Maths Expertes Ex 19 12 2023

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Exercice 18 p43

1. $$z_1 = -i$$

$$|z_1| = \sqrt{(-1)^2} = 1$$

2. $$z_2 = -2$$

$$|z_2| = \sqrt{(-2)^2} = 2$$

3. $$z_3 = 3$$

$$|z_3| = \sqrt{(3)^2} = 3$$

4. $$z_4 = 18$$

$$|z_4| = \sqrt{(18)^2} = 18$$

Exercice 19 p43

$$|z_A| = 3$$

$$|z_B| = 2$$

$$|z_C| = 3$$

$$|z_D| = 4$$

$$|z_E| = 2$$

$$|z_F| = \sqrt{2^2 + (-2)^2} = 2\sqrt{2}$$

Exercice 20 p43

1. $$z_1 = (5 + 2i) - 4(2 + 3i)$$

$$z_1 = 5 + 2i - 8 - 12i$$

$$z_1 = -3 - 10i$$

$$|z_1| = \sqrt{(-3)^2 + (-10)^2} = \sqrt{109}$$

2. $$z_2 = \sqrt{3} - 2i$$

$$|z_2| = \sqrt{\sqrt{3}^2 + (-2)^2} = \sqrt{7}$$

3. $$z_3 = (1 + 2i) \times 5(2 - 3i)$$

$$z_3 = (1 + 2i)(10 - 15i)$$

$$z_3 = 40 + 5i$$

$$|z_3| = \sqrt{40^2 + 5^2} = 5\sqrt{65}$$

4.

$$z_4 = -2\left(\sqrt{3} - i\right) + 4(6 - i)$$

$$z_4 = -2\sqrt{3} + 2i + 24 - 4i$$

$$z_4 = 24 - 2\sqrt{3} - 2i$$

$$|z_4| = \sqrt{\left(24 - 2\sqrt{3}\right)^2 + (-2i)^2} = \sqrt{592 - 96\sqrt{3}}$$

Exercice 21 p43

$$z_{\vec{u}} = -4 + 2i$$

$$|z_{\vec{u}}| = \sqrt{(-4)^2 + 2^2} = 2\sqrt{5}$$

$$|z_{\vec{u}}| = \| \vec{u} \|$$

$$\| \vec{u} \| = 2\sqrt{5}$$

Exercice 22 p43

$$z_A = -3 + i$$

$$z_B = 2 - 4i$$

$$AB = \| \overrightarrow{AB} \| = \left|z_{\overrightarrow{AB}}\right| = |z_B - z_A|$$

$$|z_B - z_A| = |2 + 3 - 4i - i| = |5 - 5i|$$

$$\left|z_{\overrightarrow{AB}}\right| = \sqrt{5^2 + (-5)^2} = 5\sqrt{2}$$

$$AB = 5\sqrt{2}$$

Exercice 24 p43

1.

$$z_1 = \frac{1}{4} + \frac{1}{4}i$$

$$|z_1| = \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2} = \sqrt{\frac{2}{16}} = \frac{\sqrt{2}}{4}$$

Donc $|z_1| \neq 1$ soit $z_1 \notin \mathbb{U}$

2.

$$z_2 = \frac{-3}{4} + \frac{\sqrt{7}}{4}i$$

$$|z_2| = \sqrt{\left(\frac{-3}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2} = \sqrt{\frac{16}{16}} = 1$$

Donc $|z_2| = 1$ soit $z_2 \in \mathbb{U}$

3.
$$z_3 = \frac{2\sqrt{6}}{5} + \frac{1}{5}i$$

$$|z_3| = \sqrt{\left(\frac{2\sqrt{6}}{5}\right)^2 + \left(\frac{1}{5}\right)^2}$$

$$|z_3| = \sqrt{\frac{25}{25}} = 1$$

Donc $|z_3| = 1$ soit $z_3 \in \mathbb{U}$

4.
$$z_4 = \frac{\sqrt{5}}{2} - \frac{1}{2}i$$

$$|z_4| = \sqrt{\left(\frac{\sqrt{5}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$$

$$|z_4| = \sqrt{\frac{6}{4}} = \frac{\sqrt{6}}{2}$$

Donc $|z_4| \neq 1$ soit $z_4 \notin \mathbb{U}$

Exercice 32 p43

Un losange est un quadrilatère avec ses côtés de même longeur. Il s'agit donc de montrer que

$$AB = DC$$

$$\Rightarrow \| \overrightarrow{AB} \| = \| \overrightarrow{DC} \|$$

$$\Rightarrow |z_B - z_A| = |z_C - z_D|$$

On a d'une part :

$$|z_B - z_A| = |7 + 2i - (6 + 5i)| = |1 - 3i|$$

$$|z_B - z_A| = |1 - 3i| = \sqrt{1^2 + (-3)^2} = \sqrt{10}$$

D'autre part :

$$|z_C - z_D| = |10 + i - (9 + 4i)| = |1 - 3i|$$

$$|z_C - z_D| = |1 - 3i| = \sqrt{1^2 + (-3)^2} = \sqrt{10}$$

Par conséquent

$$|z_B - z_A| = |z_C - z_D|$$

$$\Rightarrow AB = DC$$

Les côtés du quadrilatère sont de même longeur donc ABCD est un losange.