Partial Derivative

12 Juillet, 2023

Lucas

Partial Derivative

Definition

In calculus, a partial derivative of a function of several variables is the derivative of the function with respect to one of the variables, with the others treated as constants. For example, if $f(x, y)$ is a function of two variables, then the partial derivative of f with respect to x is the derivative of f with respect to x while holding y constant, and is denoted $\frac{\partial f}{\partial x}$.

Partial derivatives are useful in multivariable calculus, as they allow us to find the rates of change of a function with respect to each of its variables independently. They are also used in the gradient vector, which points in the direction of the greatest rate of increase of a function at a given point.

The partial derivative of a function $f(x, y)$ with respect to x is given by:

$$\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x + h, y) - f(x, y)}{h}$$

Similarly, the partial derivative of f with respect to y is given by:

$$\frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x, y + h) - f(x, y)}{h}$$

These expressions can be used to find the partial derivatives of a function by directly computing the limits, or by using the rules of calculus to simplify the expressions.

Example

To find the partial derivatives of the function $f(x, y) = \sin x \cos y^2$, we need to use the rules of calculus to take the derivatives with respect to x and y while holding the other variable constant.

The partial derivative of f with respect to x is given by:

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(\sin x \cos y^2\right) = \frac{\partial}{\partial x}(\sin x)\cos y^2 + \sin x \frac{\partial}{\partial x}\left(\cos y^2\right) = \cos x \cos y^2$$

The partial derivative of f with respect to y is given by:

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(\sin x \cos y^2\right) = \frac{\partial}{\partial y}(\sin x)\cos y^2 + \sin x \frac{\partial}{\partial y}\left(\cos y^2\right) = -2y \sin x \sin y^2$$

Links

- Derivative