

Simple Ordinary DE

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Lucas

• link:

<https://www.youtube.com/watch?v=BjvkbLfvkqY&list=PLMrJAKhIeNNTYaOnVI3QpH7jgUL-nAmvPA&index=4>

1. Simple Ordinary Differential Equation

1.1. First Order Linear Ordinary D.E

Let's say that bunnies are procreating

let x = bunny population

and the population grows at a rate λ

$$\dot{x} = \frac{dx}{dt} = \lambda x, x(0) \text{ initial population}$$

We ask the question what is the population as a function of time ?

$x(t)$?

1.1.1. Method 1:

$$\frac{dx}{dt} = \lambda x$$

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$$\int \frac{dx}{x} = \int \lambda dt$$

$$\Rightarrow \ln(x(t)) = \lambda t + C$$

$$\Rightarrow x(t) = e^{\lambda t + C} = e^{\lambda t} e^C = e^{\lambda t} K$$

$$x(t) = e^{\lambda t} + K$$

$$K ? X(0) = e^0 K \Rightarrow K = x(0)$$

1.2. Second Order ODE

1.2.1. Example

The differential equation you've written down is a second-order linear differential equation with constant coefficients, which can be written in the standard form $\frac{d^2x}{dt^2} + \frac{k}{m} \frac{dx}{dt} + \frac{g}{m} x = 0$. To solve this type of differential equation, you can use the characteristic equation, which is given by the equation $\lambda^2 + \frac{k}{m} \lambda + \frac{g}{m} = 0$. The solutions to this equation are the so-called characteristic roots, which are the values of λ that satisfy the equation. In this case, the characteristic roots are given by the quadratic formula:

$$\lambda = \frac{-\frac{k}{m} \pm \sqrt{\left(\frac{k}{m}\right)^2 - 4 \cdot \frac{g}{m}}}{2}.$$

Once you have the characteristic roots, you can find the general solution to the differential equation by writing it in the form $x(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$, where c_1 and c_2 are constants and λ_1 and λ_2 are the characteristic roots. To find the specific solution to the equation, you need to use initial conditions, which specify the values of x and \dot{x} at a particular time t_0 . Using these initial conditions, you can solve for the constants c_1 and c_2 and obtain the specific solution to the differential equation.