SimpleOrdinaryDE

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- link:

https://www.youtube.com/watch?v=BjvkBLfvkqY&list=PLMrJAkhIeNNTYaOnVI3QpH7j-gULnAmvPA&index=4

1. Simple Ordinary Differential Equation

1.1. First Order Linear Ordinary D.E

Let's say that bunnies are procreating
let x = bunny population
and the population grows at a rate λ

$\dot{x} = \frac{dx}{dt} = \lambda x, x(0)\ initial\ pop\ \ ize$

We ask the question what is the population as a function of time ?
x(t) ?

1.1.1. Method 1:

$\frac{dx}{dt} = \lambda x$
$\frac{dx}{x} = \lambda dt$
$\int \frac{dx}{x} = \int \lambda dt$
=> $ln(x(t)) = \lambda t + C$
=> $x(t) = e^{\lambda t + C}\ e^{a+b} = e^a e^b$
$x(t) = e^{\lambda t} + K$
K ? $X(0) = e^0 K \Rightarrow K = x(0)$

1.2. Second Order ODE

1.2.1. Example

The differential equation you've written down is a second-order linear differential equation with constant coefficients, which can be written in the standard form $\frac{d^2x}{dt^2} + \frac{k}{m}\frac{dx}{dt} + \frac{g}{m}x = 0$. To solve this type of differential equation, you can use the characteristic equation, which is given by the equation $\lambda^2 + \frac{k}{m}\lambda + \frac{g}{m} = 0$. The solutions to this equation are the so-called characteristic roots, which are the values of λ that satisfy the equation. In this case, the characteristic roots are given by the quadratic formula:

$$\lambda = \frac{-\frac{k}{m} \pm \sqrt{\left(\frac{k}{m}\right)^2 - 4 \cdot \frac{g}{m}}}{2}.$$

Once you have the characteristic roots, you can find the general solution to the differential equation by writing it in the form $x(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$, where c_1 and c_2 are constants and λ_1 and λ_2 are the characteristic roots. To find the specific solution to the equation, you need to use initial conditions, which specify the values of x and $\dot{x}$ at a particular time t_0. Using these initial conditions, you can solve for the constants c_1 and c_2 and obtain the specific solution to the differential equation.