Example 2.3: Consider the set of empirical measurements of P_r/P_t given in the table below for an indoor system at 2 GHz. Find the path loss exponent γ that minimizes the MSE between the simplified model (2.29) and the empirical dB power measurements, assuming that $d_0 = 1$ m and K is determined from the free space path loss formula at this d_0. Find the received power at 150 m for the simplified path loss model with this path loss exponent and a transmit power of 1 mW (0 dBm).

Distance from Transmitter	$M = P_r/P_t$
10 m	-70 dB
20 m	-75 dB
50 m	-90 dB
100 m	-110 dB
300 m	-125 dB

Table 2.2: Path Loss Measurements

Solution: We first set up the MMSE error equation for the dB power measurements as

$$F(\gamma) = \sum_{i=1}^{5}[M_{\text{measured}}(d_i) - M_{\text{model}}(d_i)]^2,$$

where $M_{\text{measured}}(d_i)$ is the path loss measurement in Table 2.2 at distance d_i and $M_{\text{model}}(d_i) = K - 10\gamma\log_{10}(d)$ is the path loss based on (2.29) at d_i. Using the free space path loss formula, $K = -20\log_{10}(4\pi)/.3333 = -31.54$ dB. Thus

$$\begin{aligned} F(\gamma) &= (-70+31.54+10\gamma)^2 + (-75+31.54+13.01\gamma)^2 + (-90+31.54+16.99\gamma)^2 \\ &+ (-110+31.54+20\gamma)^2 + (-125+31.54+24.77\gamma)^2 \\ &= 21676.3 - 11654.9\gamma + 1571.47\gamma^2. \end{aligned} \tag{2.31}$$

Differentiating $F(\gamma)$ relative to γ and setting it to zero yields

$$\frac{\partial F(\gamma)}{\partial \gamma} = -11654.9 + 3142.94\gamma = 0 \rightarrow \gamma = 3.71.$$

To find the received power at 150 m under the simplified path loss model with $K = -31.54$, $\gamma = 3.71$, and $P_t = 0$ dBm, we have $P_r(dBm) = P_t(dBm) + K(dB) - 10\gamma\log_{10}(d/d_0) = 0 - 31.54 - 10 * 3.71\log_{10}(150) = -112.27$ dBm. Clearly the measurements deviate from the simplified path loss model: this variation can be attributed to shadow fading, described in Section 2.7