RLC

substitute $V_c(t) = e^{st}$

$$\frac{d^2 V_c(t)}{dt} + 2\alpha \frac{dV_c(t)}{dt} + \omega_0^2 V_c(t) = 0$$

$$s^2 e^{st} + 2\alpha s e^{st} + \omega_0^2 e^{st}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

α: dampening
ω_0: resonant frequency
ζ: dampening ratio

$$\zeta = \frac{\alpha}{\omega_0}$$

$$Q = \sqrt{\frac{L}{CR^2}}$$

find later

$$V_c(t) = K_1 e^{(-\alpha + \sqrt{\alpha^2 - \omega_0^2})t} + K_2 e^{(-\alpha - \sqrt{\alpha^2 - \omega_0^2})t}$$

$K_1 + K_2 = V_{DD}$
$K_1 s_1 + K_2 s_2 = 0$

$$K_1 = \frac{s_2}{s_2 - s_1} V_{DD}$$
$$K_2 = -\frac{s_1}{s_2 - s_1} V_{DD}$$

if $\zeta > 1$ (overdampened): $K_1 e^{s_1 t} + K_2 e^{s_2 t}$

if $\zeta < 1$ (underdampened): $K_1 e^{-\alpha t}\cos(\omega_0 t) + K_2 e^{-\alpha t}\sin(\omega_0 t)$

if $\zeta = 1$ (critically dampened): $K_1 e^{s_1 t} + K_2 t e^{s_1 t}$ $(s_1 = s_2)$

$$V_{DD} e^{-\alpha t}\left(\cos(\omega t) + \frac{\alpha}{\omega_n}\sin(\omega t)\right) \qquad V_{DD}\left(e^{s_1 t} - s_1 t e^{s_1 t}\right)$$

Capacitors

$$C = \frac{Q}{V} \quad V = \frac{Q}{C} \quad Q = CV \quad C = \frac{\varepsilon_0 A}{d}$$

$$i = C\frac{dv}{dt} \quad \frac{dv(t)}{dt} = -\frac{1}{RC} v(t) \quad P = \frac{1}{2} QV$$

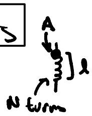
discharging: $V(t) = V_0 e^{-\frac{1}{RC}t}$

charging: $V(t) = V_{DD}(1 - e^{-\frac{1}{RC}t})$

$\tau = RC$

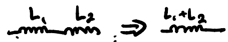
Inductors

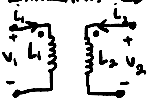
$$L = \frac{N^2 \mu A}{l}$$



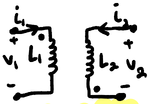
$$V(t) = L\frac{di(t)}{dt}$$

$$E = \frac{1}{2} L i^2$$





$$V_1 = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt}$$
$$V_2 = M\frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$



$$V_1 = L_1 \frac{di_1}{dt} - M\frac{di_2}{dt}$$
$$V_2 = -M\frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

$$i(t) = I_s e^{-\frac{R}{L}t}$$ (inductor + current source)

$\tau = \frac{L}{R}$

Phasors

Time Domain	Phasor Domain
$v(t) = v_0 \cos(\omega t + \theta)$ (voltage)	$\tilde{V} = V_0 e^{j\theta}$
$i(t) = i_0 \cos(\omega t + \theta)$ (current)	$\tilde{i} = i_0 e^{j\theta}$

ω = frequency

L (inductors) $Z_L = j\omega L$

C (capacitors) $Z_C = \frac{1}{j\omega C}$

R (resistors) $Z_R = R$

$\tilde{V} = |V| e^{j\theta}$

$\tilde{V}_1 \tilde{V}_2 = |V_1||V_2| e^{j(\theta_1 + \theta_2)}$

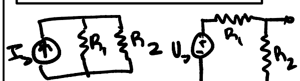
$\frac{\tilde{V}_1}{\tilde{V}_2} = \frac{|V_1|}{|V_2|} e^{j(\theta_1 - \theta_2)}$

$j\tilde{V} = \tilde{V} e^{j\frac{\pi}{2}}$

$\frac{\tilde{V}}{j} = \tilde{V} e^{-j\frac{\pi}{2}}$

$\cos(x - \frac{\pi}{2}) = \sin(x)$ $\quad v(t) = V_0 \sin(\omega t + \phi) \Rightarrow \tilde{V} = \frac{V_0 e^{j\phi}}{j}$

Current Divider



$$R_1 = \frac{R_2}{R_1 + R_2} I_s \qquad V_{out} = \frac{R_2}{R_1 + R_2} V_s$$

RLC (Phasors)

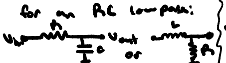
In series: $\omega_0 = \frac{1}{\sqrt{LC}}$ $\quad Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{1}{R}\sqrt{\frac{L}{C}}$

In parallel: $\omega_0 = \frac{1}{\sqrt{LC}}$ $\quad Q = \frac{R}{\omega_0 L} = \omega_0 RC = R\sqrt{\frac{C}{L}}$

Filters

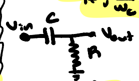
low pass
$$H(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_c}}$$

for an RC low-pass:



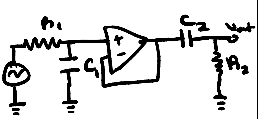
$$\tilde{V}_{out} = \frac{1}{1 + j\omega RC}\tilde{V}_{in} \qquad \omega_c = R/L$$
$$\omega_c = \frac{1}{RC}$$

high pass
$$H(j\omega) = \frac{j\frac{\omega}{\omega_c}}{1 + j\frac{\omega}{\omega_c}}$$



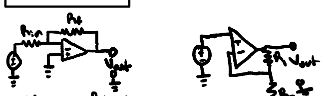
$$V_{out} = \frac{j\omega RC}{1 + j\omega RC}$$
$$\omega_c = \frac{1}{RC}$$

2nd order band pass



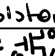
good for $\left[\frac{1}{R_2 C_2}, \frac{1}{R_1 C_1}\right]$

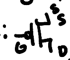
Feedback

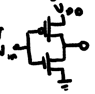


$$V_{out} = -\frac{R_f}{R_{in}} V_{in} \qquad V_{out} = V_{in}\left(1 + \frac{R_1}{R_2}\right)$$

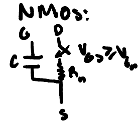
Transistors

NMOS:  on when $V_{GS} \geq V_{t_n}$

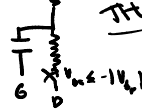
PMOS:  , on when $V_{GS} \leq -|V_{t_p}|$

inverter  (just an RC)

NMOS:



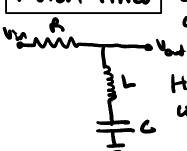
PMOS:



IGNORE THE CAPS!

Notch Filter

opposite of band pass



$H = 0$ at $\omega = \frac{1}{\sqrt{LC}}$

$$H = \frac{j\omega L + \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{1 + j\frac{\omega RC}{1 - \omega^2 LC}}$$

Poles / Zeros

$$H(j\omega) = K\frac{(j\omega)^{N_z}(1 \pm j\frac{\omega}{\omega_{z}})(1 \pm \ldots)}{(j\omega)^{N_p}(1 \pm j\frac{\omega}{\omega_{p}})(1 \pm \ldots)}$$

zeroes
poles

Complex Numbers

$a = x + jy$

$a^* = x - jy$

$\mathcal{R}\{a\} = \frac{1}{2}(a + a^*) = x$

$\mathcal{I}\{a\} = \frac{1}{2}(a - a^*) = y$

$aa^* = |a|^2 = |x|^2 + |y|^2$

$\theta = \angle a = \mathrm{atan}\left(\frac{y}{x}\right)$

$a = |a| e^{j\theta} = |a|(\cos(\theta) + j\sin(\theta))$

$|e^{j\theta}| = 1$

$\mathcal{R}\{e^{j\omega t}\} = \cos(\omega t)$

$\mathcal{R}\{e^{j\omega t - \frac{\pi}{2}}\} = \sin(\omega t)$

$\frac{1}{2}(e^{j\omega t} + e^{-j\omega t}) = \cos(\omega t)$

Trig

	$0°$	$30°$	$45°$	$60°$	$90°$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞