CS 70

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CONTENTS

CHAPTER 1

PROPOSITIONAL LOGIC

1.1 Propositions

Definition 1.1.1 (*Propositions*)

Statements that are true or false.

Some example propositions are $\sqrt{2}$ is irrational, $2+2=4$, and $2+2=3$. Some non examples are $4+5$ and $x+x$.

1.2 Propositional Forms

Propositions can be put together to make another proposition. Some examples of this are conjunction ($P \wedge Q$), disjunction ($P \vee Q$), and negation ($\neg P$).

Example : Sample propositional forms:

1. $\neg(2+2=4)$ is false
2. $(2+2=3) \wedge (2+2=4)$ is false

1.3 Implication

Definition 1.3.1 (*Implication*)

$P \Rightarrow Q$ can be read as "If P, then Q."
Implications are only false if P is true and Q is false. When P is false, the implication will be true, however this is a meaningless statement.

Example : If you stand in the rain, then you'll get wet.
$P =$ "you stand in the rain"
$Q =$ "you get wet"
$P \Rightarrow Q$

Note. If $P \Rightarrow Q$, that does not say anything about the opposite of $Q \Rightarrow P$.

Definition 1.3.2 (*Contrapositive*)

The contrapositive of $P \Rightarrow Q$ is $\neg Q \Rightarrow \neg P$. They are logically equivelent to the implication.

Definition 1.3.3 (*Converse*)

The converse of $P \Rightarrow Q$ is $Q \Rightarrow P$. They are NOT logically equivelent to the implication.

Definition 1.3.4 (*If and only if*)

If $P \Rightarrow Q \wedge Q \Rightarrow P$, then P if and only if (iff) Q or $P \Leftrightarrow Q$.

1.4 Truth Tables

Truth tables can simplify doing calculations with propositions.

Truth tables are also able to be used to prove logical equivalence. If two statements have the same truth tables, they are logically equivelent.

1.5 Quantifiers

None of the below are propositions as they have a free variable.

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- $x > 2$
- n is even and the sum of two primes

These are all called predicates. They are similar to a function which returns true or false.

To turn them into a predicate we need a quantifier.

Definition 1.5.1 (*Universe*)

A universe is the type of a variable. Some examples are:
- $N = \{0, 1, 2, \ldots\}$
- $N^+ = \{1, 2, 3, \ldots\}$

Definition 1.5.2 (*There Exists*)

$(\exists x \in S)(P(x))$ means $P(x)$ is true for some x in S.
Example: $(\exists x \in N)(x = x^2)$ True because $0\dot{0} = 0$ and is equivelent to $(0 = 0) \wedge (1 = 1) \wedge (2 = 4) \wedge \ldots$.

Definition 1.5.3 (*For all*)

$(\forall x \in S)(P(x))$ means $P(x)$ is true for all x in S.
Example: $(\forall x \in N)(x+1 > x)$ True because adding one to a number will always be larger.

Note. Quantifiers are not communitive.