On the Learning Parity with Noise Problem

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Scenario

Cryptography schemes

- address the security of communication across an insecure medium
- are usually based only on complexity assumptions (standard model)

Near Future:

- **Problem:** What if someone constructs large quantum computers?
- Cryptography world may fall apart:
  1. cryptographic assumptions broken by efficient quantum algorithms
     e.g. *factoring problem broken by Shor's algorithm (RSA is defeated)*
  2. proofs of security (or *reductions*) become unuseful

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Post-Quantum cryptography

Schemes that are believed to resist classical & quantum computers

- **Code-based cryptography**
- **Lattice-based cryptography**

Our contribution

- We investigate about the Learning Parity with Noise (LPN) problem
- We propose a Threshold Public-Key Encryption scheme based on LPN
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- Dimension ℓ (security parameter), q samples where $q \gg \ell$, $\tau \in \left(0, \frac{1}{2}\right)$
- **Search**: find $\boldsymbol{s} \in \mathbb{Z}_2^\ell$ given "noisy random inner products"

Errors $e_i \leftarrow \mathrm{Ber}_\tau$, i.e. $\Pr(e_i = 1) = \tau$

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Hardness of LPN

The best known attacks against search LPN problem takes

- $2^{\Theta(\ell/\log \ell)}$ having the same number of samples q
- $2^{\Theta(\ell/\log\log \ell)}$ having $q = poly(\ell)$ samples
- $2^{\Theta(\ell)}$ having $q = \Theta(\ell)$ samples

where ℓ is the security parameter

Interesting features

- Efficiency $\rightarrow$ suitable for limited computing power devices (e.g. RFID)
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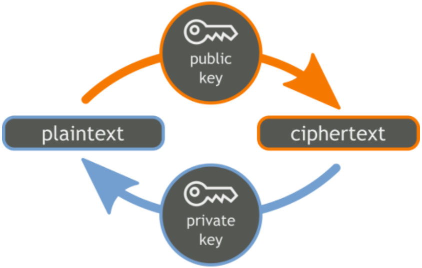

Public-key cryptography

- The ability of decrypting or signing is restricted to the owner of the secret key.
- $\Rightarrow$ only one person has all the power

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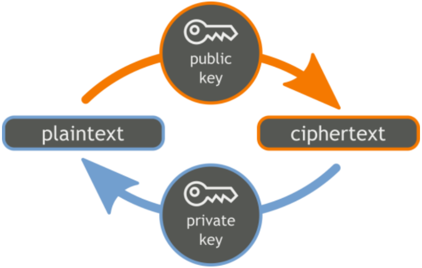

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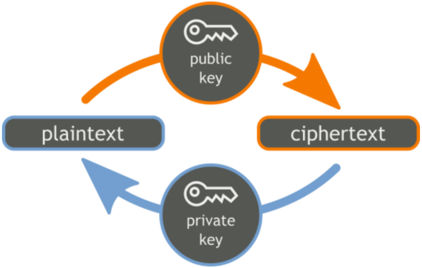

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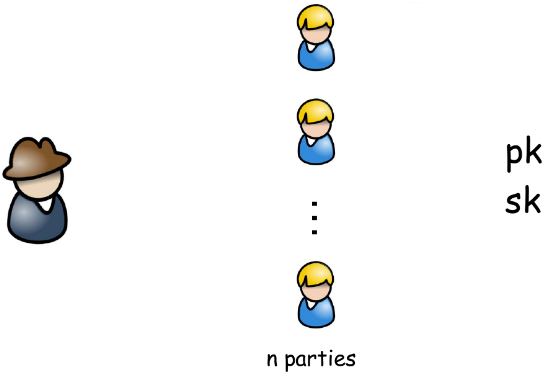

Solution: Threshold PKE

- The secret key is split into shares and each share is given to a group of parties.
- Parties can decrypt only if enough, a threshold, cooperate

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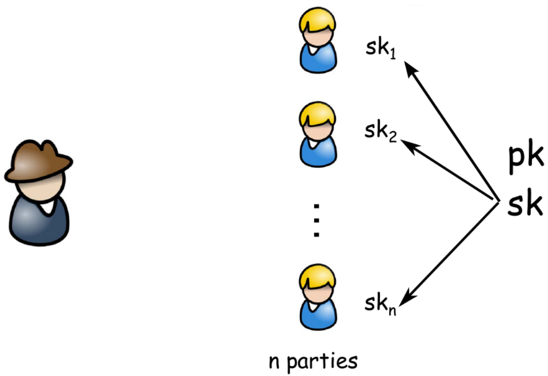

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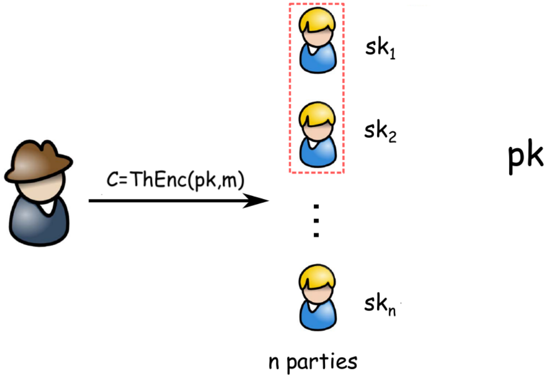

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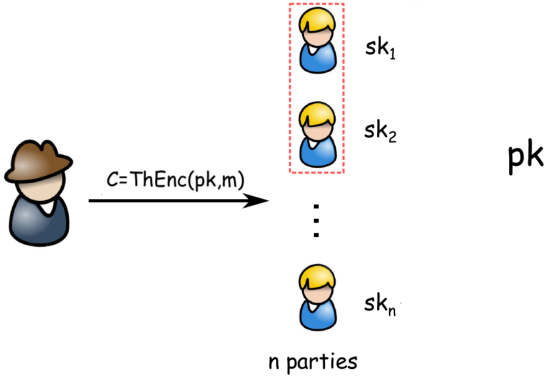

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A Threshold Public-Key Encryption scheme which is:

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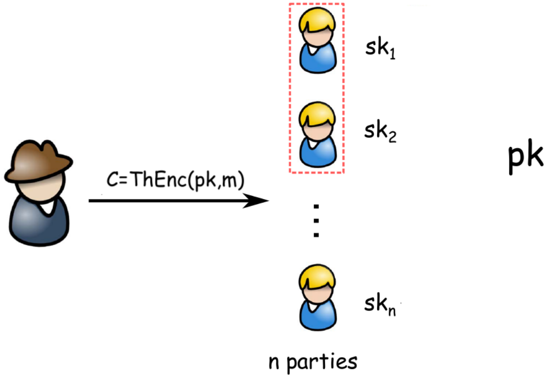

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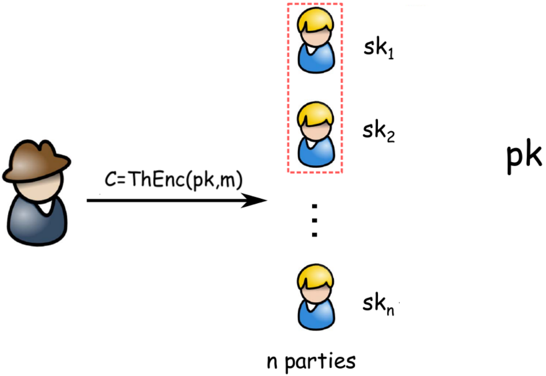

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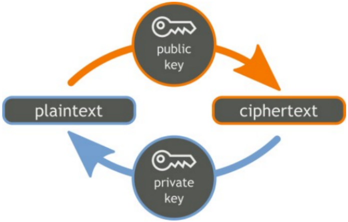

Key Generation

The receiver R chooses

- a secret key $\boldsymbol{s} \xleftarrow{R} \mathbb{Z}_2^{\ell}$
- $\boldsymbol{A} \xleftarrow{R} \mathbb{Z}_2^{q \times \ell}$ and the error $\boldsymbol{e} \leftarrow \mathrm{Ber}_{\tau}^{q}$, where $\tau \in \Theta(\frac{1}{\sqrt{\ell}})$ and computes the pk as $(\boldsymbol{A}, \boldsymbol{b} = \boldsymbol{A}\boldsymbol{s} \oplus \boldsymbol{e})$

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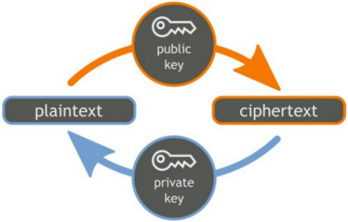

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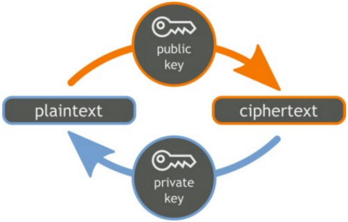
$\mathbf{pk} = (\mathbf{A}, \mathbf{b})$



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Encryption of a message bit $m \in \mathbb{Z}_2$

Sender $\underline{\mathtt{S}}$ | Receiver $\underline{\mathtt{R}}$

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noisy decryption: correct decryption $\Leftrightarrow \langle \boldsymbol{f}, \boldsymbol{e} \rangle = 0$

ThPKE: Protocol phases

- **Key Generation**
- Key Assembly
- Encryption
- Partial Decryption
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Key Assembly

The combined public key is the pair $(\boldsymbol{A}, \boldsymbol{b})$, where

$$\boldsymbol{b} = \bigoplus_{i \in I} \boldsymbol{b}_i$$

and I is the users subset

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$$\boldsymbol{C_1} = \boldsymbol{F} \cdot \boldsymbol{A}, \ \boldsymbol{c_2} = \boldsymbol{F} \cdot \boldsymbol{b} \oplus \underbrace{\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}}_{q} \cdot m \qquad \text{where } \boldsymbol{F} := \begin{bmatrix} \boldsymbol{f_1} \\ \vdots \\ \boldsymbol{f_q} \end{bmatrix}, \boldsymbol{f_i} \leftarrow \mathrm{Ber}_{\tau}^{q}$$

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Partial decryption function (Alekhnovich scheme)

$$\boldsymbol{d_i} = \boldsymbol{C_1} \cdot \boldsymbol{s_i} \oplus \boldsymbol{\nu_i}$$

where $\boldsymbol{\nu_i} \leftarrow \mathrm{Ber}_\sigma^q$

ThPKE: Protocol phases

- Key Generation
- Key Assembly
- Encryption
- **Partial Decryption**
- Finish Decryption

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- the bit in $\boldsymbol{d}$ that is in majority is separately chosen by each receiver as m

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Semi-honest model

A semi-honest party:

1. Follows the protocol properly
2. Keeps a record of all its intermediate computations

Security

- **Encryption:** from the Alekhnovich's scheme security
- **Decryption:** from the LPN hardness assumption, as each P_i is generating LPN samples

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Relaxed Semi-honest model

- Semi-honest model is not realistic
- *replay attacks*: the same message is fraudolently encrypted multiple times
- Problem: it is possible to recover information about the secret key from the ciphertexts

Proposed solutions

1. implement the receivers as stateful machines (not good in resource-constrained devices)
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- We investigate about the Learning Parity with Noise (LPN) problem
- We propose a Threshold Public-Key Encryption scheme based on LPN

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- Study the security of our Threshold Public-Key Encryption scheme in the *malicious model*

LPN open problems

- Relation between standard LPN and some variants
- Does LPN with noise rate τ imply anything about LPN with $\tau' < \tau$? Is there a threshold?
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Scenario:

Alice wants to keep a message secret from Bob for now but she intends to reveal it to Bob at some time in the future

Commitment protocol

- Alice commits the message and Bob does not learn any information about it (hiding property)
- Alice chooses to open the commitment and reveal the message but she cannot change the value committed (property)

Our contribution

We presented a Commitment protocol

- based on the commitment protocol by *Jain et al*
- based on Exact-LPN problem (where $wt(e) = \lceil \tau \cdot k \rceil$)
- does not need a trusted third party

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The commitment protocol by *Jain et al*

Setup Phase

In order to commit a message $\boldsymbol{m} \in \mathbb{Z}_2^k$ where $k \in \Theta(\ell + v)$
We state $\boldsymbol{A} = [\boldsymbol{A'} \| \boldsymbol{A''}] \in \mathbb{Z}_2^{k \times (\ell+v)}$ as the common reference string (CRS).
Finally, we set $w = \lfloor \tau k \rceil$.

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define $\boldsymbol{d} = (\boldsymbol{m'}, \boldsymbol{r'})$ $\xrightarrow{\boldsymbol{d}}$

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Opening phase

define $\boldsymbol{d} = (\boldsymbol{m'}, \boldsymbol{r'})$ $\xrightarrow{\quad \boldsymbol{d} \quad}$

computes $\boldsymbol{e'} = \boldsymbol{c} \oplus \boldsymbol{A}\,(\boldsymbol{r'} \| \boldsymbol{m'})$

$\xleftarrow{\quad \mathit{Yes}, \mathit{No} \quad}$ accepts iff $\boldsymbol{wt}(\boldsymbol{e'}) = w$

Proposed commitment protocol

Problem

We need a trusted third party for the common matrix $\boldsymbol{A} = [\boldsymbol{A}' \| \boldsymbol{A}'']$

Solution:

The Commitment and Opening phases are the same as in the original scheme

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The commitment protocol: a LPN-based variant

LPN variant

- security directly based on the standard LPN problem
- **Commit phase:** we set $w' = 2 \cdot \lfloor \tau k \rceil$ and we choose $\boldsymbol{e}$ such that $\boldsymbol{wt}(\boldsymbol{e}) \leq w'$

Choice of parameters

According to

Levieil, Éric and Fouque, Pierre-Alain
An Improved LPN Algorithm
Springer Berlin Heidelberg, 2006

we choose $\ell = 768$ and noise rate $\tau = \frac{1}{8} \Rightarrow 2^{90}$ bytes of memory to solve LPN

Theorem

Our commitment scheme is statistically binding and computationally hiding

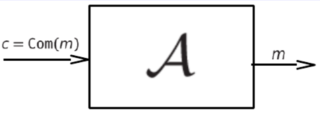
Proof

Statistically binding

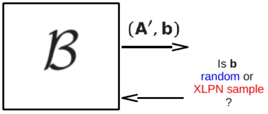
even if S is computationally unbounded she cannot cheat with probability greater than 2^{-k}

Computationally hiding

- proof for reduction (single bit message)
- we assume that $\mathcal{A}$ is able to break the commitment scheme

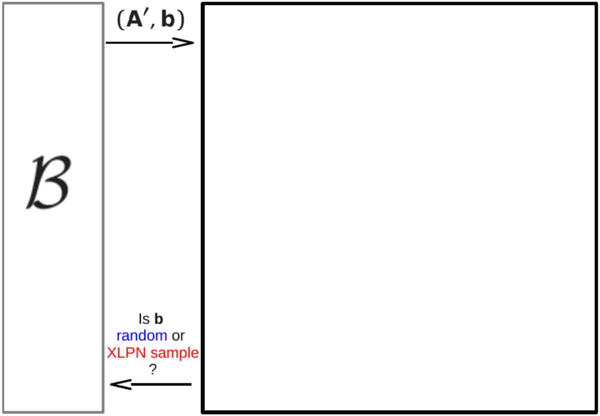


- Let $\mathcal{B}$ an oracle



where $\boldsymbol{b} = \begin{cases} \text{random} & w.p.\ 1/2 \\ \boldsymbol{A's} \oplus \boldsymbol{e} & w.p.\ 1/2 \end{cases}$

Proof



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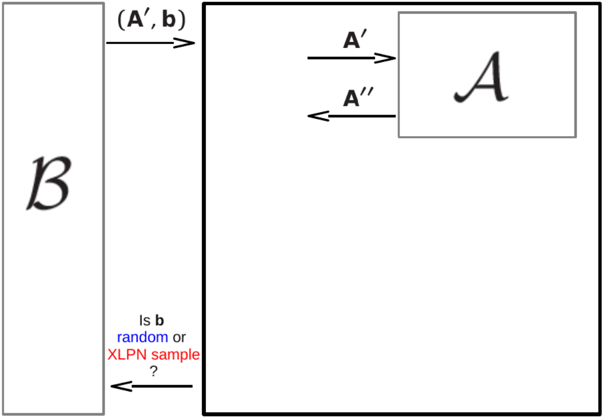
$(\mathbf{A}', \mathbf{b})$
$\mathbf{A}'$
$\mathbf{A}''$
$\mathcal{A}$
$\mathcal{B}$
Is $\mathbf{b}$
random or
XLPN sample
?

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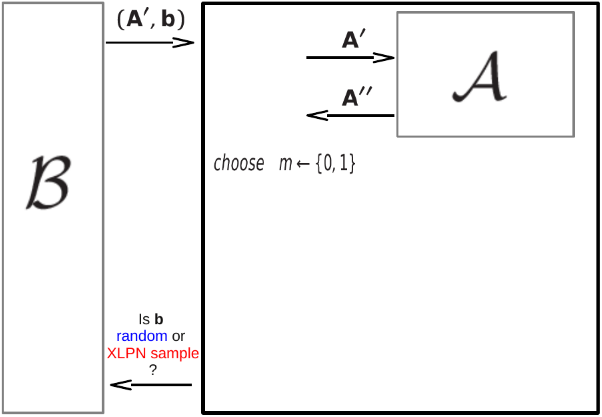

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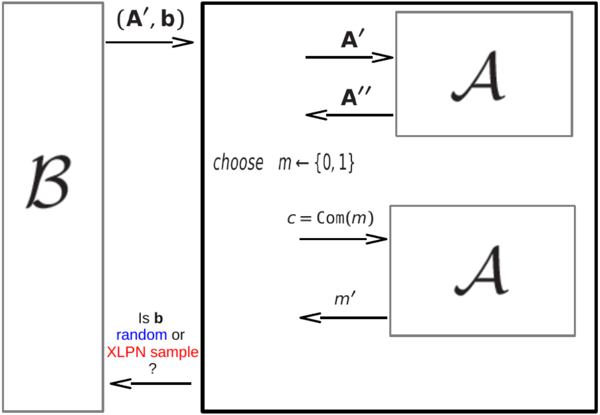
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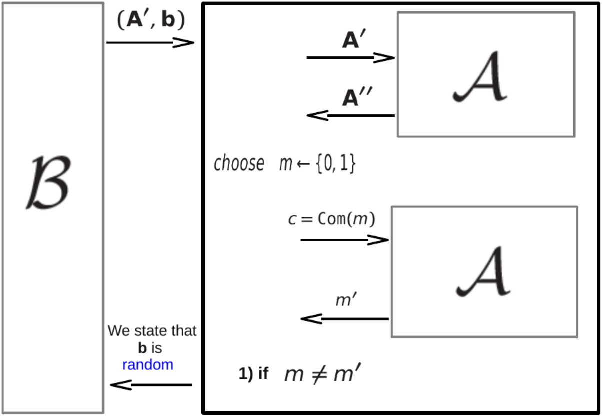
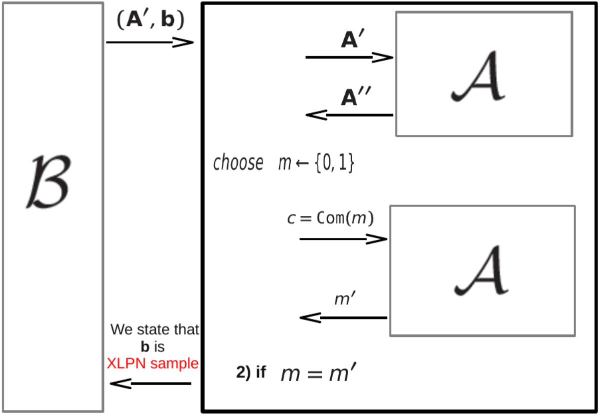


case 1)

$\boldsymbol{b}$ is random $\Rightarrow$ $\boldsymbol{c} = \boldsymbol{b} \oplus \boldsymbol{A}''m$ is a onetime-pad encryption

$\Rightarrow \mathcal{A}$ guesses w.p. $\frac{1}{2}$

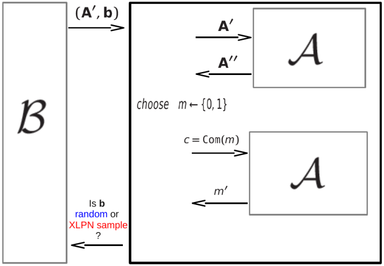
Proof



case 2)

$\boldsymbol{b}$ is a Exact-LPN sample $\Rightarrow$ $\boldsymbol{c}$ is a well formed commitment
$\Rightarrow \mathcal{A}$ guesses w.p. 1 (by hypothesis)

Proof



case 1) and 2)

Let E = the reduction breaks the Exact-LPN problem,

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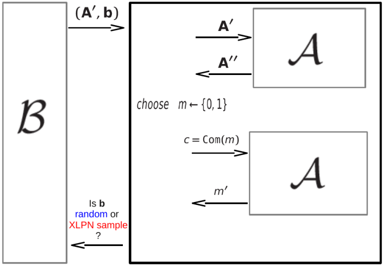

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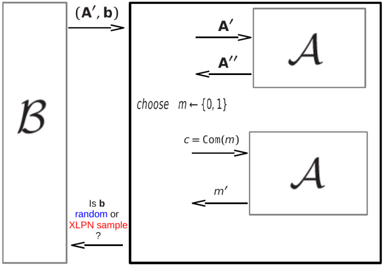

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$$= 1 \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4}$$

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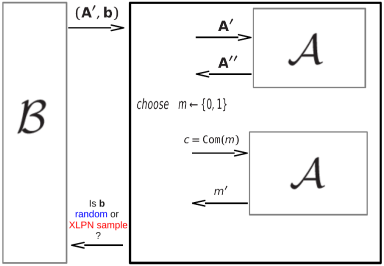

case 1) and 2)

Let E = the reduction breaks the Exact-LPN problem,

$$\Pr(E) = \Pr\left(E \middle| \, \boldsymbol{b} = \boldsymbol{A}'\boldsymbol{s} \oplus \boldsymbol{e}\right) \cdot \Pr\left(\boldsymbol{b} = \boldsymbol{A}'\boldsymbol{s} \oplus \boldsymbol{e}\right) + \Pr\left(E \middle| \, \boldsymbol{b} \text{ is random}\right) \cdot \Pr\left(\boldsymbol{b} \text{ is random}\right)$$

$$= 1 \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4}$$

Exact-LPN hardness $\Rightarrow$ Hiding commitment