FISCAL POLICY AND VOLATILITY UPON EXIT FROM A MONETARY UNION: A DSGE APPROACH

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ABSTRACT. This paper investigates volatility and fiscal policy when the government of a small-open economy leaves a monetary union. The paper builds a Dynamic Stochastic General Equilibrium model for a small-open economy without monetary policy independence and solves it to find the optimal simple fiscal rules that minimize the volatility of output, inflation, the interest rate, and the exchange rate. Exit from a monetary union is modeled by adding two key features to the underlying model: an endogenously-determined interest rate following a Taylor Rule and a floating exchange rate by imposing the uncovered interest rate parity condition. The paper draws policy-relevant conclusions by comparing fiscal rules before the small-open economy leaves the monetary union and after. Results show that upon exit from a monetary union, every fiscal instrument, with the exception of the income tax, is more responsive to gaps in government liabilities and output, as well as that the level of volatility in the economy increases. This is due to the presence of a floating exchange rate and its effects are two sided: 1) volatility in the domestic currency increases the volatility of interest rate and 2) the presence of a floating exchange rate creates a relationship between interest rate an government liabilities, which directly impacts fiscal policy.

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Contents

1. Introduction

Since the aftermath of the Second World War, several monetary unions, sometimes also referred to as currency unions, have emerged across the globe. Even though there exist different types of monetary unions, this paper considers the ones where member nations lose monetary policy independence: that is, countries that join a monetary union are willing to forego the ability to change interest rates, in favor of a united monetary policy strategy with their major trading partners and a common currency. Currently, the CFA Franc, the Multilateral Monetary Area, and the Eurozone are the largest such currency unions. In the last decade, the question of exit from a currency union, primarily posed in European member states, has interested economists and policymakers. Since the global financial crisis, skepticism towards the institutions has spread across Europe. Many of the countries that were most severely affected by the Great Recession have not yet completely recovered; they have considered the possibility of leaving the European monetary union and implementing their pre-Euro currencies in an effort to manage rising levels of government debt. These movements tend to be accompanied by populist propaganda, which often advocates for economic and political independence from international governing bodies. For example, this has been the case for Italy and Greece, both of which are still facing high levels of unemployment and sluggish economic growth. The economic consequences of an exit from a monetary union are largely speculative, primarily because no government has, to this day, formally declared its willingness to exit a monetary union and the topic has just recently begun to be explored in the economic literature.

Publications that have attempted to answer this question, at least partially, are both empirical and theoretical. Briefly in this section and in greater depth in section 2, both approaches are considered, in an effort to contextualize methodology as well as findings. Angeloni et al. find that, in an environment where fiscal policy is based on macroeconomic performance, debt accumulation, and banking system stability, fiscal rules should place a greater weight either on consumption taxes on direct government spending.[1] Gali and Monacelli study the interaction between the country-specific fiscal rules and monetary policy set by the monetary authority.[2] They find that when the government focuses on direct spending, it should do so by going beyond the "mere efficient provision of public goods" and instead considering the possible spillover effects on inflation levels. On the other hand, papers that consider a small-open economy outside of a union find that optimal monetary policy for the European Central Bank is closely related to past strategies adopted by the Bundesbank.[3]

While most papers consider a small-open economy either inside or outside a monetary union, informally referred to as the "in" or the "out" scenario in later sections, this study's contribution to the literature lies in the comparisons that it draws between them, in addition to some structural features outlined in section 3. The paper analyzes exit from a monetary union from a fiscal policy perspective: namely, it aims to define the government's best response following the acquisition of monetary policy independence by conducting the following experiment. I first consider a small-open economy Dynamic Stochastic General Equilibrium ("DSGE") modelwhich is part of a monetary union, i.e. without monetary policy independence, using a standard New Keynesian framework from Varthalitis: among other features, the most significant ones are the interest rate on government bonds, determined by the international monetary authority, and the fixed exchange rate.[4] I compute the optimal simple fiscal policy rule that minimizes the volatility of consumption, inflation, the interest rate, and the exchange rate. Each fiscal instrument, the consumption tax, the capital tax, the income tax, and government spending is a function of inherited

[1]Ignazio Angeloni, Ester Faia, and Roland Winkler, *Exit strategies*, (SSRN Electronic Journal, 2011), 19, 26.
[2]Jordi Gali and Tommaso Monacelli, *Optimal monetary and fiscal policy in a currency union*, (National Bureau of Economic Research, 2005), 29.
[3]Jurgen von Hagen and Matthias Bruckner, *Monetary and Fiscal Policy in the European Monetary Union*, (Monetary and Economic Studies, 2002), 128.
[4]Petros Varthalitis, *Fiscal and monetary policy in New Keynesian DSGE models*, (Athens University of Economics and Business, 2014).

government liabilities and the output gap. Informally, this is the control group of the policy experiment. The motivation to focus on volatility, and not the values of the indicators themselves, is two-sided: (1) most papers that compute optimal fiscal policy or the optimal simple rule do so based on the level of household welfare or on the actual values of the aforementioned variables, and (2) the media and politicians have speculated that exit from a monetary union is bound to increase volatility across the economy.[5] [6] The treatment group of the policy experiment works as follows. I mimic a country's exit from a monetary union by applying two changes to the original model: (1) I treat the interest rate as endogenous, specified following a Taylor Rule, and (2) the exchange rate as endogenous, by imposing the uncovered interest rate parity condition. When the small-open economy is outside the monetary union, I run the optimal simple rule on fiscal instruments, as well as the Taylor-Rule interest rate, since it is no longer subject to the rate set by the monetary authority. The comparison between the "in" and "out" scenarios allows me to draw policy-relevant economic conclusions.

Results show that the level of volatility increases in the small-open economy when the government adopts the consumption tax, capital tax, and government spending to minimize it. Fiscal rules also change: with the exception of the income tax, the weight placed on inherited government liabilities and the output gap increases. This means that, after exit, the same gap in either economic indicator requires a greater increase in the tax rate than it did before exit, while achieving a greater level of volatility. This is due to the presence of a floating exchange rate, which affects the prices on imports as well as the interest rate. The mechanism through which the exchange rate affects the small-open economy is discussed in detail in section 5.

[5] Naeem Aslam, *'Italexit': A Bigger Thorn Than Brexit*, (Forbes Website, 2018), https://www.forbes.com/sites/naeemaslam/2018/05/23/italexit-a-bigger-thorn-than-brexit/6b6579b16046.
[6] *What Are The Implications Of Italy Leaving The Eurozone?*, (FXCM Website, 2016), https://www.fxcm.com/uk/insights/implications-of-italy-leaving-eurozone/.

The rest of the paper is organized as follows. Section 2 provides context with an investigation of the existing literature on fiscal policy. Section 3 presents the model. Section 4 presents the data used to calibrate the parameters in the model. Section 5 presents the results. Section 6 is the conclusion. The appendix includes a summary of the equilibrium conditions.

2. Literature Review

Fiscal policy in monetary unions is a heavily-researched topic in the economic literature, as there are several empirical and theoretical papers investigating its effects on economic outcomes. The vast majority of the literature considers fiscal policy either inside or outside a monetary union.

Furceri and Mourougane examine the effects of fiscal policy on output in the euro area by developing a small-open economy DSGE model with endogenous government bond yields. Their results show that fiscal policy stimulates greater growth in key economic indicators, such as output, inflation, debt-to-GDP ratio, and sovereign spreads, in the short run.[7] Their other significant conclusion is that the fiscal multiplier is positively correlated with the level of cash that households hold.[8] A similar study considers two scenarios for a closed economy: (1) an initial debt-to-GDP ratio that aligns with historical averages, and (2) an initial debt-to-GDP ratio that is significantly higher than historical averages.[9] In "normal times", government spending is the most effective fiscal policy tool to generate better economic outcomes; if the government is instead trying to consolidate its debt, it should do so gradually and by giving preference to increases in distortionary taxes as

[7] Davide Furceri and Annabelle Mourougane, *The Effects of Fiscal Policy on Output: A DSGE Analysis*, (OECD Economics Department Working Papers No. 770, 2010), 15-16.
[8] Ibid..
[9] Cristiano Cantore, Paul Levine, Giovanni Melina, and Joseph Pearlman, *Optimal Fiscal and Monetary Rules in Normal and Abnormal Times*, (University of Surrey, 2013).

opposed to by lowering spending.[10] However, the work of Papegeorgiu, basing his parameter calibrations on the case of Greece, suggests that the beneficial effects of lower government spending or higher taxes are only observable in the medium- and long-run, since their contractionary effects are too strong in the short-run.[11] His findings also suggest that a mix of lower income taxes and higher consumption tax is the optimal fiscal bundle.[12]

Other fiscal policy papers that address a similar question to the one of this paper's are Hjortsoe, Ferrero, and Gali and Moncelli.[13] [14] [15] They all solve a Ramsey problem for fiscal policy, thus solving fiscal policy by optimizing social welfare. Hjortsoe points out that fiscal authorities in a monetary union face a trade-off between curbing intra-union imbalances and stabilizing the output gap; her main findings show that internal optimal fiscal policy hinders trade flows with the rest of the union, thus destabilizing international economic stability.[16] Therefore, when a country is inside of a currency union, it can only achieve growth through fiscal policy that is coordinated with other governments in the union. Ferrero constructs a two-country model in a monetary union to study the joint conduct of fiscal and monetary policy. His findings support the ones of Hjortsoe, as he finds that household lifetime utility is maximized when the two countries coordinate fiscal policy; moreover, he confirms the mandate of the monetary authority, by showing that it should set interest rates by maximizing each country's individual welfare and not the the one of union at large.[17] Gali and Monacelli develop a model to study interest rate

[10]Ibid., 23-24.
[11]Dimitris Papageorgiou, *Fiscal policy reforms in general equilibrium: The case of Greece*, (Journal of Macroeconomics 34, 2012), 504-505.
[12]Ibid..
[13]Ida Hjortsoe, *Imbalances and Fiscal Policy in a Monetary Union*, (European University Institute, 2011).
[14]Andrea Ferrero, *Fiscal and Monetary Rules for a Currency Union*, (European Central Bank Working Paper Series No. 502, 2005)
[15]Jordi Gali and Tommaso Monacelli, *Monetary Policy and Exchange Rate Volatility in a Small Open Economy*, (The Review of Economic Studies Limited, 2005)
[16]Hjortsoe, *Imbalances*, 13-14.
[17]Ferrero, *Fiscal and Monetary*, 39-40.

and exchange rate volatility in a small-open economy that has monetary independence: they find that acquisition of monetary policy independence generates a stabilization role for fiscal policy.[18] The relevance of their findings lies in the fact that they find that exchange rate volatility varies significantly as the target of the monetary authority changes and I consider exchange rate volatility in my definition of economic volatility. Another publication that studies optimal fiscal policy by considering Ramsey-optimal fiscal and monetary policy in a medium-scale model of the U.S. business cycle.[19] They find that if taxes on capital and on labor income are different, optimal fiscal policy is characterized by a large capital subsidy of over 40% and with a volatility of about 150%.[20] Perotti estimates the effects of fiscal policy in OECD countries and finds that the multiplier on direct government spending is relatively small, no evidence to suggest that government spending is preferable to tax cuts, or vice-versa, and that inflation increases significantly as a result of higher government spending only in specific favorable conditions of price elasticity.[21]

Conen et al. specify seven different DSGE models, each one focusing on one type of distortionary tax, to understand the government's best response to serious economic downturns, such as the 2008-2009 financial crisis.[22] They find that the fiscal multipliers are significantly higher if (1) the current policy of the monetary authority is supportive by holding interest rates constant and (2) changes to fiscal policy are not permanent.[23] If on the one hand their findings suggest that communication between fiscal and monetary authorities can be beneficial, they are not translatable to most developed economies, as central banks are typically independent agencies. In the context of this paper, Cohen et

[18] Gali and Monacelli, *Monetary Policy*, 727-728.

[19] Stephanie Schmitt-Grohe and Martin Uribe, *Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model*, (European Working Paper Series No. 612, 2006).

[20] Ibid., 42.

[21] Roberto Perotti, *Estimating the effects of fiscal policy in OECD countries*, (IGIER - Universita Bocconi and Centre for Economic Policy Research, 2004), 2.

[22] Gunter Cohen et al., *Effects of Fiscal Stimulus in Structural Models*, (American Economic Association Vol. 4 No. 1, 2012).

[23] Ibid., 60-65.

al. are relevant to the extent that I consider a small-open economy with monetary policy independence that solves for volatility-minimizing fiscal rules. On the contrary, the work of Wren-Lewis shows that aggressive expansionary fiscal policy is required, regardless of the actions of the monetary authority, by arguing that open-market operations like quantitative easing (QE) are not enough to end the recessionary period and that not all fiscal adjustments require additional borrowing.[24] Similarly, Gordon and Leeper study the impact of countercyclical fiscal policy measures on employment and total income, since some economists agree that they have stabilizing effects through automatic stabilizers and discretionary actions. They find that, if households expect higher taxes in expansionary periods, investment decreases by a "non-trivial factor" and that expected future fiscal policy is the sole factor that generate persistence in key macro variables.[25] The results of Buiter show that in order for fiscal policy to tackle a recession successfully, (1) there must be no other expansionary instrument (including monetary policy), (2) there must be no complete financial crowding out, and (3) there must be cross-border externalities from fiscal stimuli, such as foreign direct investment.[26] These provide further insights on the interaction between fiscal and monetary authorities, which are discussed in section 5 in the context of the "out" scenario.

Varthalitis shows that, in a closed economy, it is preferable to decrease government spending for shock stabilization or debt consolidation as well as that, except for a labor tax, distorting fiscal policy instruments help maximize household welfare when attempting to consolidate government debt, other things equal.[27] At the same time, he finds that

[24]Simon Wren-Lewis, *Macroeconomic Policy in Light of the Credit Crunch: the Return of Counter-cylical Fiscal Policy?*, (Oxford Review of Economic Policy Vol. 26 No. 1 Spring, 2010), 71-73.

[25]David B. Gordon and Eric M. Leeper, *Are Countercyclical Fiscal Policies Counterproductive?*, (NBER Working Paper Series, 2005), 2-3.

[26]Willem H. Buiter, *The Limits to Fiscal Stimulus*, (Oxford Review of Economic Policy Vol. 26 No. 1 Spring, 2010), 48-59.

[27]Varthalitis, *Fiscal and monetary*, 89.

tax-based debt consolidations are preferable, if the environment is one of a closed economy.[28] His work is relevant to this paper's as it provides the baseline model that I adopt and expand upon to answer my own research question. This paper's contribution to the fiscal policy literature is two sided: 1) instead of solving a Ramsey problem, I adopt an optimal simple rule approach to derive fiscal rules on the consumption tax, capital tax, income tax, and government spending and 2) I minimize the volatility of five key economic indicators: output, consumption, inflation, the interest rate, and the exchange rate. This choice is motivated by the notion that, considering a relatively simple small-open economy model in a context of exit from a monetary union, the optimal simple rule is a good approximation for *true* optimal policy, i.e. Ramsey policy, and because it is a proxy for a government's attempt to ensure a smooth transition to an independent monetary policy regime.[29]

3. Model

To study a small-open economy inside of a monetary union, I develop a theoretical model drawing from Varthalitis.[30] Each subsection below describes a different agent in the economic system

3.1. **Households.**

3.1.1. *Household's problem.* Household i maximizes lifetime utility, which is a function of consumption $c_{i,t}$, hours of labor $n_{i,t}$, real money holdings $m_{i,t}$, and government spending $g_{i,t}$. Utility in period t is given by

[28]Ibid., 89, 97-98.

[29]John B. Taylor and John C. Williams, *Simple and Robust Rules for Monetary Policy*, (NBER Working Paper Series, 2010), 11.

[30]Varthalitis, *Fiscal and monetary*, 77-80.

$$u_{i,t}(c_{i,t}, n_{i,t}, m_{i,t}, g_t) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_{i,t}^{1+\eta}}{1+\eta} + \chi_m \frac{m_{i,t}^{1-\mu}}{1-\mu} + \chi_g \frac{g_t^{1-\zeta}}{1-\zeta}, \tag{1}$$

where χ_n, χ_m, χ_g, σ, η, μ, ζ are the preference parameters.[31] Each period t, household i faces the following budget constraint

$$\begin{aligned}
&(1+\tau_t^c)\left[\frac{P_t^H}{P_t}c_{i,t}^H + \frac{P_t^F}{P_t}c_{i,t}^F\right] + \frac{P_t^H}{P_t}x_{i,t} + b_{i,t} + m_{i,t} + \frac{S_t P_t^*}{P_t} f_{i,t}^h + \frac{\phi^h}{2}\left(\frac{S_t P_t^*}{P_t} f_{i,t}^h - \frac{SP^*}{P} f^h\right)^2 \\
&= (1-\tau_t^k)\left[r_t^k \frac{P_t^H}{P_t} k_{i,t-1} + d_{i,t}\right] + (1-\tau_t^n) w_t n_{i,t} + R_{t-1}\frac{P_{t-1}}{P_t} b_{i,t-1} + \frac{P_{t-1}}{P_t} m_{i,t-1} \\
&\quad + Q_{t-1}\frac{P_{t-1}}{P_t}\frac{S_t P_t^*}{P_t^*} f_{i,t-1}^h - \tau_{i,t}^l \; ;
\end{aligned} \tag{2}$$

The variables $c_{i,t}^H$ and $c_{i,t}^F$ are domestically-produced consumption goods and consumption goods produced abroad, respectively, $x_{i,t}$ is household i's level of investment, $b_{i,t}$ is household i's end-of-period real domestic government bonds, $m_{i,t}$ is household i's end-of-period real money holdings, $f_{i,t}^h$ is i's end-of-period internationally traded assets denominated in foreign currency, r_t^k is the rental rate of capital, $k_{i,t-1}$, $d_{i,t}$ is real dividends received by domestic firms, w_t is the wage rate. R_{t-1} is the gross nominal interest rate on domestic government bonds between $t-1$ and t, Q_{t-1} is the gross nominal interest rate on international assets between $t-1$ and t, $\tau_{i,t}^l$ is a lump tax received by household i, and τ_t^c, τ_t^k, τ_t^n are taxes on consumption, capital, and labor income, respectively. The parameter ϕ^h is the transaction costs associated with foreign assets.[32] In each period t, household i chooses consumption $c_{i,t}$, the end-of-period level of physical capital $k_{i,t}$, number of hours to work $n_{i,t}$, government bonds $b_{i,t}$, amount of real money holdings $m_{i,t}$, and the end-of-period

[31] Ibid., 78.
[32] Ibid..

amount of internationally-traded assets $f_{i,t}^{h}$. The variables P_t, P_t^H, P_t^F, and P^* are price indeces, explained in detail below.

3.1.2. *Physical capital law of motion.*

$$k_{i,t} = (1-\delta)k_{i,t-1} + x_{i,t} \tag{3}$$

where δ is the depreciation rate of capital.[33]

3.1.3. *Price and aggregation.* The quantity of variety h produced at home by firm h and consumed by domestic household i is denoted as $c_{i,t}^{H}(h)$. The population size is constant and equal to N; thus, there are N identical households indexed $i = 1, \dots, N$, N domestic firms indexed $h = 1, \dots, N$ each producing a differentiated product, and N firms abroad, indexed $f = 1, \dots, N$, each producing a differentiated product. The composite of domestic goods consumed by household i is given by

$$c_{i,t}^{H} = \left[\sum_{h=1}^{N} \gamma [c_{i,t}^{H}(h)]^{\frac{1-\phi}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{4}$$

where the parameter $\gamma = 1/N$ avoids scale effects and ϕ is the price elasticity of demand.[34] The composite of foreign goods consumed by household i is given by

$$c_{i,t}^{F} = \left[\sum_{f=1}^{N} \gamma [c_{i,t}^{F}(f)]^{\frac{1-\phi}{\phi}}\right]^{\frac{\phi}{\phi-1}}; \tag{5}$$

the γ parameter is similar to the one in equation (4).[35] Since I model a small-open economy, household's total level of consumption is a function of domestic and foreign consumption.

[33]Ibid., 79.
[34]Ibid., 77.
[35]Ibid..

Household i's consumption bundle $c_{i,t}$ is therefore given by

$$c_{i,t} = \frac{(c_{i,t}^H)^\nu \, (c_{i,t}^F)^{1-\nu}}{\nu^\nu (1-\nu)^{1-\nu}}, \tag{6}$$

where the parameter ν indicates the level of bias for consumption goods produced at home.[36] This means that if $\nu = 0.5$ and holding prices constant, household i is indifferent between a domestically-produced good and a good produced abroad. Similarly, if $\nu > 0.5$ and holding prices constant, household i has a personal preference for domestically-produced goods. Household i's total cost on consumption goods is

$$\begin{aligned} P_t c_{i,t} &= \sum_{h=1}^{N} \lambda P_t^H(h) c_{i,t}^H(h) + \sum_{f=1}^{N} \lambda P_t^F(f) c_{i,t}^F(f) \\ &= P_t^H c_{i,t}^H + P_t^F c_{i,t}^F \end{aligned} \tag{7}$$

where P_t^H and P_t^F are inflation on domestically-produced consumption goods and inflation on consumption goods produced abroad, and

$$P_t = \left(P_t^H\right)^\nu \left(P_t^F\right)^{1-\nu}, \tag{8}$$

respectively.[37] The interpretation of ν is similar to its interpretation for equation (6).

I assume that the law of one price holds. This means that identical tradeable goods sell at the same price domestically and abroad, once one accounts for the exchange rate. Formally, $P_t^F(f) = S_t P_t^{H*}(f)$, where S_t is the nominal exchange rate, defined as the amount of domestic currency necessary to purchase one unit of foreign currency. The quantity $P_t^{H*}(f)$ is defined as the price of variety f produced abroad denominated in the foreign currency [38]. Therefore, an increase in S_t implies a depreciation in the domestic currency.

[36]Ibid..

[37]Ibid., 79.

[38]In this paper a star denotes the counterpart of a variable or a parameter in the rest of the world, a variable without a time subscript represent its long-run value, lower-case letters denote variables expressed in real terms, and upper-case letters denote variables in nominal terms.

From this, I derive the terms of trade, that are defined as

$$\frac{P_t^F}{P_t^H} = \frac{S_t P_t^{H*}}{P_t^H}. \tag{9}$$

The terms of trade imply no arbitrage between the domestic and foreign currency. For example, consider the United States as the domestic country and Mexico as the foreign one. The law of one price means that, if a US consumer purchases one unit of consumption goods produced in Mexico, he pays P_t^F dollars. Selling that same good in Mexico would yield him P_t^{H*} Pesos, which correspond to the same P_t^F dollars. The law of one price thus determines the price of imports.

3.2. **Firm.**

3.2.1. *Firm's problem (demand side).* For each product h that the firm produces, the demand comes from households' consumption and investment, government direct spending, and foreign consumers. The variable $c_t^{F*}(h)$ indicates the level of foreign consumption of foreign households, which means that $c_t^{F*}(h)$ is the amount of imports. Gross domestic product is thus given by

$$y_t^H(h) = c_t^H(h) + x_t(h) + g_t(h) + c_t^{F*}(h) \tag{10}$$

where $y_t^H(h) = \sum_{i=1}^N y_{i,t}(h)$, $c_t^H(h) = \sum_{i=1}^N c_{i,t}^H(h)$, $x_t(h) = \sum_{i=1}^N x_{i,t}(h)$, and $c_t^{F*}(h) = \sum_{i=1}^{N*} c_{i,t}^{F*}(h)$.[39] Households thus face another problem: minimizing total cost on consumption subject to the available quantity of consumption goods $c_t^H(h)$. This leads to

$$c_t^H(h) = \left(\frac{P_t^H(h)}{P_t^H}\right)^{-\phi} c_t^H \ , \tag{11}$$

[39]Varthalitis, *Fiscal and monetary*, 80.

where ϕ has the same interpretation as in section 3.1.3.[40] The argument for $x_t(h)$, $g_t(h)$, and $c_t^{F*}(h)$ is similar, so we have

$$x_t(h) = \left(\frac{P_t^H(h)}{P_t^H}\right)^{-\phi} x_t \tag{12}$$

$$g_t(h) = \left(\frac{P_t^H(h)}{P_t^H}\right)^{-\phi} g_t \tag{13}$$

$$c_t^{F*}(h) = \left(\frac{P_t^{F*}(h)}{P_t^{F*}}\right)^{-\phi} c_t^{F*} \ . \tag{14}$$

Note that the terms of trade specified in equation (9) imply that

$$\frac{P_t^{F*}(h)}{P_t^{F*}} = \frac{\frac{P_t^H(h)}{S_t}}{\frac{P_tH}{S_t}} = \frac{P_t^H(h)}{P_t^H} \ . \tag{15}$$

Combining equations (12-15) and (10), we have

$$y_t^H(h) = c_t^H(h) + x_t(h) + g_t(h) + c_t^{F*}(h) = \left(\frac{P_t^H(h)}{P_t^H}\right)^{-\phi} y_t^H . \tag{16}$$

Taking the summation over h, we have the aggregate demand for domestically produced goods

$$y_t^H = c_t^H + x_t + g_t + c_t^{F*} \ . \tag{17}$$

3.2.2. *Firm's problem (supply side).* In each period t, domestic firm h maximizes nominal profits given by

$$D_t(h) = P_t^H(h)y_t^H(h) - r_t^k P_t^H(h)k_{t-1}(h) - W_t n_t(h). \text{ [41]} \tag{18}$$

[40] Ibid., 100.

Furthermore, all firms are subject to the same production constraint given by a standard Cobb-Douglas production function

$$y_t^H(h) = A_t[k_{t-1}(h)]^{\alpha}[n_t(h)]^{1-\alpha}. \tag{19}$$ [42]

The variable A_t is an exogenous stochastic process whose motion is described below. Firms maximize profits by minimizing nominal costs of production. I further assume that, as in Calvo, in each period, a θ fraction of the firms in the economy is able to reset its price to $P_t^{\#}(h)$ to maximize the sum of discounted expected nominal profits,

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} D_{t+k}(h) = \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} \{P_t^{\#}(h) y_{t+k}^H(h) - \Psi_{t+k}(y_{t+k}^H(h))\} \tag{20}$$

where $\Omega_{t,t+k} = \beta^k \frac{c_{t+k}^{-\sigma}}{c_t^{-\sigma}} \frac{P_t}{P_{t+k}} \frac{\tau_t^c}{\tau_{t+1}^c}$ is the stochastic discount factor, $y_{t+k}^H(h) = \left[\frac{P_t^{\#}(h)}{P_{t+k}^H}\right]^{-\phi} y_{t+k}^H$ and $\Psi_t(.)$ denotes the lowest nominal cost for generating $y_t^H(h)$ units of output at period t so that the associated marginal cost is equal to $\Psi_t'(.)$.[43] [44] The remaining $1-\theta$ fraction of firms takes the prices from the previous period. This added component to the model ensures that not all firms are price-takers and it introduces the notion of price rigidity. Observe further that all firms h that can reset prices in period t solve an identical problem; so $P_t^{\#}(h) = P_t^{\#}$ is independent of h and each firm h, which cannot reset its price, just sets its previous period price $P_t^H(h) = P_{t-1}^H(h)$.[45] Gali shows that the evolution of the aggregate price level is given by:

$$(P_t^H)^{1-\phi} = (1-\theta)(P_{t-1}^H)^{1-\phi} + \theta\left(P_t^{\#}\right)^{1-\phi}. \tag{21}$$ [46]

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[41] Ibid., 79.

[42] Ibid..

[43] Ibid., 80

[44] Guillermo Calvo, *Staggered prices in a utility-maximizing framework*, (Journal of Monetary Economics 12, 1983), 383-398.

[45] Varthalitis, *Fiscal and monetary*, 80.

3.3. **Price Dispersion.** Section 3.3.1 includes two expressions for $y_t^H(h)$, equations (17) and (19). If we take the summation over h of equation (19), we have the supply curve

$$\sum_{h=1}^{N} y_t^H(h) = \sum_{h=1}^{N} \left(\frac{P_t^H(h)}{P_t^H}\right)^{-\phi} y_t^H = \left(\frac{1}{P_t^H}\right)^{-\phi} y_t^H \sum_{h=1}^{N} (P_t^H(h))^{-\phi}$$

$$\Rightarrow y_t^H = \frac{1}{\left(\frac{\widetilde{P_t}^H}{P_t^H}\right)^{-\phi}} A_t k_{t-1}^{\alpha} n_t^{1-\alpha} \tag{22}$$

where $\left(\sum_{h=1}^{N}(P_t^H(h))^{-\phi}\right)^{-\frac{1}{\phi}} = \widetilde{P_t}^H$ and the quantity $\left(\frac{\widetilde{P_t}^H}{P_t^H}\right)^{-\phi}$ is a measure of price dispersion.

3.4. **Government.** The government follows the following budget constraint:

$$b_t + m_t + \frac{S_t P_t^*}{P_t} f_t^g = \frac{\phi^g}{2}\left(\frac{S_t P_t^*}{P_t} f_t^g - \frac{SP^*}{P} f^g\right)^2 + R_{t-1}\frac{P_{t-1}}{P_t} b_{t-1} + \frac{P_{t-1}}{P_t} m_{t-1} +$$
$$Q_{t-1}\frac{S_t P_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*} f_{t-1}^g + \frac{P_t^H}{P_t} g_t - \tau_t^c\left(\frac{P_t^H}{P_t} c_t^H + \frac{P_t^F}{P_t} c_t^F\right) - \tau_t^k\left(r_t^k \frac{P_t^H}{P_t} k_{t-1} + d_t\right) - \tau_t^n w_t n_t - \tau^l. \text{ [47]} \tag{23}$$

I assume that the government always runs on a balanced budget but levying a non-distortionary lump tax τ_t^l on households. The variable Q_t is the nominal interest rate on foreign assets given by

$$Q_t = Q_t^* + \psi\left(e^{\frac{B_t + S_t F_t^g}{P_t^H Y_t^H}} - 1\right), \tag{24}$$

[46] Jordi Gali and Tommaso Monacelli, *Optimal monetary and exchange rate policy in a currency union*, (Journal of International Economics 76, 2008), 116-132.
[47] Varthalitis, *Fiscal and monetary*, 80.

the variable f_t^g is the level of external debt, ϕ^g is a parameter that captures the cost associated with international transactions, and ψ is the risk premium parameter.[48] Equation (24) says that the interest rate that the domestic government pays to foreign lenders, Q_t, is equal to the world interest rate (set by the monetary union) plus a premium. For example, using the same US-Mexico example as above, Q_t is the interest rate that the US government pays to Mexican debt holders. The premium is an increasing function of total government debt, $B_t + S_t F_t^g$, and a decreasing function of real gross domestic product.

3.5. **Exogenous shocks and rest-of-the-world variables.** The productivity process is stochastic and given by

$$log(A_t) = (1-\rho^a)log(A) + \rho^a log(A_{t-1}) + \kappa_t^a. \quad (25)$$

[49]

Lastly, we express domestic exports as a function of the terms of trade

$$\frac{c_t^{F*}}{c^{F*}} = \left(\frac{TT_t}{TT}\right)^{\gamma} \quad (26)$$

where $TT_t = \frac{S_t P_t^{*H}}{P_t^H}$ and $0 < \gamma < 1$.[50] Intuitively, equation (25) says that exports rise as the domestic currency depreciates, or that the rest of the world are more willing to purchase domestic goods than they otherwise would be. I set $c^{F*} = 0.9c^F$, which means that the country's imports are 90% of its exports. This is to assume that the country is a net exporter, which is a common feature in small-open economies.

3.6. **Exit from a monetary union.** The model described until now describes the representative country while it is still inside a monetary union. As introduced in section 1, I mimic exit from a currency union by (1) I treating the interest rate as endogenous, specified following a Taylor Rule, and (2) the exchange rate as endogenous, specified using

[48]Ibid., 81.
[49]Ibid., 83.
[50]Ibid..

the uncovered interest rate parity condition. These two features are necessary to consider acquisition of monetary policy independence. Once a country exits a monetary union, it is able to conduct its own monetary policy and adopts a national currency, which implies a floating exchange rate. Equation (27) below describe a standard Taylor rule, in which the interest rate in period t is a function of the interest rate in period $t-1$, the output gap, the inflation gap, and the exchange rate gap. Intuitively, this means that, once the country has monetary policy independence, the monetary authority responds the changes in inflation, output, and the exchange rate from their respective targets. The interest rate being a function of its one-period lag implies that the monetary authority is also mindful of household's expectation of interest rates in period t, proxied with the interest rate in period $t-1$:

$$\left(\frac{R_t}{R}\right) = \left(\frac{R_{t-1}}{R}\right)^{\phi_R} \left(\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{y_t^H}{y^H}\right)^{\phi_y} \left(\frac{S_t}{S}\right)^{\phi_s}\right)^{1-\phi_R} . \tag{27}$$

[51]

Equation (28) instead describes the relationship between the two real interest rates that the government pays to its debt holders, also known as uncovered interest rate parity.[52]

$$r_t = E_t \beta \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{P_t}{P_{t+1}} \frac{\tau_t^c}{\tau_{t+1}^c} q_t \left(\frac{s_{t+1}}{s_t}\right). \tag{28}$$

Similar to the law of one price above, this condition implies no arbitrage between interest rates. Let us consider the US-Mexico example one more time. For simplicity, suppose that an American investor decides to exchange US dollars for Pesos to buy a one-year Mexican government bond. If he held it until maturity, he would receive $q_t \left(\frac{S_{t+1}}{S_t}\right)$ dollars, times the discount factor, in period $t+1$, which is the same dollar amount that he would

[51]Michael Kumhof, Douglas Laxton, and Kanda Naknoi, *Does the Exchange Rate Belong in Monetary Policy Rules? New Answers from a DSGE Model with Endogenous Tradability and Trade Frictions*, (Macroeconomic Performance in a Globalising Economy, 2007), 14.

[52]Gali, *Monetary policy*, (2005), 714.

have received, had he invested the same amount of dollars in a one-year US government bond. Observe further that a floating exchange rate implies that prices on imports P_t^F are no longer fixed. Before exit, we have $P_t^F = S_t P_t^{H*}$ (from the law of one price), where S_t and P_t^{H*} are both constants. On the other hand, once the small-open economy leaves the monetary union, S_t is determined within the model and therefore so is P_t^F.

4. Methodology

To answer my research question, I study the relationship between fiscal policy and economic volatility in this small-open economy under two different monetary policy regimes. That is, I first consider the model up to, and excluding, section 3.6 and then considering the model including equations (27) and (28). This can be thought of as the control gruop of the experiment. In this scenario, the government is able to conduct fiscal policy through four channels, government spending s_t^g, a consumption tax τ_t^c, a capital tax τ_t^k, and an income tax τ_t^n. Furthermore, in each case, I consider one fiscal instrument as endogenous at a time and set the other three equal to their long-run values. This enables me to isolate the effect of each fiscal policy tool on economic volatility and avoid interaction effects. The fiscal rules are as follows.

$$\frac{\tau_t^n}{\tau^n} = \left(\frac{l_{t-1}}{l}\right)^{\gamma_l^n} \left(\frac{y_{t-1}^H}{y^H}\right)^{\gamma_y^n} \tag{29}$$

$$\frac{\tau_t^c}{\tau^c} = \left(\frac{l_{t-1}}{l}\right)^{\gamma_l^c} \left(\frac{y_{t-1}^H}{y^H}\right)^{\gamma_y^c} \tag{30}$$

$$\frac{\tau_t^k}{\tau^k} = \left(\frac{l_{t-1}}{l}\right)^{\gamma_l^k} \left(\frac{y_{t-1}^H}{y^H}\right)^{\gamma_y^k} \tag{31}$$

$$\frac{s_t^g}{s^g} = \left(\frac{l_{t-1}}{l}\right)^{-\gamma_l^g} \left(\frac{y_{t-1}^H}{y^H}\right)^{\gamma_y^g} \tag{32}$$

where the γ-coefficients are the feedback fiscal policy coefficients on the output gap and the level of inherited public liabilities as deviations from their long-run values. Informally, these coefficients represent the extent to which inherited public liabilities and the output gap affect the given fiscal instrument. Public liabilities l_t are given by

$$l_t = \frac{R_t B_t + Q_t S_{t+1} F_t^g}{P_t^H y_t^H}. \tag{33}$$

The variable l_t thus captures the government's real interest payments towards domestic (B_t) and foreign (F_t^g) lenders.

For each fiscal instrument, I solve for the optimal simple rule. That consists in finding values for the γ_l and γ_y coefficients so that the volatility of output, inflation, the interest rate, and the exchange rate is minimized. I run the four iterations for both the "in" scenario and the "out" scenario. In each scenario where the small-open economy has monetary independence, I also solve for the the optimal simple rule for monetary policy, i.e. equation (27), *in addition to* the given fiscal instrument. Since the adoption of a national currency is a distinctive characteristic of a country that has exited a monetary union, the weights on the output gap and the inflation gap remain constant, adopting parameter values used by Kumhof et al., and find solutions for the weights on the persistence term and the exchange rate gap.[53] Differences in the γ-coefficients on fiscal rules allow me to draw policy conclusions on the way in which exiting a monetary union affects the response of a small-open economy government to gaps in inherited liabilities and output to minimize economic volatility. It is important to note that this methodology does not constitute *optimal* policy, which would instead be determined by maximizing social welfare, also known as a Ramsey problem.[54] However, for small-enough models, such as the one developed in this paper, the optimal simple rule may be used as an approximation for a Ramsey problem.[55]

[53]Kumhof, *Does the exchange*, 14.
[54]Lawrence J. Cristiano, Roberto Motto, and Massimo Rostagno, *Notes on Ramsey-Optimal Monetary Policy*, (Northwestern University, 2007).
[55]Taylor, *Simple and Robust* , 11.

4.1. **Interpretation of Coefficients.** Let us consider the income tax as an example. We know that

$$\frac{\tau_t^n}{\tau^n} = \left(\frac{l_{t-1}}{l}\right)^{\gamma_l^n} \left(\frac{y_{t-1}^H}{y^H}\right)^{\gamma_y^n}$$

$$\Leftrightarrow$$

$$log\left(\frac{\tau_t^n}{\tau^n}\right) = \gamma_l^n log\left(\frac{l_{t-1}}{l}\right) + \gamma_y^n log\left(\frac{y_{t-1}^H}{y^H}\right).$$

Therefore, if there is a 1% positive output gap, then the government should raise the income tax by γ_y^n%, and vice-versa for a negative gap. In terms of γ_l^g, a positive coefficient implies that a higher-than-expected amount of liabilities in the government balance sheet should lead to an increase in the income tax, and vice-versa for a negative coefficient.

On the other hand, the monetary policy rule is interpreted as follows. We know that

$$\left(\frac{R_t}{R}\right) = \left(\frac{R_{t-1}}{R}\right)^{\phi_R} \left(\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{y_t^H}{y^H}\right)^{\phi_y} \left(\frac{S_t}{S}\right)^{\phi_s}\right)^{1-\phi_R}$$

$$\Leftrightarrow$$

$$log\left(\frac{R_t}{R}\right) = \phi_R log\left(\frac{R_{t-1}}{R}\right) + \phi_\pi(1-\phi_R) log\left(\frac{\Pi_t}{\Pi}\right) + \phi_y(1-\phi_R) log\left(\frac{y_t^H}{y^H}\right) + \phi_s(1-\phi_R) log\left(\frac{S_t}{S}\right).$$

Therefore, a 1% positive interest rate gap in the last period should be countered with a ϕ_R% increase in the interest rate in the current period. Furthermore, a 1% positive exchange rate gap in the last period should be countered with a $\phi_s(1-\phi_R)$% increase in the interest rate in the current period. The interpretation of the coefficients on the output gap and inflation gap is similar.

In terms of policy implications, consider an example where, after running all of the various simulations, the coefficient γ_y^c is 0.5 in the "in" scenario and 0.8 in the "out" scenario. That implies that the volatility-minimizing weight on the income gap for consumption taxes is greater when the small-open economy has monetary independence. Therefore, in order to close a 1% negative income gap by increasing the consumption tax, that increase is greater under monetary independence than it is inside a monetary union.

Figure 1 below shows a graphical interpretation of the optimal simple rule of the consumption tax. Given initial values for the weight on inherited liabilities γ_l^c and the weight on the output gap γ_y^c, I look for the solution (γ_l^c, γ_y^c) that minimizes the volatility of output, inflation, the interest rate, and the exchange rate, plotted on the z-axis.

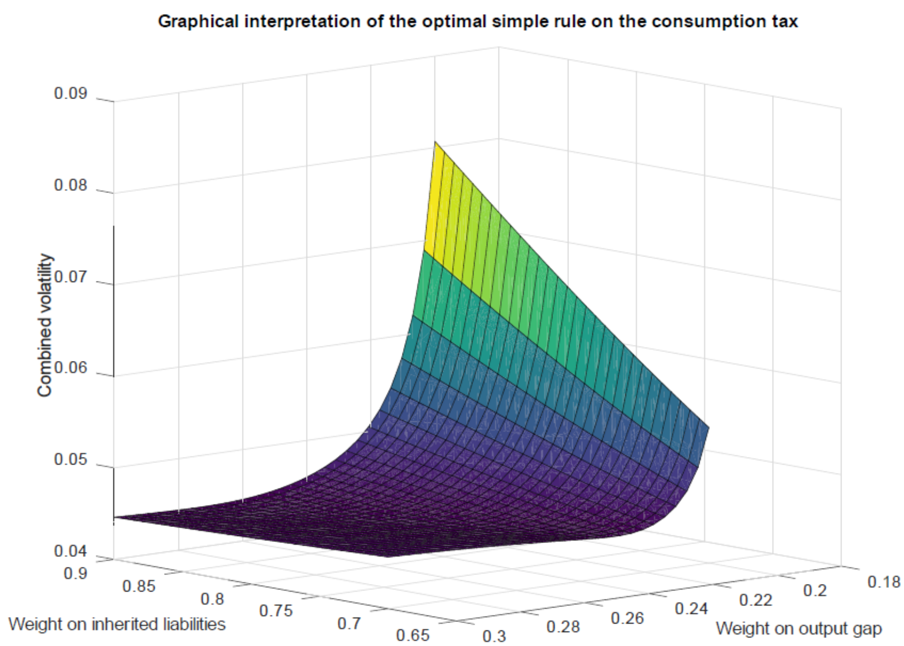

Figure 1. Optimal simple rule example.

Consistently with macroeconomic theory, I expect the weights on inherited liabilities and output to be positive for tax instruments and negative for government spending. If the government has greater-than-expected financial obligations towards its creditors or is experiencing a positive output gap, it conduct contractionary fiscal policy. On the other hand, the same positive gaps should be countered with decreases in direct government spending. Those expectations are the same in both the "in" and "out" scenarios. In terms of differences between before and after exit from the monetary union, I expect the weights on fiscal rules as well as economic volatility to be lower in the "out" scenario. Because the government effectively has an additional policy tool, i.e. the interest rate, which is not subject to exogenous shocks, it can be used, in addition to the four fiscal policy instruments, to minimize economic volatility.

5. Calibration

This sections outlines the parameter values adopted in the model.

Table 1: Description of the parameters.

Parameter	**Value**	**Description**
α	0.42	share of capital
β	0.9603	household discount factor
ν	0.5	home good bias parameter
μ	3.42	parameter related to money holding elasticity
δ	0.1	depreciation rate of capital
ϕ	6	price elasticity of demand
η	1	Frisch labor elasticity
σ	1	elasticity of intertemporal substitution
ν^*	0.5	foreign goods bias parameter
θ	0.5	price rigidity parameter
ψ	0.048	risk premium parameter
χ_m	0.001	preference parameter related to money holdings
χ_n	7	preference parameter related to hours worked

Continued on next page

Parameter	**Value**	**Description**
χ_g	0.1	preference parameter related to public spending
ρ^a	0.92	persistence of TFP
σ_a	0.17	standard deviation of TFP
γ	0.9	foreign imports parameter
ϕ^g	2	adjustment cost on foreign public debt
ϕ^h	2	adjustment cost related to international assets
R	1.0413	gross nominal interest rate
τ^c	0.17	consumption tax rate
τ^n	0.22	income tax rate
τ^k	0.42	capital tax rate
s^g	0.22	government spending share of output
s^b	0.6	domestic public debt share of output
s^f	0.5	foreign public debt share of output
ϕ_y	0.14	weight on the output gap in Taylor Rule
ϕ_π	1.09	weight on the inflation gap in Taylor Rule

All the parameter values, with the exception of the fiscal parameters τ^c, τ^k, τ^n, s^g, s^b, s^f, and monetary policy parameters ϕ_y, ϕ_π follow standard small-open economy model conventions set in the literature.[56] [57] [58] [59] The fiscal and monetary policy parameters instead require further explanation, as they are central to the question this paper addresses.

[56]Varthalitis, *Fiscal and monetary*, 85.
[57]Evi Nappa and Katherine Neiss, *Persistence without too much price stickness*, (Review of Economics Dynamics, 2005).
[58]Gali, *Optimal monetary*, 2008.
[59]Stephanie Schmitt-Grohe and Michael Uribe, *Optimal simple and implementable monetary and fiscal rules*, (Journal of Monetary Economics, 2007), 1702-1725.

Because small-open economy models are often unstable, even small variations in the long-run values of the fiscal parameters could significantly affect the final results or generate an unfeasible solution. For the purpose of this paper, these parameters are averages from Italian data taken from 2001 to 2011.[60] Notice that $s^b + s^f = 1.1$, which means that the government has a steady state debt-to-GDP ratio of 110%. The model thus assumes that the country is a net-borrower in order to operate at full employment, which is relatively common among most small-open economies. The monetary policy parameters that are held constant while solving for the optimal simple rule are instead taken from Kumhof et al.[61]

6. Results

In this section, I present and interpret the fiscal rules and then discuss their broad economic implications with respect to volatility. Table 2 shows results in the "in" scenario, Tables 3 and 4 show results in the "out" scenario.

Table 2. Weights on fiscal rules and associated volatility before exit.

	Weight		Volatility				
Fiscal instrument	γ_l	γ_y	y_t^H	Π_t	R_t	S_t	total
consumption tax	0.7904	0.2481	1.813E-3	1.4E-6	3.8E-7	0	1.8145E-3
capital tax	0.1000	0.2903	1.734E-3	3.4E-5	1.9E-7	0	1.7685E-3
income tax	5.1316	1.6803	1.738E-3	6.3E-6	5.4E-8	0	1.7439E-3
government spending	0.3317	-0.1065	1.812E-3	1.4E-6	4.5E-7	0	1.8145E-3

First and foremost, it is important to emphasize that the coefficients in the Tables 2-4 *are not* estimates. Given the assumptions outlines in sections 3 and 5, the results of the optimal simple rule iterations are the *true* coefficients in a given scenario, which is the reason why there are no confidence intervals associated with them. Note further that all

[60]Varthalitis, *Fiscal and monetary*, 85.
[61]Kumhof et al., *Does the exchange*, 14-16.

of the γ-coefficients make economic sense. That is, all of the weights on the tax instruments are positive, whereas the ones on government spending are negative (recall that γ_l^g has a negative sign in front). Moreover, Table 1 shows that, in the event the government wants to use the tax on consumption goods to minimize economic volatility, a 1% positive gap in inherited liabilities should lead to a 0.7904% increase in the consumption tax and a 1% positive output gap should be countered with a 0.2481% increase in the consumption tax. The interpretation of the other fiscal instrument is similar. In regards to volatility, it is apparent that output volatility drives overall economic volatility, being at least three orders of magnitude greater than the volatility of inflation, the interest rate, and the exchange rate. Note that the volatility of the exchange rate is zero because the small-open economy operates under a fixed exchange rate regime.

Table 3. Weights after exit.

Fiscal instrument	Weight			
	γ_l	γ_y	ϕ_R	ϕ_s
consumption tax	2.4363	2.3309	0.8053	15.0975
capital tax	0.7945	0.3689	0.0000	29.2106
income tax	3.5259	1.6872	0.8289	46.3358
government spending	0.1939	0.0540	0.7199	12.8812

Table 4. Volatility after exit.

Fiscal instrument	Volatility				
	y_t^H	Π_t	R_t	S_t	total
consumption tax	1.817E-3	1.2E-6	9.7E-7	3.7E-7	1.8192E-3
capital tax	1.898E-3	3.3E-6	4.2E-7	4.8E-8	1.9016E-3
income tax	1.721E-3	5.0E-6	1.4E-6	8.6E-8	1.7283E-3
government spending	1.821E-3	1.4E-6	6.8E-7	3.7E-7	1.8235E-3

Similar to above, Table 2 also shows fiscal coefficients that, with the exception of the weight on the output gap in the government spending rule, make economic sense for the same reason that the γ-coefficients make sense in the "before" case. Since after exit from the monetary union the small-open economy can conduct independent monetary policy, Tables 3 and 4 also present the decision coefficients on the rule that determines the interest rate R_t, the weight on the persistence term ϕ_R, and the weight on the exchange rate gap ϕ_s. Under monetary independence, a 1% positive interest rate gap in the last period requires an increase in the interest rate of 0.8053%. On the other hand, a 1% positive exchange rate gap should be countered with an increase in the interest rate of $15.0975(1 - 0.8053) = 2.9395\%$. Therefore, while the ϕ_s weights in the Taylor rule are large in magnitude, their interpretations fall within an economically-plausible range.

The results on the income tax rule are different than other fiscal rules: the weight on inherited government liabilities and overall volatility decrease when the small-open economy is outside the monetary union. The solutions from the optimal simple rule in the "out" case are somewhat ambiguous: output, inflation, and overall volatility, as well as the weight on inherited liabilities, decrease, whereas interest rate and exchange rate volatility, along with the weight on the output gap, increase.

Let us now discuss differences in fiscal rules and associated economic volatility. All of the γ-coefficients, with the exception of the weights on government liabilities in the government spending rule, increase in magnitude. Let us consider the capital tax as an example. Before exit from the monetary union, a 1% positive output gap could be countered with a 0.2903% increase in the capital tax and the economy would have experienced a level of volatility of 1.768E-3. On the other hand, under monetary independence, that same output gap requires an increase in the capital tax of 0.3689%, while achieving a volatility of 1.902E-3. Observe further that overall volatility increases when the small-open economy is outside of the monetary union. Comparing Table 1 and Table 3, it is clear that increases in output, interest rate, and exchange volatility outweigh the decrease in inflation volatility for all three fiscal instruments. This can be attributed to the presence of a floating exchange rate, that itself no longer has a zero volatility. Furthermore, it contributes to increasing overall volatility by increasing the volatility of the interest rate through the uncovered interest rate parity condition, see equation (28). Intuitively, under monetary policy independence, the interest rate is a function of the exchange rate, in addition to other variables that appear in the "in" case as well. In the "out" case, the exchange rate is floating and its non-zero volatility increases the volatility of the interest rate as well.

While these results do not align with the theoretical expectations outlined in section 4, they do have a number of noteworthy policy implications for a small-open economy. Under monetary independence, fiscal instruments are more sensitive to discrepancies in inherited government liabilities and output from their respective long-run values. This is likely due to the presence of a floating exchange rate. Recalling from section 3, the exchange rate appears in the law of one price, the rule that determines interest rates, and the uncovered interest rate parity condition. This implies that prices on imports are no longer fixed, as per equation (9), which in turn means that inflation is no longer solely a function of inflation on domestic consumption goods, see equation (8). Evidence to this is the magnitude of the solutions on ϕ_s and ϕ_R, the weight on the exchange rate and persistence term, respectively, in the rule that determines the interest rate. Even after adjusting for the correct economic interpretation, all of the ϕ_s coefficients are relatively large, which means that, under an individual national currency, the independent monetary authority is very responsive to deviations of the exchange rate from its target. Having to increase the interest rate significantly in the event, for instance, of a positive exchange rate gap, the government increases its liabilities at the same time, since the exchange rate is positively correlated to interest rates. In turn, according to the four fiscal rules, an increase in liabilities must be countered with an increase in government revenue, which would not otherwise be there before exit from a monetary union. Thus, while the ability to set interest rates acts as an additional policy instrument at the government's disposal, the presence of a floating exchange rate affects those very policy decisions and creates a relationship with government liabilities that was not there before exit.

7. Conclusion

This paper studies fiscal policy and economic volatility upon exit from a monetary union. I adopt a small-open economy model and estimate the optimal simple rule without and with monetary policy independence by minimizing the volatility of output, inflation, the interest rate, and the exchange rate. I mimic exit from the currency union by adding a Taylor Rule specification for interest rates as well as an endogenous exchange rate including the uncovered interest rate parity condition. I solve for one fiscal rule at a time and let the other fiscal instruments be constant at their steady state values. My results show that the combined volatility of three out of the four aforementioned indicators increases after the government adopts monetary independence. I find that the weights on inherited government liabilities and the output gap increase when the small-open economy is outside the monetary union. That implies a greater fiscal burden on households for the same liabilities or output gap, as well as a significant weight on the exchange rate. These results are largely due to the very nature of independent monetary policy depending on the exchange rate gap. The exchange rate is correlated with import prices as well as the interest rate. The combined effect creates a relationship that does not otherwise exist when the economy is inside the monetary union.

At the same time, my results do have some intrinsic level of bias. First and foremost, in solving for the optimal simple rule, I only apply a technology shock, while a portion of the literature also includes an interest rate or an exchange rate shock. Given the context of my research question, either shock is a possibility to expand upon my findings. Furthermore, I only consider four fiscal instruments. In order to more accurately represent a small-open economy, it would be interesting to include a tax on foreign consumption goods, effectively a tariff, or a tax on cash holdings above a given threshold. My results are also likely conditional on the exchange rate mechanism I implement. As a robustness check, or future research, one should look at alternative specifications for the exchange rate. Lastly, and probably most importantly, the results of the optimal simple rule do not constitute *optimal* fiscal policy, which would be derived from a typical Ramsey problem. However, the results of my paper do provide some level of insight in the way in which fiscal policy changes if a small-open economy decided to leave a monetary union. It is an undoubtedly rapidly-growing topic in the fiscal policy literature and, as the economic and political landscape inside monetary unions change, it is crucial to expand upon it.

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Appendix

I. **Solving the household's problem.** The household chooses $b_{i,t}$, $c_{i,t}^H$, $c_{i,t}^F$, $f_{i,t}^h$, $k_{i,t}$, $n_{i,t}$, $m_{i,t}$.

$$\begin{aligned}\max \mathcal{L} \;=\; & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_{i,t}, n_{i,t}, m_{i,t}, g_t) + \lambda_t \Bigg\{ (1+\tau_k) \left[r_t^k \frac{P_t^H}{P_t} k_{i,t-1} + d_{i,t} \right] + (1-\tau_t^n) w_t n_{i,t} \\ & + R_{t-1} \frac{P_{t-1}}{P_t} b_{i,t-1} + \frac{P_{t-1}}{P_t} m_{i,t-1} + Q_{t-1} \frac{P_{t-1}}{P_t} \frac{S_t P_t^*}{P_t^*} f_{i,t-1}^h - \tau_{i,t}^l - (1+\tau_c) \left[\frac{P_t^H}{P_t} c_{i,t}^H + \frac{P_t^F}{P_t} c_{i,t}^F \right] \\ & - \frac{P_t^H}{P_t} x_{i,t} - b_{i,t} - m_{i,t} - \frac{S_t P_t^*}{P_t} f_{i,t}^h - \frac{\phi^h}{2} \left(\frac{S_t P_t^*}{P_t} f_{i,t}^h - \frac{S P^*}{P} f^h \right)^2 \Bigg\}. \end{aligned} \tag{34}$$

Therefore, household i's first order conditions are as follows. [1] Optimality condition on domestic bonds, b_t:

$$\begin{aligned} -\lambda_t + \mathbb{E}_t \lambda_{t+1} R_t \frac{P_t}{P_{t+1}} &= 0 \\ \Rightarrow \lambda_t &= \mathbb{E}_t \lambda_{t+1} R_t \frac{P_t}{P_{t+1}}. \end{aligned} \tag{35}$$

[2] Optimality condition on consumption of domestic goods, c_t^H:

$$\begin{aligned} \beta^t \frac{\partial U_{i,t}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}^H} - \mathbb{E}_t \lambda_t (1+\tau_t^c) \frac{P_t^H}{P_t} &= 0 \\ \Rightarrow \lambda_t = \beta^t \mathbb{E}_t \frac{\partial U_{i,t}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}^H} \frac{P_t}{(1+\tau_t^c) P_t^H} \\ \Rightarrow \frac{\partial U_{i,t}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}^H} \frac{P_t}{(1+\tau_t^c) P_t^H} = \beta \mathbb{E}_t \frac{\partial U_{i,t+1}}{\partial c_{i,t+1}} \frac{\partial c_{i,t+1}}{\partial c_{i,t+1}^H} \frac{P_{t+1}}{(1+\tau_{t+1}^c) P_{t+1}^H} R_t \frac{P_t}{P_{t+1}}. \end{aligned} \tag{36}$$

[3] Optimality condition on consumption of foreign goods, c_t^F:

$$\beta^t \frac{\partial U_{i,t}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}^F} - \mathbb{E}_t \lambda_t (1+\tau_t^c) \frac{P_t^F}{P_t} = 0$$
$$\Rightarrow \; \lambda_t = \beta^t \mathbb{E}_t \frac{\partial U_{i,t}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}^F} \frac{P_t}{(1+\tau_t^c) P_t^F}$$
$$\Rightarrow \frac{\partial U_{i,t}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}^F} \frac{P_t}{(1+\tau_t^c) P_t^F} = \beta \mathbb{E}_t \frac{\partial U_{i,t+1}}{\partial c_{i,t+1}} \frac{\partial c_{i,t+1}}{\partial c_{i,t+1}^F} \frac{P_{t+1}}{(1+\tau_{t+1}^c) P_{t+1}^F} R_t \frac{P_t}{P_{t+1}}. \tag{37}$$

[4] Optimality condition on foreign assets, f_t^h:

$$-\lambda_t \left[\frac{S_t P_t^*}{P_t} + \phi^h \left(\frac{S_t P_t^*}{P_t} f_{i,t}^h - \frac{SP^*}{P} f^h \right) \frac{S_t P_t^*}{P_t} \right] + \lambda_{t+1} Q_t \frac{P_t}{P_{t+1}} \frac{S_{t+1} P_{t+1}^*}{P_{t+1}^*} = 0$$
$$\Rightarrow \; \frac{\partial U_{i,t}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}^H} \frac{P_t}{(1+\tau_t^c) P_t^H} \frac{S_t P_t^*}{P_t} \left[1 + \phi^h \left(\frac{S_t P_t^*}{P_t} f_{i,t}^h - \frac{SP^*}{P} f^h \right) \right]$$
$$= \beta \mathbb{E}_t \frac{\partial U_{i,t+1}}{\partial c_{i,t+1}} \frac{\partial c_{i,t+1}}{\partial c_{i,t+1}^H} \frac{P_{t+1}}{(1+\tau_{t+1}^c) P_{t+1}^H} Q_t \frac{P_t}{P_{t+1}} \frac{S_{t+1} P_{t+1}^*}{P_{t+1}^*}. \tag{38}$$

[5] Optimality condition on capital, k_t:

$$-\lambda_t \frac{P_t^H}{P_t} \frac{\partial x_{i,t}}{\partial k_{i,t}} + \lambda_{t+1} \left[(1+\tau_{t+1}^k) r_{t+1}^k \frac{P_{t+1}^H}{P_{t+1}} - \frac{P_{t+1}^H}{P_{t+1}} \frac{\partial x_{i,t+1}}{\partial k_{i,t}} \right] = 0 \; ; \tag{39}$$

from equation (3), we have the partial derivatives

$$\frac{\partial x_{i,t}}{\partial k_{i,t}} = 1 \tag{40}$$

and

$$\frac{\partial x_{i,t+1}}{\partial k_{i,t}} = (\delta - 1); \tag{41}$$

plugging equations (40) and (41) into equation (39), we have

$$\frac{\partial U_{i,t}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}^H} \frac{P_t}{(1+\tau_t^c)P_t^H} \frac{P_t^H}{P_t} = \beta \mathbb{E}_t \frac{\partial U_{i,t+1}}{\partial c_{i,t+1}} \frac{\partial c_{i,t+1}}{\partial c_{i,t+1}^H} \frac{P_{t+1}}{(1+\tau_{t+1}^c)P_{t+1}^H} \frac{P_{t+1}^H}{P_{t+1}} \left[(1+\tau_{t+1}^k) r_{t+1}^k + (1-\delta) \right]. \tag{42}$$

[6] Optimality condition on work hours, n_t:

$$\begin{aligned} \beta^t \frac{\partial U_{i,t}}{\partial n_{i,t}} + \lambda_t (1-\tau_t^n) w_t &= 0 \\ \Rightarrow\ -\beta^t \frac{\partial U_{i,t}}{\partial n_{i,t}} &= \beta^t \frac{\partial U_{i,t}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}^H} \frac{P_t}{(1+\tau_t^c)P_t^H} (1-\tau_t^n) w_t \\ \Rightarrow\ -\frac{\partial U_{i,t}}{\partial n_{i,t}} &= \frac{\partial U_{i,t}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}^H} \frac{P_t}{(1+\tau_t^c)P_t^H} (1-\tau_t^n) w_t. \end{aligned} \tag{43}$$

[7] Optimality condition on real money holdings, m_t:

$$\begin{aligned} \beta^t \frac{\partial U_{i,t}}{\partial m_{i,t}} - \lambda_t + \lambda_{t+1} \frac{P_t}{P_{t+1}} &= 0 \\ \Rightarrow\ \beta^t \frac{\partial U_{i,t}}{\partial m_{i,t}} - \beta^t \mathbb{E}_t \frac{\partial U_{i,t}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}^H} \frac{P_t}{(1+\tau_t^c)P_t^H} + \beta^{t+1} \mathbb{E}_{t+1} \frac{\partial U_{i,t+1}}{\partial c_{i,t+1}} \frac{\partial c_{i,t+1}}{\partial c_{i,t+1}^H} \frac{P_{t+1}}{(1+\tau_{t+1}^c)P_{t+1}^H} \frac{P_t}{P_{t+1}} &= 0 \\ \Rightarrow\ \frac{\partial U_{i,t}}{\partial m_{i,t}} = \frac{\partial U_{i,t}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}^H} \frac{P_t}{(1+\tau_t^c)P_t^H} - \beta \mathbb{E}_t \frac{\partial U_{i,t+1}}{\partial c_{i,t+1}} \frac{\partial c_{i,t+1}}{\partial c_{i,t+1}^H} \frac{P_{t+1}}{(1+\tau_{t+1}^c)P_{t+1}^H} \frac{P_t}{P_{t+1}}. & \end{aligned} \tag{44}$$

In each period t, household i also solves a second problem: it chooses an optimal bundle of domestic and foreign consumption goods. It maximizes lifetime utility subject to the total value of consumption goods available. Formally, that is

$$\max \mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_{i,t}, n_{i,t}, m_{i,t}, g_t) + \lambda_t (P_t^H c_{i,t}^H + P_t^F c_{i,t}^F) \tag{45}$$

with first order conditions:

$$(c_{i,t}^H) : \beta^t \frac{\partial U_{i,t}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}^H} + \lambda_t P_t^H = 0 \tag{46}$$

$$(c_{i,t}^F) : \beta^t \frac{\partial U_{i,t}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}^F} + \lambda_t P_t^F = 0. \tag{47}$$

This implies that the optimal allocation is given by

$$\begin{aligned}
-\lambda_t &= \beta^t \frac{\partial U_{i,t}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}^H} \frac{1}{P_t^H} = \beta^t \frac{\partial U_{i,t}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}^F} \frac{1}{P_t^F} \\
&\Rightarrow \frac{\partial c_{i,t}}{\partial c_{i,t}^H} \frac{\partial c_{i,t}^F}{\partial c_{i,t}} = \frac{P_t^H}{P_t^F} \\
&\Rightarrow \frac{v (c_{i,t}^H)^{v-1} \, (c_{i,t}^F)^{1-v}}{v^v (1-v)^{1-v}} \frac{v^v (1-v)^{1-v}}{(c_{i,t}^H)^v (1-v) (c_{i,t}^F)^{-v}} = \frac{P_t^H}{P_t^F} \\
&\Rightarrow \frac{c_{i,t}^F}{c_{i,t}^H} = \frac{1-v}{v} \frac{P_t^H}{P_t^F}.
\end{aligned} \tag{48}$$

Given that there are two choices for consumption, household i also minimizes total cost on consumption. Formally,

$$\begin{aligned}
\min_{c_{i,t}^H(h),\ c_{i,t}^F(f)} \mathcal{L} \ &= \ \sum_{h=1}^{N} \gamma P_t^H(h) c_{i,t}^H(h) + \sum_{f=1}^{N} \gamma P_t^F(f) c_{i,t}^F(f) \\
+\lambda_t^1 &\left\{ c_{i,t}^H - \left[\sum_{h=1}^{N} \gamma [c_{i,t}^H(h)]^{\frac{1-\phi}{\phi}} \right]^{\frac{\phi}{\phi-1}} \right\} + \lambda_t^2 \left\{ c_{i,t}^F - \left[\sum_{f=1}^{N} \gamma [c_{i,t}^F(f)]^{\frac{1-\phi}{\phi}} \right]^{\frac{\phi}{\phi-1}} \right\}.
\end{aligned} \tag{49}$$

Observe that one-unit increases in $c_{i,t}^H$ and $c_{i,t}^F$ correspond to P_t^H- and P_t^F-unit increases in the objective function. Therefore, we can use P_t^H and P_t^F as λ_t^1 and λ_t^2, respectively. First order conditions:

$$[c_{i,t}^H(h)] : \gamma P_t^H(h) - P_t^H \frac{\phi}{\phi - 1} \left[\sum_{h=1}^{N} \gamma [c_{i,t}^H(h)]^{\frac{1-\phi}{\phi}} \right]^{\frac{1}{\phi-1}} \left(\frac{\phi - 1}{\phi} \right) c_{i,t}^H(h)^{-1/\phi} = 0$$

$$\Rightarrow P_t^H(h) = P_t^H (c_{i,t}^H)^{1/\phi} \, [c_{i,t}^H(h)]^{-1/\phi}$$

$$\Rightarrow c_{i,t}^H(h) = \left(\frac{P_t^H(h)}{P_t^H} \right)^{-\phi} c_{i,t}^H \tag{50}$$

$$[c_{i,t}^F(f)] : \gamma P_t^F(f) - P_t^F \frac{\phi}{\phi - 1} \left[\sum_{f=1}^{N} \gamma [c_{i,t}^F(f)]^{\frac{1-\phi}{\phi}} \right]^{\frac{1}{\phi-1}} \left(\frac{\phi - 1}{\phi} \right) c_{i,t}^F(f)^{-1/\phi} = 0$$

$$\Rightarrow P_t^F(f) = P_t^F (c_{i,t}^F)^{\phi} \, [c_{i,t}^F(f)]^{-1/\phi}$$

$$\Rightarrow c_{i,t}^F(f) = \left(\frac{P_t^F(f)}{P_t^F} \right)^{-\phi} c_{i,t}^F \, . \tag{51}$$

Finally, equations (22), (24), and (25) combined with equation (7) imply the three price indexes:

$$P_t = (P_t^H)^{v} (P_t^F)^{1-v} \tag{52}$$

$$P_t^H = \left[\sum_{h=1}^{N} \gamma_h [P_t^H(h)]^{1-\phi} \right]^{\frac{1}{1-\phi}} \tag{53}$$

$$P_t^F = \left[\sum_{f=1}^{N} \gamma_f [P_t^F(f)]^{1-\phi} \right]^{\frac{1}{1-\phi}} . \tag{54}$$

II. **Firm's first order conditions.** We first solve for the cost-minimizing levels of w_t and r_t^k. From equation (41), we have

$$W_t - MC_t(1-\alpha)A_t[k_{t-1}(h)]^{\alpha}[n_t(h)]^{-\alpha} = 0$$
$$\Rightarrow W_t = MC_t(1-\alpha)\frac{y_t^H(h)}{n_t(h)}$$

so in real terms we have

$$w_t = mc_t(1-\alpha)\frac{y_t^H(h)}{n_t(h)} \tag{55}$$

where $mc_t = \frac{MC_t}{P_t}$ is the real marginal cost.

Similarly, for r_t^k, we have

$$r_t^k P_t^H(h) - MC_t\alpha A_t[k_{t-1}(h)]^{\alpha-1}[n_t(h)]^{1-\alpha} = 0$$
$$\Rightarrow \frac{P_t^H(h)}{P_t} r_t^k = mc_t\alpha\frac{y_t^H(h)}{k_{t-1}(h)}. \tag{56}$$

To solve for the first order conditions of the Calvo-pricing problem, we differentiate

$$\mathbb{E}_t\sum_{k=0}^{\infty}\theta^k\Omega_{t,t+k}\left\{P_t^{\#}(h)\left[\frac{P_t^{\#}(h)}{P_{t+k}^H}\right]^{-\phi}y_{t+k}^H - \Psi_{t+k}(y_{t+k}^H(h))\right\}$$
$$= \mathbb{E}_t\sum_{k=0}^{\infty}\theta^k\Omega_{t,t+k}\left\{P_t^{\#}(h)^{1-\phi}\left[\frac{1}{P_{t+k}^H}\right]^{-\phi}y_{t+k}^H - \Psi_{t+k}(y_{t+k}^H(h))\right\}$$

with respect to $P_t^{\#}(h)$ and obtain

$$\begin{aligned}
&\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} \left\{ (1-\phi) P_t^{\#}(h)^{-\phi} \left[\frac{1}{P_{t+k}^H} \right]^{-\phi} y_{t+k}^H - \Psi'_{t+k}(y_{t+k}^H(h)) \cdot (-\phi) \left[\frac{P_t^{\#}(h)}{P_{t+k}^H} \right]^{-\phi-1} \frac{1}{P_{t+k}^H} y_{t+k}^H \right\} \\
&\quad = \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} \left[\frac{P_t^{\#}(h)}{P_{t+k}^H} \right]^{-\phi} y_{t+k}^H \left\{ (1-\phi) - \Psi'_{t+k}(y_{t+k}^H(h)) \cdot (-\phi) \left[\frac{P_t^{\#}(h)}{P_{t+k}^H} \right]^{-1} \frac{1}{P_{t+k}^H} \right\} \\
&\quad = \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} \left[\frac{P_t^{\#}(h)}{P_{t+k}^H} \right]^{-\phi} y_{t+k}^H \left\{ \frac{P_t^{\#}(h)(1-\phi) - \Psi'_{t+k}(y_{t+k}^H(h)) \cdot (-\phi)}{P_t^{\#}(h)} \right\} = 0.
\end{aligned}$$

We take $P_t^{\#}(h)$ out of the summation, since it does not have a k subscript, and cancel it, which yields

$$\begin{aligned}
&\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} \left[\frac{P_t^{\#}(h)}{P_{t+k}^H} \right]^{-\phi} y_{t+k}^H \left\{ P_t^{\#}(h)(1-\phi) - \Psi'_{t+k}(y_{t+k}^H(h)) \cdot (-\phi) \right\} \\
&\quad = \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} \left[\frac{P_t^{\#}(h)}{P_{t+k}^H} \right]^{-\phi} y_{t+k}^H \left\{ \frac{P_t^{\#}(h) - \Psi'_{t+k}(y_{t+k}^H(h)) \cdot (-\phi)}{1-\phi} \right\} \\
&\quad = \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} \left[\frac{P_t^{\#}(h)}{P_{t+k}^H} \right]^{-\phi} y_{t+k}^H \left\{ P_t^{\#}(h) - \frac{\phi}{\phi-1} \Psi'_{t+k}(y_{t+k}^H(h)) \right\} = 0. \qquad (57)
\end{aligned}$$

Finally, we transform equation (45) by dividing by the aggregate domestic price index, P_t^H:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} \left[\frac{P_t^{\#}(h)}{P_{t+k}^H} \right]^{-\phi} y_{t+k}^H \left\{ \frac{P_t^{\#}(h)}{P_t^H} - \frac{\phi}{\phi-1} \frac{\Psi'_{t+k}(y_{t+k}^H(h))}{P_t^H} \right\} = 0. \qquad (58)$$

Recall that $\Psi'_{t+k}(y_{t+k}^H(h))$ is the nominal marginal cost of production, therefore we can re-write equation (45) as

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} \left[\frac{P_t^{\#}(h)}{P_{t+k}^H} \right]^{-\phi} y_{t+k}^H \left\{ \frac{P_t^{\#}(h)}{P_t^H} - \frac{\phi}{\phi-1} mc_{t+k} \frac{P_{t+k}}{P_t^H} \right\} = 0. \qquad (59)$$

We manipulate equation (45) by letting

$$z_t^1 = \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} \left[\frac{P_t^\#(h)}{P_{t+k}^H} \right]^{-\phi} y_{t+k}^H \frac{P_t^\#(h)}{P_t^H} \tag{60}$$

and

$$z_t^2 = \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} \left[\frac{P_t^\#(h)}{P_{t+k}^H} \right]^{-\phi} y_{t+k}^H mc_{t+k} \frac{P_{t+k}}{P_t^H} \tag{61}$$

so that

$$z_t^1 = \frac{\phi}{\phi - 1} z_t^2. \tag{62}$$

III. **Model Solution.** Before I solve the model I apply the following transformations. Domestic inflation Π_t, foreign inflation Π_t^*, domestic goods inflation Π_t^H, the price dispersion index Δ_t, exchange rate depreciation ϵ_t, and the terms of trade TT_t are all expressed as ratios: $\Pi_t = \frac{P_t}{P_{t-1}}$, $\Pi_t^* = \frac{P_t^*}{P_{t-1}^*}$, $\Pi_t^H = \frac{P_t^H}{P_{t-1}^H}$, $\Delta_t = \left[\frac{\widetilde{P}_t^H}{P_t^H} \right]^{-\phi}$, $\epsilon_t = \frac{S_t}{S_{t-1}}$, $TT_t = \frac{P_t^F}{P_t^H} = \frac{S_t P_t^{*H}}{P_t^H}$. I also adopt the auxiliary variable $\Theta_t = \frac{P_t^\#}{P_t^H}$. This implies that we get $f^{TT} = TT_t^{\nu^*} f_t^g$, where $f_t^{TT} = s^f y_t^H$, $g_t = s_t^g y_t^H$, $\tau_t^l = s_t^l y_t^H TT_t^{-1}$, and $b_t = s_t^b y_t^H TT_t^{\nu-1}$. Finally, from equations (47-50), we write the solution the the Calvo-pricing portion of the firm's problem in recursive form:

$$z_t^1 = \Theta_t^{1-\phi} y_t^H TT_t^{\nu-1} + \beta\theta \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \left(\frac{\Theta_t}{\Theta_{t+1}} \right)^{1-\phi} \left(\frac{1}{\Pi_{t+1}^H} \right)^{1-\phi} z_{t+1}^1 \tag{63}$$

$$z_t^2 = \Theta_t^{-\phi} y_t^H mc_t + \beta\theta \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \left(\frac{\Theta_t}{\Theta_{t+1}} \right)^{-\phi} \left(\frac{1}{\Pi_{t+1}^H} \right)^{-\phi} z_{t+1}^2. \tag{64}$$

The final system of equations is therefore:

$$c_t^{-\sigma}\frac{1}{(1+\tau_t^c)} = \beta\mathbb{E}_t c_{t+1}^{-\sigma}\frac{1}{(1+\tau_{t+1}^c)}R_t\frac{1}{\Pi_{t+1}} \tag{65}$$

$$\begin{aligned} c_t^{-\sigma}\frac{1}{(1+\tau_t^c)}TT_t^{\nu^*+\nu-1}\left[1+\phi^h\left(TT_t^{\nu^*+\nu-1}f_t^h - TT^{\nu^*+\nu-1}f^h\right)\right] \\ = \beta\mathbb{E}_t c_{t+1}^{-\sigma}\frac{1}{(1+\tau_{t+1}^c)}Q_t TT_{t+1}^{\nu^*+\nu-1}\frac{1}{\Pi_t^*} \end{aligned} \tag{66}$$

$$c_t^{-\sigma}\frac{1}{(1+\tau_t^c)}TT_t^{\nu-1} = \beta\mathbb{E}_t c_{t+1}^{-\sigma}\frac{1}{(1+\tau_{t+1}^c)}TT_{t+1}^{\nu-1}\left[(1-\tau_{t+1}^k)r_{t+1}^k + (1-\delta)\right] \tag{67}$$

$$\chi_m m_t^{-\mu} = c_t^{-\sigma}\frac{1}{1+\tau_t^c} - \beta\mathbb{E}_t c_{t+1}^{-\sigma}\frac{1}{1+\tau_{t+1}^c}\frac{1}{\Pi_{t+1}} \tag{68}$$

$$\chi_n n_t^{\eta} = (1-\tau^n)w_t c_t^{-\sigma}\frac{1}{1+\tau_t^c} \tag{69}$$

$$\frac{c_t^H}{c_t^F} = \frac{\nu}{1-\nu}TT_t \tag{70}$$

$$k_t = (1-\delta)k_{t-1} + x_t - \frac{\xi}{2}\left(\frac{k_t}{k_{t-1}} - 1\right)^2 k_{t-1} \tag{71}$$

$$c_t = \frac{(c_t^H)^{\upsilon}\,(c_t^F)^{1-\upsilon}}{\upsilon^{\upsilon}(1-\upsilon)^{1-\upsilon}} \tag{72}$$

$$w_t = mc_t(1-\alpha)\frac{y_t^H}{n_t} \tag{73}$$

$$\frac{1}{TT_t^{1-\nu}}r_t^k = mc_t\alpha\frac{y_t^H}{k_{t-1}} \tag{74}$$

$$d_t = \frac{1}{TT_t^{1-\nu}}y_t^H - \frac{1}{TT_t^{1-\nu}}r_t^k k_{t-1} - w_t n_t \tag{75}$$

$$y_t^H = \frac{1}{\Delta_t}A_t k_{t-1}^{\alpha}n_t^{1-\alpha}y_t^H = c_t^H + x_t + g_t + c_t^{F*} \tag{77}$$

$$\begin{aligned} s_t^b y_t^H TT_t^{\nu-1} + m_t + TT_t^{\nu^*+\nu-1}\frac{f_t^{TT}}{TT_t^{\nu^*}} = \frac{\phi^g}{2}\left(TT_{t+1}^{\nu^*+\nu-1}\frac{f_t^{TT}}{TT_{t+1}^{\nu^*}} - TT_{t+1}^{\nu^*+\nu-1}\frac{f^{TT}}{TT^{\nu^*}}\right)^2 \\ +R_{t-1}\frac{1}{\Pi_t}s_{t-1}^b y_{t-1}^H TT_{t-1}^{\nu-1} + \frac{1}{\Pi_{t-1}}m_{t-1} + Q_{t-1}TT_{t-1}^{\nu^*+\nu-1}\frac{f_{t-1}^{TT}}{TT_{t-1}^{\nu^*}}\frac{1}{\Pi_{t-1}^*} + \frac{1}{TT_t^{1-\nu}}s_t^g y_t^H \\ -\tau_t^c\left(\frac{1}{TT_t^{1-\nu}}c_t^H + TT_t^{\nu}c_t^F\right) - \tau_t^k\left(r_t^k\frac{1}{TT_t^{\nu-1}}k_{t-1} + d_t\right) - \tau_t^n w_t n_t - s_t^l y_t^H TT_t^{\nu-1} \end{aligned} \tag{78}$$

$$TT_t^{\nu^*+\nu-1}\left(\frac{f_t^{TT}}{TT_t^{\nu^*}}-f_t^h\right)=-TT_t^{\nu-1}c_t^{F*}+TT_t^{\nu}c_t^F+Q_{t-1}TT_t^{\nu^*+\nu-1}\frac{1}{\Pi_t^*}\left(\frac{f_{t-1}^{TT}}{TT_{t-1}^{\nu^*}}-f_{t-1}^h\right)$$

$$+\frac{\phi^h}{2}\left(TT_t^{\nu^*+\nu-1}f_t^h-TT^{\nu^*+\nu-1}f^h\right)^2+\frac{\phi^g}{2}\left(TT_{t+1}^{\nu^*+\nu-1}\frac{f_t^{TT}}{TT_{t+1}^{\nu^*}}-TT_{t+1}^{\nu^*+\nu-1}\frac{f^{TT}}{TT^{\nu^*}}\right)^2 \quad (79)$$

$$\left(\Pi_t^H\right)^{1-\phi}=\theta+(1-\theta)\left(\Theta_t\Pi_t^H\right)^{1-\phi} \quad (80)$$

$$\frac{\Pi_t}{\Pi_t^H}=\left(\frac{TT_t}{TT_{t-1}}\right)^{1-\nu} \quad (81)$$

$$\frac{TT_t}{TT_{t-1}}=\frac{\epsilon_t\Pi_t^{H*}}{\Pi_t^H} \quad (82)$$

$$\frac{\Pi_t^*}{\Pi_t^{H*}}=\left(\frac{TT_t}{TT_{t-1}}\right)^{1-\nu^*} \quad (83)$$

$$\Delta_t=\theta\Delta_{t-1}\left(\Pi_t^H\right)^{\phi}+(1-\theta)\left(\Theta_t\right)^{-\phi} \quad (84)$$

$$Q_t=Q_t^*+\psi\left(e^{(\frac{f_t^{TT}}{y_t^H}+s_t^b-\bar{d})}-1\right). \quad (85)$$

$$z_t^1=\Theta_t^{1-\phi}y_t^HTT_t^{\nu-1}+\beta\theta\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\left(\frac{\Theta_t}{\Theta_{t+1}}\right)^{1-\phi}\left(\frac{1}{\Pi_{t+1}^H}\right)^{1-\phi}z_{t+1}^1 \quad (86)$$

$$z_t^2=\Theta_t^{-\phi}y_t^Hmc_t+\beta\theta\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\left(\frac{\Theta_t}{\Theta_{t+1}}\right)^{-\phi}\left(\frac{1}{\Pi_{t+1}^H}\right)^{-\phi}z_{t+1}^2 \quad (87)$$

$$z_t^1=\frac{\phi}{\phi-1}z_t^2. \quad (88)$$

We can immediately derive the steady-state values of some of the ratios presented above. From equation (73), we have $\Theta = 1$. From equation (77), we have $\Delta = 1$. By definition, Π, Π^H, and Π^{H*} are all equal to 1. To solve the model analytically we start with equation (59). In steady state, we have $1 = \beta Q$, so

$$Q = \frac{1}{\beta}. \tag{89}$$

From equation (60), we have $1 = \beta\left[(1+\tau^c)r^k + 1 - \delta\right]$, which implies that

$$r^k = \frac{1}{1+\tau^k}\left(\frac{1}{\beta} - 1 + \delta\right). \tag{90}$$

Dividing equation (79) by equation (80), we have $\frac{z^1}{z^2} = \frac{TT^{\nu-1}}{mc}$. Combining this result with equation (81), we have

$$\frac{z^1}{z^2} = \frac{TT^{\nu-1}}{mc} = \frac{\phi}{\phi-1}.$$

Turning our attention to the firm's first order conditions (66-67) and (69), we have

$$TT^{\nu-1}r^k = \alpha \cdot mc\frac{y^H}{k} \;\Rightarrow\; \frac{y^H}{k} = \frac{\phi}{\phi-1}\frac{r^k}{\alpha}.$$

Dividing equation (69) by k, we have

$$\frac{y^H}{k} = \left(\frac{n}{k}\right)^{1-\alpha} \;\Rightarrow\; \frac{n}{k} = \left(\frac{y^H}{k}\right)^{\frac{1}{1-\alpha}}$$

and therefore $\frac{y^H}{n} = \frac{y^H}{k}\frac{k}{n}$. We now apply the following manipulations to obtain $\frac{c^H}{y^H}$:

- From equation (63), $c^F = \frac{1-\nu}{\nu}TT^{-1}c^H$.
- From equation (66), $w = TT^{\nu-1}\frac{\phi-1}{\phi}(1-\alpha)\frac{y^H}{n} \Rightarrow wn = TT^{\nu-1}\frac{\phi-1}{\phi}(1-\alpha)y^H$. This implies that $d = TT^{\nu-1}y^H - TT^{\nu-1}r^k k - TT^{\nu-1}\frac{\phi-1}{\phi}(1-\alpha)y^H$.
- By definition, $\frac{f^{TT}}{y^H} = s^f$.

We substitute these values into equation (71) and have

$$
\begin{gathered}
s^b y^H TT^{\nu-1} + TT^{\nu-1} f^{TT} = Rs^b y^H TT^{\nu-1} + QTT^{\nu-1} f^{TT} + TT^{\nu-1} s_t^g y^H \\
-\tau^c \left(TT^{\nu-1} c^H + \frac{1-\nu}{\nu} TT^{\nu-1} c^H \right) - \tau^k \left[TT^{\nu-1} y^H - TT^{\nu-1} \frac{\phi-1}{\phi}(1-\alpha) \right] \\
-\tau^n TT^{\nu-1} \frac{\phi-1}{\phi}(1-\alpha) y^H - s^l y^H TT^{\nu-1}.
\end{gathered}
$$

We divide both sides by $TT^{\nu-1} y^H$ and have

$$
\begin{gathered}
s^b + \frac{f^{TT}}{y^H} = Rs^b + QTT^{\nu-1} f^{TT} + s^g - \tau^c \left(\frac{c^H}{y^H} + \frac{1-\nu}{\nu} \frac{c^H}{y^H} \right) - \tau^k \left[1 - \frac{\phi-1}{\phi}(1-\alpha) \right] \\
-\tau^n \frac{\phi-1}{\phi}(1-\alpha) - s^l
\end{gathered}
$$

which implies that

$$
\frac{c^H}{y^H} = \frac{\nu}{\tau^c} \left[(R-1)s^b + (Q-1)s^f - \tau^k [1 - \frac{\phi-1}{\phi}(1-\alpha)] - \tau^n \frac{\phi-1}{\phi}(1-\alpha) - s^l \right].
$$

Dividing both sides of equation (70) by y^H we have

$$
\frac{c^{F*}}{y^H} = 1 - \frac{c^H}{y^H} - \delta \frac{y^H}{k} - s^g,
$$

and since $c^{F*} = 0.9c^F$, by assumption, we also have $\frac{c^H}{c^F}$. We can now solve for TT from equation (63):

$$
TT = \frac{c^H}{c^F} \frac{1-\nu}{\nu}. \tag{91}
$$

Then, we also have

$$
mc = TT^{\nu-1} \frac{\phi-1}{\phi} \tag{92}
$$

$$
w = mc(1-\alpha) \frac{y^H}{n}. \tag{93}
$$

From equation (65), we obtain the $\frac{c}{y^H} = \frac{(\frac{c^H}{y^H})^v \ (\frac{c^F}{y^H})^{1-v}}{v^v(1-v)^{1-v}}$, which gives us $\frac{c}{n} = \frac{c}{y^H}\frac{y^H}{n}$. We can now divide equation (62) by $n^{-\sigma}$ and have

$$
\begin{aligned}
{}_n\frac{n^\eta}{n^{-\sigma}} &= (1-\tau^n)w\frac{c^{-\sigma}}{n^{-\sigma}}\frac{1}{1+\tau_t^c} \\
n &= \left[\frac{1}{{}_n}(1-\tau^n)w\left(\frac{c}{n}\right)^{-\sigma}\frac{1}{1+\tau^k}\right]^{\frac{1}{\eta+\sigma}}. \qquad (94)
\end{aligned}
$$

Having this, we can find the rest of the endogenous variables:

$$y^H = \frac{y^H}{n}n \qquad (95)$$

$$c^H = \frac{c^H}{y^H}y^H \qquad (96)$$

$$c^F = \frac{c^F}{y^H}y^H \qquad (97)$$

$$c = \frac{c}{n}n \qquad (98)$$

$$k = \frac{k}{y^H}y^H \qquad (99)$$

$$x = \delta k \qquad (100)$$

$$f^{TT} = \frac{f^{TT}}{y^H}y^H \qquad (101)$$

$$z^1 = \frac{1}{1-\beta\theta} y^H TT^{\nu-1} \tag{102}$$

$$z^2 = \frac{1}{1-\beta\theta} y^H mc \tag{103}$$

$$d = TT^{\nu-1} y^H - TT^{\nu-1} r^k k - wn \tag{104}$$

$$m = \left[\frac{1}{m} c^{-\sigma} \frac{1}{1+\tau^c} (1-\beta) \right]^{-1/\mu} \tag{105}$$

$$b = s^b y^H TT^{\nu-1}. \tag{106}$$

Finally, from equation (72), we obtain

$$f^h = \frac{TT^{\nu-1} f^{TT} + TT^{\nu^*} c^F * - TT^{\nu-1} c^F - QTT^{\nu-1} f^{TT}}{TT^{\nu^*+\nu-1}(Q-1)}. \tag{107}$$

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