Study of a nonlinear finance system

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Introduction

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It is important studying internal structural characteristics to reveal the reasons why such complicated phenomena occur and to provide theory basis and practical ways for the analysis, prediction and control of the complicated continuous economic esystem.

Mathematical Model

Financial model composed by:

- parts of product;
- money;
- bound;
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model is extremely sensitivity to the initial value and changing of parameters.
We consider:

- x: interest rate;
- y: investment demand;
- z : price exponent.

So that we are focus in the sensitivity of parameter, we consider changing rates about time : $\dot{x} = dx/dt, \dot{y} = dy/dt, \dot{z} = dz/dt$.

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- it is the structure adjustment from good prices.

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$$\dot{x} = f_1(y - SV)x + f_2 z \tag{1}$$

SV is the amount of saving, f_1, f_2 constants.

y: Investment demand

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$$\dot{y} = f_3(BEN - \alpha y - \beta x^2) \tag{2}$$

BEN is benefit rate of investment, f_3,α and β are all constants (considering benefit rate of investment costant in a certain period of time)

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- influenced by inflation rate

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$$\dot{z} = -f_4 z - f_5 x \tag{3}$$

Dynamic System

Changing coordinate system and simplifying:

$$\begin{cases} \dot{x} = z + (y - a)x \\ \dot{y} = 1 - by - x^2 \\ \dot{z} = -x - cz \end{cases} \tag{4}$$

with:

- $a \geq 0$ saving amount;
- $b \geq 0$ is the per-investiment cost;
- $c \geq 0$ is the elasticity of demands.

Model analysis

First of all we have to find the stationary point:

$$F(x, y, z) = \begin{pmatrix} -ax + xy + z \\ -x^2 - by + 1 \\ -x - cz \end{pmatrix} = 0 \Rightarrow \begin{cases} z + (y - a)x = 0 \\ 1 - by - x^2 = 0 \\ -x - cz = 0 \end{cases} \tag{5}$$

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Se:

- se $(c - b - abc \leq 0)$, we have a single stationary point $V_s = (0, 1/b, 0)$
- se $c - b - abc \geq 0$, we have 3 stationary point:

$$V_{s1} = (0, 1/b, 0)$$
$$V_{s2} = (\sqrt{(c-b-abc)/c}, (1+ac)/c, -1/c\sqrt{(c-b-abc)/c})$$
$$V_{s3} = (-\sqrt{(c-b-abc)/c}, (1+ac)/c, 1/c\sqrt{(c-b-abc)/c})$$

Study of the local topological structure

Let's study $V_s = (0, 1/b, 0)$ for $(c - b - abc \leq 0)$, translating the coordinate:

$$X = x, \quad Y = y - 1/b \quad Z = z \quad \Longrightarrow \quad V_s^*(0,0,0)$$

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$$\begin{cases} \dot{X} = (1/b - a)X + Z + XY \\ \dot{Y} = -bY - X^2 \\ \dot{Z} = -X - cZ \end{cases} \tag{6}$$

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Evaluate the Jacobian of the system in V_s*:

$$J(V_s^*) = \begin{pmatrix} -a + \frac{1}{b} & 0 & 1 \\ 0 & -b & 0 \\ -1 & 0 & -c \end{pmatrix} \tag{7}$$

Study of the local topological structure

Characteristic polynomial:

$$P(\lambda) = (\lambda + b)(\lambda^2 + \lambda(c + a - 1/b) + ac - c/b + 1) \tag{8}$$

A first eigenvalue is $\lambda = -b$, the other 2 comes from:

$$\lambda^2 + \lambda \underbrace{(c + a - 1/b)}_{T} + \underbrace{ac - c/b + 1}_{D} = 0 \tag{9}$$

Solutions can be divided into 3 cases

Case 1) $c - b - abc < 0$, $T > 0$

$$\lambda^2 + \lambda \underbrace{(c + a - 1/b)}_{T} + \underbrace{ac - c/b + 1}_{D} = 0$$

If $c - b - abc < 0 \Rightarrow D > 0$ and $T > 0$, we find $\lambda_1, \lambda_2, \lambda_3 < 0$ so $V_s(0, 1/b, 0)$ is a sink, the point is stable.

Case 2) $c - b - abc < 0$, $T < 0$

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Case 3.1) $c - b - abc = 0, (1 - c^2)/c > 0$

$$\lambda^2 + \lambda \underbrace{(c + a - 1/b)}_{T} + \underbrace{ac - c/b + 1}_{D} = 0$$

If $c - b - abc = 0 \Rightarrow \lambda_2 = 0$, $\lambda_3 = -(c + a - 1/b)$ and $\lambda_1 = -b < 0$, we need to study 2 different case:

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If $(1 - c^2)/c > 0 \rightarrow 0 < c < 1$ than $\lambda_1 < 0, \lambda_2 = 0, \lambda_3 > 0$ and for the theorem of existence of stable, central and unstable manifold, we can conclude $V_s(0, 1/b, 0)$ is unstable.

Case 3.2) $c - b - abc = 0, (1 - c^2)/c < 0$

$$\lambda^2 + \lambda \underbrace{(c + a - 1/b)}_{T} + \underbrace{ac - c/b + 1}_{D} = 0$$

If $(1 - c^2)/c < 0 \to c > 1$ than we can use central manifold theorem:

$$\lambda_1 = -b < 0 \to \bar{v_1} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\lambda_2 = 0 \to \bar{v_2} = \begin{pmatrix} 1 \\ 0 \\ -1/c \end{pmatrix}$$

$$\lambda_3 = (1 - c^2)/c < 0 \to \bar{v_3} = \begin{pmatrix} -1/c \\ 0 \\ 1 \end{pmatrix}$$

Case 3.2) $c - b - abc = 0, (1 - c^2)/c < 0$

$\bar{v}_1, \bar{v}_3$ are extended into the stable subspace E^s and $\bar{v}_2$ is extended into the central subspace E^c

$$T = \begin{pmatrix} 1 & -1/c & 0 \\ 0 & 0 & 1 \\ -1/c & 1 & 0 \end{pmatrix} \quad T^{-1} = \begin{pmatrix} c^2/(c^2-1) & 0 & c/(c^2-1) \\ c/(1-c^2) & 0 & c^2/(1-c^2) \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = T \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

Put this in (6) we find

Case 3.2) $c - b - abc = 0, (1 - c^2)/c < 0$

$$\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & (c^2 - 1)/c & 0 \\ 0 & 0 & -b \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} + \begin{pmatrix} c^2 w(u - v/c)/(c^2 - 1) \\ cw(u - v/c)/(1 - c^2) \\ -(u - v/c)^2 \end{pmatrix} \tag{10}$$

Therefore $E^c = u$ axis, $E^s =$ span $\{(v, w)\}$,$E^u = \emptyset$

Case 3.2) $c - b - abc = 0, (1 - c^2)/c < 0$

We find:

$$\dot{u} = \frac{c^2}{c^2 - 1}(u - v/c)w \tag{11}$$

$$\begin{pmatrix} \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} (c^2 - 1)/c & 0 \\ 0 & -b \end{pmatrix} \begin{pmatrix} \dot{v} \\ \dot{w} \end{pmatrix} + \begin{pmatrix} \frac{c}{(1-c^2)} w(u - v/c) \\ -(u - v/c)^2 \end{pmatrix} \tag{12}$$

Central manifold W^c is the curve tangent the central subspace E^c at V_s^* and passing at it.

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Central manifold W^c is the curve tangent the central subspace E^c at V_s^* and passing at it.
To find the equation of W^c we can use the method of form power series:

$$\begin{pmatrix} v \\ w \end{pmatrix} = \bar{h}(U) = \begin{pmatrix} a_1 u^2 + b_1 u^3 + c_1 u^4 \\ a_2 u^2 + b_2 u^3 + c_2 u^4 \end{pmatrix}$$

Coefficient of u and costant term is 0 due to $\bar{h} = 0, D\bar{h}(0) = 0$

Case 3.2) $c - b - abc = 0, (1 - c^2)/c < 0$

So that $\dot{h}_i = \partial h_i(u)/\partial u = 2a_i u + 3b_i u^2 + 4c_i u^3 + \cdots$

$$\begin{pmatrix} \dot{h_1}(u) \\ \dot{h_2}(u) \end{pmatrix} \left(\frac{c^2}{c^2-1}(u - v/c)h_2(u) \right) = \begin{pmatrix} \frac{c^2-1}{c}h_1(u) + \frac{c}{1-c^2}(u - h_1(u)/c)h_2(u) \\ -bh_2(u) - (u - h_1(u)/c)^2 \end{pmatrix} \tag{13}$$

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Comparing the coefficients we obtain:

$$\begin{cases} a_1 = 0, \quad b_1 = -\frac{c^2}{b(c^2-1)^2}, \quad c_1 = 0 \\ a_2 = -\frac{1}{b}, \quad b_2 = 0, \quad c_2 = -\frac{2c}{b^3(c^2-1)^2}(b + c(c^2 - 1)) \end{cases} \quad (14)$$

Case 3.2) $c - b - abc = 0, (1 - c^2)/c < 0$

Putting this in (11) and consider only the first term:

$$\dot{u} = -\frac{c^2}{b(c^2 - 1)} u^3 + o(u^4) \tag{15}$$

We can conclude that the stationary point V_s^* and also V_s is gradually inclined to be stable.

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Synthesizing this 2 cases, we can conclude that if $c = 1$ a bifurcation occurs at V_s

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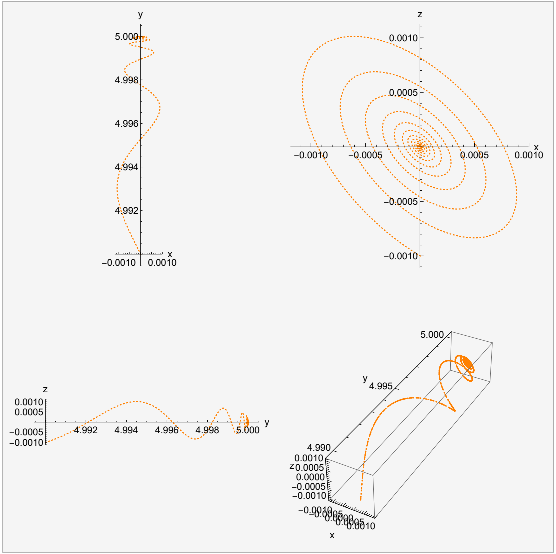
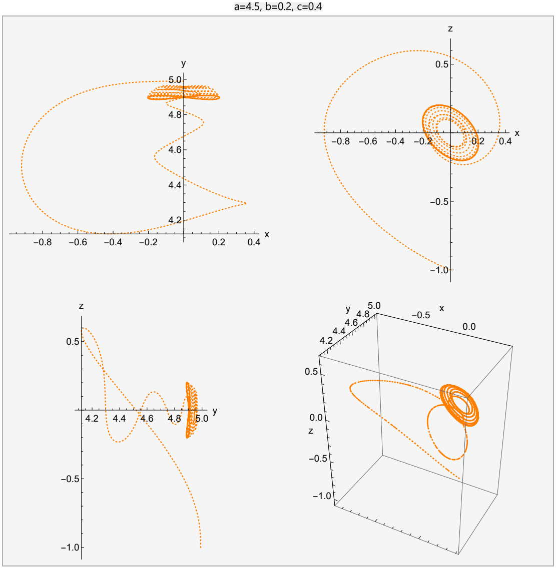
We get $c^2 < 1$, λ_2, λ_3 are purely imaginary roots, with $\lambda_1 = -b < 0$. Considering $\alpha = -(c + a - 1/b)$, than $\partial_\alpha / \partial a|_{a=a_0} = -1 \neq 0$, the conditions of Hopf-bifurcation are satisfied, so that at the point V_s Hopf-bifurcation occurs and exists periodic solution group

Case 1

a=4.5, b=0.2, c=0.6

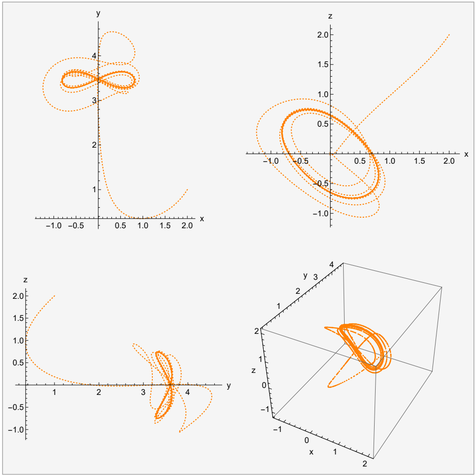


Case 2

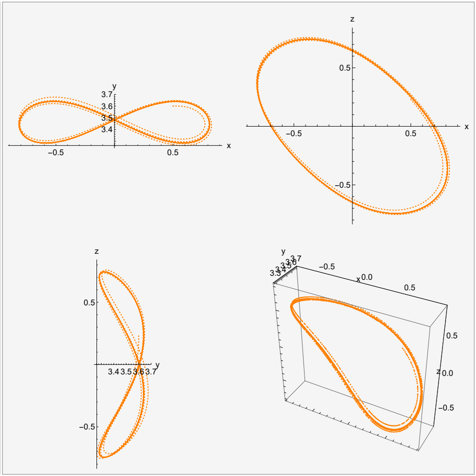

a=4.5, b=0.2, c=0.4
y
5.0
4.8
4.6
4.4
4.2
x
−0.8
−0.6
−0.4
−0.2
0.2
0.4
z
0.5
−0.5
−1.0
4.2
4.4
4.6
4.8
5.0
−0.5
0.0

Case 3.1

a=3, b=0.2, c=0.5,x[0]=2,y[0]=1,z[0]=2

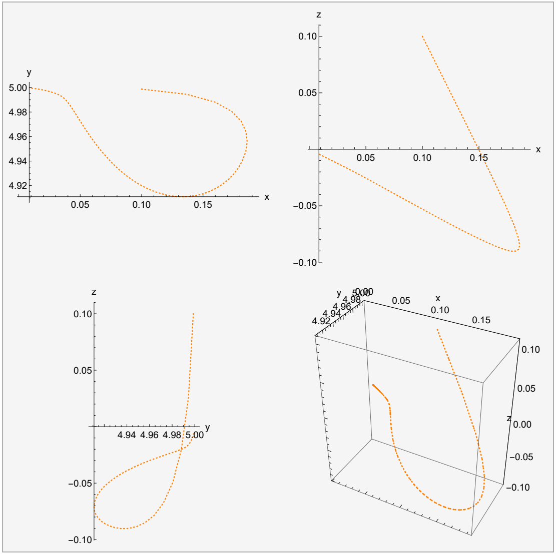


a=4.5, b=0.2, c=0.5,x[0]=0.5,y[0]=3.6,z[0]=0.23



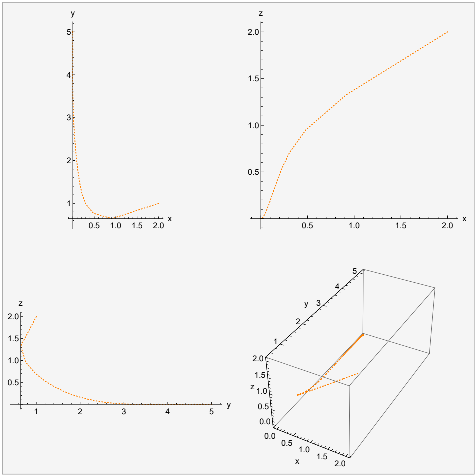
Case 3.2

a=4.5, b=0.2, c=2,

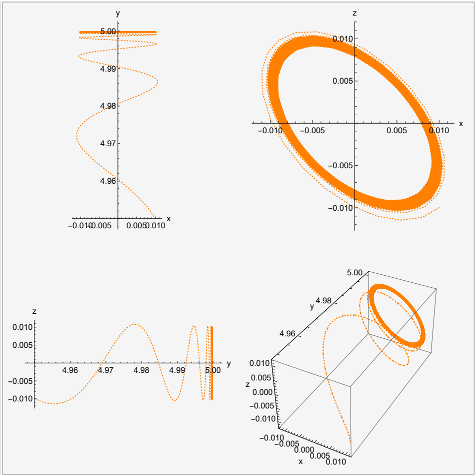


Case 4

a=4.5, b=0.2, c=2,x[0]=0.5,y[0]=3.6,z[0]=0.23



a=4.5, b=0.2, c=2,x[0]=0.1,y[0]=4.95,z[0]=−0.1



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- The inappropriate combination of the parameters in the system is the source that causes chaotic, it is likely to make system inclined to chaotic and lose control, or make it into the stagnant state;

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- The elasticity deficiency of variables will cause the lagging down of the information feedback, therfore strengthening the elasticity of variables will help stabilize economy and help operation of the financial system;
- Saving amount variable must be kept in an appropriate level, smaller is, greater the fluctuation of the system is; if is too small, it will cause chaotic situation, if too large will cause economy to lack vigor

Bibliography

- Study for the bifurcation topological structure and the global complicated character of a kind of nonlinear finance system (Ma Jun-hai, Chen Yu-shu)