Final Project Report

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Contents

1 Introduction

The purpose of the following report is to describe the gravitational effects on a Free-Flyer, taking into account both its rendezvous with a Chief (Task 1) and its internal rotational motion (Task 2).

1.1 Team-Member's Role and Contributions

The Work Breakdown Structure in Figure 1 illustrates the main tasks of the project and their distribution among the team members, with an indication of the time assigned to each one.

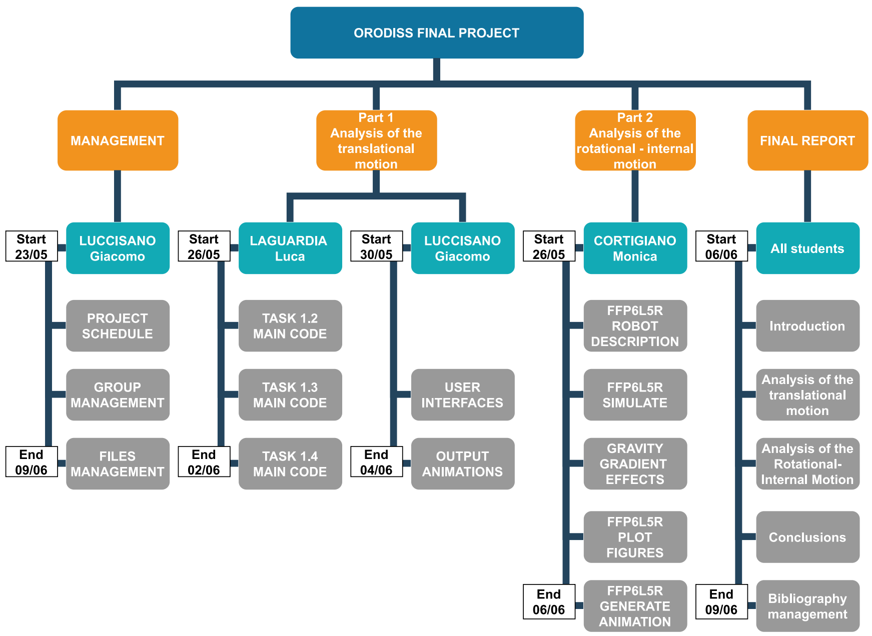

Figure 1: Project Work Breakdown Structure

This approach turned out to be a crucial tool for the project: it enabled the team to operate with clear goals for each member and, as a result, to accurately track the tasks' progress at all times.

1.2 Modelling Assumptions

It is appropriate to approach the study with the following assumptions:

- Motion happens in the plane of orbit.
- The Chief is moving along a circular orbit, at 500 km of altitude, according to the KR2BP mode.
- The base body and the manipulator have similar mass value.
- The free-flyer has a square body with a protuding 5 revolute DOF robotic manipulator.
- The bodies involved are isolated in the space considered.
- Earth is spherical and homogeneous.

- Gravity is Newtonian and is the only force acting on the bodies.
- The distance between the Target and the Chief is much smaller that their distance from the center of the Earth.

1.3 Quantities Involved

It's crucial to take into account several mathematical parameters to appropriately analyze the modeled scenario. The primary factors for the Translational Motion Analysis are:

- Mass of the chief (target): represents the mass of the object moving along the circular orbit.
- Circular orbit radius: the radius of the circular orbit can be calculated by adding the radius of the Earth to the altitude of the orbit.
- Earth gravity constant: approximately 3.986 km^3/s^2.
- Orbital period: calculated by using Kepler's Third Law.
- Inertial properties: they are referred to the the free-flyer and include the moment of inertia and mass distribution, necessary to analyze the dynamics of the system.
- Robot manipulator kinematics: the dimensions and joint parameters of the robotic manipulator, such as link lengths or widths and joint angles

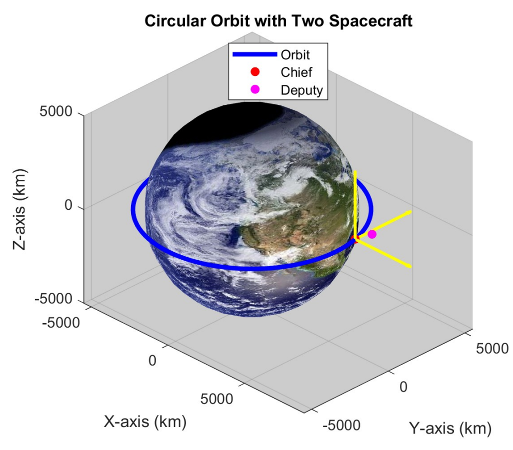

Figure 2: Translational Motion

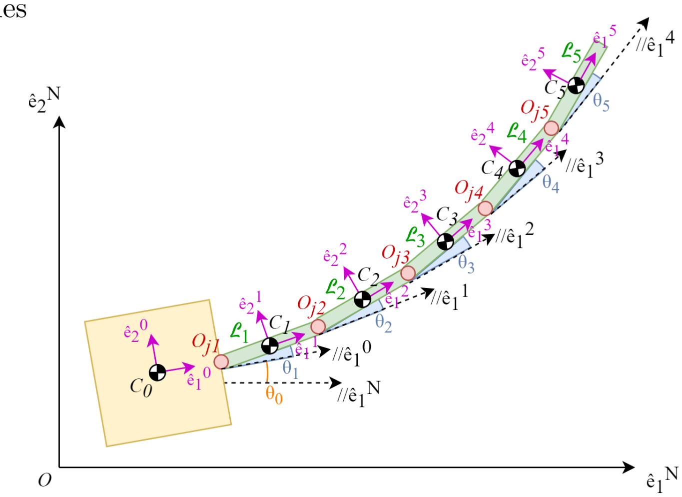

Figure 3: Internal Rotational Motion

For the Rotational-Internal Motion, instead, the followings are stated:

- Mass of the free-flyer: it includes both the manipulator and the base body. As mentioned, the two masses are chosen similar in value.
- Type of Joints, revolute in our case study.
- Parameters of each link considering the Denavit-Hartenberg convention.
- Equations of motion: accounting for gravitational forces, inertial forces, and any external forces or torques acting on the system.

- Control strategy: desired tasks and eventual constraints.
- The system is analyzed by means of the recursive approach (Netwon-Euler), as it enables to solve complex translational and rotational equations. In addition it is more efficient than Lagrangian Approach for systems composed by many bodies.

2 Analysis of the translational motion

The primary objective of this analysis is to examine the translational behavior of the Free-Flyer, considering it as a point mass. This simplification allows us to focus solely on its overall motion, without considering its internal structure and rotations.

2.1 Analysis of the spontaneous motion of deputy relative to target for different I.C.

Using Hill's equations, we can determine what happens to the Interceptor's orbit - which is elliptical shaped - by changing its initial position and velocity with respect to the Target.

$$\begin{cases} x\left(t\right)=\frac{\dot{x}_0}{\omega}sin\left(\omega t\right)-\left(3x_0+\frac{2\dot{y}_0}{\omega}\right)cos\left(\omega t\right)+\left(4x_0+\frac{2\dot{y}_0}{\omega}\right) \\ y(t)=\left(6x_0+\frac{4\dot{y}_0}{\omega}\right)sin(\omega t)+\frac{2\dot{x}_0}{\omega}cos\left(\omega t\right)-(6\omega x_0+3\dot{y}_0)t+\left(y_0-\frac{2\dot{x}_0}{\omega}\right) \\ z(t)=z_0cos\left(\omega t\right)+\frac{\dot{z}_0}{\omega}sin\left(\omega t\right) \\ \dot{x}\left(t\right)=\dot{x}_0cos\left(\omega t\right)+\left(3\omega x_0+2\dot{y}_0\right)sin\left(\omega t\right) \\ \dot{y}\left(t\right)=\left(6\omega x_0+4\dot{y}_0\right)cos\left(\omega t\right)-2\dot{x}_0sin\left(\omega t\right)-\left(6\omega x_0+3\dot{y}_0\right) \\ \dot{z}\left(t\right)=-z_0\omega sin\left(\omega t\right)+\dot{z}_0cos\left(\omega t\right) \end{cases}$$

To begin with, changes in the initial position x_0 entail orbits with a different period because the semi-major axis changes. The initial velocity is the same as the Target's, but the Deputy is in a higher (or lower) orbit, meaning it's in a slightly eccentric orbit, which creates the wobbles. Since an elliptical orbit doesn't have a constant velocity throughout the orbit, the Interceptor will appear to move faster or slower with respect to the Target's constant motion; thus, the loops.

As for changes in velocity, any non zero value of $\dot{x}_0$ produces an osculating ellipse, changing the eccentricity of the Interceptor's orbit but not the period. Because of this, both satellites rendezvous every period. Non zero values of $\dot{y}_0$ also produce an elliptical motion, changing both the eccentricity and the semi-major axis (period). Because the period has changed, the Interceptor rendezvous with the Target's altitude in one period of the Interceptor.

Coupled variations of x_0 and $\dot{x}_0$ are a bit more challenging: the initial x_0 displacement means the Interceptor is in a slightly eccentric orbit, while the variation of $\dot{x}_0$ simply magnifies the change in eccentricity. More interesting is the combined effect of the variations of x_0 and $\dot{y}_0$, which turns out to solely determine the direction the Interceptor drifts with respect to the Target. Depending on the value of $\dot{y}_0$, the resulting motion may be either to the right, left, or in "orbit" about the Target.

Finally, any z values superimpose an oscillatory motion over the x-y motion, which remains unaffected. Any initial $\dot{z}_0$ increases the amount of the variation in z and any initial displacement z_0 has a corresponding oscillation.

2.2 Minimum total ΔV two-impulse randezvous maneuver

The aim of this work is to find the precise rendezvous maneuver that guarantees the lowest cost in terms of total ΔV needed. The two-impulse linear rendezvous maneuver is carried out in two stages as follows:

1. We initially impose an impulsive maneuver, known as *targeting*, that nullifies the final relative distance between the Chief and the Deputy.
2. Once the relative distance has been nullified, we impose a second impulsive maneuver to cancel the relative velocity and complete the rendezvous.

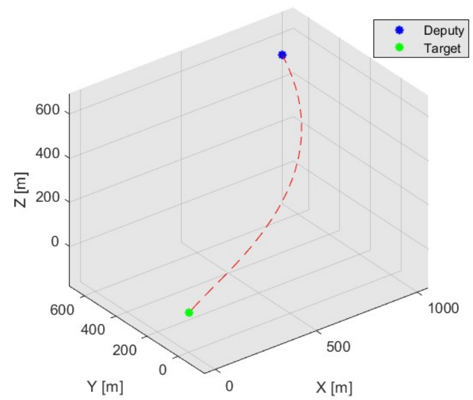

Figure 4: Minimum ΔV two-impulse rendezvous

In order to achieve a more accurate result, a loop has been implemented to compute the ΔV values for each time step within the specified range, with maximum final time equal to half orbital period of the Target. This allows to find the minimum ΔV value and its corresponding time index.

An example is given in Figure 4, assuming the following set of initial position and velocity of the Deputy with respect to the Target:

$$x_0(t) = 1000\ m \qquad \dot{x}_0(t) = 1\ m/s$$
$$y_0(t) = 700\ m \qquad \dot{y}_0(t) = -0.5\ m/s$$
$$z_0(t) = 500\ m \qquad \dot{z}_0(t) = -0.2\ m/s$$

In this particualar case, the minimum total impulse computed is equal to 2.971 m/s, with a required time for rendezvous of 27.716 min.

3 Analysis of the Rotational-Internal Motion

A Free Flyer rotating in the plane of motion is the subject of this section's analysis, as previously stated.

3.1 Simulation

Different initial conditions may be used in the analysis. The initial geometric configuration is shown in Figure 12, and the system is assessed in the cases the joint velocities are equal or not to zero. The latter emphasizes a crucial topic that will be covered later. In the study, no torques are acting on the MBS, except for gravity gradient forces and torques.

3.2 Results and Figures

In order to validate and verify the simulation code, we are looking for quantities that are conserved. The first proof comes from the graphs that show the motion of the satellite in orbit: for one-fourth of a period, as seen in Figure 6, the evolution along the x-axis of the system CoM is lowering, while along y is increasing, according to the motion along the circular orbit.

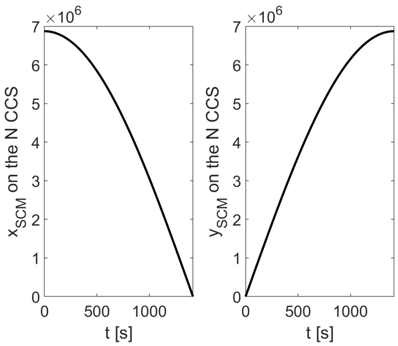

Figure 5: Evolution of Center of Mass of the System

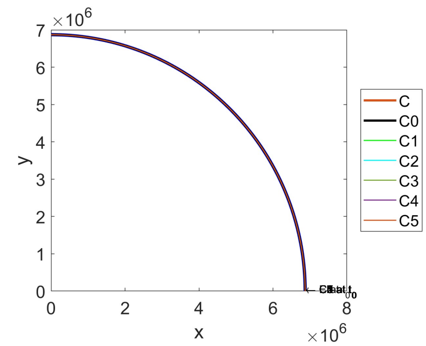

Figure 6: Birdview

It is evident from Figure 7, Figure 8 and Figure 9 that the MoM with respect to any reference point is not conserved, because the system is not at rest at the beginning of the propagation, due to the initial velocity not equal to zero - necessary for the system to maintain itself around the circular orbit - and gravity. In fact, the gravitational wrenches applied to the base link produce the motion of the entire system, whereas the internal ones cause only an internal reconfiguration of the links.

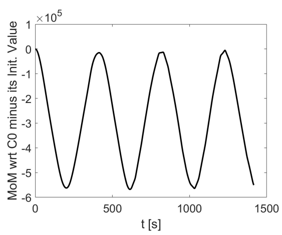

Figure 7: MoM wrt C0

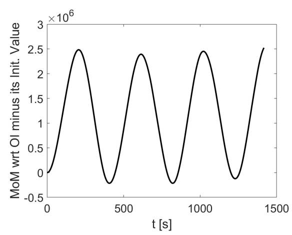

Figure 8: MoM wrt OI

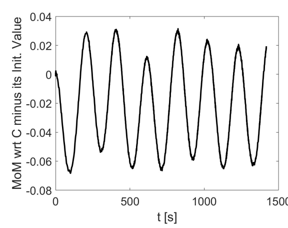

Figure 9: MoM wrt C

Indeed, Figure 10 and Figure 11 demonstrate this final pattern as a semi-periodic trend despite what appears to be some perturbations spreading from one link to the next. This final characteristic could be related to the fact that links have inertias to the movement.

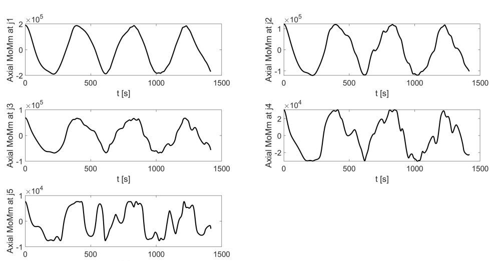

Figure 10: Axial Moment wrt Ji

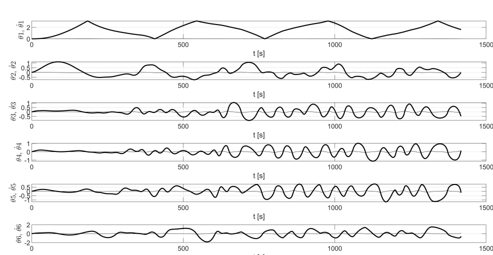

Figure 11: Thetas and Dot Thehas

3.3 Final Considerations

The last and more straightforward verification is carried out by comparing the initial and final configuration of the system. It is generally known that the gravitational field created by the Earth, which is the primary perturber of the motion analyzed, exerts torques on satellites with different principal moments of inertia. The latter feature is clear from Figure 13, where these torques have the tendency to align the local vertical with the axis of inertia. This is equivalent to the situation where the joints' starting velocities are fixed to zero (Figure 14). The first one tends to take into account the links' inertia, which means that they do not allow a perfect alignment with the local vertical, even though the second one is fully aligned because the MBS acts as a point mass.

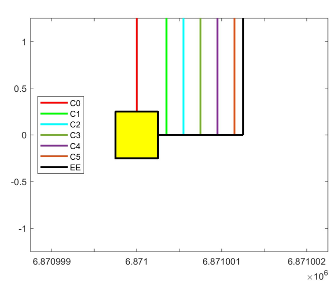

Figure 12: Initial Configuration of the simulation

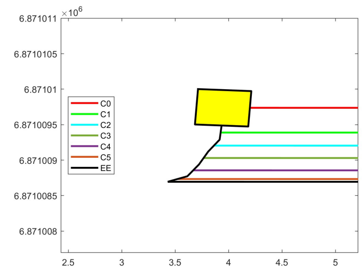

Figure 13: Final Configuration with $\dot{q}_m(t_0) = 0.01$ rad/s

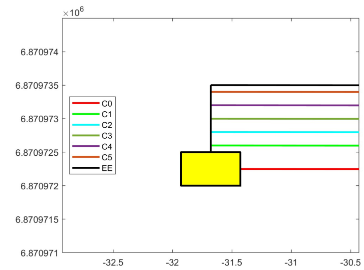

Figure 14: Final Configuration with $\dot{q}_m(t_0) = 0$ rad/s

4 Conclusions

4.1 Comments on the Results Obtained

To sum up, the results seem to be in accordance with theory, although numerical propagation was the only possible approach, given the complexity of the system.
Additional studies could be conducted in Task 1 to identify potential bottlenecks or areas where the produced code's performance could be enhanced. This could involve optimizing loops, utilizing parallel processing or leveraging built-in MATLAB functions for specific tasks.
As for the study carried out in Task 2, it would be more accurate if some neglected effects were also considered, such as motion coupling - although the masses are nearly similiar - and the reaction at the joints. However, it has enabled us to recognize and validate several theoretical findings, such as the alignment along the local vertical, which is frequently employed for space applications and occasionally further benefited by the addition of magnetic equipment.

4.2 Main Challenges and Problems

The main issues were encountered during the implementation of Task 2, particularly after the addition of the gravitational gradient. This initially led to unwarranted trends in the graphs as well as a complete blockage of the propagation when transitioning from quaternions to DCM, which was solved by using the direct transition instead of the suggested function.
Regarding teams assignment, given the smaller number of people, some minor difficulties were encountered in terms of time management and roles distribution.

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