On the Learning Parity with Noise Problem

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Scenario

Cryptography schemes

- address the security of communication across an insecure medium
- are usually based only on complexity assumptions (standard model)

Near Future:

- **Problem:** What if someone constructs large quantum computers?
- Cryptography world may fall apart:
  1. cryptographic assumptions broken by efficient quantum algorithms
     e.g. *factoring and discrete-logarithm broken by Shor's algorithm*
  2. proofs of security (or *reductions*) become unuseful

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Post-Quantum cryptography

Schemes that are believed to resist classical & quantum computers

- **Code-based cryptography**
- **Lattice-based cryptography**

Our contribution

- We investigate about the Learning Parity with Noise (LPN) problem
- We propose a Threshold Public-Key Encryption scheme based on LPN
- We propose a Commitment protocol based on LPN

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Learning Parity with Noise Problem LPN

- Dimension ℓ (security parameter), $q \gg \ell$, $\tau \in \left(0, \frac{1}{2}\right)$
- **Search**: find $\boldsymbol{s} \in \mathbb{Z}_2^\ell$ given "noisy random inner products"

Errors $e_i \leftarrow \mathrm{Ber}_\tau$, i.e. $\Pr(e_i = 1) = \tau$

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- **Decisional**: distinguish (A, b) from uniform (A, b)
- decisional and search LPN are “polynomially equivalent”

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Hardness of LPN

The best known attacks against search LPN problem takes

- $2^{\Theta(\ell/\log \ell)}$ having the same number of samples q
- $2^{\Theta(\ell/\log \log \ell)}$ having $q = poly(\ell)$ samples
- $2^{\Theta(\ell)}$ having $q = \Theta(\ell)$ samples

where ℓ is the security parameter

Interesting features

- **Efficiency – suitable for limited computing power devices (e.g. RFID)**
- **Quantum algorithms resistance**

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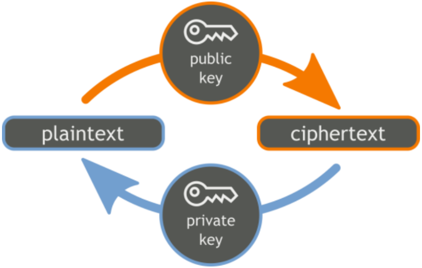
Public-Key Encryption schemes



Public-key cryptography

- The ability of decrypting or signing is restricted to the owner of the secret key.
- $\Rightarrow$ only one person has all the power

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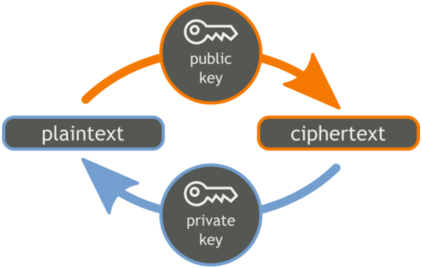

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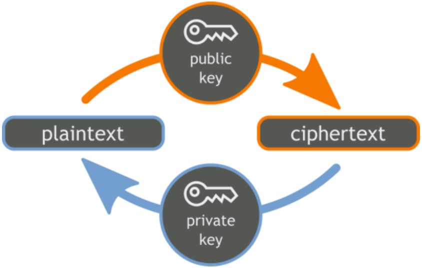

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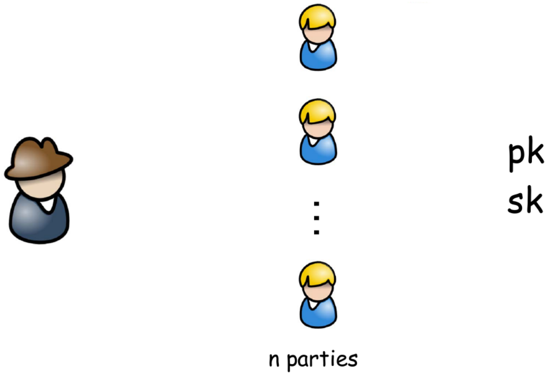

Solution: Threshold PKE

- The secret key is split into shares and each share is given to a group of parties.
- Parties can decrypt or sign only if enough, a threshold, cooperate

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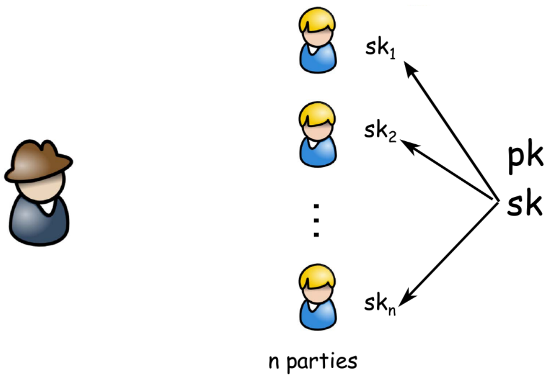

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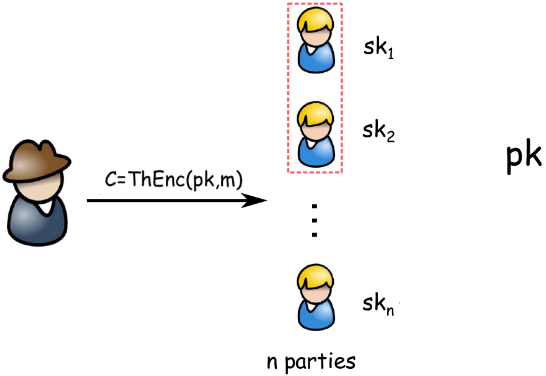

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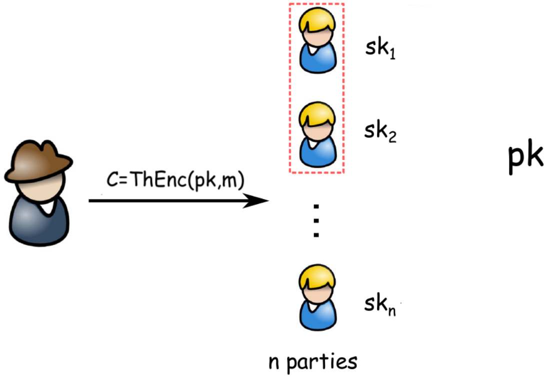

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A Threshold Public-Key Encryption scheme which is:

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- secure in the *Semi-honest* model

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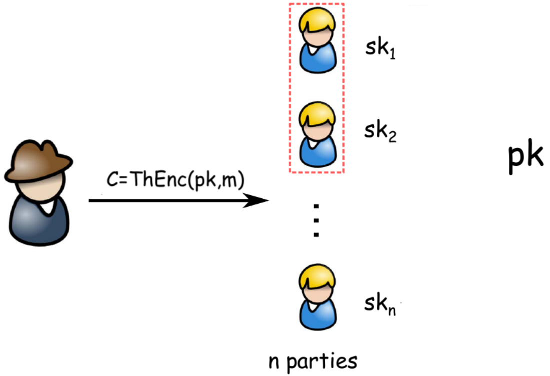

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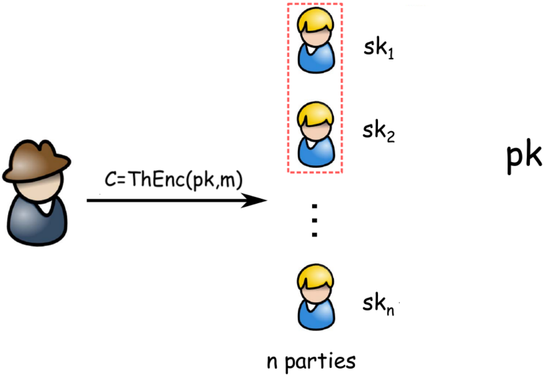

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Alekhnovich PKE scheme

Key Generation

The receiver R chooses

- a secret key $\boldsymbol{s} \xleftarrow{R} \mathbb{Z}_2^{\ell}$
- $\boldsymbol{A} \xleftarrow{R} \mathbb{Z}_2^{q\times\ell}$ and the error $\boldsymbol{e} \leftarrow \mathrm{Ber}_{\tau}^{q}$, where $\tau \in \Theta(\frac{1}{\sqrt{\ell}})$ and computes the pk as $(\boldsymbol{A}, \boldsymbol{b} = \boldsymbol{A}\boldsymbol{s} \oplus \boldsymbol{e})$

Encryption of a message bit $m \in \mathbb{Z}_2$

Sender S | Receiver R

choose a vector $\boldsymbol{f} \leftarrow \mathrm{Ber}_{\tau}^{q}$

compute $\boldsymbol{u} = \boldsymbol{f} \cdot \boldsymbol{A}$

$c = \langle \boldsymbol{f}, \boldsymbol{b} \rangle \oplus m \xrightarrow{(\boldsymbol{u}, c)}$

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The receiver R computes $d = c \oplus \langle \boldsymbol{s}, \boldsymbol{u} \rangle$

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ThPKE: Protocol phases

- **Key Generation**
- Key Assembly
- Encryption
- Partial Decryption
- Finish Decryption

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Key Generation

- All the receivers share a matrix $\boldsymbol{A} \xleftarrow{R} \mathbb{Z}_2^{q\times\ell}$
- Each receiver $\mathtt{R_i}$ indipendently choose a secret key $\boldsymbol{s}_i \xleftarrow{R} \mathbb{Z}_2^{\ell}$ and an error $\boldsymbol{e}_i \leftarrow \mathrm{Ber}_\tau^q$
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Key Assembly

The combined public key is the pair $(\boldsymbol{A}, \boldsymbol{b})$, where

$$\boldsymbol{b} = \bigoplus_{i \in I} \boldsymbol{b}_i$$

and I is the users subset

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Sender $\underline{\mathtt{S}}$ Receivers $\underline{\mathtt{R_i}, \mathtt{R_j}}$

$$(\boldsymbol{C_1}, \boldsymbol{c_2}) \leftarrow \mathtt{ThLPN.Enc}(m, \boldsymbol{b})$$

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Encryption function (Alekhnovich scheme)

$$\boldsymbol{C_1} = \boldsymbol{F} \cdot \boldsymbol{A}, \; \boldsymbol{c_2} = \boldsymbol{F} \cdot \boldsymbol{b} \oplus \underbrace{\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}}_{q} \cdot m \qquad \text{where } \boldsymbol{F} := \begin{bmatrix} \boldsymbol{f_1} \\ \vdots \\ \boldsymbol{f_q} \end{bmatrix}, \; \boldsymbol{f_i} \leftarrow \mathrm{Ber}_{\tau}^{q}$$

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Receiver $\underline{\mathtt{R_i}}$

Receiver $\underline{\mathtt{R_j}}$

$$\boldsymbol{d_i} \leftarrow \mathtt{ThLPN.Pdec}(\boldsymbol{C_1}, \boldsymbol{c_2}, \boldsymbol{s_i})$$

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Finish decryption

- Each receiver indipendently computes the vector

$$\boldsymbol{d} = \boldsymbol{c_2} \bigoplus_{i \in I} (\boldsymbol{d_i})$$

- the bit in $\boldsymbol{d}$ that is in majority is separately chosen by each receiver as m

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- the bit in $\boldsymbol{d}$ that is in majority is separately chosen by each receiver as m

ThPKE: Protocol phases

- Key Generation
- Key Assembly
- Encryption
- Partial Decryption
- **Finish Decryption**

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Protocol Security Analysis

Semi-honest model

A semi-honest party:

1. Follows the protocol properly
2. Keeps a record of all its intermediate computations

Security

- **Encryption:** from the Alekhnovich's scheme security
- **Decryption:** from the LPN hardness assumption, as each P_i is generating LPN samples

$$d_i = C_1 \cdot s_i \oplus \nu_i$$

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Relaxed Semi-honest model

- Semi-honest model is not realistic
- *replay attacks*: the same message is fraudolently encrypted multiple times
- Problem: it is possible to recover information about the secret key from the ciphertexts

Proposed solutions

1. implement the receivers as stateful machines (not good in resource-constrained devices)
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Conclusions and Open Problems

Our contribution

- study the security of our Threshold Public-Key Encryption scheme in the *malicious model*

LPN open problems

- relation between standard LPN and some variants
- Does LPN with noise rate τ imply anything about LPN with $\tau' < \tau$? Is there a threshold?
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Scenario:

Alice wants to keep a message secret from Bob for now but she intends to reveal it to Bob at some time in the future

Commitment protocol

- Alice commits the message and Bob does not learn any information about it (hiding property)
- Alice chooses to open the commitment and reveal the message but she cannot change the value committed (property)

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We presented a Commitment protocol

- based on the commitment protocol by *Jain et al.*
- based on Exact-LPN problem (where $wt(e) = \lfloor \tau \cdot \ell \rceil$)
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The commitment protocol by *Jain et al*

Setup Phase

In order to commit a message $\boldsymbol{m} \in \mathbb{Z}_2^k$ where $k \in \Theta(\ell + v)$
We state $\boldsymbol{A} = [\boldsymbol{A'} \| \boldsymbol{A''}] \in \mathbb{Z}_2^{k \times (\ell+v)}$ as the common reference string (CRS).
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Opening phase

define $\boldsymbol{d} = (\boldsymbol{m'}, \boldsymbol{r'})$ $\xrightarrow{\boldsymbol{d}}$
computes $\boldsymbol{e'} = \boldsymbol{c} \oplus \boldsymbol{A}\left(\boldsymbol{r'} \| \boldsymbol{m'}\right)$
$\xleftarrow{\textit{Yes, No}}$ accepts iff $\boldsymbol{wt}(\boldsymbol{e'}) = w$

Proposed commitment protocol

Problem

We need a trusted third party for the common matrix $\boldsymbol{A} = [\boldsymbol{A}' \| \boldsymbol{A}'']$

Solution:

The Commitment and Opening phases are the same as in the original scheme

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$$\begin{array}{ccc} \text{Sender } \underline{\mathtt{S}} & & \text{Receiver } \underline{\mathtt{R}} \\ \text{chooses } \boldsymbol{A'} \xleftarrow{R} \mathbb{Z}_2^{k\times \ell} & \xrightarrow{\quad \boldsymbol{A'} \quad} & \\ & \xleftarrow{\quad \boldsymbol{A''} \quad} & \text{chooses } \boldsymbol{A''} \xleftarrow{R} \mathbb{Z}_2^{k\times v} \end{array}$$

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The commitment protocol: a LPN-based variant

LPN variant

- security directly based on the standard LPN problem
- **Commit phase:** we set $w' = 2 \cdot \lfloor \tau k \rceil$ and we choose $\boldsymbol{e}$ such that $\boldsymbol{wt}(\boldsymbol{e}) \leq w'$

Choice of parameters

According to

Levieil, Éric and Fouque, Pierre-Alain
An Improved LPN Algorithm
Springer Berlin Heidelberg, 2006

we choose $\ell = 768$ and noise rate $\tau = \frac{1}{8} \Rightarrow 2^{90}$ bytes of memory to solve LPN

Theorem

Our commitment scheme is statistically binding and computationally hiding

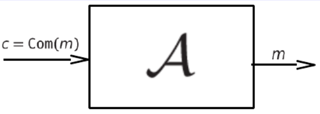
Proof

Statistically binding

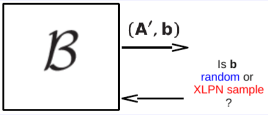
even if S is computationally unbounded she cannot cheat with probability greater than 2^{-k}

Computationally hiding

- proof for reduction (single bit message)
- we assume that $\mathcal{A}$ is able to break the commitment scheme

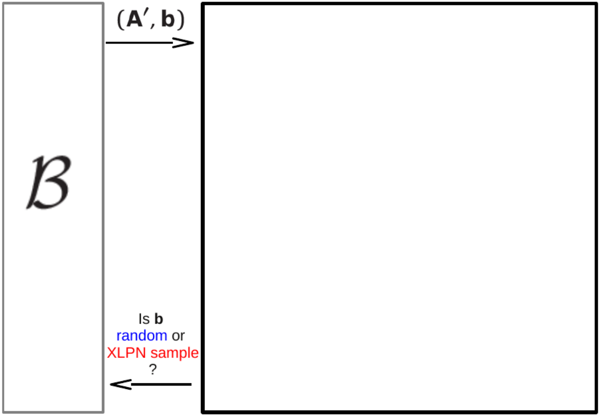


- Let $\mathcal{B}$ an oracle



where $\boldsymbol{b} = \begin{cases} \text{random} & w.p.\ 1/2 \\ \boldsymbol{A's} \oplus \boldsymbol{e} & w.p.\ 1/2 \end{cases}$

Proof



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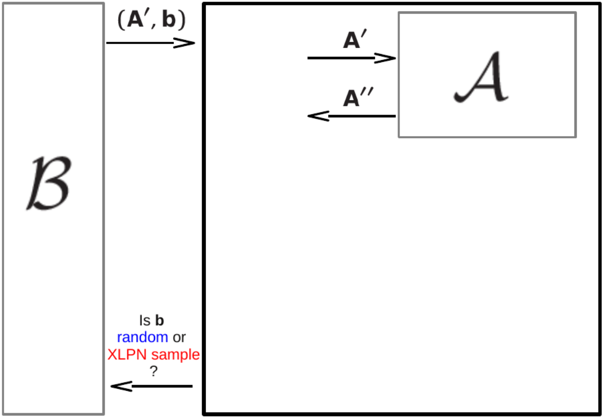

$(\mathbf{A}', \mathbf{b})$
$\mathbf{A}'$
$\mathbf{A}''$
$\mathcal{A}$
$\mathcal{B}$
Is **b**
random or
XLPN sample
?

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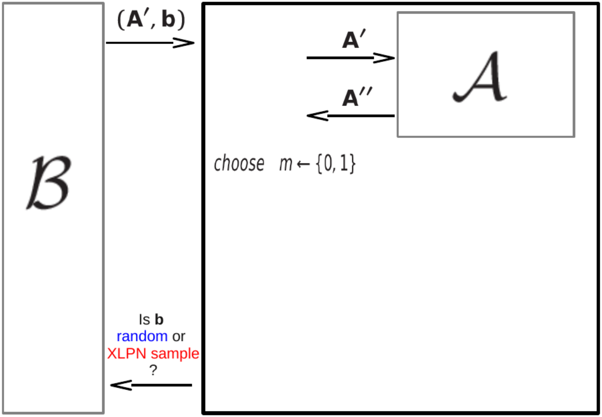

(A′, b)
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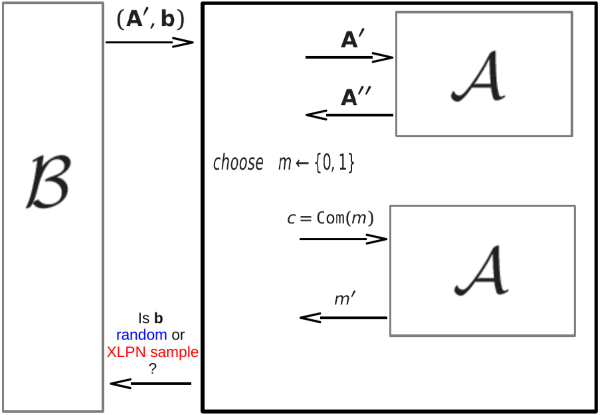
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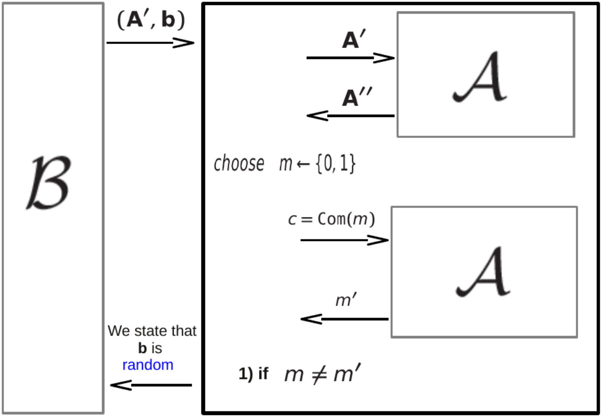
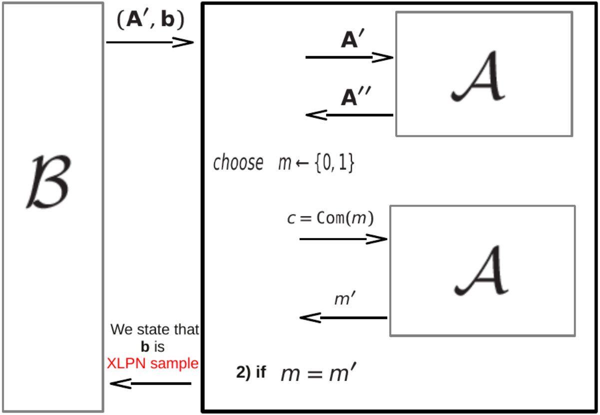


case 1)

$\boldsymbol{b}$ is random $\Rightarrow \boldsymbol{c} = \boldsymbol{b} \oplus \boldsymbol{A}'' m$ is a onetime-pad encryption

$\Rightarrow \mathcal{A}$ guesses w.p. $\frac{1}{2}$

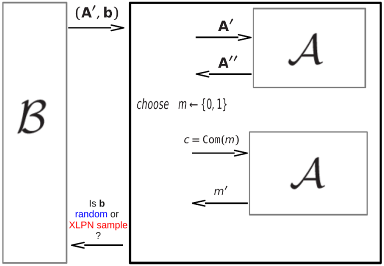
Proof



case 2)

$\boldsymbol{b}$ is a Exact-LPN sample $\Rightarrow$ $\boldsymbol{c}$ is a well formed commitment
$\Rightarrow$ $\mathcal{A}$ guesses w.p. 1 (by hypothesis)

Proof



case 1) and 2)

Let E = the reduction breaks the Exact-LPN problem,

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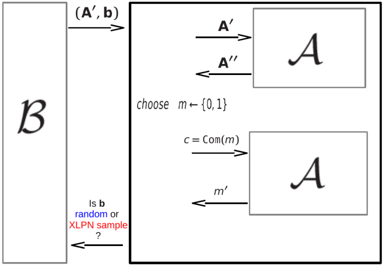

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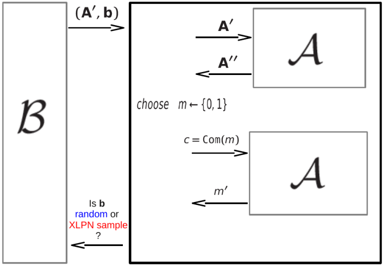
Proof



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$$
\begin{aligned}
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&= 1 \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4}
\end{aligned}
$$

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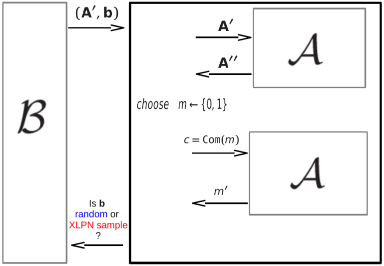

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Exact-LPN hardness $\Rightarrow$ Hiding commitment