

# Gradient Vector

24 Septembre, 2023

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## Gradient Vector

### Definition

In multivariable calculus, the gradient vector of a scalar-valued function is a vector-valued function that points in the direction of the greatest rate of increase of the function at a given point. It is a generalization of the concept of the derivative of a function of one variable, which points in the direction of the greatest rate of increase of the function at a given point.

The gradient vector of a function  $f(x, y)$  at a point  $(x_0, y_0)$  is given by:

$$\nabla f(x_0, y_0) = \left( \frac{\partial f}{\partial x}(x_0, y_0) \frac{\partial f}{\partial y}(x_0, y_0) \right)$$

Where  $\frac{\partial f}{\partial x}(x_0, y_0)$  and  $\frac{\partial f}{\partial y}(x_0, y_0)$  are the partial derivatives of the function with respect to  $x$  and  $y$  respectively.

The gradient vector is a useful concept in multivariable calculus, as it allows us to find the direction of maximum increase of a function at a given point, and to calculate the directional derivative of a function in a particular direction.

### Example

To find the gradient vector of the function  $f(x, y) = y \ln x + xy^2$  at a point  $(x_0, y_0)$ , we need to take the partial derivatives of the function with respect to  $x$  and  $y$  and evaluate them at the point  $(x_0, y_0)$ .

The partial derivatives of  $f$  with respect to  $x$  and  $y$  are:

$$\frac{\partial f}{\partial x} = y^2 + y \frac{1}{x}$$

$$\frac{\partial f}{\partial y} = \ln x + 2xy$$

The gradient vector of the function at the point  $(x_0, y_0)$  is then given by:

$$\nabla f(x_0, y_0) = \left( \frac{\partial f}{\partial x}(x_0, y_0) \frac{\partial f}{\partial y}(x_0, y_0) \right) = \left( y_0^2 + \frac{y_0}{x_0} \ln x_0 + 2x_0 y_0 \right)$$

For example, if  $(x_0, y_0) = (1, 1)$ , then the gradient vector is:

$$\nabla f(1, 1) = \left( 1^2 + \frac{1}{1} \ln 1 + 2 \cdot 1 \cdot 1 \right) = (2 \ 2)$$

Evaluating these partial derivatives at the point  $(1, 2)$  gives us:

$$\frac{\partial f}{\partial x}(1, 2) = 2^2 + 2 \cdot \frac{1}{1} = 6$$

$$\frac{\partial f}{\partial y}(1, 2) = \ln 1 + 2 \cdot 1 \cdot 2 = 4$$

The gradient vector at the point  $(1, 2)$  is then given by:

$$\nabla f(1, 2) = \left( \frac{\partial f}{\partial x}(1, 2) \quad \frac{\partial f}{\partial y}(1, 2) \right) = (6 \ 4)$$

Therefore, the gradient vector at the point  $(1, 2)$  is  $(6 \ 4)$ .

Link

- [Partial Derivative](#)