Technical report: Deadline-TSN worst-case queuing time calculation

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***Abstract*—This technical report deals with the calculation of the worst-case queeuing time in the worst-case response-time analysis of Deadline-TSN presented in [1], following the Spuri's analysis provided in [2].**

While the core of the worst-case response-time analysis of Deadline-TSN is found in the submitted manuscript (Sect. V), this report addresses the mere application of Spuri's methodology to calculate the frame worst-case queuing time. For the sake of double-blind review, this report is currently submitted with the manuscript as a separate file. In case of acceptance of our submission, this report will be made publicly available as reference [31].

TABLE I
SUMMARY OF NOTATION

Symbol	Description
F_i	The i-th flow.
P_i	F_i period.
D_i	F_i relative deadline.
Q	Number of transmission queues in each Ethernet port used in D-TSN.
N	Number of stream gates in each switch used in D-TSN.
u	Time unit, a constant configurable interval at which the IPV of the stream gates is recalculated.
$f_{i,j}$	The j-th frame of F_i.
$d_{i,j}$	Absolute deadline of $f_{i,j}$.
$a_{i,j}$	Arrival time of $f_{i,j}$.
W_i	Waiting time of F_i at the source to meet Condition (3) in [1].
C_i	Transmission time of the largest Ethernet frame of F_i.
TQi^{L_y}	Maximum queuing time of a frame belonging to F_i in the transmission port of the y-th link.
$Conf\_d_i(t)$	Function used to calculate the blocking and the interference of a frame with absolute deadline equal to t.
I_k	Maximum time the channel is busy due to the transmission of a frame belonging to F_k
$B_i^{L_y}(t)$	Maximum blocking of a frame of F_i with arrival time t at the queue of the L_y transmission port.
$nI_i(a)$	Contribution to the busy period due to the frames belonging to the same flow of $f_{i,j}$ with arrival time $a_{i,j} = a$.
$nLP_i(a,j)$	Number of frames of F_j with deadline lower than or equal to Max_d$(a + D_i)$.
$WK_i(a,t)$	Interference due to the frames with priority lower than or equal to the frame $f_{i,j}$ with $a_{i,j} = a$ in the interval [0, t].
$BP_i(a)$	Busy period of F_i calculated with a frame at the arrival time a.
BP_{max}	Length of the largest busy period.

The blocking component for a frame $f_{i,j}$, with arrival time $t = a_{i,j}$ at the queue of the transmission port of L_y can be calculated as the maximum time the channel is busy for the transmission of a frame with a lower priority than $f_{i,j}$, i.e.,

$$B_i^{L_y}(t) = \max_{\substack{\forall f_{k,j}:(a_{k,j}+D_k)>\text{Conf\_d}(t+D_i)\\ a_{k,j}<t}} (I_k) \tag{1}$$

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where $a_{k,j}$ is the arrival time of $f_{k,j}$ and I_k is the maximum time duration the channel is found busy due to the transmission of a frame belonging to F_k. Such a time duration is the sum of C_k and the Inter-Frame Gap time, i.e., $I_k = C_k + (96/\delta)$. This means that the blocking is due to all the flows with relative deadline D_k larger than Conf_d$(d_{i,j}) - a_{i,j}$, as in [3].

In Deadline-TSN the completion time of the frame $f_{i,j}$ with absolute deadline $d_{i,j}$ must be the end of a busy period in which all transmitted frames have deadlines lower than or equal to Conf_d$(d_{i,j})$. If we are able to examine all such periods, taking the maximum length we can find the worst-case queuing time of a flow. According to [2] also the frames of F_i with absolute deadline lower than Conf_d$(d_{i,j})$ may contribute to the busy period.

In the analysis here we focus on the frame $f_{i,j}$ arrived at time a, with $a \geq -W_i$, and possibly preceded by other frames of F_i, where W_i is waiting time of F_i at the source to meet Condition (3) in [1]. For this reason, we assume that the first frame of F_i, i.e., $f_{i,1}$, has the lowest arrival time, so that there is one arrival at time a.

This way, the contribution to the busy period due to the frames belonging to the same flow of $f_{i,j}$ with arrival time a is given by,

$$\text{nI}_i(a) = \left(1 + \left\lfloor \frac{a + W_i}{P_i} \right\rfloor\right) \times I_i \tag{2}$$

The number of frames with deadline lower than or equal to Conf_d$(a + D_i)$ is calculated as

$$\text{nLP}_i(a,j) = 1 + \left\lfloor \frac{\text{Conf\_d}(a + D_i) + W_j - D_j}{P_j} \right\rfloor \tag{3}$$

Hence, the interference due to the frames with priority higher than or equal to the frame of the flow F_i, arrived at a in the interval [0, t] is calculated as

$$WK_i(a,t) = \sum_{\substack{j\neq i\\ D_j \leq W_j +\\ \text{Conf\_d}(a+D_i)}} \min\left[\left\lceil \frac{t + W_j}{P_j} \right\rceil, \text{nLP}_i(a,j)\right] \times I_j \tag{4}$$

However, we are interested in calculating the busy period BP for each arrival time a. This can be done by processing the following iteration until the result does not change, i.e., $BP_i^{(n)}(a) = BP_i^{(n+1)}(a)$,

$$\begin{cases} BP_i^{(n)}(a) = \sum_{D_j \leq \text{Conf\_d}(a+D_i)+W_j}^{j\neq i} I_j, \text{with } n = 0 \\ BP_i^{(n+1)}(a) = B_i(a) + WK_i(a, BP_i^{(n)}(a)) + \text{nI}(a,i) \end{cases} \tag{5}$$

In this way, the $TQ_i^{L_y}(a)$ is calculated as,

$$TQ_i^{L_y}(a) = \max\left[B_i(a), BP_i(a) - a - I_i\right] \tag{6}$$

Finally, $TQ_i^{L_y}$ is calculated as the maximum value of $TQ_i^{L_y}(a)$ obtained for each value a in the range $[0, BP_{max} - I_i]$, i.e.,

$$TQ_i^{L_y} = \max_{\forall a \in [0, BP_{max} - I_i]} \left[TQ_i^{L_y}(a)\right] \tag{7}$$

where BP_{max} is the length of the largest busy period calculated processing the following iteration until the result does not change, i.e., $BP_{max}^{(n)} = BP_{max}^{(n+1)}$

$$\begin{cases} BP_{max}^{(n)} = \sum_{\forall F_i} I_i, \text{with } n = 0 \\ BP_{max}^{(n+1)} = \sum_{\forall F_i} \left\lceil \dfrac{BP_{max}^{(n)} + W_i}{P_i} \right\rceil \times I_i. \end{cases} \tag{8}$$

References

[1] G. Patti, L. Lo Bello, and L. Leonardi, "Deadline-aware Online Scheduling of TSN Flows for Automotive Applications," *IEEE Transactions on Industrial Informatics*, 2022.

[2] M. Spuri, "Holistic Analysis for Deadline Scheduled Real-Time Distributed Systems," *INRIA Rapport de recherche*, no. 2873, Apr. 1996.

[3] L. George, N. Rivierre, and M. Spuri, "Preemptive and Non-Preemptive Real-Time UniProcessor Scheduling," *INRIA Rapport de recherche*, no. 2966, Sept. 1996.