Business Analytics - Homework

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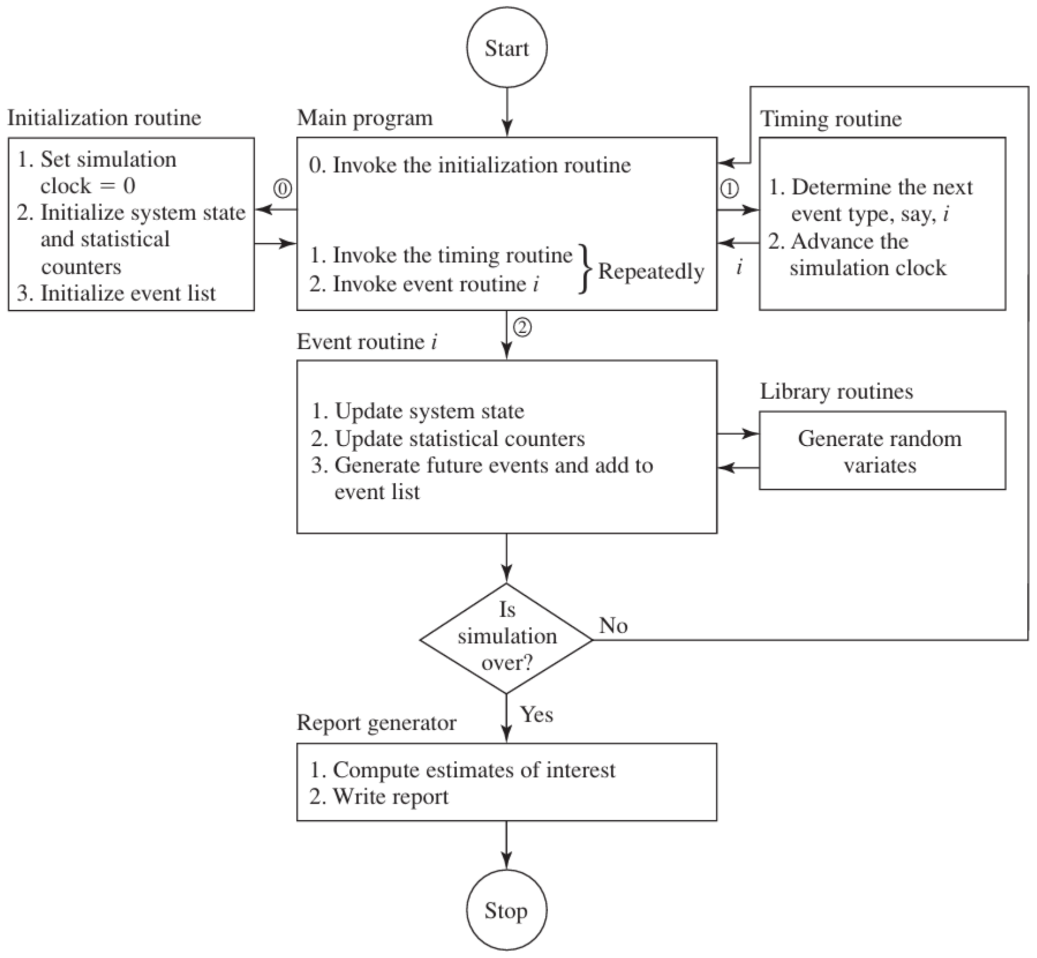
The goal of the homework was to build an object-oriented library for **Discrete Event Simulation.**

The library is mainly composed by 5 classes, namely:

- `SimulationEngine`
- `event`
- `state`
- `futureEventsList`
- `statsManager`

```
addpath("MATDES\")
addpath("MATDES\Statistics\")
addpath("Manifacturing Example\")
```

The logic behind discrete event simulation can be summarized in the figure below, taken from [1].


Start
Initialization routine
1. Set simulation clock = 0
2. Initialize system state and statistical counters
3. Initialize event list
Main program
0. Invoke the initialization routine
1. Invoke the timing routine
2. Invoke event routine i
Repeatedly
⓪
①
②
i
Timing routine
1. Determine the next event type, say, i
2. Advance the simulation clock
Event routine i
1. Update system state
2. Update statistical counters
3. Generate future events and add to event list
Library routines
Generate random variates
Is simulation over?
No
Yes
Report generator
1. Compute estimates of interest
2. Write report
Stop

SimulationEngine

The SimulationEngine class (subclass of handle) defines a framework for discrete event simulation. In particular, the simulation we will run will be instances of this class.

The properties of the SimulationEngine class are the following:

- state, a state object. This will represent the state of the system we want to simulate;
- clock, the simulation clock;
- eventsList, a futureEventsList object. It will contain the scheduled future events.
- stats, a statsManager object. It will be used to track some statistics during the simulation run.

The constructor for this class relies on the constructors for state, futureEventsList and statsManager. The syntax is the following:

```
mySim = SimulationEngine(stateStruct, statStruct)
```

The state will be initialized using the argument stateStruct. On the other hand, stats (which is an instance of the statsManager class) will be initialized using the argument statStruct. The idea is to track some quantities specified in statStruct. In particular, the field names will correspond to the quantity we want to track and the corresponding value will be a subclass of the Statistic class (more on this below).

The methods (other than the constructor) provided by this class are the following:

- initEventsList, which initializes the futureEventsList property. In particular, this methods takes as input an array of event objects. This method **must** be called before starting the simulation run. If this is not the case, the simulation will start with an empty futureEventsList and an error will occur.
- terminationCondition, which returns a flag that indicates whether the simulation must stop or not. This method is abstract (since different simulations may have very different stopping conditions) and takes as input stoppingParameters, useful to define stopping conditions that depend on some parameters.
- run, which runs the simulation. This method takes stoppingParameters as the only argument.
- clear, which clears the state, the statistical counters and sets the simulation clock to zero. Specifically, this method takes as input DataStruct, a struct used to update the state to the desired default value.
- generateReport, which prints a summary of the statistics collected during the simulation run. This method does not take arguments.

event

The event class is the core of discrete event simulation. It is defined as a subclass of handle and matlab.mixin.Heterogeneous. From the documentation, matlab.mixin.Heterogeneous is an abstract class that supports forming heterogeneous arrays. A heterogeneous array is an array of objects that differ in their specific class, but are all derived from or are instances of a root class. The root class derives directly from matlab.mixin.Heterogeneous. This choice was made for a clear reason: we would like to build arrays of specific events which are subclasses of the event class.

The properties of this class are the following:

- `label`, which is a string that represents the kind of event we are modelling.
- `parameters`, which is (typically) a row (or column) vector containing the parameters necessary to generate random variables.
- `clock`, which is the event clock.

The constructor for this class is the obvious one, setting the properties to the desired values.

As far as the other methods are concerned, only abstract methods are defined. Specifically, the methods defined are:

- `rnd`, which returns a random variable from some distribution using the parameters defined in `parameters`.
- `manageEvent`, which takes as input a `SimulationEngine` object and updates it.

To run a specific simulation, we will need to define these two methods for every type of event that can occur during the simulation run.

`state`

The `state` class will represent the state of the system we are simulating. It is defined as a subclass of the `dynamicprops` class, which is in turn a class derived from the `handle` class. The reason for this is quite simple: we want to define the properties when we actually build the `state` object. In particular, with the following syntax

```
s = state(DataStruct)
```

we instantiate a `state` object whose properties (and the corresponding values) are defined by the struct `DataStruct`.

Other than the constructor, this class provides only one method, which is `update`. This method takes as inputs two row cell arrays, which correspond to the properties we want to update and the corresponding new values. This method can be used to clear the state, too (provided that the passed argument is the desired default value).

For example,

```
DataStruct = struct( ...
    'A', zeros(10,10), ...
    'B', zeros(10,1), ...
    'C', 0);
s = state(DataStruct)
```

```
s = 
  state with properties:

    B: [10×1 double]
    A: [10×10 double]
    C: 0
```

`futureEventsList`

The `futureEventsList` class is a handle class which implements a priority queue to manage an array of `event` objects (actually, subclasses of the `event` superclass). The priority queue is implemented using a binary heap. This allows the `Enqueue` and `Dequeue` operations to be performed with a complexity of $\mathcal{O}(logn)$, where n is the number of elements inside the heap. The implementation corresponds to the one presented in [2].

The methods provided by this class do not differ by the methods provided by a basic priority queue (namely, `Enqueue`, `Dequeue`, `First`, `Empty`). The priority of an event is simply its simulation clock, so that the earliest event is always the one that is dequeued first. No secondary criterion was considered in the case of two events with the same clock.

`statsManager`

The `statsManager` class is a handle class responsible of collecting some desired statistics during the simulation run.

The properties of this class are the following:

- `statistics`, a struct whose fields correspond to the quantities we are tracking and the corresponding values are subclasses of the `Statistic` class. The `Statistic` class is an abstract class with no pre-defined properties and with three abstract methods: `update`, `clear`, `getResult`. This choice was made in order to make the implementation as modular as possible. In fact, the statistics that we might want to collect may be of very different nature. Some possible subclasses of the `Statistic` class have been implemented, namely `CountStatistic`, `SumStatistic`, `AverageStatistic`, `TimeAverageStatistic`.
- `clock`, the simulation clock.

The constructor for this class is quite basic. With this syntax

```
myStatsManager = statsManager(statStruct)
```

we are creating a `statsManager` object whose `statistics` properties is initialized using `statStruct`. For example,

```
statStruct = struct( ...
    'numInQueue', SumStatistic, ...
    'average_queue_length', TimeAverageStatistic);
myStatsManager = statsManager(statStruct)
```

```
myStatsManager = 
  statsManager with properties:

    statistics: [1×1 struct]
         clock: 0
```

```
myStatsManager.statistics
```

```
ans = struct with fields:
              numInQueue: [1×1 SumStatistic]
    average_queue_length: [1×1 TimeAverageStatistic]
```

```
myStatsManager.statistics.average_queue_length
```

```
ans = 
  TimeAverageStatistic with properties:

       weightedSum: 0
         totalTime: 0
    lastUpdateTime: 0
```

Other than the constructor, only two methods are provided: `update` and `clear`. Both of these methods rely on the respective `update` and `clear` methods for the `Statistic` subclasses.

For example,

```
myStatsManager.update('numInQueue', 10);
myStatsManager.statistics.numInQueue
```

```
ans = 
  SumStatistic with properties:

    sum: 10
```

```
myStatsManager.clock = 10;
myStatsManager.update('average_queue_length', 5)
myStatsManager.statistics.average_queue_length
```

```
ans = 
  TimeAverageStatistic with properties:

       weightedSum: 50
         totalTime: 10
    lastUpdateTime: 10
```

Important: when we update a `TimeAverageStatistic` object, we must provide the **old** value, not the new.

```
myStatsManager.clear()
myStatsManager.statistics.numInQueue
```

```
ans = 
  SumStatistic with properties:

    sum: 0
```

```
myStatsManager.statistics.average_queue_length
```

```
ans = 
  TimeAverageStatistic with properties:

       weightedSum: 0
         totalTime: 0
    lastUpdateTime: 0
```

A (simple) example (1.22 from [1])

A manufacturing system contains m machines, each subject to randomly occuring breakdowns. A machine runs for an amount of time that is an exponential random variable with mean 8 hours before breaking down. There are s (where s is a fixed, positive integer) repairmen to fix broken machines, and it takes one repairman an exponential amount of time with mean 2 hours to complete the repair of one machine; no more than one repairman can be assigned to work on a broken machine even if there are other idle repairmen. If more than s machines are broken down at a given time, they form a FIFO "repair" queue and wait for the first available repairman. Further, a repairman works on a broken machine until it is fixed, regardless of what else is happening in the system. Assume that it costs the system $50 for each hour that each machine is broken down and $10 an hour to employ each repairman.

Simulate the system for 500 hours (basically, three weeks) for each of the employment policies $s = 1, \ldots, 10$ to determine which policy results in the smallest expected average cost per hour. Assume that at time 0 all machines have just been "freshly" repaired.

```
numMachines =10
```

```
numMachines =
    10
```

The code for this example can be found inside the folder `Manifacturing Example`. This could take some minutes to run, and using the MATLAB profiler it is possible to see that the most expensive operations are the ones that modify the future events list (and it makes sense).

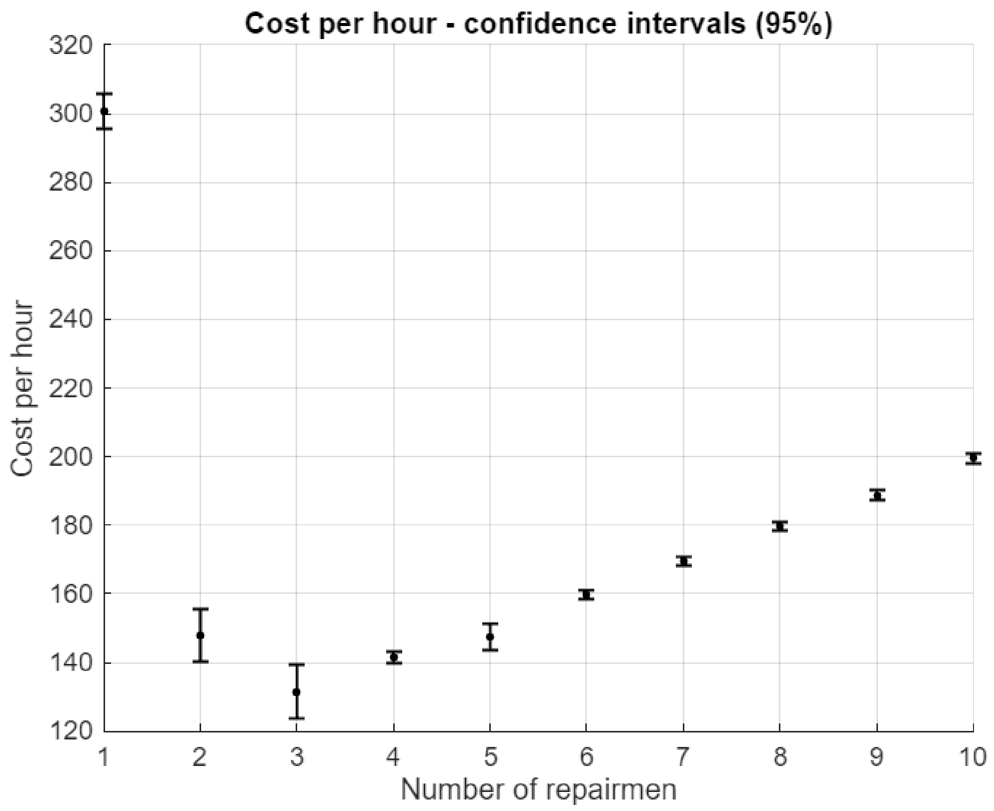
The cost could have been modeled using some subclass of `Statistic` too and the code would be very similar.

```
rng 'default'
% Data
numWorkers = 1:10;
breakdownRate = 1/8;
repairRate = 1/2;
numHours = 500;

% Number of repetitions per configuration
numRep = 50;

% Quantities of interest
costCI = zeros(2, length(numWorkers));
numIdleCI = zeros(2, length(numWorkers));

for i=1:length(numWorkers)
    % Run the simulation
    [costCI(:,i), numIdleCI(:,i)] = ...
        runManifacturingSystem(numRep, numHours, numWorkers(i), numMachines,
breakdownRate, repairRate);
end
```

Let's visualize the costs.

```
costMean = (costCI(1,:) + costCI(2,:)) / 2;
neg = costMean - costCI(1,:);
pos = costCI(2,:) - costMean;
figure;
hold on;
errorbar(numWorkers, costMean, neg, pos, 'ko', 'MarkerSize', 2, 'MarkerFaceColor',
'k', 'LineStyle', 'none', 'LineWidth', 1);
xlabel('Number of repairmen');
```

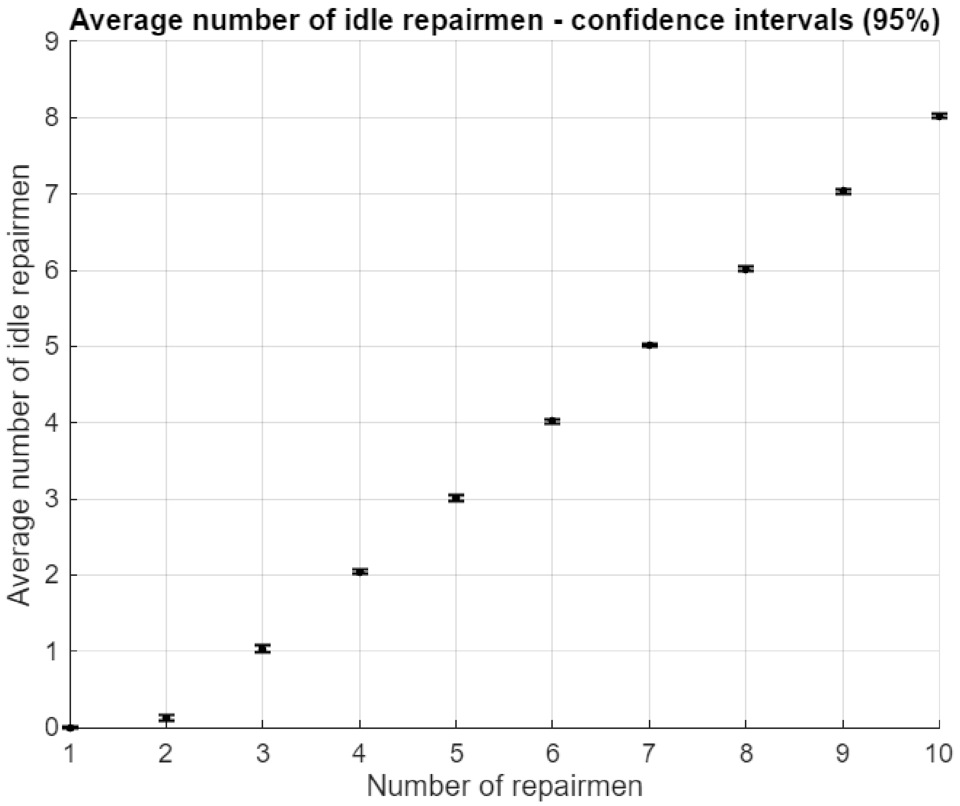
```
ylabel('Cost per hour');
title('Cost per hour - confidence intervals (95%)');
grid on;
```



From the plot, it looks like the optimal choice is $s = 3$ (if the number of machines is 10). We can see that as the number of workers increases, the confidence intervals shrink around the mean point. To give an interpretation to this phenomenon, this might be the caused by the fact that having more workers simply implies that it is more unlikely that, by chance, the repair queue gets large, and hence it is unlikely that very costly breakdowns occur.

We can take a look at the mean number of idle repairmen (computed as a time average, as for the queue length, for example).

```
numIdleMean = (numIdleCI(1,:) + numIdleCI(2,:)) / 2;
neg = numIdleMean - numIdleCI(1,:);
pos = numIdleCI(2,:) - numIdleMean;
figure;
hold on;
errorbar(numWorkers, numIdleMean, neg, pos, 'ko', 'MarkerSize', 2,
'MarkerFaceColor', 'k', 'LineStyle', 'none', 'LineWidth', 1);
xlabel('Number of repairmen');
ylabel('Average number of idle repairmen');
title('Average number of idle repairmen - confidence intervals (95%)');
grid on;
```



The example considered is very simple. It seems unlikely, for example, that only one repairman is allowed to work on a broken machine. Moreover, it seems unlikely that all the repairmen have the ability to repair all the machines, requiring always the same mean amount of time. In addition, it might not be true that a repairman starts repairing another machine as soon as he is done fixing another one. To model this, we might introduce a `Break` event in which the repairman gets idle independently of the size of the repair queue.

This simple example was, however, enough to demonstrate the capabilities of the simulation library developed. Of course, by defining more complex events, more complex systems can be modelled.

References

[1] Averill M. Law and David M. Kelton. *Simulation Modeling and Analysis*. McGraw-Hill Higher Education, 5th edition, 2015.

[2] P. Crescenzi et al., *Strutture di dati e algoritmi. Progettazione, analisi e visualizzazione*. Pearson, 2012