

# Simple Ordinary DE

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• link:

<https://www.youtube.com/watch?v=BjvkBLfvkqY&list=PLMrJAKhIeNNTYaOnVI3QpH7jgULnAmv-PA&index=4>

## Simple Ordinary Differential Equation

### First Order Linear Ordinary D.E

Let's say that bunnies are procreating

let  $x$  = bunny population

and the population grows at a rate  $\lambda$

$$\dot{x} = \frac{dx}{dt} = \lambda x, x(0) \text{ initial pop size}$$

We ask the question what is the population as a function of time ?

$x(t)$  ?

Method 1:

$$\frac{dx}{dt} = \lambda x$$

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$$\int \frac{dx}{x} = \int \lambda dt$$

$$\Rightarrow \ln(x(t)) = \lambda t + C$$

$$\Rightarrow x(t) = e^{\lambda t + C} = e^{\lambda t} e^C = e^{\lambda t} K$$

$$x(t) = e^{\lambda t} + K$$

$$K ? x(0) = e^0 K \Rightarrow K = x(0)$$

### Second Order ODE

#### Example

The differential equation you've written down is a second-order linear differential equation with constant coefficients, which can be written in the standard form  $\frac{d^2x}{dt^2} + \frac{k}{m} \frac{dx}{dt} + \frac{g}{m} x = 0$ . To solve this type of differential equation, you can use the characteristic equation, which is given by the equation  $\lambda^2 + \frac{k}{m} \lambda + \frac{g}{m} = 0$ . The solutions to this equation are the so-called characteristic roots, which are the values of  $\lambda$  that satisfy the equation. In this case, the characteristic roots are given by the quadratic formula:

$$\lambda = \frac{-\frac{k}{m} \pm \sqrt{\left(\frac{k}{m}\right)^2 - 4 \cdot \frac{g}{m}}}{2}.$$

Once you have the characteristic roots, you can find the general solution to the differential equation by writing it in the form  $x(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$ , where  $c_1$  and  $c_2$  are constants and  $\lambda_1$  and  $\lambda_2$

are the characteristic roots. To find the specific solution to the equation, you need to use initial conditions, which specify the values of  $x$  and  $\dot{x}$  at a particular time  $t_0$ . Using these initial conditions, you can solve for the constants  $c_1$  and  $c_2$  and obtain the specific solution to the differential equation.

Maths