \lambda_n^l n_1^\star$. The solution coefficients are found by substituting the actual average posterior beliefs given by $\int_0^1 E^i[a_1|v^i, z^l] di = \frac{(1+\delta)\beta_v a_1 + \beta_z z^l}{\beta_v + \beta_z + \beta_a}$ and the coefficients $(\tilde{\lambda}_a^l, \tilde{\lambda}_n^l)$ in the actual market clearing condition (18). We collect these results in the following proposition.

Proposition 1. *Under the assumed information structure, there exists a unique symmetric linear equilibrium. In equilibrium, the exchange rate is perceived to follow:*

$$q_0 = \underbrace{-\frac{\omega_2}{\Gamma \tilde{\theta} \omega_1 + \omega_3} \frac{\beta_v + \beta_z}{\beta_a + \beta_v + \beta_z}}_{\tilde{\lambda}_a^l} a_1 + \underbrace{\frac{\Gamma \omega_1}{\Gamma \tilde{\theta} \omega_1 + \omega_3} \frac{\beta_v + \beta_z}{\beta_v}}_{\tilde{\lambda}_n^l} n_1^\star, \tag{27}$$

with $\beta_z \equiv \Lambda_l^2 \beta_n$ *and a unique* $\Lambda_l^2 \equiv \left(\frac{\tilde{\lambda}_a^l}{\tilde{\lambda}_n^l} \right)^2$ *implicitly defined by:*

$$\Lambda_l^2 = \left(\frac{\omega_2}{\Gamma \omega_1} \right)^2 \frac{\beta_v^2}{(\beta_a + \beta_v + \Lambda_l^2 \beta_n)^2}. \tag{28}$$

The actual exchange rate process is given by:

$$q_0 = \underbrace{(1+\delta)\tilde{\lambda}^l_a}_{\lambda^l_a} a_1 + \underbrace{\tilde{\lambda}^l_n}_{\lambda^l_n} n^\star_1. \tag{29}$$

Proof. See Appendix E. $\square$

The role of endogenous extrapolation It is useful to compare the perceived and actual exchange rate processes to understand the role played by the misperception of others' beliefs. From eq. (29), consider the actual, rather than perceived, properties of public signal that agents use to form their beliefs

$$z^l|a_1 \equiv \left.\frac{q_0}{\tilde{\lambda}^l_a}\right| a_1 \sim \mathcal{N}((1+\delta)a_1, (\tilde{\lambda}^l_n/\tilde{\lambda}^l_a)^2\beta_n^{-1}) \tag{30}$$

where $\sim$ introduces the distribution that the signal actually follows.

If $\delta = 0$, there is no extrapolation and the public signal used by agents is interpreted correctly, *i.e.*, eq. (25) coincides with eq. (30). However, $\delta > 0$ leads to a misinterpretation of the endogenous public signal z^l.

First, agents fail to internalize that they are using a public signal that is biased. As agents overreact to their own *private* signal, the average belief loads more on the average private signal (equal to a_1) by a factor of $1+\delta$. Since agents are unaware of others' overreaction, a gap emerges between perceived and actual average beliefs: $\bar{\mathbb{E}}[a_1|v^i, z^l] - \bar{E}[a_1|v^i, z^l] = {}^{-\delta\beta_v a_1}/_{(\beta_v+\beta_z+\beta_a)}$. As a result, individuals perceive the public signal, *i.e.* the equilibrium exchange rate, to load less on productivity a_1 than it actually does. It follows that for any given movement in the exchange rate, agents mistakenly attribute a larger-than-rational fraction of that movement to fluctuations in underlying productivity. The end result is that agents endogenously extrapolate the information about productivity contained in the exchange rate.

Second, the perceived accuracy of the public signal equals the actual accuracy: while agents misperceive the mean of the public signal, they correctly perceive its accuracy. This result is due to the particular structure of the diagnosticity bias (Bordalo et al., 2020).[11]

To sum up, since agents are unaware that the other agents in the economy are subject to extrapolative beliefs, they underestimate the covariance between exchange

[11] A belief bias on the perceived accuracy of private signals, *e.g.* overconfidence, would affect also the perceived accuracy of the endogenous public signal, as in Broer and Kohlhas (2022).

rate and productivity, and thus overreact to its informational content.

2.2 Informational role of the exchange rate

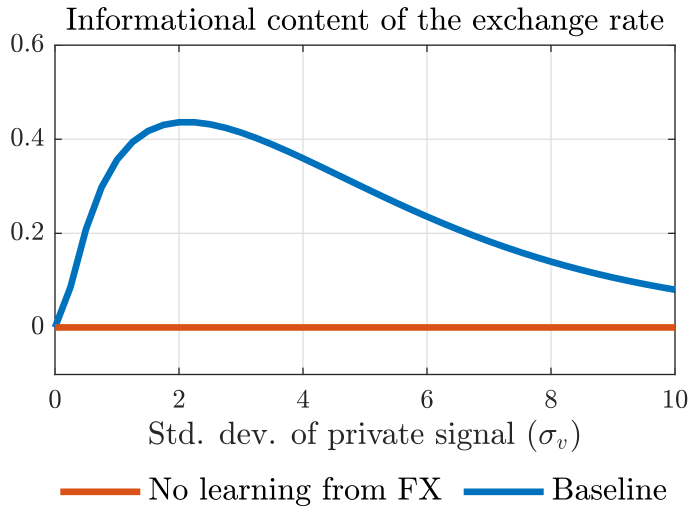
By aggregating private information about aggregate productivity, the exchange rate in our model plays a novel important role in agents' expectations formation and decisions. The strength of this informational role depends on how much weight agents put on the exchange rate as a signal. The following definition highlights that this weight in turn depends on how informative the equilibrium exchange rate is relative to private signals and to the common prior.

Definition 1 (Informational content of the exchange rate)**.** *In the laissez-faire economy, the informational content of the exchange rate corresponds to the Bayesian weight on public signal:* $\mathcal{I}_R = \frac{\Lambda^2 \beta_n}{\beta_a + \beta_v + \Lambda^2 \beta_n}$.

Figure 1 depicts the informational content of the exchange rate for different degrees of dispersion of private information under an illustrative calibration of the model with $\delta = 0$. When private information in infinitely precise ($\sigma_v^2 \equiv \beta_v^{-1} = 0$), the exchange rate does not carry any additional information, and agents place zero weight on it. At the other extreme, when private information is infinitely noisy ($\sigma_v^2 = \infty$), agents place no weight on their private signals. Thus, the exchange rate does not aggregate private information and also receives a zero weight in agents' posterior beliefs. For intermediate values of dispersion in private signals, the informational content is positive as the exchange rate meaningfully aggregates private information about future productivity.

By directly influencing agents' expectations in these intermediate cases, the exchange rate affects macroeconomic outcomes through a novel channel. To see this, examine the effects of an increase in noise-trading demand for home-currency bonds when agents learn from the exchange rate (Figure 2b, blue line). In each island, agents observe an exchange rate appreciation, but do not know whether it stems from an expected improvement in future productivity ($a_1 \uparrow$) or from higher noise-trading demand for home currency ($n_1^\star \downarrow$). They confound, at least in part, the effect of the noise-trading shock on the exchange rate with the effect of higher future productivity, thereby revising their beliefs about future productivity upward. Higher expected productivity leads households to increase their consumption and external borrowing, relative to the no-learning-from-FX case (red line), which further appreciates the ex-

Figure 1: Informational content of exchange rate under dispersed information



Notes: This figure reports the equilibrium value of the informational content of the exchange rate for different levels of the noise in private signal, σ_v, under laissez faire, and without belief extrapolation ($\delta = 0$). The rest of parameters are set according to Table A.1.

change rate.[12] The effect is analogous to an exogenous productivity news shock, but it is due to the endogenous response of the exchange rate to higher home-currency demand from noise traders. Note that this channel is in place every time agents use the exchange rate as a public signal, including in response to an increase in future productivity (see Figure 2a).

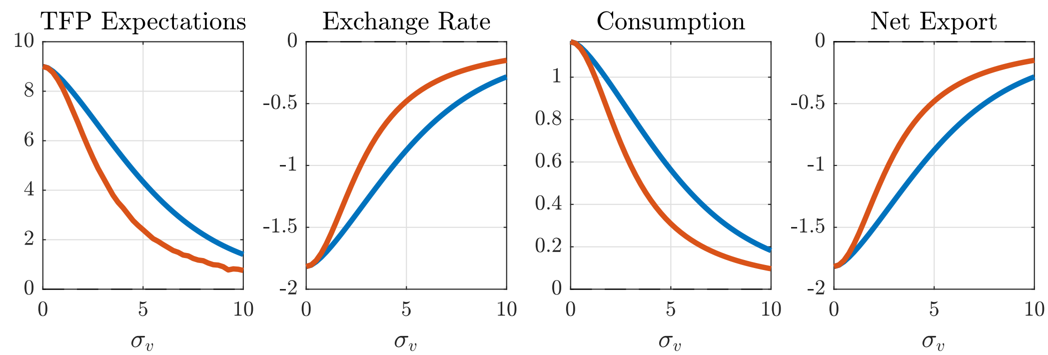
We make two related observations. First, the informational channel of the exchange rate just described does not rely on expectations' overreaction, as it operates even under rational expectations (indeed, $\delta = 0$ in Figure 2). Yet, expectations overreaction changes the quantitative role of the informational channel. When $\delta > 0$, the exchange rate is a *biased* public signal, resulting in amplified responses to productivity shocks (as depicted in Figure A.3, where $\delta = 0.5$). This feature will turn out to be a key source of inefficiency that the policymaker considers when optimally choosing FX policies. Second, learning from the exchange rate blurs the relationship between productivity *expectations* and their subsequent *realizations*: because noise trading affects the exchange rate by altering the balance sheet of financiers, it also influences agents' expectations of productivity. This aligns with Chahrour et al.'s (2024) evidence that

[12]The rational confusion between noise trading and expected productivity improvements amplifies the equilibrium effects of noise-trading shocks on the exchange rate, as originally highlighted in Bacchetta and Wincoop (2006), and on macro aggregates, as we emphasize using our fully-specified macro model.

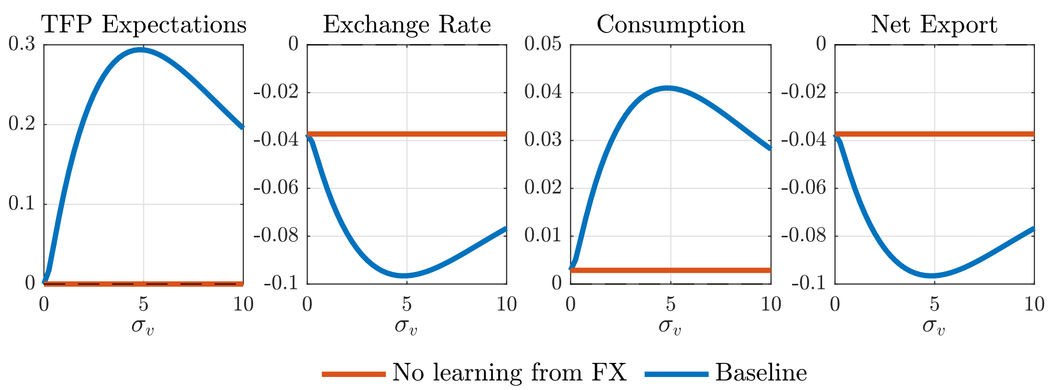
a substantial portion of the exchange rate variation can be attributed to both correctly anticipated changes in productivity and expectational "noise," which influences expectations of productivity but not the actual realization (see Appendix B).

Figure 2: Equilibrium responses to shocks under dispersed information

(a) Response to one-standard deviation productivity shock ($a_1 \uparrow$)



(b) Response to one-standard deviation noise-trading shock ($n_1^\star \downarrow$)



Notes: This figure reports the equilibrium response of model variables to productivity and noise-trading shocks for different levels of the noise in private signal, σ_v, under laissez faire, and without belief extrapolation ($\delta = 0$). The rest of parameters are set according to Table A.1.

2.3 Examples of informational role of FX in policy and media

To conclude this section, we provide three recent examples in which the informational role of the exchange rate is explicitly discussed by policy makers or commentators. During a recent meeting of African central bank governors, IMF African Department Director Abebe Aemro Selassie highlights that "The exchange rate in most developing countries is the most visible and important price in the economy and so helps to anchor expectations, facilitate planning, as well as investment, and consumption decisions." (Selassie, 2023). This observation aligns with the survey evidence that the nominal exchange rate receives substantial attention by firms in emerging economies (Delgado et al., 2024). Recently, after the Euro depreciated following a increase in the ECB policy rate, *Financial Times* Markets Editor Katie Martin writes that "The euro's latest wobble also forms yet another big signal that investors think Europe's luck has run out." (Martin, 2023). In a recent report to the Bank of International Settlements, economists at the Central Bank of Korea verbally describe how they think depreciation pressure on the Korean Won transmit through financial markets, and highlight that "The depreciation of the won in turn sends negative signals about the Korean economy and can make banks' FX funding more difficult again." (Ryoo et al., 2013).

While anecdotal, these pieces of evidence reveal that the informational role of the exchange rate is often taken into consideration in the media and policy circles.[13] We formalized this channel in a general equilibrium model, and we now turn to study its implications for FX interventions and their communication.

3 Foreign exchange interventions

We now introduce the possibility for the central bank to intervene in the foreign exchange market by purchasing foreign-currency bonds $f_1^\star$ according to:

$$f_1^\star = \kappa_n n_1^\star + \kappa_a \bar{E}_0[a_1]. \tag{31}$$

We assume throughout that the intervention form (κ_a, κ_n) is known to agents while the actual quantity of foreign-currency bonds $f_1^\star$ purchased by the central bank may or may not be known, as we detail below.[14]

The assumption underlying equation (31) is that the central bank is able to observe aggregate financial flows, such as $n_1^\star$, and the exchange rate. This idea is consistent with the fact that many monetary authorities serve supervisory and regulatory roles over local financial institutions, providing them with information about aggregate patterns

[13] Gholampour and van Wincoop (2019) use Twitter data to compute a measure of investors' private information about the fundamentals driving the Euro-Dollar exchange rate, and use it in a structural estimation of the dispersed information model of Bacchetta and Wincoop (2006). One of their main findings is that Twitter data imply a sizable degree of dispersion in private information.

[14] In Appendix C, we outline the case in which FX interventions follow an exogenous process.

of bond holdings.[15] Observing financial flows, such as $n_1^\star$, and the exchange rate, the central bank in the model can back out the average expectation $\bar{E}_0[a_1]$.[16] The assumption that private agents know that the central bank responds to average beliefs implies that their higher-order belief biases influence their interpretation of the central bank's FX interventions as well. It is not necessary to specify whether the central bank is subject to behavioral biases at this stage. We make explicit assumptions about the central bank's beliefs when analyzing optimal FX intervention policy in Section 4.

In our model, FX interventions are allocative for two reasons. As in standard portfolio-balance models, FX interventions can affect the exchange rate, because they alter the balance-sheet position of financiers. The novelty of our model is that FX interventions may alter the information available to agents.

We consider two types of FX intervention communication policy: publicly announced and secretly conducted FX interventions. We define them below.

Public FX intervention. The central bank communicates the size of the intervention to the public, and thus $f_1^\star$ becomes common knowledge.

Secret FX intervention. The central bank does not reveal the size of the intervention to the public, and thus $f_1^\star$ is unobserved.

3.1 Public foreign exchange interventions

When the central bank communicates the volume of bond purchase $f_1^\star$, the intervention becomes a public signal about average expectations. To see this, rewrite eq. (31) as:

$$\frac{f_1^\star}{\kappa_a} = \bar{E}_0[a_1] + \frac{\kappa_n}{\kappa_a} n_1^\star. \tag{32}$$

Agents can now access two public signals, the exchange rate (26) and the FX intervention (32), which contain independent information about the same two components, $\bar{E}_0[a_1]$ and $n_1^\star$. As a result, agents are able to perfectly back out average expectations,

[15] In many emerging economies, in addition, monetary authorities mandate that they report details of their FX and bond transactions, such as counterparties and amounts, for regulatory purposes.

[16] One could alternatively express the FX intervention rule in terms of the actual productivity, $f_1^\star = \hat{\kappa}_n n_1^\star + \hat{\kappa}_a a_1$, with linear mapping between the two sets of parameters, that is $\hat{\kappa}_a = \kappa_a (1+\delta) \frac{\beta_v + \Lambda^2 \beta_n}{\beta_a + \beta_v + \Lambda^2 \beta_n}$ and $\hat{\kappa}_n = \kappa_n + \kappa_a \frac{\Lambda \beta_n}{\beta_a + \beta_v + \Lambda^2 \beta_n}$. However, since agents misperceive the mapping between average expectation and actual productivity, they also misperceive the mapping between $f_1^\star$ and productivity.

$\bar{E}_0[a_1]$. The sufficient statistic for all the public information about $\bar{E}_0[a_1]$ under public FX interventions can be formally defined as z^p, which follows:

$$z^p | a_1 \sim \mathcal{N}((1+\delta)a_1, \beta_z^{-1}), \qquad \text{with} \qquad \beta_z = \infty \tag{33}$$

If agents are rational ($\delta = 0$), they correctly understand the expectations mapping and therefore can perfectly back out productivity from average expectations. If $\delta > 0$, average beliefs exhibit extrapolation ($\bar{E}_0[a_1] = (1+\delta)a_1$) and agent misinterpret the public signal by δa_1, as we explain in Appendix G. Either way, under public communication, the intervention has a *signaling* effect that expands agents' information set by revealing average expectations.

Proposition 2 (Public FX intervention)**.** *Suppose that $f_1^\star$ is observable. The actual exchange rate follows*

$$q_0 = \underbrace{-\frac{\omega_2 - \Gamma\omega_1\kappa_a}{\Gamma\tilde{\theta}\omega_1 + \omega_3}}_{\tilde{\lambda}_a^p}(1+\delta)a_1 + \underbrace{\frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(1+\kappa_n)}_{\tilde{\lambda}_n^p} n_1^\star. \tag{34}$$

The parameters κ_n and κ_a do not directly affect the accuracy of public information. However, the combined informational content of the FX intervention and the exchange rate, summarized by z^p, perfectly reveals the average expectations $\bar{E}_0[a_1]$.

Proof. See Appendix E. □

By announcing the quantity of bonds purchased, the central bank reveals its information set, which, together with knowledge of the exchange rate, fully discloses average expectations. Consequently, the actual choices of κ_a and κ_n influence equilibrium exchange rates and overall allocation solely through a portfolio balance channel. The only exception is the special case of "exchange rate smoothing." Formally, $f_1^\star = \kappa_q q_0$ is a particular case of the general FX intervention rule (31) where the loadings of $f_1^\star$ on average expectations and noise trading match those of the exchange rate, q_0, up to a constant κ_q. In this case, as $f_1^\star$ is not independent from the other public signal, q_0, and thus it does not convey any additional information.

Discussion Note that the result that public interventions fully reveal average expectations is due to our assumption that the central bank observes and acts on average beliefs. We make this assumption to simplify the signal extraction problem and the exposition. If the central bank had not *superior* but more generally *different* information with respect to agents, then public interventions would still increase agents' information set, albeit only partially. Either way, transparent communication about FX interventions gives the private sector more information about productivity by revealing, at least in part, the central bank's information.

3.2 Secret foreign exchange interventions

Now consider the case in which the central bank does *not* reveal the size of FX interventions.[17] Substituting the central bank's reaction function (31) in the exchange rate equation (18), one obtains

$$q_0 = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(1+\kappa_n)n_1^\star - \frac{\omega_2 - \Gamma\omega_1\kappa_a}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\bar{E}_0 a_1. \tag{35}$$

Equation (35) already suggests that, when the exchange rate is the only source of public information, secret FX interventions can alter the informational content of the exchange rate by changing the equilibrium relationship between the exchange rate and the two shocks. We formally prove this result in the following proposition.

Proposition 3. *(Secret FX Interventions) Suppose the central bank adopts a secret FX intervention, i.e.,* $f_1^\star = \kappa_n n_1^\star + \kappa_a \bar{E}_0 a_1$ *and* $f_1^\star$ *is not directly observed. Then*

$$q_0 = \underbrace{-\frac{\omega_2 - \Gamma\omega_1\kappa_a}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\frac{\beta_v + \beta_z}{\beta_a + \beta_v + \beta_z}(1+\delta)}_{\tilde{\lambda}_a^s} a_1 + \underbrace{\frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(1+\kappa_n)\frac{\beta_v + \beta_z}{\beta_v}}_{\tilde{\lambda}_n^s} n_1^\star, \tag{36}$$

with $\beta_z \equiv \Lambda_s^2 \beta_n$ *and a unique* $\Lambda_s^2 \equiv \left(\frac{\tilde{\lambda}_a^s}{\tilde{\lambda}_n^s}\right)^2$ *implicitly defined by*

$$\Lambda_s^2 = \left(\frac{\omega_2 - \Gamma\omega_1\kappa_a}{\Gamma\omega_1(1+\kappa_n)}\right)^2 \frac{\beta_v^2}{(\beta_a + \beta_v + \Lambda_s^2\beta_n)^2}, \tag{37}$$

Under secret FX interventions, the relative informational content of the exchange rate and the overall posterior accuracy depend on the intervention rule via eq. (37)*. A rule*

[17] Inferring the size of interventions from publicly available data is hard. Central banks publish data about international reserves only infrequently, and factors such as valuation effects and investment income flows make international reserves a poor proxy of FX interventions (Adler et al., 2021). We also note that central banks maintain secrecy around interventions by imposing a confidentiality requirement on counterparties.

that strengthens the correlation between the exchange rate and productivity or weakens its correlation with noise-trading shocks increases the exchange rate informativeness.

Proof. See Appendix E. □

Proposition 3 frames our key positive result. Like in the public intervention case, FX interventions affects the equilibrium relationship between exchange rate and underlying shocks. Differently from the public communication case, however, FX interventions are unobserved and thus alter the informational content of the exchange rate.

Formally, now the public signal sufficient statistics, defined as z^s, is only comprised of exchange rate information, and follows:

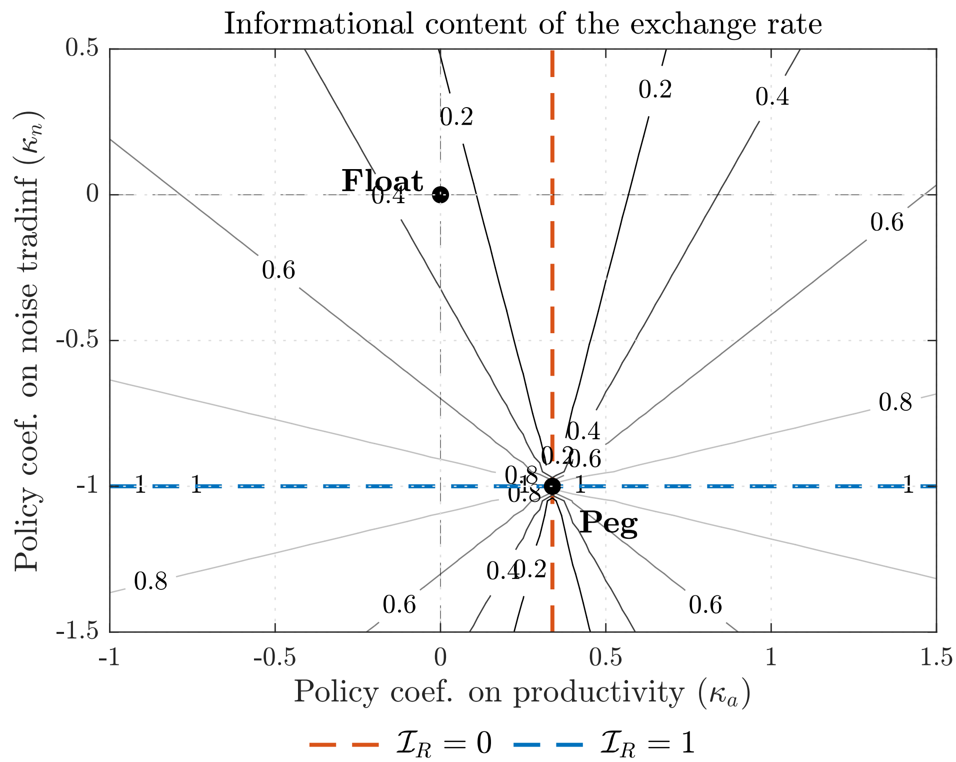
$$z^s|a_1 \equiv \frac{q_0}{\tilde{\lambda}_a^s}\bigg|a_1 \sim \mathcal{N}((1+\delta)a_1, \beta_z^{-1}) \tag{38}$$

where $\tilde{\lambda}_a^s$, $\tilde{\lambda}_n^s$ and $\beta_z = \Lambda_s \beta_n$ follow from equations (36)-(37). Differently from the public communication case, the precision of this public signal depends on policy via (36)-(37).

When the exchange rate is more closely tied to future productivity rather than noise-trading, it becomes a more reliable signal of future productivity. Through appropriate choice of FX intervention policy coefficients κ_a and κ_n, which are known to agents, the central bank can affect the equilibrium relationship between exchange rate and productivity, thereby changing its informational content.

Figure 3 displays the equilibrium informational content of the exchange rate for different configurations of the central bank's reaction function (κ_a, κ_n) under an illustrative calibration of the model. It is useful to highlight a number of interesting cases. First, secret intervention policy can make the exchange rate perfectly informative of average expectations if it completely offsets the noise trading variation in the exchange rate ($\kappa_n = -1$). By doing so, the central banks renders the exchange rate a perfectly informative signal of average expectations. Second, secret interventions can lead to an equilibrium in which the exchange rate is uninformative, by completely eliminating the effect of productivity shocks on the exchange rate (under $\delta = 0$, this requires $\kappa_a = {}^{\omega_2}/_{\Gamma\omega_1}$). Note that a special case of an uninformative equilibrium exchange rate is an *exchange rate peg* obtained when the secret FX policy enforces a constant exchange rate. Outside of these two limit cases, there is a spectrum of cases in which the exchange rate is an *imperfect* signal of future productivity, consistent with

Figure 3: Secret FX intervention and the informational context of the exchange rate



Notes: The figure reports values of the informational content of the exchange rate ($\mathcal{I}_R$) for different values of the central bank's reaction function (κ_a, κ_n) under secret FX intervention policy, for an illustrative calibration of the model under rational expectations (that is, $\delta = 0$). The rest of the parameters are set according to Table A.1.

a corresponding choice of its reaction function (κ_a, κ_n).[18]

Note that *freely floating* (*i.e.*, laissez-faire) is a particular case of imperfectly informative exchange rate with $\kappa_a = \kappa_n = 0$. Under free floating, exchange rate fluctuations reflect both productivity and noise-trading, and thus their informational content is limited by the relative amount of noise-trading in exchange rate fluctuations.

Finally, an intervention rule that only depends on the exchange rate, *i.e.* $f_1^\star = \kappa_q q_0$, a form of "exchange rate smoothing," does not alter its informational content. Intuitively, as the intervention depends only on publicly available information, smoothing is intrinsically a form of public intervention that does convey information that is in-

[18] Hassan et al. (2022) explore how exchange rate policy influences the riskiness of that country's currency, by altering the stochastic properties of the exchange rate, and derive the implications for optimal exchange rate policy. We also emphasize that FX policy affects the macroeconomic allocation by altering the stochastic properties of the exchange rate, yet through a distinct, complementary channel: the informativeness of the exchange rate.

dependent of the one contained in the exchange rate. Therefore, aside for the limit case of $\kappa_q = -\infty$, which corresponds to a peg, this intervention lies on the same iso-information curve of the free floating case.

In summary, we emphasize that secret FX interventions uniquely enable a central bank to "manage" the informativeness of the exchange rate. Next, we examine whether the central bank may find it preferable to intervene publicly or secretly.

4 Welfare and optimal policies

We now turn to the normative implications of FX interventions when agents learn from the exchange rate. Section 4.1 introduces the welfare function and describes the sources of inefficiencies relative to the frictionless allocation. Section 4.2 outlines the policy problem faced by central banks in their choice of FX intervention size and communication, and highlights the trade-offs involved in implementing optimal policies.

4.1 Welfare and sources of inefficiency

To characterize optimal policy we need a welfare criterion. Let the welfare of the small open economy be defined as the household's expected utility (1) under the objective (*i.e.*, rational) expectations.[19,20] We evaluate welfare by taking a quadratic approximation of the welfare function around the first-best allocation, that is the allocation that would arise under Full Information Rational Expectation (FIRE, $\delta = 0$ and $\beta_v = \infty$) absent any international asset market segmentation ($\Gamma = 0$).

Welfare function *Welfare in deviation from the first best ($\widetilde{\mathbb{W}} = \mathbb{W} - \overline{\mathbb{W}}$) can be written as:*

$$\widetilde{\mathbb{W}} = -\left[\Omega_1 \operatorname{var}(q_0^i - \bar{q}_0) + \Omega_2 \operatorname{var}(E_0^i a_1 - a_1)\right] \tag{39}$$

where $\Omega_1 > 0$ and $\Omega_2 \geq 0$ are convolutions of parameters independent of policy, reported in Appendix D.

[19]Equation (1) is the island-level expected utility, which coincide with aggregate expected utility since all islands are ex-ante identical.

[20]This choice follows Gabaix (2020) and most of the behavioral literature, which thinks of behavioral agents as using heuristics but experiencing utility from consumption like rational agents.

Equation (39) reveals that there are two margins that are distorted in our small open economy. First, welfare losses arise from inefficient fluctuations in external borrowing, reflected in deviations of the island-level relative price of tradable goods from its first best, $q_0^i - \bar{q}_0$, that determines domestic absorption. Second, welfare losses arise from misalignment in expectations of future productivity, that is forecast errors, $E_0^i a_1 - a_1$. Misaligned expectations emerge when departing from the FIRE assumption. Under FIRE, expectations would instead be aligned to future realizations and welfare losses would solely reflect expected excess currency returns due to intermediation frictions (as, for example, in Itskhoki and Mukhin, 2022).[21]

Misaligned expectations of future productivity are detrimental because they lead to a suboptimal allocation of the final good into consumption and investment, conditional on a given level of external borrowing.[22] Optimistic expectations of future tradable-sector productivity influence the consumption/investment allocation through both wealth and substitution effects, primarily shaped by the intertemporal elasticity of substitution, $1/\sigma$, and the price elasticity of tradables, θ.

Interestingly, wealth and substitution effects cancel each other out when $\sigma = \theta = 1$ (Cole and Obstfeld, 1991; Corsetti et al., 2008). Under the Cole-Obstfeld calibration, for given domestic absorption, productivity expectations do not alter the consumption/investment allocation, and thus drop out of the welfare function (i.e., $\Omega_2 = 0$). Under this knife-edge case, public or secret FX intervention policy can achieve the first-best allocation by appropriate intervention in the foreign exchange market that eliminate the external borrowing gap, $q_0^i - \bar{q}_0$. Outside of this knife-edge case, inefficient fluctuations in external borrowing and forecast errors have independent welfare effects, giving rise to a non-trivial trade off.

4.2 Optimal foreign exchange interventions

The optimal FX intervention policy is described by the values of (κ_a, κ_n) in the central bank's reaction function (31) that maximize welfare (39) under either public or secret interventions. The communication choice, public or secret, effectively consists in choosing the type of public signal z that agents have access to. Under a public communication, agents access public signal z^p in equation (33); under secret interventions,

[21] That is, under FIRE, $E_0^i a_1 - a_1 = 0$ and $q_0^i - \bar{q}_0 = \frac{\omega_1}{\omega_3}(r_0 - \Delta q_1)$.

[22] In an alternative model without capital accumulation but with endogenous non-tradable goods, misaligned expectations would imply a suboptimal choice of the sum of tradable and non-tradable goods.

agents access the public signal z^s in equation (38).

Planner's problem *The central bank's problem is formally defined as:*

$$\max_{\kappa_a,\kappa_n,z} \qquad \widetilde{\mathbb{W}} = -\left[\Omega_1 \operatorname{var}(q_0^i - \bar{q}_0) + \Omega_2 \operatorname{var}(E_0^i a_1 - a_1)\right]$$

$$\text{subject to:} \qquad q_0^i - \bar{q}_0 = \frac{\omega_1}{\omega_3}(r_0 - \Delta E_0^i q_1) - \frac{\omega_2}{\omega_3}(E_0^i a_1 - a_1); \tag{40}$$

$$E_0^i a_1 \equiv E[a_1 | v^i, z] = \frac{(1+\delta)\beta_v v^i + \beta_z z}{\beta_v + \beta_z + \beta_a}; \qquad \textit{with } z = \{z^p, z^s\}; \tag{41}$$

$$f_1^\star = \kappa_a \bar{E}_0 a_1 + \kappa_n n_1^\star, \tag{42}$$

where $\bar{E}_0 a_1 = \int E_0^i a_1 \mathrm{d}i$ *are average expectations, and expected excess (home) currency returns are:*

$$(r_0 - \Delta E_0^i q_1) = \underbrace{\frac{\Gamma}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\left(\tilde{\theta}\omega_2 \bar{E}_0 a_1 + \omega_3(n_1^\star + f_1^\star)\right)}_{(r_0 - \Delta \bar{E}_0 q_1)} - (E_0^i - \bar{E}_0) q_1; \tag{43}$$

$$\text{with: } (E_0^i - \bar{E}_0) q_1 = \frac{1-\gamma}{\theta}\left[(E_0^i - \bar{E}_0) a_1 + \frac{\alpha(\omega_1 - \omega_2)\Gamma}{(1-\alpha)\omega_2}(E_0^i - \bar{E}_0)(n_1^\star + f_1^\star)\right].$$

A key aspect of the planner's problem lies in the dual effects of FX interventions. First, these interventions influence the flows in the foreign exchange market, which financiers must intermediate, thereby affecting both the equilibrium exchange rate and expected excess currency returns. These effects depend on the choice of (κ_a, κ_n) and operate through standard portfolio balance channels (see, *e.g.* Gabaix and Maggiori, 2015). Second, interventions have informational effects that vary based on whether they are publicly announced or conducted in secret, shaping how the private sector forms expectations. This is a distinctive feature of our model, described in details in Section 3. On the one hand, publicly announced intervention reveal average expectations, regardless of the specific choice of (κ_a, κ_n). Agents rely solely on the public signal, which is a combination of the exchange rate and the intervention itself, though it is biased and perceived as infinitely precise, as outlined in Proposition 2. On the other hand, secret interventions affect the informational content of the exchange rate, which varies with the specific (κ_a, κ_n) chosen, as outlined in Proposition 3. In this case, agents combine both private and public signals—the equilibrium exchange rate—based on their relative informational content.

Furthermore, welfare can be expressed solely as a function of the choice of policy coefficients (κ_a, κ_n), communication format—public (z^p) or secret (z^s)—and the precision of the ensuing public signal, denoted by β_z. That is, welfare can be expressed as $\widetilde{\mathbb{W}}(\kappa_a, \kappa_n, \beta_z)$, with a closed-form expression for this function provided in Appendix D.1.[23] The planner's problem can thus be reformulated as follows:

Auxiliary planner's problem

$$\max_{\kappa_a, \kappa_n, \beta_z} \qquad \widetilde{\mathbb{W}}(\kappa_a, \kappa_n, \beta_z) = -\left[\Omega_1 \underbrace{\tilde{q}(\kappa_a, \kappa_n, \beta_z)}_{\text{var}(q_0^i - \bar{q}_0)} + \Omega_2 \underbrace{\tilde{a}(\beta_z)}_{\text{var}(E_0^i a_1 - a_1)}\right] \tag{44}$$

$$\text{subject to:} \qquad \begin{cases} \beta_z = \infty, & \text{if } z = z^p \\ (1+\kappa_n)^2 = \frac{\beta_n}{\beta_z}\left(\frac{\omega_2 - \Gamma\omega_1\kappa_a}{\Gamma\omega_1}\right)^2 \frac{\beta_v^2}{(\beta_a + \beta_v + \beta_z)^2}, & \text{if } z = z^s \end{cases}$$

where closed-form expressions for $\tilde{q}(\kappa_a, \kappa_n, \beta_z)$ and $\tilde{a}(\beta_z)$ are provided in Appendix D.1.

This auxiliary welfare function makes clear that (κ_a, κ_n) do not affect the variance of forecast errors for a given β_z. Intuitively, different combinations of (κ_a, κ_n) that deliver the same public signal precision β_z imply the same information structure and variance of productivity forecast errors. Moreover, the planner's constraints in the auxiliary problem capture the distinct informational effects of public and secret interventions discussed earlier. Public interventions provide an infinitely precise signal of average productivity expectations, while the precision of the public signal under secret interventions is determined by the sensitivity of the exchange rate to productivity, which in turn depends on κ_a and κ_n, as established in Proposition 3.

In what follows, we show that it is impossible to eliminate both sources of welfare loss in (39), and thus attain the first best, except in the specific case of rational expectations where a "divine coincidence" occurs. In the general case, publicly announced policy cannot use (κ_a, κ_n) to influence the variance of forecast errors, and thus focuses its interventions on mitigating inefficient fluctuations in the foreign exchange market.

[23] The auxiliary welfare function, denoted by $\widetilde{\mathbb{W}}(\kappa_a, \kappa_n, \beta_z)$, is also influenced by the covariance between noise trading shocks and the noise in the public signal. Under secret and public communication regimes, this covariance term can be expressed as a distinct function of $(\kappa_a, \kappa_n, \beta_z)$. We thus write the welfare function as $\widetilde{\mathbb{W}}(\kappa_a, \kappa_n, \beta_z)$, with a slight abuse of notation.

In contrast, secret interventions can leverage the informational content of the exchange rate to balance the trade-off between inefficient fluctuations in external borrowing and productivity forecast errors (that imply misallocations of consumption and investment).

We first characterize the optimal choice of publicly announced FX interventions.

Proposition 4. *Under publicly announced interventions, the optimal policy is characterized by*

$$\kappa_n^p = -1; \qquad \kappa_a^p = -\frac{\omega_2}{\omega_3}\frac{\tilde{\theta}\Gamma\omega_1 - \delta\omega_3}{\Gamma\omega_1(1+\delta)}.$$

At the optimum:

$$\text{var}(q_0^i - \bar{q}_0) = 0; \qquad \text{var}(E_0^i a_1 - a_1) = \frac{\delta^2}{\beta_a} \geq 0, \qquad \widetilde{\mathbb{W}}^p = -\Omega_2 \frac{\delta^2}{\beta_a} \leq 0.$$

Proof. See Appendix E. □

When interventions are publicly announced, the actual choice of policy parameters κ_a and κ_n does not influence the accuracy of public information, or expectations more generally, but only the flows in the FX market. For this reason, under optimal public communication the central bank completely offsets noise-trading fluctuations in exchange rates (*i.e.*, $\kappa_n^p = -1$). Likewise, the response to productivity shocks, governed by κ_a^s, eliminates inefficient fluctuations in external borrowing: expected excess currency returns optimally counteract the effects of misaligned expectations on external borrowing, as per equation (40). At the optimum, however, because average productivity expectations remain misaligned, the allocation of domestic absorption into consumption and investment is distorted and results in a welfare loss.[24]

We now turn to examining the welfare properties of secretly conducted FX interventions. A full analytical characterization of the optimal secret intervention policy is made difficult by the fact that now the policy parameters also influence the informativeness of the exchange rate, and thus the properties of productivity expectations. Nevertheless, we analytically establish a number of important results. To this end, we break down the problem of optimal secret interventions in two steps:

Two-step approach (Secret foreign exchange interventions)

[24] As outlined above, the $\sigma = \theta = 1$ calibration represents the exception since misaligned productivity expectations do not have independent welfare effects.

Step 1. *Choose (κ_a, κ_n) to maximize welfare $\widetilde{\mathbb{W}}(\kappa_a, \kappa_n, \beta_z)$ subject to $\beta_z = \bar{\beta}_z$ and $(1+\kappa_n)^2 = \frac{\beta_n}{\bar{\beta}_z}\left(\frac{\omega_2 - \Gamma\omega_1\kappa_a}{\Gamma\omega_1}\right)^2 \frac{\beta_v^2}{(\beta_a+\beta_v+\bar{\beta}_z)^2}$. Define $(\kappa_a^s(\bar{\beta}_z), \kappa_n^s(\bar{\beta}_z))$ as the solution to this step of the problem.*

Step 2. *Choose $\bar{\beta}_z$ to maximize welfare $\widetilde{\mathbb{W}}(\kappa_a^s(\bar{\beta}_z), \kappa_n^s(\bar{\beta}_z), \bar{\beta}_z)$. Define β_z^s as the solution to this step of the problem.*

Because (κ_a, κ_n) do not affect the variance of productivity forecast errors for a given $\beta_z = \bar{\beta}_z$, in the first step the planner effectively chooses the combination of (κ_a, κ_n) that minimizes inefficient fluctuations in external borrowing subject to maintaining a given informational content of the exchange rate. In the second step of the problem, the planner then optimizes over the informational content of the exchange rate, β_z. The second step is what distinguishes the secret intervention from the public intervention. As established in Proposition 3, secret conduct of FX interventions allows the planner to leverage the informational content of the exchange rate and thus influence the variance of productivity forecast errors, effectively altering the agents' information structure. The next proposition analytically establishes that, under certain conditions, secret policy can achieve a superior allocation than public policy.

Proposition 5. *Secret policy can always attain the level of welfare of the optimal public policy.*

In addition, if $\delta > \hat{\delta}$, where $\hat{\delta}$ is defined in Appendix E, optimal secret FX intervention policy attains a higher level of welfare than optimal public intervention policy.

The optimum can be attained with:

$$\kappa_a^s(\beta_z^s) = \underbrace{-\frac{\omega_2}{\omega_3}\frac{\tilde{\theta}\Gamma\omega_1 - \delta\omega_3}{\Gamma\omega_1(1+\delta)}}_{\kappa_a^p} - \frac{\omega_2}{\omega_3}\frac{(\Gamma\tilde{\theta}\omega_1 + \omega_3)}{\Gamma\omega_1(1+\delta)}\frac{\beta_a[(1+\delta)^2\beta_z^s - (\beta_v + \beta_z^s)]}{(\beta_a + (1+\delta)^2\beta_z^s)(\beta_v + \beta_z^s)} \tag{45}$$

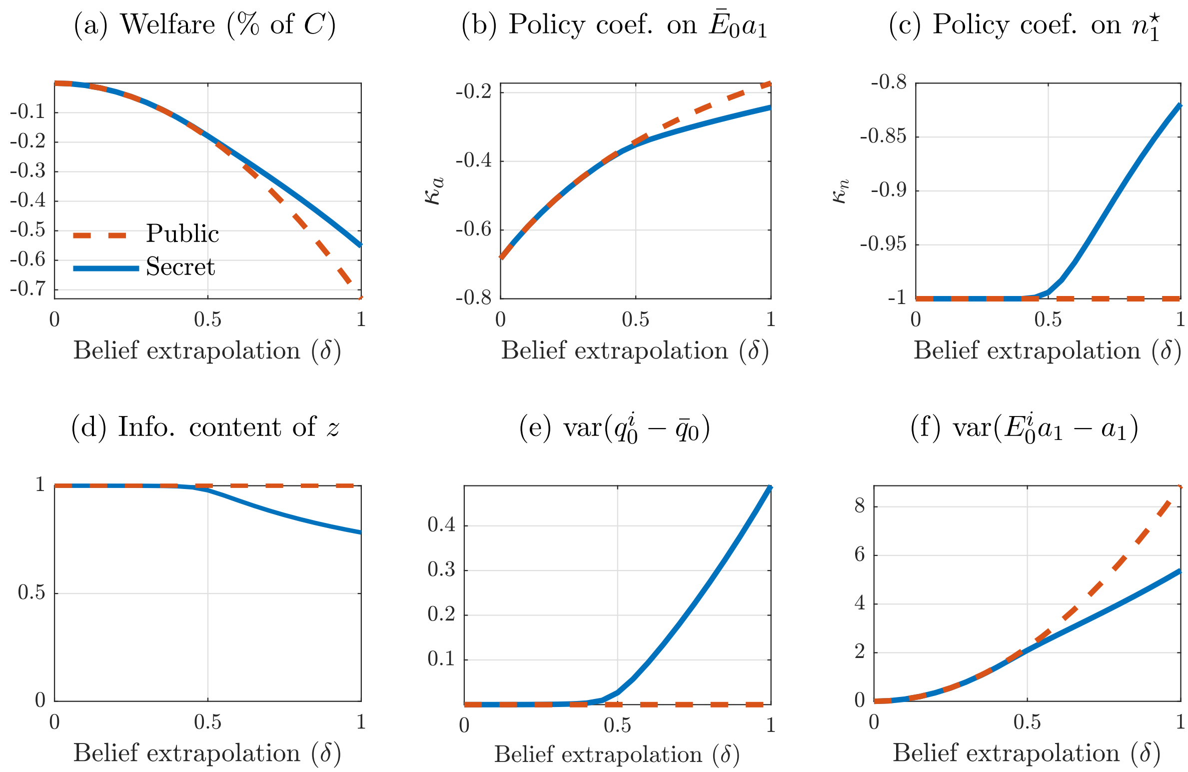
$$\kappa_n^s(\beta_z^s) = \underbrace{-1}_{\kappa_n^p} + \frac{\omega_2(\Gamma\tilde{\theta}\omega_1 + \omega_3)}{\Gamma\omega_1\omega_3}\frac{(1+\delta)\beta_z^s\beta_v}{(\beta_a + (1+\delta)^2\beta_z^s)(\beta_v + \beta_z^s)}\left|\sqrt{\frac{\beta_n}{\beta_z^s}}\right| \tag{46}$$

and β_z^s solves

$$\frac{\partial\widetilde{\mathbb{W}}(\kappa_a^s(\beta_z^s), \kappa_n^s(\beta_z^s), \beta_z^s)}{\partial\beta_z^s} = 0 \implies -\frac{\Omega_1}{\Omega_2} = \frac{\partial(\text{var}(E_0^i a_1 - a_1))/\partial\beta_z^s}{\partial(\text{var}(q_0^i - \bar{q}_0))/\partial\beta_z^s} \tag{47}$$

Proof. See Appendix E. □

Figure 4: Outcomes under optimal policies



Notes: The figure reports values of different variables under optimal FX intervention policies for different levels of belief extrapolation (δ). The rest of parameters are set according to Table A.1.

Proposition 5 provides several key insights, which we will now explore in more detail, while Figure 4 aids in visualizing the characteristics of both the optimal secret and public intervention policies under an illustrative calibration of the model.

A first insight of Proposition 5 is that secret FX intervention policy can always implement the allocation of the *optimal* public FX intervention policy. It can do so by setting $\kappa_a = \kappa_a^p$ and $\kappa_n = \kappa_n^p$, where (κ_a^p, κ_n^p) are the policy coefficients chosen by the planner under public communication (see Proposition 4). By fully eliminating noise-trading fluctuations ($\kappa_n = \kappa_n^p = -1$), the equilibrium exchange rate reflects only productivity, regardless of whether agents observe the quantity of bonds purchased by the central bank ($\beta_z = \infty$). In turn, by setting $\kappa_a = \kappa_a^p$, the secret policy can also replicate its overall allocation, and thus attain the same level of welfare.

The key insight of Proposition 5 is that a secret FX intervention policy can yield a *higher* level of welfare compared to the *optimal* public FX intervention policy. Additionally, Proposition 5 makes clear that the policy maker achieves a superior allocation over public policy only if the degree of overreaction is sufficiently strong. To understand this result, we introduce the following proposition, establishing that, when overreaction is sufficiently strong, the policy maker can reduce inefficient fluctuations in productivity expectations by making the exchange rate *less* informative.

Proposition 6. *If* $\delta \leq \bar{\delta}$ *then* $\frac{\partial \operatorname{var}(E_0^i a_1 - a_1)}{\partial \beta_z} < 0$. *If instead* $\delta > \bar{\delta}$, $var(E_0^i a_1 - a_1)$ *is non-monotonic in* β_z. *It reaches its minimum at* $\hat{\beta}_z \equiv (\beta_a + \beta_v) \frac{1-2(1+\delta)}{1-2\delta(1+\delta)}$ *and* $\frac{\partial \operatorname{var}(E_0^i a_1 - a_1)}{\partial \beta_z} < 0$ *for* $\beta_z \in (\hat{\beta}_z, \infty)$.

Proof. See Appendix E. □

Recall that in our model expectations are imperfect due to both noisy private signals and to extrapolative beliefs. These frictions lead to underreaction and overreaction, respectively. When extrapolation is low, the marginal benefit of higher coordination on a public signal (even if biased) is always greater than the cost of beliefs being more anchored to the prior or dispersed due to idiosyncratic noise. When extrapolation is stronger this is no longer the case. Minimizing the variance of the forecast errors requires balancing the (now higher) costs of extrapolative beliefs with the benefits of moving expectations away from the prior and from noisy private signal. This optimal balance is achieved at a finite $\hat{\beta}_z$. For levels of public signal precision above this value, lowering the informational content of the public signal reduces, rather than increases, the variance of productivity forecast errors. Our paper formalizes thus a new setting in which public information can be socially harmful, namely in presence of extrapolation of endogenous public signals.[25]

Recall now the two-step auxiliary problem, whereby the planner first chooses (κ_a, κ_n) for given $\beta_z = \bar{\beta}_z$ and then it optimizes over $\bar{\beta}_z$. For any given information structure encoded in $\bar{\beta}_z$, we learnt that eliminating inefficient fluctuations in external borrowing maximizes allocative efficiency. Yet, when overreaction is sufficiently strong, optimal secret policy deviates from this benchmark: it trades-off less allocative efficiency, or more inefficient external borrowing, for the sake of limiting the amount of mis-learning through exchange rate. Optimal policy thus chooses the informativeness of the exchange rate such that the welfare benefit of making the exchange rate marginally less informative (thus taming overreaction) exactly equals its marginal welfare cost, *i.e.*,

[25] The threshold $\bar{\delta} \approx 0.35$ above which more information increases the variance of forecast errors is below the empirical estimates of δ. Estimates from forecast data are 0.91 for credit spreads (Bordalo et al., 2018), 1.22 for stock prices (Bordalo et al., 2019) and 0.53 on average for several macroeconomic and financial variables (Bordalo et al., 2020). Estimates from macroeconomic models range from around 0.75 (L'Huillier et al., 2024; Na and Yoo, 2024) to around 2 (Bianchi et al., 2024).

the cost of inefficient external borrowing. We stress that only the policymaker that conducts secret interventions can exploit this trade off, and thus achieve a higher level of welfare than the public intervention policy.

Discussion An interesting implication of our analysis is that whether exchange rates reflect the economy's fundamental or noise is an equilibrium outcome that depends on the design and communication of FX interventions. In our model, if public interventions are optimally designed, they should, *ceteris paribus*, result in an exchange rate that is closely tied to future economic conditions. In contrast, optimally secret interventions can lead to an exchange rate that is partly influenced by noise trading. Letting some degree of noise trading is *the* mean to reduce the informational content of the exchange rate. Thus, contrary to models solved under FIRE (see, for example, Itskhoki and Mukhin, 2022), eliminating noise-trading fluctuations is not always optimal.

Importantly, even though optimal secret interventions preserve some noise-trading fluctuations into the exchange rate, the resulting exchange rate volatility need not be higher than under the public communication policy. A less informative exchange rate mitigates the impact of expectations overreaction, which can reduce the overall volatility of macroeconomic aggregates and exchange rates due to productivity shocks.[26]

Finally, both secret and public optimal FX intervention policies involve intervening against inflows in the FX market to some extent, aligning with the empirical observation of "systematic managed floating." This practice sees central banks consistently responding to changes in total market pressure, with part of the adjustment reflected in the exchange rate and the remainder absorbed through changes in foreign exchange reserves (Frankel, 2019).

5 Conclusions

The debate on foreign exchange interventions presents two seemingly conflicting features. The consensus view among policymakers—as shown in surveys of central bankers—suggests that FX interventions primarily influence markets by shaping expectations through a signaling channel (Patel and Cavallino, 2019). At the same time, somewhat paradoxically, most intervention activities in the FX market have historically been, and

[26] Under the calibration in Table A.1, the optimal secret intervention policy results in lower exchange rate volatility compared to the optimal public intervention policy.

continue to be, largely secret. This paper demonstrates that these observations can co-exist without contradiction. Our formal analysis is based on two intuitive premises: (i) exchange rates serve an informational role by aggregating beliefs about economic fundamentals, and their informational content is influenced by how exchange rate policy is conducted; and (ii) households and financial market participants tend to overreact to news embedded in exchange rate movements. Together, these elements make exchange rate policy a complex task with non-trivial trade-offs. In particular, the tendency for expectations to overreact naturally implies that central banks should exercise caution in using publicly announced interventions—precisely *because* of their signaling effects. In addition, it may be optimal for the central bank to influence the weight that the private sector places on exchange rate information by secretly intervening in the market, and preserving some amount of noise in exchange rate fluctuations.

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Appendix

A Derivations of the small open economy model

A.1 Financiers' demand for foreign currency bonds

We follow Fanelli and Straub (2021) and assume that there exists a continuum of risk-neutral financiers, labeled by $j \in [0, \infty)$, in each island i. Financiers also hold a zero-capital portfolio in home and foreign bonds denoted $(d^i_{j,1}, {d^i_{j,1}}^\star)$. Financier's investment decisions are subject to two important restrictions. First, each intermediary is subject to a net open position limit of size $D > 0$. Second, intermediaries face heterogeneous participation costs, as in Alvarez et al. (2009). In particular, each intermediary j active in the foreign bond market at time t is obliged to pay a participation cost of exactly j per unit of foreign currency invested.[27]

Putting these ingredients together, intermediary j on island i chooses ${d^i_{j,1}}^\star$ that solves

$$\max_{\frac{{d^i_{j,1}}^\star}{R^\star_0} \in [-D, D]} \frac{{d^i_{j,1}}^\star}{R^\star_0} E^i_0\left(\tilde{R}^\star_1\right) - j\left|\frac{{d^i_{j,1}}^\star}{R^\star_0}\right|,$$

where $\tilde{R}^\star_1 \equiv R^\star_0 - R_0 \frac{\mathcal{S}_0}{\mathcal{S}_1}$ is the return on one foreign-currency unit holding expressed in foreign currency and E^i_0 is the same expectation operator as the island-i household's.

Intermediary j's expected cash flow conditional on investing is $D\left|E^i_0\left(\tilde{R}^\star_1\right)\right|$ while participation costs are jD. Thus, investing is optimal for all intermediaries $j \in [0, \bar{j}]$, with the marginal active intermediary $\bar{j}$ given by $\bar{j} = \left|E^i_0\left(\tilde{R_1}^\star\right)\right|$. The aggregate investment volume is then

$$\frac{{D^i_1}^\star}{R^\star_0} = \bar{j} D \operatorname{sign}\left\{E^i_0\left(\tilde{R_1}^\star\right)\right\}.$$

Defining $\hat{\Gamma} \equiv D^{-1}$ and substituting out $\bar{j}$, we obtain the total demand for foreign-currency bonds on island i, ${D^i_1}^\star = \int {d^i_{j,1}}^\star \mathrm{d}j$:

$$\frac{{D^i_1}^\star}{R^\star_0} = \frac{1}{\hat{\Gamma}} E^i_0\left(R^\star_0 - R_0 \frac{\mathcal{S}_0}{\mathcal{S}_1}\right). \tag{A.1}$$

[27] We also assume that participation costs constitute transfers to households in the home island economy. Thus, no extra cost terms enter the household's budget constraint.

which is equation (12) in the text.

A.2 Island equilibrium

The log-linearized version of the household's optimality conditions (7), (9), and (10) are:

$$\sigma(E_0^i c_1^i - c_0^i) = r_0 - (E_0^i p_1^i - p_0^i), \tag{A.2}$$

$$(1-\alpha)k_1^i = E_0^i(s_1 - p_1^i) + E_0^i a_1 - r_0 + (E_0^i p_1^i - p_0^i), \tag{A.3}$$

$$q_t^i \equiv s_t - p_t^i = -\frac{1-\gamma}{\theta} y_{T,t}^i, \tag{A.4}$$

Island-i budget in (11) can be combined and loglinearized as:[28]

$$\frac{1+\phi}{\beta} y_{T,0}^i = a_1 + \alpha k_1^i - y_{T,1}^i \tag{A.5}$$

where $\phi = \beta\alpha\gamma$. The final good aggregator in (3) yields:

$$\frac{1}{1+\phi} c_0^i + \frac{\phi}{1+\phi} k_1^i = \gamma y_{T,0}^i \qquad c_1^i = \gamma y_{T,1}^i. \tag{A.6}$$

The log-linear optimality condition of financiers (13):

$$\Gamma \int d_1^{i\star} \, \mathrm{d}i = \bar{E}_0 s_1 - s_0 - (r_0 - r_0^\star) \tag{A.7}$$

where $d_1^{i\star} \equiv \frac{\mathrm{d}D_1^{i\star}}{Y_{T,1}^{ss}}$ and $\Gamma \equiv \hat{\Gamma} \cdot Y_{T,1}^{ss} \cdot \beta^2$.

Finally, bond market clearing, (17) implies:

$$\int d_1^{i\star} \, \mathrm{d}i = -\frac{1+\phi}{\beta} \int y_{T,0}^i \, \mathrm{d}i - n_1^\star - f_1^\star \tag{A.8}$$

We also normalize the average price of the consumption basket, such that $\int p_t^i \, \mathrm{d}i = 0$, for $t = [0, 1]$. This implies that the aggregate real exchange rate equals the nominal exchange rate, that is:

$$q_t = s_t$$

[28] The log-linearized budget constraint is not affected by the size of the tax on financiers and noise traders' carry-trade profits are taxed, nor on how they are distributed across islands, as these represent second-order terms.

A.3 Equilibrium exchange rate

Consider the following set of island-i equilibrium equations:

$$s_0 - p_0^i = -\frac{1-\gamma}{\theta} y_{T,0}^i \tag{A.9}$$

$$s_1 - p_1^i = -\frac{1-\gamma}{\theta} y_{T,1}^i \tag{A.10}$$

$$r_0 - (E_0^i p_1^i - p_0^i) = \sigma\gamma E_0^i y_{T,1}^i - (\sigma\gamma)(1+\phi) y_{T,0}^i + \sigma\phi k_1^i \tag{A.11}$$

$$\frac{(1+\phi)}{\beta} y_{T,0}^i = a_1 + \alpha k_1^i - y_{T,1}^i \tag{A.12}$$

$$k_1^i = \frac{1}{1-\alpha} E_0^i \left(s_1 - p_1^i\right) + \frac{1}{1-\alpha} E_0^i a_1 - \frac{1}{1-\alpha}\left(r_0 - (E_0^i p_1^i - p_0^i)\right) \tag{A.13}$$

where eqs. (A.9) and (A.9) represent island-i's demand for tradables in period 0 and 1, respectively (c.f. (A.4)); eq. (A.11) is obtained by combining the Euler equation and island-i resource constraint and (cf. (A.2) and (A.6)); eq (A.12) is island-i budget constraint (cf. (A.5)); (A.13) is island-i's demand for capital (c.f. (A.3))

Using eqs. (A.9)-(A.13), one can express island-i price level and tradable demand as:

$$p_0^i = s_0 - \frac{\omega_1}{\omega_3}\left(r_0 - (E_0^i s_1 - s_0)\right) + \frac{\omega_2}{\omega_3} E_0^i a_1 \tag{A.14}$$

$$y_{T,0}^i = -\frac{\theta}{1-\gamma}\frac{\omega_1}{\omega_3}\left(r_0 - (E_0^i s_1 - s_0)\right) + \frac{\theta}{1-\gamma}\frac{\omega_2}{\omega_3} E_0^i a_1 \tag{A.15}$$

where $\omega_1 > 0, \omega_2 > 0, \omega_3 > 0$ are all convolution of parameters:

$$\omega_1 \equiv [\theta\sigma\alpha\gamma(1+\beta) + (1-\alpha)\theta + (1-\gamma)\alpha] \qquad \omega_2 \equiv [(1-\gamma) + \sigma\gamma\theta(1+\alpha\beta)] \tag{A.16}$$

$$\omega_3 \equiv \frac{\theta(1-\alpha)(1+\phi)}{\beta(1-\gamma)}\left[\sigma\gamma\theta(1+\beta) + (1-\gamma)\right] + \theta\sigma\alpha\gamma(1+\beta) + (1-\alpha)\theta + (1-\gamma)\alpha$$

Sum (A.14) across islands and use $\int p_t^i \, \mathrm{d}i = 0$, for $t = [0,1]$, to express the small open economy (real and nominal) exchange rate as a function of average-expected excess currency returns and average-expected productivity:

$$q_0 = s_0 = \frac{\omega_1}{\omega_3}\left(r_0 - (\bar{E}_0 s_1 - s_0)\right) - \frac{\omega_2}{\omega_3}\bar{E}_0 a_1 \tag{A.17}$$

Consider now the modified UIP condition for the small open economy bond, along with the market clearing condition in the financial market:

$$r_0 = \bar{E}_0 s_1 - s_0 - \Gamma\left(\int d_i^\star \, \mathrm{d}i\right) \qquad \text{w/} \qquad d_1^\star = -n_1^\star - f_1^\star - \frac{(1+\phi)}{\beta}\int y_{T,0}^i \, \mathrm{d}i, \tag{A.18}$$

and note that, by using (A.15):

$$\int y_{T,0}^i \, \mathrm{d}i = -\frac{\theta}{1-\gamma}\frac{\omega_1}{\omega_3}\left(r_0 - (\bar{E}_0 s_1 - s_0)\right) + \frac{\theta}{1-\gamma}\frac{\omega_2}{\omega_3}\bar{E}_0 a_1 \tag{A.19}$$

Using (A.18) and (A.19), one can express the average expectation of aggregate excess home-currency returns as:

$$r_0 - \left(\bar{E}_0 s_1 - s_0\right) = \frac{\Gamma\tilde{\theta}\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\bar{E}_0 a_1 + \frac{\Gamma\omega_3}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(n_1^\star + f_1^\star) \tag{A.20}$$

where $\tilde{\theta} \equiv \frac{(1+\alpha\beta\gamma)\theta}{\beta(1-\gamma)} > 0$. Use (A.20) in (A.17), we obtain the aggregate real exchange rate:

$$q_0 = s_0 = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(n_1^\star + f_1^\star) - \frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\bar{E}_0 a_1, \tag{A.21}$$

which is eq. (18) in the paper.

A.3.1 Expression for $E_0^i q_1 - \bar{E}_0 q_1$

It is useful to express $E_0^i q_1 - \bar{E}_0 q_1$ as a function of shocks and expectations thereof. First recall that:

$$k_1^i = \frac{1}{1-\alpha}\left[\left(\frac{\omega_1}{\omega_3} - 1\right)\left(r_0 - (E_0^i s_1 - s_0)\right) - \left(\frac{\omega_2}{\omega_3} - 1\right)E_0^i a_1\right] \tag{A.22}$$

$$y_{T,1}^i = a_1 + \alpha k_1^i - \frac{1+\phi}{\beta}y_{T,0}^i \tag{A.23}$$

$$y_{T,0}^i = -\frac{\theta}{1-\gamma}\left[\frac{\omega_1}{\omega_3}\left(r_0 - (E_0^i s_1 - s_0)\right) - \frac{\omega_2}{\omega_3}E_0^i a_1\right] \tag{A.24}$$

$$s_1 - p_1^i = \frac{1-\gamma}{\theta}y_{T,1}^i \tag{A.25}$$

Use equations (A.22)-(A.25):

$$s_1 - p_1^i = \frac{1-\gamma}{\theta}a_1 + \left[\frac{1-\gamma}{\theta}\frac{\alpha}{1-\alpha}\left(\frac{\omega_1}{\omega_3}-1\right) + \frac{1+\phi}{\beta}\frac{\omega_1}{\omega_3}\right]\big(r_0 - (E_0^i s_1 - s_0)\big) + \\ - \left[\frac{1-\gamma}{\theta}\frac{\alpha}{1-\alpha}\left(\frac{\omega_2}{\omega_3}-1\right) + \frac{1+\phi}{\beta}\frac{\omega_2}{\omega_3}\right] E_0^i a_1 \tag{A.26}$$

The aggregate version of eq. (A.26) implies:

$$q_1 = s_1 = \frac{1-\gamma}{\theta}a_1 + \left[\frac{1-\gamma}{\theta}\frac{\alpha}{1-\alpha}\left(\frac{\omega_1}{\omega_3}-1\right) + \frac{1+\phi}{\beta}\frac{\omega_1}{\omega_3}\right]\big(r_0 - (\bar{E}_0 s_1 - s_0)\big) + \\ - \left[\frac{1-\gamma}{\theta}\frac{\alpha}{1-\alpha}\left(\frac{\omega_2}{\omega_3}-1\right) + \frac{1+\phi}{\beta}\frac{\omega_2}{\omega_3}\right] \bar{E}_0 a_1 \tag{A.27}$$

Use equations (A.20) and (A.21) in equation (A.27)

$$q_1 = s_1 = \frac{1-\gamma}{\theta}a_1 + \left[\frac{1-\gamma}{\theta}\frac{\alpha}{1-\alpha}\left(\frac{\omega_1-\omega_2}{\omega_1}\right)\right]\bar{E}_0 a_1 \\ + \left[\frac{1-\gamma}{\theta}\frac{\alpha}{1-\alpha}\left(\frac{\omega_1}{\omega_3}-1\right) + \frac{1+\phi}{\beta}\frac{\omega_1}{\omega_3}\right]\frac{\omega_3}{\omega_1}q_0$$

Thus:

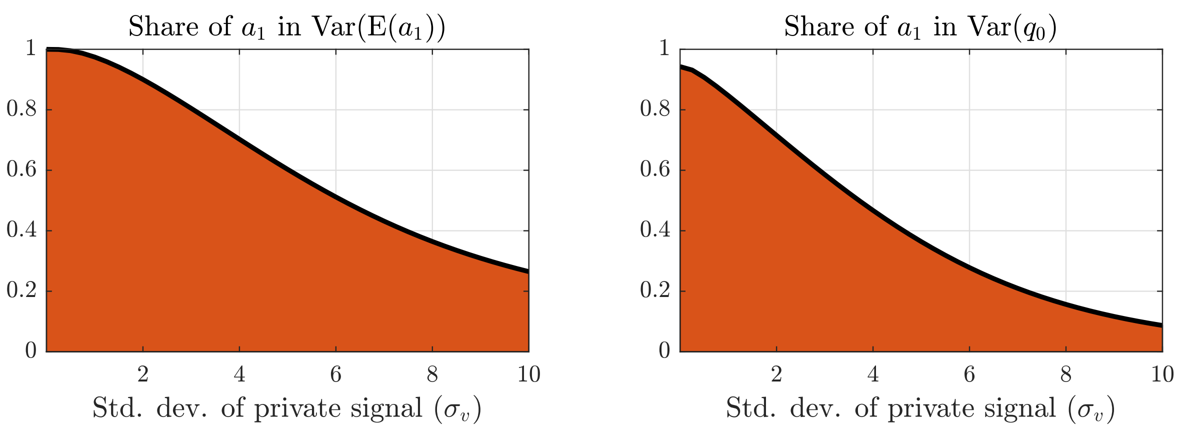
$$E_0^i q_1 - \bar{E}_0 q_1 = \frac{1-\gamma}{\theta}(E_0^i a_1 - \bar{E}_0 a_1) + \left[\frac{1-\gamma}{\theta}\frac{\alpha}{1-\alpha}\left(\frac{\omega_1-\omega_2}{\omega_1}\right)\right](E_0^i - \bar{E}_0)\bar{E}_0[a_1] \tag{A.28}$$

B On exchange rates and fundamentals

What fraction of exchange rate fluctuations reflects productivity—the economy's fundamental—relative to independent noise trading? It turns out that the answer to this question is subtle in our model when agents learn from the exchange rate. When the exchange rate is a public signal, noise-trading shock become the noise in the public signal, blurring the relationship between expectations of productivity and their subsequent realization (Bacchetta and Wincoop, 2006): noise trading affects the exchange rate by altering the balance sheet position of financiers, *and*, through the exchange rate, influences agents' expectations of productivity. As evident from the left panel of Figure A.1, whenever information is dispersed ($\sigma_v \in (0, \infty)$) and thus agents learn from the exchange rate, only a fraction of fluctuations in productivity *expectations* reflects subsequent productivity *realizations*. Accordingly, the share of exchange rate fluctuations reflecting future realized productivity represents a lower bound on the overall effect of productivity expectations on the exchange rate (right panel of Figure A.1).

Chahrour et al. (2024) investigates the sources of fluctuations in the USD exchange rate. It reveals that a substantial portion of the exchange rate variation can be attributed to both correctly anticipated changes in productivity and expectational "noise," which influences expectations of productivity but not the actual realization. Our model aligns well with these findings, provided that information is dispersed, and thus agents learn from exchange rates, offering novel insights into the relationship between exchange rates and macroeconomic fundamentals.

Figure A.1: Variance decomposition under dispersed information



Notes: This figure reports the decomposition of the exchange rate variation (left panel) and productivity expectations (right panel) for different levels of the noise in private signal, σ_v, under laissez faire, and without extrapolative beliefs ($\delta = 0$). The rest of parameters are set according to Table A.1.

C Exogenous foreign exchange interventions

In this Appendix, we outline the case in which FX interventions is completely exogenous, *i.e.*, $f_1^\star = \varepsilon_1^{f^\star}$ with $\varepsilon_1^{f^\star} \sim N\left(0, \beta_\varepsilon^{-1}\right)$. Under exogenous interventions, the equilibrium exchange rate follows:

$$q_0 = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(n_1^\star + \varepsilon_1^{f^\star}) - \frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\bar{E}_0 a_1, \tag{A.29}$$

highlighting that exogenous FX interventions are an additional shock to the foreign exchange market.

Publicly announced exogenous FX interventions Let us first consider the case in which agents are able to observe the aggregate volume of the FX intervention, $\varepsilon_1^{f^\star}$. Guess a linear solution for the *perceived* exchange rate process:

$$q_0 = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3} f_1^\star + \tilde{\lambda}_a^p a_1 + \tilde{\lambda}_n^p n_1^\star, \tag{A.30}$$

where $f_1^\star = \varepsilon_1^{f^\star}$. Define $\hat{q}_0 \equiv q_0 - \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1+\omega_3} f_1^\star$, as the equilibrium exchange *after the effect of the FX intervention is "partialed out."* Agents use the exchange rate as signal

$$z^p|a_1 \equiv \left.\frac{\hat{q}_0}{\tilde{\lambda}_a^p}\right| a_1 \overset{p}{\sim} \mathcal{N}(a_1, \beta_z^{-1})$$

with a error variance of $\beta_z^{-1} \equiv \frac{1}{(\tilde{\Lambda}^p)^2}\beta_n^{-1}$ with $(\Lambda^p)^2 \equiv \frac{(\tilde{\lambda}_a^p)^2}{(\tilde{\lambda}_n^p)^2}$, the same as in the laissez-faire economy.

Since the intervention is public, agents can partial it out from the exchange rate when they solve their signal extraction problem. It follows that the intervention does not affect the informational content of the exchange rate. Moreover, since the intervention is random, it only adds non-fundamental variation to the exchange rate. Substituting the actual average belief in (A.29), one gets the actual exchange rate

$$q_0 = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3} f_1^\star + (1+\delta)\tilde{\lambda}_a^p a_1 + \tilde{\lambda}_n^p n_1^\star,$$

Secretly conducted exogenous FX interventions Consider now the case in which the central bank does not reveal the aggregate volume of the FX intervention. Notice that the intervention $\varepsilon_1^{f^\star}$ and $n_1^\star$ are both unobservable exogenous shocks to the exchange rate (A.29). Guess a linear solution for the perceived exchange rate process

$$q_0 = \tilde{\lambda}_a^s a_1 + \tilde{\lambda}_n^s (n_1^\star + \varepsilon^{f^\star}). \tag{A.31}$$

Agents use the exchange rate as signal

$$z^s|a_1 \equiv \left.\frac{q_0}{\tilde{\lambda}_a^s}\right| a_1 \overset{p}{\sim} \mathcal{N}(a_1, \beta_z^{-1}) \tag{A.32}$$

with a error variance of $\beta_z^{-1} \equiv \frac{1}{(\tilde{\Lambda}^s)^2}(\beta_n^{-1} + \beta_\varepsilon^{-1})$ with $(\tilde{\Lambda}^s)^2 \equiv \frac{(\tilde{\lambda}_a^s)^2}{(\tilde{\lambda}_n^s)^2}$. Since the FX intervention is unobserved, it increases non-fundamental volatility of the exchange rate analogously to noise trading, and therefore decreases the informational content of the exchange rate $\mathcal{I}_R$.

One can show that, when implemented secretly, FX interventions have an informational effect. In particular, secret exogenous FX interventions alter agents' expectations of productivity by reducing the informativeness of the exchange rate. Substituting the actual average belief in (A.29), one gets the actual exchange rate

$$q_0 = (1+\delta)\tilde{\lambda}_a^s a_1 + \tilde{\lambda}_n^s (n_1^\star + \varepsilon^{f\star}).$$

D Welfare criterion

We evaluate welfare using an utilitarian criterion. Since all islands are ex-ante identical, we can simply maximize the welfare of an individual island i:

$$\mathbb{W} = \mathbb{E}\left[\frac{C_0^{i\,1-\sigma}}{1-\sigma} + \beta\left(\frac{C_1^{i\,1-\sigma}}{1-\sigma}\right)\right]. \tag{A.33}$$

We consider a second-order approximation of the above welfare function around the steady state:[29]

$$\mathbb{W} = C^{1-\sigma}\mathbb{E}\left\{[\hat{c}_0^i + \frac{1}{2}(1-\sigma)(\hat{c}_0^i)^2] + \beta[\hat{c}_1^i + \frac{1}{2}(1-\sigma)(\hat{c}_1^i)^2]\right\} + t.i.p. + \mathcal{O}(||\xi||^3), \tag{A.34}$$

where hatted variables are expressed in log-deviations from steady state, $t.i.p.$ stands for terms independent of policy, and $\mathcal{O}(||\xi||^3)$ denotes terms that are of third or higher order. Utility is maximized when consumption takes on its efficient values

$$\overline{\mathbb{W}} \approx \mathbb{E}\left[[\bar{c}_0 + \frac{1}{2}(1-\sigma)\bar{c}_0^2] + \beta[\bar{c}_1 + \frac{1}{2}(1-\sigma)\bar{c}_1^2]\right] \tag{A.35}$$

where barred variables are log-deviations from the steady state in the efficient allocation (which is the same for every island). In general, this maximum may not be attainable.

To lighten notation we drop the i subscript in the following derivations. Take the

[29] The steady state coincides with the steady state of the frictionless economy, meaning with no intermediation frictions $\Gamma = 0$ and with perfect information $E_0^i a_1 = a_1$

second-order approximation of the resource constraint at period 1 (eq. (3)):

$$\hat{c}_1 + \frac{1}{2}\hat{c}_1^2 = \gamma \hat{y}_{T,1} + \frac{1}{2}\frac{\gamma}{\theta}(\gamma + \theta - 1)\hat{y}_{T,1}^2 \tag{A.36}$$

Take the square of equation (A.36), while keeping only terms that are at most of order 2:

$$c_1^2 = \gamma^2 y_{T,1}^2 \tag{A.37}$$

Take the second-order approximation of the resource constraint at period 0 (eq. (3)):

$$\hat{c}_0 + \frac{1}{2}\hat{c}_0^2 + \phi(\hat{k}_1 + \frac{1}{2}\hat{k}_1^2) = (1+\phi)\gamma\hat{y}_{T,0} + \frac{1}{2}(1+\phi)\frac{\gamma}{\theta}(\gamma + \theta - 1)\hat{y}_{T,0}^2 \tag{A.38}$$

Take the square of equation (A.38), while keeping only terms that are at most of order 2:

$$\hat{c}_0^2 = \phi^2 \hat{k}_1^2 + (1+\phi)^2\gamma^2\hat{y}_{T,0}^2 - 2\gamma(1+\phi)\phi\hat{k}_1\hat{y}_{T,0} \tag{A.39}$$

Take the second-order approximation of the intertemporal budget constraint and the tradable-sector production function (equations (2) and (11)):

$$\hat{y}_{T,1} + \frac{1}{2}\hat{y}_{T,1}^2 = -\frac{1}{\beta}(1+\phi)\left(\hat{y}_{T,0} + \frac{1}{2}\hat{y}_{T,0}^2\right) + \left(\hat{a}_1 + \alpha\hat{k}_1 + \frac{1}{2}\left(\alpha^2\hat{k}_1^2 + 2\alpha\hat{a}_1\hat{k}_1 + \hat{a}_1^2\right)\right) \tag{A.40}$$

Take the square of equation (A.40), while keeping only terms that are at most of order 2:

$$\hat{y}_{T,1}^2 = \frac{1}{\beta^2}(1+\phi)^2\hat{y}_{T,0}^2 + \hat{a}_1^2 + \alpha^2\hat{k}_1^2 - 2\frac{1}{\beta}(1+\phi)\hat{y}_{T,0}\hat{a}_1 - 2\frac{1}{\beta}(1+\phi)\alpha\hat{y}_{T,0}\hat{k}_1 + 2\alpha\hat{a}_1\hat{k}_1 \tag{A.41}$$

Combine (A.36) with (A.37):

$$\hat{c}_1 = \gamma\hat{y}_{T,1} + \frac{1}{2}\gamma(1-\gamma)\frac{\theta - 1}{\theta}\hat{y}_{T,1}^2 \tag{A.42}$$

Combine (A.38) with (A.39):

$$\hat{c}_0 = \gamma(1+\phi)\hat{y}_{T,0} - \phi k_1 - \frac{1}{2}\phi(1+\phi)\hat{k}_1^2 + \frac{1}{2}(1+\phi)\gamma\left[(1-\gamma)\frac{\theta-1}{\theta} - \gamma\phi\right]\hat{y}_{T,0}^2 + \gamma(1+\phi)\phi\hat{k}_1\hat{y}_{T,0}. \tag{A.43}$$

Multiply (A.42) by β and substitute $y_{T,1}$ from eq. (A.40) as well as $\hat{y}_{T,1}^2$ from equation

(A.41) to get:

$$\beta\hat{c}_1 = -\gamma(1+\phi)\hat{y}_{T,0} + \gamma\beta\hat{a}_1 + \gamma\alpha\beta\hat{k}_1 + \frac{\beta\gamma}{2}\left\{\alpha^2\hat{k}_1^2 + 2\alpha\hat{a}_1\hat{k}_1 + \hat{a}_1^2 - \frac{(1+\phi)}{\beta}\hat{y}_{T,0}^2\right\}$$
$$+\frac{1}{2}\beta\gamma\left[(1-\gamma)\frac{\theta-1}{\theta}-1\right]\left(\frac{1}{\beta^2}(1+\phi)^2\hat{y}_{T,0}^2 + \hat{a}_1^2 + \alpha^2\hat{k}_1^2 - 2\frac{1}{\beta}(1+\phi)\hat{y}_{T,0}\hat{a}_1 - 2\frac{1}{\beta}(1+\phi)\alpha\hat{y}_{T,0}\hat{k}_1 + 2\alpha\hat{a}_1\hat{k}_1\right) \tag{A.44}$$

Summing (A.43) and (A.44) one obtains:

$$\hat{c}_0 + \beta\hat{c}_1 = \gamma\beta\hat{a}_1 + \frac{\beta\gamma}{2}(1+\chi)\hat{a}_1^2 - \frac{1}{2}\phi(1+\phi-\alpha(1+\chi))\hat{k}_1^2 + \frac{1}{2}\Omega\hat{y}_{T,0}^2$$
$$+\phi(1+\chi)\hat{a}_1\hat{k}_1 - \gamma\chi(1+\phi)\hat{y}_{T,0}\hat{a}_1 + \gamma(1+\phi)(\phi-\chi\alpha)\hat{k}_1\hat{y}_{T,0} \tag{A.45}$$

where $\Omega = \gamma(1+\phi)\left[\chi\left(1+\frac{1+\phi}{\beta}\right)-\gamma\phi\right]$ and $\chi = \left[(1-\gamma)\frac{\theta-1}{\theta}-1\right]$.

Now recognize that for any variable $\hat{x} = \tilde{x}+\bar{x}$ we have $\hat{x}^2 = \tilde{x}^2 + 2\tilde{x}\bar{x} + \bar{x}^2$, where $\bar{x}^2$ can be ignored because it is independent of policy. Use these relationships in (A.45), and drop terms independent of policy (t.i.p.):

$$\hat{c}_0 + \beta\hat{c}_1 = -\frac{1}{2}\phi(1+\phi-\alpha(1+\chi))(\tilde{k}_1^2 + 2\bar{k}_1\tilde{k}_1) + \frac{1}{2}\Omega(\tilde{y}_{T,0}^2 + 2\bar{y}_{T,0}\tilde{y}_{T,0})$$
$$+\phi(1+\chi)\hat{a}_1(\tilde{k}_1) - \gamma\chi(1+\phi)\tilde{y}_{T,0}\hat{a}_1 + \gamma(1+\phi)(\phi-\chi\alpha)(\tilde{k}_1+\bar{k}_1)(\tilde{y}_{T,0}+\bar{y}_{T,0}) \tag{A.46}$$

Now expand and use efficient allocation relationships:

$$\bar{y}_{T,0} = \frac{\theta}{1-\gamma}\frac{\omega_2}{\omega_3}\hat{a}_1 \tag{A.47}$$

$$\bar{k}_1 = \frac{1}{1-\alpha}\left(\frac{\omega_3-\omega_2}{\omega_3}\right)\hat{a}_1 \tag{A.48}$$

The two imply:

$$\bar{k}_1 = \frac{1-\gamma}{\theta(1-\alpha)}\left(\frac{\omega_3-\omega_2}{\omega_2}\right)\bar{y}_{T,0} \tag{A.49}$$

to get:

$$\hat{c}_0 + \beta\hat{c}_1 = -\frac{1}{2}\phi(1+\phi-\alpha(1+\chi))(\tilde{k}_1^2 + 2\bar{k}_1\tilde{k}_1) + \frac{1}{2}\Omega(\tilde{y}_{T,0}^2 + 2\bar{y}_{T,0}\tilde{y}_{T,0})$$
$$+\phi(1+\chi)(1-\alpha)\left(\frac{\omega_3}{\omega_3-\omega_2}\right)\bar{k}_1\tilde{k}_1 - \gamma\chi(1+\phi)\frac{(1-\gamma)}{\theta}\frac{\omega_3}{\omega_2}\tilde{y}_{T,0}\bar{y}_{T,0}$$

$$+\gamma(1+\phi)(\phi-\chi\alpha)\tilde{k}_1\tilde{y}_{T,0}+$$
$$\gamma(1+\phi)(\phi-\chi\alpha)\frac{\theta(1-\alpha)}{1-\gamma}\left(\frac{\omega_2}{\omega_3-\omega_2}\right)\tilde{k}_1\bar{k}_1+$$
$$\gamma(1+\phi)(\phi-\chi\alpha)\frac{1-\gamma}{\theta(1-\alpha)}\left(\frac{\omega_3-\omega_2}{\omega_2}\right)\bar{y}_{T,0}\tilde{y}_{T,0} \tag{A.50}$$

Using eq. (A.39), as well as eqs. (A.37) and (A.41):

$$\frac{1}{2}(1-\sigma)(\hat{c}_0)^2+\beta\frac{1}{2}(1-\sigma)(\hat{c}_1)^2=$$
$$\frac{1}{2}(1-\sigma)\left[\phi^2\hat{k}_1^2+(1+\phi)^2\gamma^2\hat{y}_{T,0}^2-2\gamma(1+\phi)\phi\hat{k}_1\hat{y}_{T,0}\right]+$$
$$\frac{\beta}{2}(1-\sigma)\gamma^2\left(\frac{1}{\beta^2}(1+\phi)^2\hat{y}_{T,0}^2+\hat{a}_1^2+\alpha^2\hat{k}_1^2-2\frac{1}{\beta}(1+\phi)\hat{y}_{T,0}\hat{a}_1-2\frac{1}{\beta}(1+\phi)\alpha\hat{y}_{T,0}\hat{k}_1+2\alpha\hat{a}_1\hat{k}_1\right)$$

which gives (excluding t.i.p.):

$$\frac{1}{2}(1-\sigma)(\hat{c}_0)^2+\beta\frac{1}{2}(1-\sigma)(\hat{c}_1)^2=$$
$$\frac{1-\sigma}{2}\left[\phi(\phi+\gamma\alpha)\hat{k}_1^2+(1+\frac{1}{\beta})(1+\phi)^2\gamma^2\hat{y}_{T,0}^2-2\gamma(1+\phi)(\phi+\alpha\gamma)\hat{k}_1\hat{y}_{T,0}-2(1+\phi)\gamma^2\hat{y}_{T,0}\hat{a}_1+2\phi\gamma\hat{a}_1\hat{k}_1\right] \tag{A.51}$$

Recognize that for any variable $\hat{x}=\tilde{x}+\bar{x}$ we have $\hat{x}^2=\tilde{x}^2+2\tilde{x}\bar{x}+\bar{x}^2$, where $\bar{x}^2$ can be ignored because it is independent of policy, one can rewrite (A.51) as:

$$\frac{1}{2}(1-\sigma)(\hat{c}_0)^2+\beta\frac{1}{2}(1-\sigma)(\hat{c}_1)^2=$$
$$\frac{1-\sigma}{2}\left[\phi(\phi+\gamma\alpha)(\tilde{k}_1^2+2\bar{k}_1\tilde{k}_1)+(1+\frac{1}{\beta})(1+\phi)^2\gamma^2(\tilde{y}_{T,0}^2+2\bar{y}_{T,0}\tilde{y}_{T,0})\right]$$
$$\frac{1-\sigma}{2}\left[-2\gamma(1+\phi)(\phi+\alpha\gamma)(\tilde{k}_1+\bar{k}_1)(\tilde{y}_{T,0}+\bar{y}_{T,0})-2(1+\phi)\gamma^2(\tilde{y}_{T,0}+\bar{y}_{T,0})\hat{a}_1+2\phi\gamma\hat{a}_1(\tilde{k}_1+\bar{k}_1)\right]$$

Applying the substitutions from the efficient allocation in eqs. (A.47)-(A.49):

$$\frac{1}{2}(1-\sigma)(\hat{c}_0)^2+\beta\frac{1}{2}(1-\sigma)(\hat{c}_1)^2=$$
$$\frac{1-\sigma}{2}\left[\phi(\phi+\gamma\alpha)(\tilde{k}_1^2+2\bar{k}_1\tilde{k}_1)+(1+\frac{1}{\beta})(1+\phi)^2\gamma^2(\tilde{y}_{T,0}^2+2\bar{y}_{T,0}\tilde{y}_{T,0})\right]$$

$$
\begin{aligned}
&- (1-\sigma)\gamma(1+\phi)(\phi+\alpha\gamma)(\tilde{k}_1\tilde{y}_{T,0}) \\
&- (1-\sigma)\gamma(1+\phi)(\phi+\alpha\gamma)\frac{\theta(1-\alpha)}{1-\gamma}\left(\frac{\omega_2}{\omega_3-\omega_2}\right)\tilde{k}_1\bar{k}_1 \\
&- (1-\sigma)\gamma(1+\phi)(\phi+\alpha\gamma)\frac{1-\gamma}{\theta(1-\alpha)}\left(\frac{\omega_3-\omega_2}{\omega_2}\right)\tilde{y}_{T,0}\bar{y}_{T,0} \\
&- (1-\sigma)(1+\phi)\gamma^2\frac{1-\gamma}{\theta}\frac{\omega_3}{\omega_2}\tilde{y}_{T,0}\bar{y}_{T,0} \\
&+ (1-\sigma)\phi\gamma(1-\alpha)\left(\frac{\omega_3}{\omega_3-\omega_2}\right)\tilde{k}_1\bar{k}_1
\end{aligned}
\tag{A.52}
$$

Sum eq. (A.50) and eq. (A.52):

$$
\begin{aligned}
&\hat{c}_0+\beta\hat{c}_1+\frac{1}{2}(1-\sigma)(\hat{c}_0)^2+\beta\frac{1}{2}(1-\sigma)(\hat{c}_1)^2 = \\
&-\frac{1}{2}\phi\left[1+\phi-\alpha(1+\chi)-(1-\sigma)(\phi+\gamma\alpha)\right](\tilde{k}_1^2+2\bar{k}_1\tilde{k}_1) \\
&+\frac{1}{2}\left[\Omega+(1-\sigma)(1+\frac{1}{\beta})(1+\phi)^2\gamma^2\right](\tilde{y}_{T,0}^2+2\bar{y}_{T,0}\tilde{y}_{T,0}) \\
&+\phi[(1+\chi)+(1-\sigma)\gamma](1-\alpha)\left(\frac{\omega_3}{\omega_3-\omega_2}\right)\bar{k}_1\tilde{k}_1 \\
&-\gamma(1+\phi)[\chi+(1-\sigma)\gamma]\frac{(1-\gamma)}{\theta}\frac{\omega_3}{\omega_2}\bar{y}_{T,0}\tilde{y}_{T,0} \\
&+\gamma(1+\phi)[(\phi-\chi\alpha)-(1-\sigma)(\phi+\alpha\gamma)]\tilde{k}_1\tilde{y}_{T,0}+ \\
&+\gamma(1+\phi)[(\phi-\chi\alpha)-(1-\sigma)(\phi+\alpha\gamma)]\frac{\theta(1-\alpha)}{1-\gamma}\left(\frac{\omega_2}{\omega_3-\omega_2}\right)\tilde{k}_1\bar{k}_1+ \\
&+\gamma(1+\phi)[(\phi-\chi\alpha)-(1-\sigma)(\phi+\alpha\gamma)]\frac{1-\gamma}{\theta(1-\alpha)}\left(\frac{\omega_3-\omega_2}{\omega_2}\right)\bar{y}_{T,0}\tilde{y}_{T,0}
\end{aligned}
\tag{A.53}
$$

Note that the following two set of relationships among parameters hold:

$$
\begin{aligned}
\phi\left[1+\phi-\alpha(1+\chi)-(1-\sigma)(\phi+\gamma\alpha)\right] &= \phi[(1+\chi)+(1-\sigma)\gamma](1-\alpha)\left(\frac{\omega_3}{\omega_3-\omega_2}\right) \\
&\quad+\gamma(1+\phi)[(\phi-\chi\alpha)-(1-\sigma)(\phi+\alpha\gamma)]\frac{\theta(1-\alpha)}{1-\gamma}\left(\frac{\omega_2}{\omega_3-\omega_2}\right) \\
-\left[\Omega+(1-\sigma)(1+\frac{1}{\beta})(1+\phi)^2\gamma^2\right] &= -\gamma(1+\phi)[\chi+(1-\sigma)\gamma]\frac{(1-\gamma)}{\theta}\frac{\omega_3}{\omega_2} \\
&\quad+\gamma(1+\phi)[(\phi-\chi\alpha)-(1-\sigma)(\phi+\alpha\gamma)]\frac{1-\gamma}{\theta(1-\alpha)}\left(\frac{\omega_3-\omega_2}{\omega_2}\right)
\end{aligned}
$$

Use these two conditions in eq. (A.53), and plug it in (A.34):

$$
\begin{aligned}
(C^{\sigma-1})\mathbb{W} = &-\frac{1}{2}\phi\left[1+\phi-\alpha(1+\chi)-\phi(1-\sigma)(1+1/\beta)\right]\mathbb{E}(\tilde{k}_1^i)^2 \\
&-\frac{1}{2}\gamma(1+\phi)\left[\gamma\phi-\chi\left(1+\frac{1+\phi}{\beta}\right)-(1-\sigma)(1+1/\beta)(1+\phi)\gamma\right]\mathbb{E}(\tilde{y}_{T,0}^i)^2 \\
&+\gamma(1+\phi)[(\phi-\chi\alpha)-\phi(1-\sigma)(1+1/\beta)]\mathbb{E}\tilde{k}_1^i\tilde{y}_{T,0}^i+t.i.p.+\mathcal{O}(||\xi||^3),
\end{aligned}
\tag{A.54}
$$

where $\chi = \left[(1-\gamma)^{\frac{\theta-1}{\theta}}-1\right]$ and $C^{\sigma-1} = \left(\frac{1}{\gamma}\right)^{\sigma-1}(\alpha\beta)^{\frac{\alpha(\sigma-1)}{1-\alpha}}$. Rewrite (A.54) as:

$$
\begin{aligned}
(C^{\sigma-1})\mathbb{W} = &-\frac{1}{2}\frac{\phi}{\theta}\omega_1\mathbb{E}(\tilde{k}_1^i)^2-\frac{1}{2}(1+\phi)\frac{\gamma}{(1-\alpha)(1-\gamma)}\left[(\omega_3-\omega_1)+(1-\alpha)\theta\right]\mathbb{E}(\tilde{q}_0^i)^2 \\
&-\frac{\phi}{(1-\alpha)\theta}(\omega_3-\omega_1)\mathbb{E}\tilde{k}_1^i\tilde{q}_0^i+t.i.p.+\mathcal{O}(||\xi||^3),
\end{aligned}
\tag{A.55}
$$

The island-level real exchange rate and capital relative to their frictionless counterparts are:

$$
\tilde{q}_0^i = \frac{\omega_1}{\omega_3}\left(r_0-(E_0^iq_1-q_0)\right)-\frac{\omega_2}{\omega_3}(E_0^ia_1-a_1) \tag{A.56}
$$

$$
\tilde{k}_1^i = \frac{1}{1-\alpha}\left[\left(\frac{\omega_1}{\omega_3}-1\right)\left(r_0-(E_0^iq_1-q_0)\right)-\left(\frac{\omega_2}{\omega_3}-1\right)(E_0^ia_1-a_1)\right] \tag{A.57}
$$

Equations (A.56)-(A.57) imply:

$$
\tilde{k}_1^i = \frac{1}{1-\alpha}\left[-\left(\frac{\omega_3-\omega_1}{\omega_1}\right)\tilde{q}_0^i+\left(\frac{\omega_1-\omega_2}{\omega_1}\right)(E_0^ia_1-a_1)\right] \tag{A.58}
$$

Using (A.58) inside (A.55), and noting that welfare (A.55) is equal to zero when evaluated at the first best, since, by definition, all gaps are zero. Thus:

$$
\mathbb{W}-\overline{\mathbb{W}} = -\Omega_1\mathbb{E}(\tilde{q}_0^i)^2-\Omega_2\mathbb{E}(E_0^ia_1-a_1)^2+t.i.p.+\mathcal{O}(||\xi||^3), \tag{A.59}
$$

where

$$
\Omega_1 \equiv \frac{1}{2C^{\sigma-1}}\frac{\gamma}{(1-\gamma)}(1+\phi)\theta\frac{\omega_3}{\omega_1}; \qquad \Omega_2 \equiv \frac{1}{2C^{\sigma-1}}\frac{\phi}{\theta}\left(\frac{1}{1-\alpha}\right)^2\frac{(\omega_1-\omega_2)^2}{\omega_1} \tag{A.60}
$$

Equation (A.59) is equation (39) in Section 4.

D.1 Auxiliary welfare representation

This appendix derives the auxiliary welfare representation, presented in Section 4.2. In doing so, it uses the fact that the public signal z satisfies the following representation (regardless of whether FX interventions are conducted publicly or secretly):

$$z|a_1 \sim \mathcal{N}\left((1+\delta)a_1, \beta_z^{-1}\right) \tag{A.61}$$

Auxiliary expression for $\mathrm{var}(E_0^i a_1 - a_1)$ Using equations (A.61) and (22), one can obtain:

$$\mathrm{var}(E_0^i a_1 - a_1) = \left(\frac{1}{\beta_v + \beta_a + \beta_z}\right)^2 \left[(1+\delta)^2\beta_v + \beta_z + (\delta\beta_z + \delta\beta_v - \beta_a)^2 \frac{1}{\beta_a}\right] \tag{A.62}$$

Auxiliary expression for $\mathrm{var}(q_0^i - \bar{q}_0)$ First, find an auxiliary expression for island-expected excess currency returns. To do so, use eqs. (A.28) and

$$(E_0^i - \bar{E}_0)\bar{E}_0[a_1] = \frac{\beta_v}{\beta_v + \beta_a + \beta_z}(E_0^i a_1 - \bar{E}_0 a_1); \qquad E_0^i a_1 - \bar{E}_0 a_1 = \frac{(1+\delta)\beta_v}{\beta_v + \beta_a + \beta_z}\epsilon^i$$

into eq. (43) to obtain:

$$\begin{aligned} r_0 - \left(E_0^i s_1 - s_0\right) = &-\frac{1-\gamma}{\theta}\left[1 + \frac{\alpha}{1-\alpha}\left(\frac{\omega_1 - \omega_2}{\omega_1}\right)\frac{\beta_v}{\beta_a + \beta_v + \beta_z}\right]\frac{\beta_v}{\beta_v + \beta_a + \beta_z}(1+\delta)\epsilon^i \\ &+ \frac{\Gamma\tilde{\theta}\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\bar{E}_0 a_1 + \frac{\Gamma\omega_3}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(n_1^\star + f_1^\star). \end{aligned} \tag{A.63}$$

Use (31) in (A.63):

$$\begin{aligned} r_0 - \left(E_0^i s_1 - s_0\right) = &-\frac{1-\gamma}{\theta}\left[1 + \frac{\alpha}{1-\alpha}\left(\frac{\omega_1 - \omega_2}{\omega_1}\right)\frac{\beta_v}{\beta_a + \beta_v + \beta_z}\right]\frac{\beta_v}{\beta_v + \beta_a + \beta_z}(1+\delta)\epsilon^i \\ &+ \frac{\Gamma\tilde{\theta}\omega_2 + \Gamma\omega_3\kappa_a}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(\bar{E}_0 a_1) + \frac{\Gamma\omega_3}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(1+\kappa_n)n_1^\star. \end{aligned} \tag{A.64}$$

Plug (A.64) into (40):

$$\begin{aligned} q_0^i - \bar{q}_0 = &-\frac{1-\gamma}{\theta}\frac{\omega_1}{\omega_3}\left[1 + \frac{\alpha}{1-\alpha}\left(\frac{\omega_1 - \omega_2}{\omega_1}\right)\frac{\beta_v}{\beta_a + \beta_v + \beta_z}\right]\frac{\beta_v}{\beta_v + \beta_a + \beta_z}(1+\delta)\epsilon^i \\ &+ \frac{\Gamma\tilde{\theta}\omega_2 + \Gamma\omega_3\kappa_a}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\frac{\omega_1}{\omega_3}(\bar{E}_0 a_1) + \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}(1+\kappa_n)n_1^\star - \frac{\omega_2}{\omega_3}(E_0^i a_1 - a_1) \end{aligned} \tag{A.65}$$

Using equations (A.61) and (22) into equation (A.65), one obtains:

$$
\begin{aligned}
q_0^i - \bar{q}_0 = & -\frac{1}{\omega_3}\left\{\frac{1-\gamma}{\theta}\omega_1\left[1+\frac{\alpha}{1-\alpha}\left(\frac{\omega_1-\omega_2}{\omega_1}\right)\frac{\beta_v}{\beta_a+\beta_v+\beta_z}\right]+\omega_2\right\}\frac{\beta_v}{\beta_v+\beta_a+\beta_z}(1+\delta)\epsilon^i \\
& \left(\frac{1}{\omega_3}\right)\left[\frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1+\omega_3}\left(\tilde{\theta}\omega_2+\omega_3\kappa_a\right)(1+\delta)(\beta_v+\beta_z)-\omega_2(\delta\beta_z+\delta\beta_v-\beta_a)\right]\frac{1}{(\beta_v+\beta_z+\beta_a)}a_1 \\
& +\frac{\Gamma\kappa_a\omega_1-\omega_2}{\Gamma\tilde{\theta}\omega_1+\omega_3}\frac{\beta_z}{\beta_v+\beta_z+\beta_a}\epsilon^z+\frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1+\omega_3}(1+\kappa_n)n_1^\star
\end{aligned}
\tag{A.66}
$$

and therefore:

$$
\mathrm{var}(q_0^i-\bar{q}_0) = \left[\begin{array}{l}
\left(\frac{1}{\omega_3}\right)^2\left\{\frac{1-\gamma}{\theta}\omega_1\left[1+\frac{\alpha}{1-\alpha}\left(\frac{\omega_1-\omega_2}{\omega_1}\right)\frac{\beta_v}{\beta_a+\beta_v+\beta_z}\right]+\omega_2\right\}^2\frac{1}{(\beta_v+\beta_a+\beta_z)^2}(1+\delta)^2\beta_v+ \\
\left(\frac{1}{\omega_3}\right)^2\left[\frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1+\omega_3}\left(\tilde{\theta}\omega_2+\omega_3\kappa_a\right)(1+\delta)(\beta_v+\beta_z)-\omega_2(\delta\beta_z+\delta\beta_v-\beta_a)\right]^2\frac{1}{(\beta_v+\beta_z+\beta_a)^2}\frac{1}{\beta_a}+ \\
+\frac{(\Gamma\kappa_a\omega_1-\omega_2)^2}{(\Gamma\tilde{\theta}\omega_1+\omega_3)^2}\frac{1}{(\beta_v+\beta_z+\beta_a)^2}\beta_z \\
+\frac{(\Gamma\omega_1)^2}{(\Gamma\tilde{\theta}\omega_1+\omega_3)^2}(1+\kappa_n)^2\frac{1}{\beta_n}+2\frac{(\Gamma\kappa_a\omega_1-\omega_2)\Gamma\omega_1}{(\Gamma\tilde{\theta}\omega_1+\omega_3)^2}\frac{\beta_z}{\beta_v+\beta_z+\beta_a}(1+\kappa_n)\,\mathrm{cov}(n_1^\star,\epsilon^z)
\end{array}\right]
\tag{A.67}
$$

where ϵ^z denotes the noise in the public signal z.

Auxiliary welfare expression Having derived the auxiliary expressions for the welfare components in equations (A.62) and (A.67), one can define the auxiliary expression for overall welfare as:

$$
\widetilde{\mathbb{W}}(\kappa_a,\kappa_n,\beta_z) = -\left[\begin{array}{l}
\Omega_1\underbrace{\left[\begin{array}{l}
\left(\frac{1}{\omega_3}\right)^2\left\{\frac{1-\gamma}{\theta}\omega_1\left[1+\frac{\alpha}{1-\alpha}\left(\frac{\omega_1-\omega_2}{\omega_1}\right)\frac{\beta_v}{\beta_a+\beta_v+\beta_z}\right]+\omega_2\right\}^2\frac{1}{(\beta_v+\beta_a+\beta_z)^2}(1+\delta)^2\beta_v+ \\
\left(\frac{1}{\omega_3}\right)^2\left[\frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1+\omega_3}\left(\tilde{\theta}\omega_2+\omega_3\kappa_a\right)(1+\delta)(\beta_v+\beta_z)-\omega_2(\delta\beta_z+\delta\beta_v-\beta_a)\right]^2\frac{1}{(\beta_v+\beta_z+\beta_a)^2}\frac{1}{\beta_a}+ \\
+\frac{(\Gamma\kappa_a\omega_1-\omega_2)^2}{(\Gamma\tilde{\theta}\omega_1+\omega_3)^2}\frac{1}{(\beta_v+\beta_z+\beta_a)^2}\beta_z \\
+\frac{(\Gamma\omega_1)^2}{(\Gamma\tilde{\theta}\omega_1+\omega_3)^2}(1+\kappa_n)^2\frac{1}{\beta_n}+2\frac{(\Gamma\kappa_a\omega_1-\omega_2)\Gamma\omega_1}{(\Gamma\tilde{\theta}\omega_1+\omega_3)^2}\frac{\beta_z}{\beta_v+\beta_z+\beta_a}(1+\kappa_n)\,\mathrm{cov}(n_1^\star,\epsilon^z)
\end{array}\right]}_{\mathrm{var}(q_0^i-\bar{q}_0)}+ \\
\Omega_2\underbrace{\left(\frac{1}{\beta_v+\beta_a+\beta_z}\right)^2\left[(1+\delta)^2\beta_v+\beta_z+(\delta\beta_z+\delta\beta_v-\beta_a)^2\frac{1}{\beta_a}\right]}_{\mathrm{var}(E_0^i a_1-a_1)}
\end{array}\right]
\tag{A.68}
$$

We observe that the auxiliary welfare function, denoted by $\widetilde{\mathbb{W}}(\kappa_a,\kappa_n,\beta_z)$, is also influenced by the covariance between noise trading shocks and the noise in the public signal, represented by $\mathrm{cov}(n_1^\star,\epsilon^z)$. Since, under both the secret and public communication regimes, this covariance term can be expressed as a distinct function of $(\kappa_a, \kappa_n, \beta_z)$, we write the welfare function as in equation (A.68), with a slight abuse of notation.

Auxiliary welfare expression under public communication Using public signal (33) in (A.67) one can express the auxiliary welfare representation under secret communication as:

$$\widetilde{\mathbb{W}}(\kappa_a, \kappa_n; \beta_z) = -\left[\Omega_1 \underbrace{\left[\frac{\left(\Gamma\tilde{\theta}\omega_2\omega_1 - \omega_3\delta\omega_2 + \Gamma\omega_3\omega_1(1+\delta)\kappa_a\right)^2 \frac{1}{\beta_a} + (\Gamma\omega_3\omega_1(1+\kappa_n))^2 \frac{1}{\beta_n}}{\omega_3^2(\Gamma\tilde{\theta}\omega_1+\omega_3)^2}\right]}_{\text{var}(q_0^i - \bar{q}_0)} + \Omega_2 \underbrace{\delta^2 \frac{1}{\beta_a}}_{\text{var}(E_0^i a_1 - a_1)}\right] \tag{A.69}$$

Auxiliary welfare expression under secret conduct of FX interventions Using public signal (38) in (A.67) one can express the auxiliary welfare representation under secret communication as:

$$\widetilde{\mathbb{W}}(\kappa_a, \kappa_n; \beta_z) = -\left[\begin{array}{l} \Omega_1 \underbrace{\left[\begin{array}{l} \left(\frac{1}{\omega_3}\right)^2 \left\{\frac{1-\gamma}{\theta}\omega_1\left[1+\frac{\alpha}{1-\alpha}\left(\frac{\omega_1-\omega_2}{\omega_1}\right)\frac{\beta_v}{\beta_a+\beta_v+\beta_z}\right]+\omega_2\right\}^2 \frac{1}{(\beta_v+\beta_a+\beta_z)^2}(1+\delta)^2\beta_v + \\ \left(\frac{1}{\omega_3}\right)^2 \left[\frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1+\omega_3}\left(\tilde{\theta}\omega_2+\omega_3\kappa_a\right)(1+\delta)(\beta_v+\beta_z) - \omega_2(\delta\beta_z+\delta\beta_v-\beta_a)\right]^2 \frac{1}{(\beta_v+\beta_z+\beta_a)^2}\frac{1}{\beta_a} + \\ \frac{(\Gamma\kappa_a\omega_1-\omega_2)^2}{(\Gamma\tilde{\theta}\omega_1+\omega_3)^2}\frac{1}{(\beta_a+\beta_v+\beta_z)^2}\frac{(\beta_v+\beta_z)^2}{\beta_z} \end{array}\right]}_{\text{var}(q_0^i-\bar{q}_0)} + \\ \Omega_2 \underbrace{\left(\frac{1}{\beta_v+\beta_a+\beta_z}\right)^2\left[(1+\delta)^2\beta_v+\beta_z+(\delta\beta_z+\delta\beta_v-\beta_a)^2\frac{1}{\beta_a}\right]}_{\text{var}(E_0^i a_1 - a_1)} \end{array}\right] \tag{A.70}$$

E Proofs

Proof of Proposition 1. Define $D \equiv \beta_v + \beta_z + \beta_v$. Plug the signal under laissez faire in the solution for the perceived exchange rate process (26):

$$q_0 = \left[1 + \frac{\omega_2}{\Gamma\tilde{\theta}\omega_1+\omega_3}\frac{\beta_z}{D\tilde{\lambda}_a^l}\right]^{-1}\left\{\frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1+\omega_3}n_1^\star - \left[\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1+\omega_3}\frac{\beta_v}{D}\right]a_1\right\} \tag{A.71}$$

To find the undetermined coefficients, set (A.71) equal to the guess (24). You get

$$\begin{aligned} -\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1+\omega_3}(1+\delta)\frac{\beta_v}{D} &= \tilde{\lambda}_a^l\left[1+\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1+\omega_3}\frac{\beta_z}{D\tilde{\lambda}_a^l}\right] \\ \tilde{\lambda}_a^l &= -\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1+\omega_3}\frac{\beta_v+\beta_z}{D} \end{aligned} \tag{A.72}$$

and

$$\begin{aligned} \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1+\omega_3} &= \tilde{\lambda}_n^l\left[1+\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1+\omega_3}\frac{\beta_z}{D\tilde{\lambda}_a^l}\right] \\ \tilde{\lambda}_n^l &= \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1+\omega_3}\frac{\beta_v+\beta_z}{\beta_v} \end{aligned} \tag{A.73}$$

Take the ratio

$$\frac{\tilde{\lambda}_a^l}{\tilde{\lambda}_n^l} = -\frac{\omega_2}{\Gamma\omega_1}\frac{\beta_v}{D} \tag{A.74}$$

Define $\Lambda_l \equiv \frac{\tilde{\lambda}_a^l}{\tilde{\lambda}_n^l}$. Then:

$$\begin{aligned} \Lambda_l &= -\frac{\omega_2}{\Gamma\omega_1}\frac{\beta_v}{\beta_v+\beta_a+\Lambda_l^2\beta_n} \\ \Lambda_l^3+\left(\frac{\beta_v}{\beta_n}+\frac{\beta_a}{\beta_n}\right)\Lambda_l+\frac{\omega_2}{\Gamma\omega_1}\frac{\beta_v}{\beta_n} &= 0 \end{aligned} \tag{A.75}$$

Define $\rho_1 \equiv \frac{(\beta_v+\beta_a)}{\beta_n}$ and $\rho_2 \equiv \frac{\omega_2}{\Gamma\omega_1}\frac{\beta_v}{\beta_n}$. Thus, rewrite (A.82) as:

$$\Lambda_l^3+\rho_1\Lambda_l+\rho_2 = 0 \tag{A.76}$$

Cubics of this form are said to be "depressed." Cardano's formula states the following. If

1. the cubic equation is of the form in (A.83)

2. ρ_1 and ρ_2 are real numbers

3. $\frac{\rho_2^2}{4}+\frac{\rho_1^3}{27}>0$ (which is satisfied in our context for any real value of $\frac{\omega_3}{\Gamma\omega_2}$)

Then, equation (A.83) has:

(i) the real root:

$$\sqrt[3]{-\frac{\rho_2}{2}+\sqrt{\frac{\rho_2^2}{4}+\frac{\rho_1^3}{27}}}+\sqrt[3]{-\frac{\rho_2}{2}-\sqrt{\frac{\rho_2^2}{4}+\frac{\rho_1^3}{27}}} \tag{A.77}$$

(ii) and two other roots that are non-real complex conjugate numbers.

□

Proof of Proposition 2. By combining the two public signals—the exchange rate given by (26) and the FX intervention given by (32)—which provide independent information about $\bar{E}_0[a_1]$ and $n_1^\star$, the resulting public signal z^p fully reveals average expectations. Specifically:

$$z^p \equiv \left(\frac{f_1^\star}{\kappa_n} - q_0\frac{\Gamma\tilde{\theta}\omega_1+\omega_3}{\Gamma\omega_1}\right) \Bigg/ \left(\frac{\kappa_a}{\kappa_n}+\frac{\omega_2}{\Gamma\omega_1}\right) = \bar{E}_0[a_1].$$

□

Proof of Proposition 3. Define $D \equiv \beta_a + \beta_v + \beta_z$. Plug the signal under secretly conducted FX interventions in the solution for the exchange rate (35):

$$q_0 = \left[1+\frac{\omega_2-\Gamma\omega_1\kappa_a}{\Gamma\tilde{\theta}\omega_1+\omega_3}(1+\delta)\frac{\beta_z}{D\tilde{\lambda}_a^s}\right]^{-1}\left\{\frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1+\omega_3}(1+\kappa_n)n_1^\star - \left[\frac{\omega_2-\Gamma\omega_1\kappa_a}{\Gamma\tilde{\theta}\omega_1+\omega_3}(1+\delta)\frac{\beta_v}{D}\right]a_1\right\} \tag{A.78}$$

To find the undetermined coefficients, set (A.78) equal to the guess (24). You get

$$\tilde{\lambda}_a^s = -\frac{\omega_2-\Gamma\omega_1\kappa_a}{\Gamma\tilde{\theta}\omega_1+\omega_3}(1+\delta)\frac{\beta_v+\beta_z}{D} \tag{A.79}$$

and

$$\tilde{\lambda}_n^s = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1+\omega_3}(1+\kappa_n)\frac{\beta_v+\beta_z}{\beta_v} \tag{A.80}$$

Take the ratio

$$\frac{\tilde{\lambda}_a^s}{\tilde{\lambda}_n^s} = -\frac{\omega_2-\Gamma\omega_1\kappa_a}{\Gamma\omega_1(1+\kappa_n)}(1+\delta)\frac{\beta_v}{D} \tag{A.81}$$

Define $\Lambda_s \equiv \frac{\tilde{\lambda}_a^s}{\tilde{\lambda}_n^s}$. Then:

$$\begin{aligned}
\Lambda_s &= -\frac{\omega_2}{\Gamma\omega_1}(1+\delta)\frac{\beta_v}{\beta_v+\beta_a+\Lambda_s^2\beta_n} \\
\Lambda_s^3 + \left(\frac{\beta_v}{\beta_n}+\frac{\beta_a}{\beta_n}\right)\Lambda_s &+ \frac{\omega_2-\Gamma\omega_1\kappa_a}{\Gamma\omega_1(1+\kappa_n)}(1+\delta)\frac{\beta_v}{\beta_n} = 0
\end{aligned} \tag{A.82}$$

Define $\rho_1 \equiv \frac{(\beta_v+\beta_a)}{\beta_n}$ and $\rho_2 \equiv \frac{\omega_2-\Gamma\omega_1\kappa_a}{\Gamma\omega_1(1+\kappa_n)}(1+\delta)\frac{\beta_v}{\beta_n}$. Thus, rewrite (A.82) as:

$$\Lambda_s^3 + \rho_1\Lambda_s + \rho_2 = 0 \tag{A.83}$$

Applying the Cardano's formula as in Proposition 1, one gets the following unique solution:

$$\sqrt[3]{-\frac{\rho_2}{2}+\sqrt{\frac{\rho_2^2}{4}+\frac{\rho_1^3}{27}}}+\sqrt[3]{-\frac{\rho_2}{2}-\sqrt{\frac{\rho_2^2}{4}+\frac{\rho_1^3}{27}}} \tag{A.84}$$

$\square$

Proof of Proposition 4. We let the planner solve the auxiliary problem as defined in Section 4, where the auxiliary representation of the welfare function under public communication is given by (A.69), which internalizes the relevant constraint in the auxiliary planner's problem (44),. The first-order conditions of the resulting problem with respect to κ_n and κ_a are:

$$\kappa_n^p = -1; \qquad \kappa_a^p = -\frac{\omega_2}{\omega_3}\frac{\tilde{\theta}\Gamma\omega_1 - \delta\omega_3}{\Gamma\omega_1(1+\delta)}.$$

Plugging (κ_a^p, κ_n^p) in the auxiliary welfare function (A.69), implies that, at the optimum:

$$\mathrm{var}(q_0^i - \bar{q}_0) = 0; \qquad \mathrm{var}(E_0^i a_1 - a_1) = \frac{\delta^2}{\beta_a} \geq 0,$$

and, as a result:

$$\widetilde{\mathbb{W}}^p = -\Omega_2\frac{\delta^2}{\beta_a} \leq 0.$$

$\square$

Proof of Proposition 5. We let the planner solve the *two-step* auxiliary problem outlined in Section 4, where the auxiliary representation of the welfare function under public communication is given by equation (A.70). Note that equation (A.70) already internalizes the relevant constraint in the auxiliary planner's problem (44), thereby expressing the welfare solely as a function of κ_a and β_z. Because of this, Step 1 of the auxiliary problem boils down to finding the welfare-maximizing value of κ_a for any given $\beta_z = \bar{\beta}_z$, that is $\kappa_a^s(\bar{\beta}_z)$. The corresponding maximizing value of $\kappa_n^s(\bar{\beta}_z)$ can be then determined by the relevant constraint in the auxiliary planner's problem (44). The first-order condition of (A.70) with respect to κ_a, implies that:[30]

$$\kappa_a^s(\bar{\beta}_z) = \underbrace{-\frac{\omega_2}{\omega_3}\frac{(\tilde{\theta}\Gamma\omega_1 - \delta\omega_3)}{(1+\delta)\Gamma\omega_1}}_{\kappa_a^p} - \frac{\omega_2}{\omega_3}\frac{(\Gamma\tilde{\theta}\omega_1 + \omega_3)}{(1+\delta)\Gamma\omega_1}\frac{\beta_a[(1+\delta)^2\bar{\beta}_z - (\bar{\beta}_z + \beta_v)]}{(\beta_v + \bar{\beta}_z)[(1+\delta)^2\bar{\beta}_z + \beta_a]}. \tag{A.86}$$

By replacing κ_a with its optimal value $\kappa_a^s(\bar{\beta}_z)$ into welfare function (A.70) we obtain:

$$\begin{aligned}\widetilde{\mathbb{W}}(\kappa_a^s(\bar{\beta}_z), \kappa_n^s(\bar{\beta}_z), \bar{\beta}_z) = & -\Omega_1 \underbrace{\begin{bmatrix} \frac{\omega_2^2}{\omega_3^2}\left\{\frac{1-\gamma}{\theta}\left[\frac{\omega_1}{\omega_2} + \frac{\alpha}{1-\alpha}\left(\frac{\omega_1 - \omega_2}{\omega_2}\right)\frac{\beta_v}{\beta_a + \beta_v + \bar{\beta}_z}\right] + 1\right\}^2 \frac{(1+\delta)^2\beta_v}{(\beta_v + \beta_a + \bar{\beta}_z)^2} \\ + \frac{\omega_2^2}{\omega_3^2}\frac{1}{\beta_a + (1+\delta)^2\bar{\beta}_z} \end{bmatrix}}_{\text{var}(\tilde{q}_0^i)} \\ & - \Omega_2 \underbrace{\left(\frac{\delta^2}{\beta_a} + \frac{(1-\delta^2)\beta_a + (1-\delta^2)\beta_v + (1-2\delta-2\delta^2)\bar{\beta}_z}{(\beta_v + \beta_a + \bar{\beta}_z)^2}\right)}_{\text{var}(E_0^i a_1 - a_1)},\end{aligned} \tag{A.87}$$

where $\Omega_1 > 0$ and $\Omega_2 \geq 0$ are defined in equation (A.60).

Equation (A.87) represents the level of welfare for any value of $\bar{\beta}_z$, where the values of (κ_a, κ_n) are set to their welfare-maximizing values $(\kappa_a^s(\bar{\beta}_z), \kappa_n^s(\bar{\beta}_z))$.

Equation (A.87) delivers two important results. First, secret policy can always attain the level of welfare of the optimal public policy. To see this, take the limit of equation (A.87) when $\bar{\beta}_z \to \infty$:

$$\lim_{\bar{\beta}_z \to \infty} \widetilde{\mathbb{W}}(\kappa_a^s(\bar{\beta}_z), \kappa_n^s(\bar{\beta}_z), \bar{\beta}_z) = -\frac{1}{2}(C^{1-\sigma})\left[\frac{\alpha\beta\gamma(\omega_2 - \omega_1)^2}{\theta(1-\alpha)^2\omega_1}\delta^2\frac{1}{\beta_a}\right].$$

---

[30] The corresponding value for $\kappa_n^s(\beta_z^s)$ is:

$$\kappa_n^s(\beta_z^s) = \underbrace{-1}_{\kappa_n^p} \pm \frac{\omega_2(\Gamma\tilde{\theta}\omega_1 + \omega_3)}{\Gamma\omega_1\omega_3}\frac{(1+\delta)\beta_z^s\beta_v}{(\beta_a + (1+\delta)^2\beta_z^s)(\beta_v + \beta_z^s)}\left|\sqrt{\frac{\beta_n}{\beta_z^s}}\right| \tag{A.85}$$

The two possible values, symmetric around -1, deliver the same allocation and level of welfare.

Note that this level of welfare is equivalent to the one attained by the optimal public FX intervention policy, $\widetilde{\mathbb{W}}^p$, reported in Proposition 4. Thus, by choosing a perfectly informative signal, while maximizing over (κ_a, κ_n) secret FX interventions can reproduce the allocation of the optimal public FX intervention policy and attain its level of welfare. One can also note that this secret policy is obtained by setting (κ_a, κ_n) according to (κ_a^p, κ_n^p) in Proposition 4. It is worth stressing that this is just one possible choice of $\bar{\beta}_z$ under secret conduct, but not necessarily the welfare-maximizing one.

Second, secret policy can attain a higher welfare than public if δ is sufficiently large. To see this, examine the asymptotic behavior of equation (A.87) as $\bar{\beta}_z$ tends to infinity. To begin with, identify and collect the leading terms in equation (A.87):

$$-\frac{\gamma\beta}{\omega_1}\frac{\alpha(\omega_2-\omega_1)^2}{\theta(1-\alpha)^2}\frac{1}{(\beta_v+\beta_a+\bar{\beta}_z)^2}(1-2\delta-2\delta^2)\bar{\beta}_z-\frac{\gamma\beta}{\omega_1}\tilde{\theta}\frac{\omega_2^2}{\omega_3}\frac{1}{\beta_a+(1+\delta)^2\bar{\beta}_z}$$

The sign of the dominant terms is positive if:

$$2\delta^4+6\delta^3+5\delta^2>1+\tilde{\theta}\frac{\omega_2^2}{\omega_3}\frac{\theta(1-\alpha)^2}{\alpha(\omega_2-\omega_1)^2} \tag{A.88}$$

The left-hand side of equation (A.88) is monotonically increasing in δ for $\delta>0$, while right-hand side of equation (A.88) is a positive constant. Define $\hat{\delta}\geq 0$ as the smallest positive root of the quartic equation (A.88). If $\delta>\hat{\delta}$, then the welfare function (A.87) reaches the optimal-public level of welfare from above as $\beta_z\to\infty$. This means that $\delta>\hat{\delta}$ is a sufficient condition for the secret FX intervention policy to dominate the optimal public FX policy in welfare terms. □

Proof of Proposition 6. First, rearrange (A.62) to express the variance of productivity forecast errors under secret FX interventions as follows:

$$\mathrm{var}(E_0^i a_1-a_1)=\left(\frac{\delta^2}{\beta_a}+\frac{(1-\delta^2)\beta_a+(1-\delta^2)\beta_v+(1-2\delta-2\delta^2)\beta_z}{(\beta_v+\beta_a+\beta_z)^2}\right) \tag{A.89}$$

Then, take the limit of eq. (A.89) for $\beta_z\to\infty$ and $\beta_z\to 0$:

$$\begin{aligned}\lim_{\beta_z\to\infty}\mathrm{var}(E_0^i a_1-a_1)&=\delta^2\frac{1}{\beta_a}\\ \lim_{\beta_z\to 0}\mathrm{var}(E_0^i a_1-a_1)&=\frac{(\delta\beta_v-\beta_a)^2\frac{1}{\beta_a}+(1+\delta)^2\beta_v}{(\beta_v+\beta_a)^2}\end{aligned} \tag{A.90}$$

Thus, $\lim_{\beta_z\to\infty} \mathrm{var}(E_0^i a_1 - a_1) < \lim_{\beta_z\to 0} \mathrm{var}(E_0^i a_1 - a_1)$ if $\delta < 1$. Moreover,

$$\frac{\partial\,\mathrm{var}(E_0^i a_1 - a_1)}{\partial \beta_z} = \frac{1}{(\beta_v + \beta_z + \beta_a)^3} \left\{(\beta_v + \beta_a)[1 - 2(1+\delta)] - \beta_z[1 - 2\delta(1+\delta)]\right\}. \tag{A.91}$$

We can distinguish two regions. If $\Leftrightarrow \delta \leq \bar{\delta} \equiv \frac{-1+\sqrt{3}}{2}$, *i.e.* $[1 - 2\delta(1+\delta)] > 0$, then ${}^{\partial\,\mathrm{var}(E_0^i a_1 - a_1)}/_{\partial\beta_z} < 0$, and the forecast error variance declines in public signal accuracy. Instead, if $\delta > \bar{\delta}$, then ${}^{\partial\,\mathrm{var}(E_0^i a_1 - a_1)}/_{\partial\beta_z} < 0$ as long as $\beta_z \leq (\beta_a + \beta_v)\frac{1-2(1+\delta)}{1-2\delta(1+\delta)}$ and ${}^{\partial\,\mathrm{var}(E_0^i a_1 - a_1)}/_{\partial\beta_z} > 0$ as long as $\beta_z > (\beta_a + \beta_v)\frac{1-2(1+\delta)}{1-2\delta(1+\delta)}$.

As a result, if $\delta \leq \frac{-1+\sqrt{3}}{2}$ the global minimum forecast error variance obtains at $\beta_z = \infty$. Instead, if $\delta > \frac{-1+\sqrt{3}}{2}$, the global minimum obtains at $\beta_z = (\beta_a + \beta_v)\frac{1-2(1+\delta)}{1-2\delta(1+\delta)}$. $\square$

F Diagnostic expectations

Diagnostic Expectations are a model of belief formation where probabilistic assessments depart from the Bayesian benchmark and are instead consistent with Kahneman and Tversky's representativeness heuristic, which they define as follows: "an attribute is representative of a class if it is very diagnostic; that is, the relative frequency of this attribute is much higher in that class than in the relevant reference class" (Tversky and Kahneman, 1983, p. 296). In this framework, individuals judge types or attributes as more likely when the are instead more representative, and provides experiments and evidence in support of this thesis. Based on this idea, Gennaioli and Shleifer (2010) construct a model in which belief biases arise because agents overweight events that are representative, *i.e.* the new information they receive. A growing literature applied this overweight bias to belief formation in macroeconomics and finance (for a review, see Bordalo et al., 2022). Here we describe this belief formation model and a related application to stereotype formation by Bordalo et al. (2016).

F.1 Representativeness

Consider a decision maker assessing the distribution of some trait T in group G, with true distribution $h(T = t|G)$. Gennaioli and Shleifer (2010) define the representativeness of trait $T = t$ in group G as

$$\frac{h(T = t|G)}{h(T = t|-G)} \tag{A.92}$$

where $-G$ indicates the relevant comparison group. That is, a trait is more representative if it is relatively more frequent in group G than $-G$. This framework assumes that representative events are easier to recall, and therefore agents with a limited working memory overweight these types in their assessment. The degree of representativeness of a trait is state-dependent, as it depends on the comparison group $-G$.

To illustrate, consider the case of an individual forming beliefs about the distribution of hair color among the Irish population, from Bordalo et al. (2019). The trait T is hair color, the group G is the Irish and the comparison group $-G$ is the rest of the world population. Suppose the true relevant distributions are

	$T =$ red	$T =$ blond/light brown	$T =$ dark brown
$G \equiv$ Irish	10%	40%	50%
$-G \equiv$ World	1%	14%	85%

Red is the most representative hair color as it is associated with the higher likelihood ratio among hair colors

$$\frac{\Pr(\text{red hair} \mid \text{Irish})}{\Pr(\text{red hair} \mid \text{World})} = \frac{10\%}{1\%} = 10 \tag{A.93}$$

The representative expectation model would therefore predict that agents overestimate the frequency of red-haired Irish. As this color is more representative of the Irish population, it is the one the comes to mind when thinking of group Irish, which results in its relative frequency being exaggerated in subjective judgments and the frequency of the other colors discounted. Gennaioli and Shleifer (2010) and Bordalo et al. (2016) show that this model rationalizes several widely documented errors in probabilistic judgment, such as base rate neglect and the conjunction fallacy, and sheds light on key features of social stereotypes and context-dependent beliefs.

F.2 Diagnostic expectations about economic conditions

We can apply the same logic to belief formation about economic conditions. Consider a case similar to Section 1.3, where agents form belief about a state a with prior distribution $a \sim N(\mu_a, \chi^{-1})$.[31] Agents observe a noisy signal about the realization, $s = a + e$, with $e \sim N(0, \tau^{-1})$.

[31] This formulation embeds an AR(1) structure $a = \rho a_0 + u$, with $\mu_a \equiv \rho a_0$ and $u \sim N(0, \chi^{-1})$.

The intuition underlying the hair color example can be applied to this setting, where the agents forms posterior belief about the state a_1. According to Bayes' rule, the probability density is

$$f(a|s) = \frac{f(s|a)f(a)}{\int f(s|a)f(a)\mathrm{d}a} \tag{A.94}$$

with $f(a)$ indicating the unconditional probability density, that is the prior in absence of additional information. With the normality assumption, the Bayesian posterior would be $a|s \sim N(\mu_a + \frac{\tau}{\chi+\tau}(s - \mu_a), (\tau + \chi)^{-1})$.

Similarly to above, agents have this true distribution in the back of their mind, but selectively retrieve and thus overweight realizations of a that are representative/diagnostic of G compared to $-G$. While before G ="Irish" and $-G$ ="Rest of the world", here, following the literature, we take a dynamic perspective and take "context" as reflecting the information held before observing the signal. In other words, $-G$ is the state prevailing if there is no news, and G is the state prevailing after observing the signal. In line with Bordalo et al.'s (2020) proposal for a diagnostic Kalman filter, we define the representativeness of a state a at as the likelihood ratio:

$$R(a) = \frac{f(a|s)}{f(a|s = \mu_a)} \tag{A.95}$$

State a is more representative if the signal s received in this period raises the probability of that state relative to the case where the news equals the *ex ante* forecast, $s = \mu_a$.

The forecaster will then overweight states that are representative by using the distorted posterior

$$f^{\delta}(a|s) = f(a|s)R(a)^{\delta}\frac{1}{Z} \tag{A.96}$$

where Z is a normalization factor ensuring that $f^{\delta}(a|s)$ integrates to one. Parameter δ denotes the extent to which beliefs depart from rational updating due to representativeness. If $\delta = 0$, beliefs are rational, and the posterior distribution is the Bayesian one, $f(a|s)$. If $\delta > 0$, the diagnostic density $f^{\delta}(a|s)$ inflates the probability of representative states and deflates the probability of unrepresentative ones (we assume that all agents have the same δ). Intuitively, future events that are relatively more associated with the signal, that is those that become more likely, are more accessible from the agent's memory and are overweighted in judgments.

As shown by Bordalo et al. (2020), with a normally distributed prior and signal, the distorted belief (A.96) is also normal.[32] The posterior variance is the same as in the Bayesian case, $\text{var}^\delta(a|s) = \text{var}(a|s) = (\tau + \chi)^{-1}$, and the posterior mean equals

$$E^\delta[a|s] = \mu_a + (1+\delta)\frac{\tau}{\chi + \tau}(s - \mu_a) \tag{A.97}$$

In the particular case of $\mu_a = 0$, then $E^\delta[a|s] = (1+\delta)\mathbb{E}[a|s]$ as in equation (21).

G Information effect of public FX interventions

In case of publicly communicated FX interventions, the agents can combine the two signals available, the exchange rate—perceived to follow eq. (26)—and the FX intervention (32), to back out the average expectations $\bar{E}_0[a_1]$, as follows:

$$z^p \equiv \left(\frac{f_1^\star}{\kappa_n} - \frac{q_0}{\frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}}\right) \Bigg/ \left(\frac{\kappa_a(1+\delta)}{\kappa_n} - \frac{\omega_2}{\Gamma\omega_1}\right) = \bar{E}_0 a_1 \tag{A.98}$$

Under rational expectations, this implies observing the average private signal and therefore perfectly back out the level of productivity a_1, such that $\bar{E}_0[a_1] = \bar{\mathbb{E}}[a_1] = a_1$. However, under the DE framework with higher order belief bias illustrated in Section 1.3, agents make mistakes in mapping the average expectations to the level of productivity. In order to find the equilibrium belief, one needs to solve a fixed point problem.

Under public communication, agents observe a signal that perfectly reveals average expectation, $z^p = \bar{E}_0[a_1]$. Moreover, they observe private signal $v^i = a_1 + \epsilon^i$, with $\epsilon^i \sim N(0, \beta_v^{-1})$ and $\int_i \epsilon^i \, \mathrm{d}i = 0$, and the common prior $a_1 \sim N(0, \beta_a^{-1})$. Agents are behaviorally biased in that (i) overreact to the exogenous private signal according to parameter $\delta > 0$ (DE1), and (ii) they think of themselves and everyone else as rational (DE2).

To solve the fixed point between individual and average expectations, first we guess a solution for the *perceived* average expectation (i.e. public signal), $\frac{z^p}{\tilde{\lambda}}\Big| a_1 \overset{p}{\sim} \mathcal{N}(a_1, 1/\beta_z)$, where $\tilde{\lambda}$ is determined as the solution of the fixed point problem. As agents think of themselves and everyone else as rational, they believe average expecta-

[32] Our framework is a simplified version of the AR(1) case outlined in Bordalo et al.'s (2020) Proposition 1, where the autoregressive parameter equals zero, *i.e.* $\rho = 0$.

tions don't exhibit overreaction. Thus, they perceive that:

$$z^p = \int^i \mathbb{E}_0^i a_1 di = \frac{\beta_v a_1 + \beta_z \frac{z^p}{\tilde{\lambda}}}{\beta_v + \beta_z + \beta_a} \tag{A.99}$$

and solving the fixed point

$$\begin{aligned}\left[1 - \frac{\beta_z \frac{1}{\tilde{\lambda}}}{\beta_v + \beta_z + \beta_a}\right] \tilde{\lambda} &= \frac{\beta_v}{\beta_v + \beta_z + \beta_a} \\ \tilde{\lambda} &= \frac{\beta_v + \beta_z}{\beta_v + \beta_z + \beta_a}\end{aligned} \tag{A.100}$$

since the signal z^p is perceived to be perfectly accurate, $\beta_z \to \infty$, which gives $\tilde{\lambda} = 1$ and therefore perceived average expectation follows

$$z^p = \bar{\mathbb{E}}_0[a_1] = a_1 \tag{A.101}$$

If agents were rational, then perceived and actual average expectations would coincide and $\bar{\mathbb{E}}_0[a_1] = a_1$ is the solution, as explained above. However, if agents are subject to the aforementioned behavioral biases, then actual average expectations equal $z^p = \lambda a_1$, where λ satisfies

$$z^p = \int^i E^i a_1 di = \frac{(1+\delta)\beta_v a_1 + \beta_z \frac{z^p}{\tilde{\lambda}}}{\beta_v + \beta_z + \beta_a}, \tag{A.102}$$

and thus:

$$\begin{aligned}\left[1 - \frac{\beta_z \frac{1}{\tilde{\lambda}}}{\beta_v + \beta_z + \beta_a}\right] \lambda &= (1+\delta)\frac{\beta_v}{\beta_v + \beta_z + \beta_a} \\ \lambda &= (1+\delta)\frac{\beta_v + \beta_z}{\beta_v + \beta_z + \beta_a}\end{aligned} \tag{A.103}$$

As $\beta_z \to \infty$, one obtains that $\lambda = (1+\delta)$. As a result, actual average expectations equal

$$\bar{E}_0[a_1] = (1+\delta)a_1.$$

H Alternative belief formation models

Our baseline framework of higher-order beliefs—encoded in assumption DE2—implies that agents systematically underestimate the response of other agents. This form of higher order belief bias leads to endogenous overreaction to public signals. In this appendix, we show that our substantive results obtain also under the following alternatives: (i) a setting without higher-order bias but in which agents are diagnostic with respect to both private and public signals; (ii) a Partial Equilibrium Thinking model as in Bastianello and Fontanier (2022), where agents use private signals optimally, but fail to realize other agents also learn from prices.

H.1 Diagnostic expectation without higher order belief bias

We explore a different version of the Diagnostic Expectation model considered in Section 1.3. First, we assume that the representativeness heuristic apply to both exogenous private signals and endogenous public signal. For simplicity, we focus on the laissez faire equilibrium where the only public signal is the equilibrium exchange rate q_0. When agents extrapolate both their private and public signals, they use distorted posterior

$$h^{\delta}(a_1|v^i, q_0) = h(a_1|v^i, q_0)\left[\frac{h(a_1|v^i, q_0)}{h(a_1|v^i = 0, q_0 = 0)}\right]^{\delta}\frac{1}{Z} \tag{A.104}$$

where $h(a_1|v^i, q_0)$ is the true Bayesian posterior distribution of a_1 after observing the signals v^i and q_0, and $h(a_1|v^i = 0, q_0 = 0)$ is the posterior if the signals observed equal the prior, which in this case is zero. For simplicity, with some abuse of terminology, we refer to this latter case as "receiving no news." The term $\frac{h(a_1|v^i,q_0)}{h(a_1|v^i=0,q_0=0)}$ reflects the representativeness of a_1, which is higher if the signals received in this period raise the probability of that state relative to the case where the news equals the prior (no news). The parameter $\delta \geq 0$ captures the strength of the impact of representativeness on judgments (if $\delta = 0$, memory is frictionless and agents are Bayesian). Finally, Z is a normalizing constant ensuring that $h^{\delta}(a_1|v^i, q_0)$ integrates to 1. Under the normality assumption, the conditional expectations of a_1 is

$$E^i[a_1|v^i, q_0] = (1 + \delta)\mathbb{E}^i[a_1|v^i, q_0]. \tag{A.105}$$

Differently from the belief formation model considered in the baseline framework, we assume that each agent knows that others are behaviorally biased (while they incorrectly think themselves as rational). As a result, agents correctly perceive the exchange rate as following

$$q_0 = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3} n_1^\star - \frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3} \bar{E}_0 a_1. \tag{A.106}$$

As agents correctly perceive the exchange rate process, we solve the fixed point problem by guessing the actual law of motion. We conjecture that the equilibrium exchange rate process depends linearly on future productivity a_1 and noise trading $n_1^\star$

$$q_0 = \lambda_a a_1 + \lambda_n n_1^\star, \tag{A.107}$$

The exchange rate process depends on aggregate expectations about productivity, but it is itself an informational source for agents when forming their beliefs. As a result, the relationship between the exchange rate and the two shocks, governed by $(\lambda_a,\, \lambda_n)$, is determined as the solution of a fixed point problem. In particular, one can rewrite eq. (A.107) as

$$\frac{q_0}{\lambda_a} = a_1 + \frac{\lambda_n}{\lambda_a} n_1^\star. \tag{A.108}$$

In this formulation, q_0/λ_a represents an unbiased signal centered around future productivity a_1 with a error variance of $\beta_z^{-1} \equiv (\lambda_n^2/\lambda_a^2)\beta_n^{-1}$.

To sum up, agent i accesses three sources of information: (i) the prior distribution of a_1; (ii) the private signal, eq. (19); (iii) the exchange rate, eq. (A.108). Following eq (A.105), their posterior is

$$E^i[a_1|v^i, q_0] = (1+\delta)\frac{\beta_v v^i + \beta_z \frac{q_0}{\lambda_a}}{\beta_v + \beta_z + \beta_a}, \tag{A.109}$$

We can average posterior beliefs $\bar{E}[a_1|v^i, q_0] \equiv \int^i E^i[a_1|v^i, q_0]\, \mathrm{d}i$ using $\int^i v^i\, \mathrm{d}i = a_1$ and substitute back in the exchange rate process, eq. (26), to verify the conjecture, eq. (A.107). The following proposition characterizes the unique equilibrium of the model economy.

Proposition H.1. *The symmetric linear market equilibrium is unique and the ex-*

change rate process is described by eq. (A.107) with coefficients

$$\begin{aligned} \lambda_a &= -(1+\delta)\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1+\omega_3}\frac{\beta_v+\Lambda^2\beta_n}{\beta_a+\beta_v+\Lambda^2\beta_n} \\ \lambda_n &= \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1+\omega_3}\frac{\beta_v+\Lambda^2\beta_n}{\beta_v} \end{aligned} \tag{A.110}$$

where $\Lambda \equiv \frac{\lambda_a}{\lambda_n}$*, and* Λ^2 *is unique and implicitly defined by*

$$\Lambda^2 = \left((1+\delta)\frac{\omega_2}{\Gamma\omega_1}\right)^2 \frac{\beta_v^2}{(\beta_a+\beta_v+\Lambda^2\beta_n)^2} \tag{A.111}$$

Proof. The proof follows from the proof of Proposition 1 in Appendix E. □

Forecast error variance Agents extrapolate both private and public signal according to parameter $\delta \geq 0$. The forecast error variance in this model equals

$$\mathrm{var}(E_0^i a_1 - a_1) = \frac{[\delta(\beta_v+\beta_z)-\beta_a]^2}{(\beta_v+\beta_z+\beta_a)^2}\frac{1}{\beta_a} + \frac{(1+\delta)^2(\beta_z+\beta_v)}{(\beta_v+\beta_z+\beta_a)^2} \tag{A.112}$$

The implications for the forecast error variance with this belief formation model are very similar to the baseline framework. If the exchange rate is perfectly accurate, $\beta_z \to \infty$, the economy operates the consensus forecast error variance remains positive, $\mathrm{var}(\bar{E}_0 a_1 - a_1) = \delta^2 \frac{1}{\beta_a}$. Even if the informational content of the exchange rate is infinitely precise, and it reveals average expectations, agents overreact to this information due to extrapolative beliefs.

More generally, for large enough extrapolative bias δ, higher public signal precision has two opposing effects on the forecast error variance. First, like in the rational expectation case, higher precision provides more information, lowering the forecast error variance. Second, higher precision increases the relative weight on the public signal in posterior belief, and therefore it amplifies the degree of extrapolation, increasing the forecast error variance. We show the first effect prevails for low values of signal accuracy, while the second effect prevails for larger values of accuracy.

Proposition H.2 (Forecast error variance with extrapolative expectations)**.** *If* $\delta < \bar{\delta}$*, then the unconditional variance of the individual forecast error on productivity is minimized with perfectly informative public signal* $\beta_z \to \infty$*. If* $\delta > \bar{\delta}$*, then the unconditional variance of the individual forecast error on productivity is minimized with perfectly un-*

informative public signal $\beta_z \to 0$, where $\bar{\delta} \equiv 1$.

Proof. Take the derivative of (A.112) with respect to public signal accuracy β_z

$$\frac{\partial \operatorname{var}(E_0^i a_1 - a_1)}{\partial \beta_z} = \frac{(\beta_v + \beta_z + \beta_a)}{(\beta_v + \beta_z + \beta_a)^4}(1+\delta)\left(2[\delta(\beta_v + \beta_z) - \beta_a] - (1+\delta)((\beta_v + \beta_z) - \beta_a)\right) \tag{A.113}$$

Therefore

$$\operatorname{sign}\left(\frac{\partial \operatorname{var}(E_0^i a_1 - a_1)}{\partial \beta_z}\right) = \operatorname{sign}(1-\delta) \tag{A.114}$$

Moreover,

$$\begin{aligned} \lim_{\beta_z \to \infty} \operatorname{var}(E_0^i a_1 - a_1) &= \frac{\delta^2}{\beta_a} \\ \lim_{\beta_z \to 0} \operatorname{var}(E_0^i a_1 - a_1) &= \frac{(\delta\beta_v - \beta_a)^2}{(\beta_a + \beta_v)^2 \beta_a} + \frac{(1+\delta)^2 \beta_v}{(\beta_a + \beta_v)^2} \end{aligned} \tag{A.115}$$

therefore, $\lim_{\beta_z \to \infty} \operatorname{var}(E_0^i a_1 - a_1) \gtreqless \lim_{\beta_z \to 0} \operatorname{var}(E_0^i a_1 - a_1) \iff \delta \gtreqless 1$. □

The qualitative implications in terms of forecast errors are therefore similar to the baseline model, *i.e.* Proposition 6. However, the threshold $\bar{\delta}$ above which the forecast error variance is increasing in public signal accuracy is higher.

H.2 Partial equilibrium thinking

We consider now a different believe updating model, where the only bias is in the formation of higher order beliefs. Similarly to the Partial Equilibrium Thinking model of Bastianello and Fontanier (2022), suppose that agents update their belief rationally with respect to the exogenous signal, but fail to realize that the other agents are learning from the exchange rate as well. As a result, they make mistakes interpreting the endogenous signal. In particular, they perceive the exchange rate to be equal to:

$$q_0 \overset{p}{=} \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3} n_1^\star - \frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3} \bar{\mathbb{E}}_0[a_1|v^i] \tag{A.116}$$

where $\bar{\mathbb{E}}_0[a_1|v^i]$ is the average Bayesian posterior from observing *only* the exogenous private signal:

$$\bar{\mathbb{E}}_0[a_1|v^i] = \frac{\beta_v}{\beta_a + \beta_v} a_1. \tag{A.117}$$

The *perceived* exchange rate thus follows

$$\tilde{q}_0 = \tilde{\lambda}_a a_1 + \tilde{\lambda}_n n_1^\star, \tag{A.118}$$

from which they extract the signal

$$\frac{\tilde{q}_0}{\tilde{\lambda}_a} = a_1 + \frac{\tilde{\lambda}_n}{\tilde{\lambda}_a} n_1^\star \tag{A.119}$$

with

$$\tilde{\lambda}_a = -\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\frac{\beta_v}{\beta_a + \beta_v} \qquad \tilde{\lambda}_n = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3} \tag{A.120}$$

where the perceived accuracy of the exchange rate is $\tilde{\beta}_z \equiv \frac{\tilde{\lambda}_a^2}{\tilde{\lambda}_n^2}\beta_n = \left(\frac{\omega_2}{\Gamma\omega_1}\frac{\beta_v}{\beta_a+\beta_v}\right)^2\beta_n$.

Agents form expectations using perceived exchange rate. Actual expectations equal

$$\bar{E}_0[a_1] = \frac{\beta_v a_1 + \tilde{\beta}_z \frac{\tilde{q}_0}{\tilde{\lambda}_a}}{\beta_a + \beta_v + \tilde{\beta}_z} \tag{A.121}$$

Substituting back in the actual exchange rate process, one can find the solution for the actual exchange rate, which equals

$$q_0 = \lambda_a a_1 + \lambda_n n_1^\star, \tag{A.122}$$

with

$$\lambda_a = -\frac{\omega_2}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\frac{\beta_v^2}{\beta_v(\beta_v + \beta_a) - \beta_a\tilde{\beta}_z} \qquad \lambda_n = \frac{\Gamma\omega_1}{\Gamma\tilde{\theta}\omega_1 + \omega_3}\frac{\beta_v(\beta_a + \beta_v + \tilde{\beta}_z)}{\beta_v(\beta_v + \beta_a) - \beta_a\tilde{\beta}_z} \tag{A.123}$$

As in the baseline model, this higher order belief bias leads to misperception of the exchange rate process, and therefore mistakes in information extraction. The signal agents actually use is

$$\frac{q_0}{\tilde{\lambda}_a} = \frac{\lambda_a}{\tilde{\lambda}_a}a_1 + \frac{\lambda_n}{\tilde{\lambda}_a}n_1^\star = \frac{\beta_v(\beta_v + \beta_a)}{\beta_v(\beta_v + \beta_a) - \beta_a\tilde{\beta}_z}a_1 - \frac{\Gamma\omega_1}{\omega_2}\frac{(\beta_v + \beta_a + \tilde{\beta}_z)(\beta_v + \beta_a)}{\beta_v(\beta_v + \beta_a) - \beta_a\tilde{\beta}_z}n_1^\star, \tag{A.124}$$

Equation (A.124) reveals that the actual accuracy of the signal they use is $\beta_z = \left(\frac{\omega_2}{\Gamma\omega_1}\frac{\beta_v(\beta_v+\beta_a)-\beta_a\tilde{\beta}_z}{(\beta_v+\beta_a+\tilde{\beta}_z)(\beta_v+\beta_a)}\right)^2\beta_n$, which is distinct from the perceived accuracy $\tilde{\beta}_z$. In particular:

$$\tilde{\beta}_z = \beta_z \left(\frac{\beta_v(\beta_v + \beta_a + \tilde{\beta}_z)}{\beta_v(\beta_v + \beta_a) - \beta_a \tilde{\beta}_z} \right)^2 \tag{A.125}$$

There are two difference between the perceived signal (A.119) and the actual signal (A.124). First, the signal is not centered around the realization a_1, but it is multiplied by the term $\frac{\beta_v(\beta_v+\beta_a)}{\beta_v(\beta_v+\beta_a)-\beta_a\tilde{\beta}_z} \equiv \Psi_1$. Agents thus misperceive the mean of the endogenous signal, similarly to our baseline model of beliefs. However, in this case agents also misperceive the accuracy of the endogenous signal: their perceived accuracy equals the actual one multiplied by $\Psi_2 \equiv \left(\frac{\beta_v(\beta_v+\beta_a+\tilde{\beta}_z)}{\beta_v(\beta_v+\beta_a)-\beta_a\tilde{\beta}_z}\right)^2$.

The forecast error variance equals

$$\operatorname{var}(E^i a_1 - a_1) = \frac{\left[\tilde{\beta}_z \left(\frac{\beta_a \tilde{\beta}_z}{\beta_v(\beta_v+\beta_a)-\beta_a\tilde{\beta}_z}\right) - \beta_a\right]^2 \beta_a^{(-1)} + \tilde{\beta}_z \left(\frac{\beta_v(\beta_v+\beta_a+\tilde{\beta}_z)}{\beta_v(\beta_v+\beta_a)-\beta_a\tilde{\beta}_z}\right)^2 + \beta_v}{(\beta_v + \beta_a + \tilde{\beta}_z)^2} \tag{A.126}$$

As described in Section 3, secret interventions can manipulate the informativeness of the exchange rate. Consider what happens when the perceived accuracy varies:

- If $\tilde{\beta}_z = 0$, the perceived signal equal the actual signal: the bias comes from agents thinking other agents do not use the public signal to update, and in this case they are correct.

$$\operatorname{var}(E^i a_1 - a_1) = \frac{1}{\beta_v + \beta_a} \tag{A.127}$$

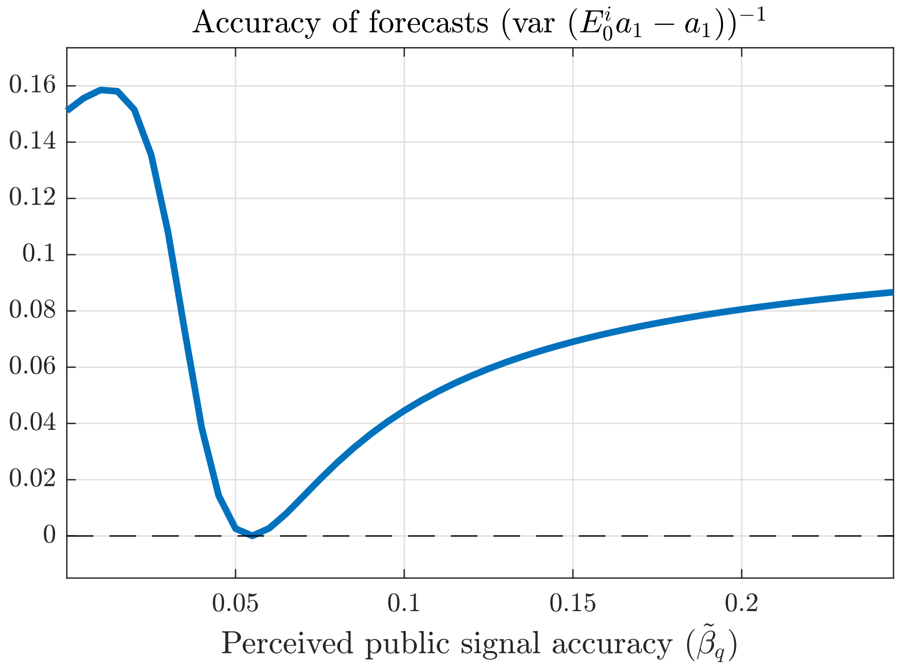
- If $\beta_v(\beta_v + \beta_a) - \beta_a\tilde{\beta}_z > 0$, agents misinterpret the endogenous signal for two reasons. First, as $\Psi_1 > 0$, agents think the public signal loads less on the shock than it actually does. Similarly to the baseline model, this leads to overreaction to the endogenous signal. Second, the term $\Psi_2 > 0$, meaning that perceived accuracy is higher than the actual, leading agents to overweight the public signal and therefore also overreacting to it.

- As the perceived precision of the public signal increases (that is, $\beta_v(\beta_v + \beta_a) - \beta_a\tilde{\beta}_z \to 0$, and both Ψ_1 and Ψ_2 go to infinity), this implies infinite overreaction and thus:

$$\operatorname{var}(E^i a_1 - a_1) \to \infty \tag{A.128}$$

- As the perceived precision of the public signal increases further (that is $\beta_v(\beta_v +$

Figure A.2: Forecast error accuracy with Partial Equilibrium Thinking



Notes: This figure reports the precision of forecasts, $(\text{var}(E_0^i a_1 - a_1))^{-1}$, where $\text{var}(E_0^i a_1 - a_1)$ is defined in (A.126), as a function of the perceived public signal accuracy $\tilde{\beta}_z$. The rest of parameters are set according to Table A.1.

$\beta_a) - \beta_a \tilde{\beta}_z < 0$ and therefore $\Psi_1 > 0$) agents incorrectly think the signal positively correlates with productivity, and thus they update in the opposite direction.

- If $\tilde{\beta}_z \to \infty$, the signal does not load anymore on productivity a_1, as the only equilibrium is one in which neither expectations nor the exchange rate respond to changes in the productivity, and:

$$\text{var}(E^i a_1 - a_1) = \frac{1}{\beta_a} \tag{A.129}$$

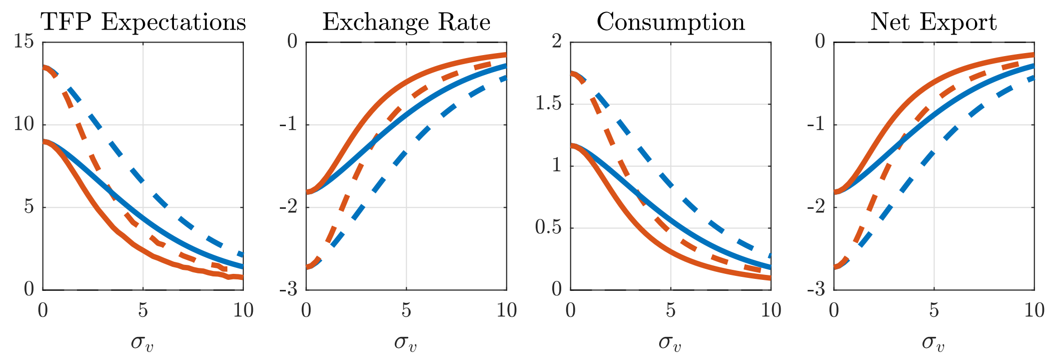
To sum up, for certain values of $\tilde{\beta}_z$, this belief formation model also leads to overreaction, not only through a misinterpretation of the signal mean, but also a misinterpretation of its accuracy. As a result, the level of exchange rate informativeness that minimizes the forecast error variance is not infinite, but finite. While this result echoes Proposition 6, its characterization is not available in closed form solution. Therefore, we provide a numerical illustration in Figure A.2.

Parameter	Interpretation	Value
β	Discount Factor	0.99
α	Share of Capital in Tradable	0.33
γ	Trade Opennes	0.33
θ	Trade Elasticity	0.67
σ	CRRA parameter	10.00
Γ	Intermediation Friction	5.00
σ_a	Std. dev. of productivity shocks	3.00
σ_v	Std. dev. of private signal	5.00
σ_n	Std. dev. of noise-trading shock	0.25

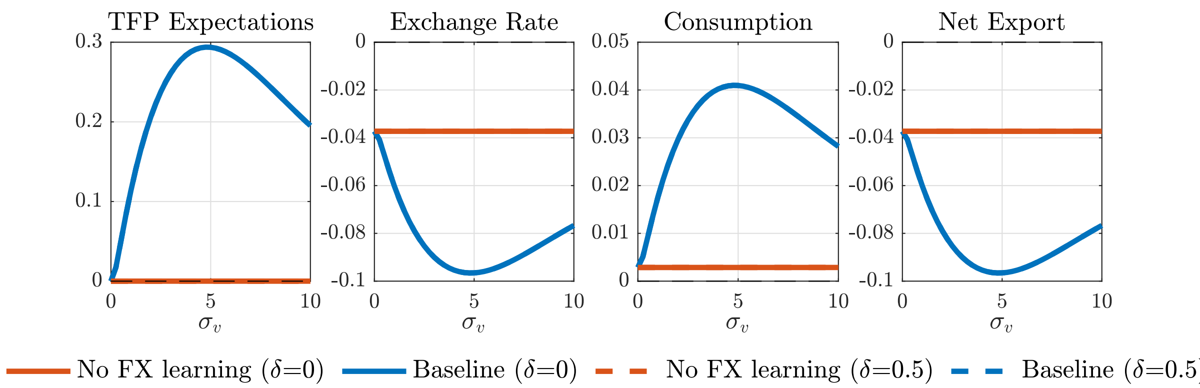
Table A.1: Baseline parameterization for numerical illustrations

Figure A.3: Equilibrium responses to shocks under dispersed information

(a) Response to one-standard deviation productivity shock ($a_1 \uparrow$)



(b) Response to one-standard deviation noise-trading shock ($n_1^\star \downarrow$)



Notes: This figure reports the equilibrium response of model variables to productivity and noise-trading shocks for different levels of the noise in private signal, σ_v, under laissez faire. The parameter governing the extent of belief extrapolation is $\delta = [0; 0.5]$. The rest of parameters are set according to Table A.1.