Derivative

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The Derivative

Definition

The Derivative is the rate of change of function f(x) with respect to an independent variable ‹ x ›.
It's the slope of the tangent line at a point x

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Example :

$$f(x) = x^2$$

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 + -x^2}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + \Delta x^2 - x^2}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{2x\Delta x}{\Delta x} + \frac{\Delta x^2}{\Delta x}$$

$$\lim_{\Delta x \to 0} 2x + \Delta x$$

$$\lim_{\Delta x \to 0} 2x$$

Power law: derivative of $f(x) = x^n = nx^{n-1}$

Chain law: Two function f(x),g(x) $\frac{d}{dx} f(g(x)) = \frac{df}{dx}(g(x)).\frac{dg}{dx}(x) = f'(g(x)).g'(x)$

Example:

$f(x) = sin(x)$
$g(x) = x^3$
$f(g(x)) = sin(x^3)$
$f'(g(x)) = 3cos(x^3)x^2$

Links

- Identities