

# Directional Derivative

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## Directional Derivative

### Definition

The directional derivative of a multivariable function at a given point is a measure of the rate of change of the function in a particular direction at that point. It is a generalization of the concept of the derivative of a function of one variable, which measures the rate of change of the function in the direction of the x-axis.

The directional derivative of a function  $f(x, y)$  at a point  $(x_0, y_0)$  in the direction of a unit vector  $\mathbf{u} = (a \ b)$  is given by:

$$\frac{\partial f}{\partial \mathbf{u}}(x_0, y_0) = \nabla f(x_0, y_0) \cdot \mathbf{u} = \left( \frac{\partial f}{\partial x}(x_0, y_0) \ \frac{\partial f}{\partial y}(x_0, y_0) \right) \cdot (a \ b)$$

Where  $\nabla f(x_0, y_0)$  is the gradient vector of the function at the point  $(x_0, y_0)$ , and  $\frac{\partial f}{\partial x}(x_0, y_0)$  and  $\frac{\partial f}{\partial y}(x_0, y_0)$  are the partial derivatives of the function with respect to  $x$  and  $y$  respectively.

The directional derivative can be thought of as the projection of the gradient vector onto the direction specified by the unit vector  $\mathbf{u}$ . It tells us the rate of change of the function in the direction of  $\mathbf{u}$  at the point  $(x_0, y_0)$ .

### Example

To find the directional derivative of the function  $f(x, y) = x^2 \sin 2y$  in the direction of the vector  $\mathbf{u} = (\cos \sin)$ , we need to first find the gradient vector of the function at the given point.

The partial derivatives of  $f$  with respect to  $x$  and  $y$  are:

$$\frac{\partial f}{\partial x} = 2x \sin 2y$$

$$\frac{\partial f}{\partial y} = 2x^2 \cos 2y$$

The gradient vector of the function at a point  $(x_0, y_0)$  is given by:

$$\nabla f(x_0, y_0) = \left( \frac{\partial f}{\partial x}(x_0, y_0) \ \frac{\partial f}{\partial y}(x_0, y_0) \right) = (2x_0 \sin 2y_0 \ 2x_0^2 \cos 2y_0)$$

The directional derivative of the function in the direction of  $\mathbf{u}$  at the point  $(x_0, y_0)$  is then given by:

$$\frac{\partial f}{\partial \mathbf{u}}(x_0, y_0) = \nabla f(x_0, y_0) \cdot \mathbf{u} = (2x_0 \sin 2y_0 \ 2x_0^2 \cos 2y_0) \cdot (\cos \sin)$$

We can evaluate this expression to find the directional derivative. For example, if  $(x_0, y_0) = (1, 1)$ , then the directional derivative is:

$$\frac{\partial f}{\partial \mathbf{u}}(1, 1) = (2 \cdot 1 \cdot \sin 2 \cdot 1 \cdot 2 \cdot 1^2 \cos 2 \cdot 1) \cdot (\cos \sin) = (\sin 2 \cos 2) \cdot (\cos \sin) = \sin^2 2 + \cos^2 2 = 1$$

Link

- [Gradient Vector](#)
- [Partial Derivative](#)