

# Derivative

15 Octobre, 2023

**Lucas Duchet-Annez**

## The Derivative

### Definition

The Derivative is the rate of change of function  $f(x)$  with respect to an independent variable  $\langle x \rangle$ .  
It's the slope of the tangent line at a point  $x$

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Example :

$$f(x) = x^2$$

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + \Delta x^2 - x^2}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} \frac{2x\Delta x}{\Delta x} + \frac{\Delta x^2}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} 2x + \Delta x$$

$$\lim_{\Delta x \rightarrow 0} 2x$$

Power law: derivative of  $f(x) = x^n = nx^{n-1}$

Chain law: Two function  $f(x), g(x)$   $\frac{d}{dx} f(g(x)) = \frac{df}{dg}(g(x)) \cdot \frac{dg}{dx}(x) = f'(g(x)) \cdot g'(x)$

Example:

$$f(x) = \sin(x)$$

$$g(x) = x^3$$

$$f(g(x)) = \sin(x^3)$$

$$f'(g(x)) = 3\cos(x^3)x^2$$

### Links

- [Identities](#)