



POLITECNICO
MILANO 1863

ALLOY 6-Syntactic overview

TEMPORAL CONNECTIVES

Authors:

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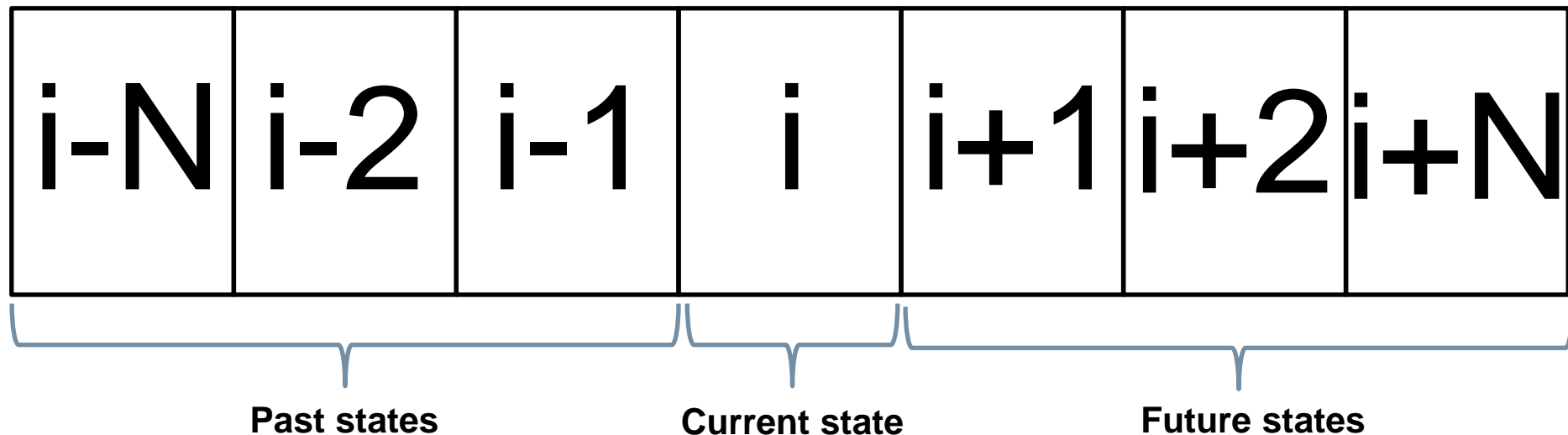
Francesco Santambrogio

LINEAR TEMPORAL LOGIC (LTL)

Definition

1

LINEAR TEMPORAL LOGIC (LTL): «an infinite sequence of states where each point in time has a unique successor, based on a linear-time perspective»^[1]



[1] Ashari, R., & Habib, S. (n.d.). *LINEAR TEMPORAL LOGIC (LTL)*.

SIGNATURES

```
sig Person {  
  var liveness: Liveness  
}
```

```
enum Liveness {Alive, Dead}
```

PREDICATES

```
pred Die [p: Person] {  
  p.liveness = Alive  
  p.liveness' = Dead }
```

Goal



To express constraints that hold at different instants of time or for a certain amount of time



- How can we express that a person is not immortal?
- How can we express that a person cannot come back to life?

Goal



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- How can we express that a person is not immortal?
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TEMPORAL CONNECTIVES

Always

Syntax: $\text{Exp} ::= \underline{\text{un0p}} \text{ expr} \mid \text{expr} \underline{\text{bin0p}} \text{ expr}$

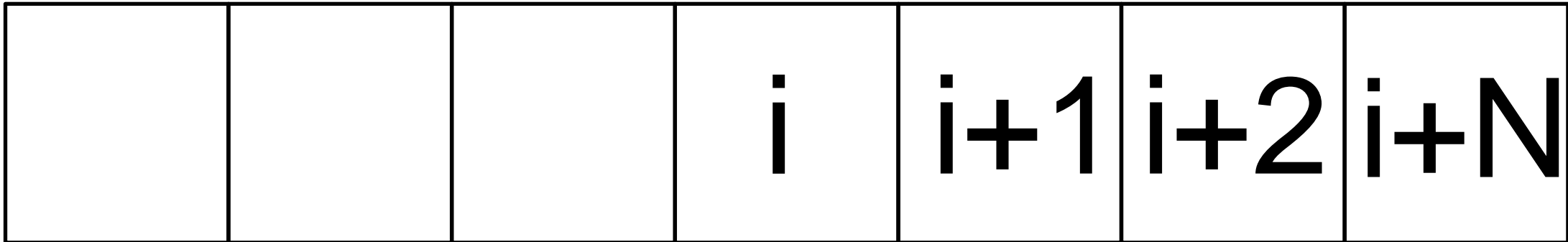
● F IS TRUE

❖ $\text{Un0p} ::= \text{always} \mid \text{after} \mid \text{eventually}$

❖ $\text{Bin0p} ::= \text{until} \mid \text{releases} \mid ;$

ALWAYS F = true in i iff $F = \text{true}$ in $k \geq i$
for each state k

ALWAYS F



ALWAYS

```
fact NoResurrection {  
  always (all p:Person |  
    p.liveness = Dead  
    implies always  
    p.liveness = Dead)}
```


After

Syntax: $\text{Exp} ::= \underline{\text{un0p}} \text{ expr} \mid \text{expr} \underline{\text{bin0p}} \text{ expr}$

● F IS TRUE

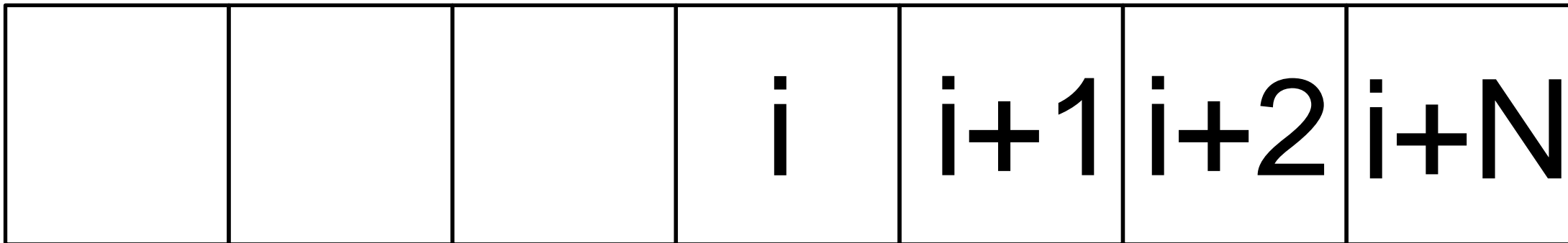
❖ $\text{Un0p} ::= \text{always} \mid \text{after} \mid \text{eventually}$

❖ $\text{Bin0p} ::= \text{until} \mid \text{releases} \mid ;$

AFTER F = true in i iff $F = \text{true}$ in $i+1$

AFTER $S = S'$

AFTER F



AFTER

```
assert NoResurrection {  
  always (all p:Person |  
    p.liveness = Dead  
    implies after  
    p.liveness = Dead)}
```

Syntax: $\text{Exp} ::= \underline{\text{un0p}} \text{ expr} \mid \text{expr} \underline{\text{bin0p}} \text{ expr}$

● F IS TRUE

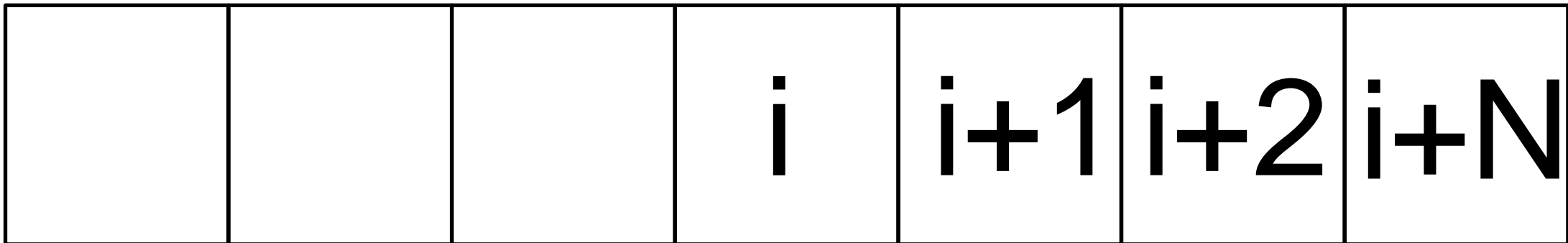
❖ $\text{Un0p} ::= \text{always} \mid \text{after} \mid \text{eventually}$

❖ $\text{Bin0p} ::= \text{until} \mid \text{releases} \mid ;$

EVENTUALLY F = true in i iff $F = \text{true}$ in $k \geq i$
for some state k

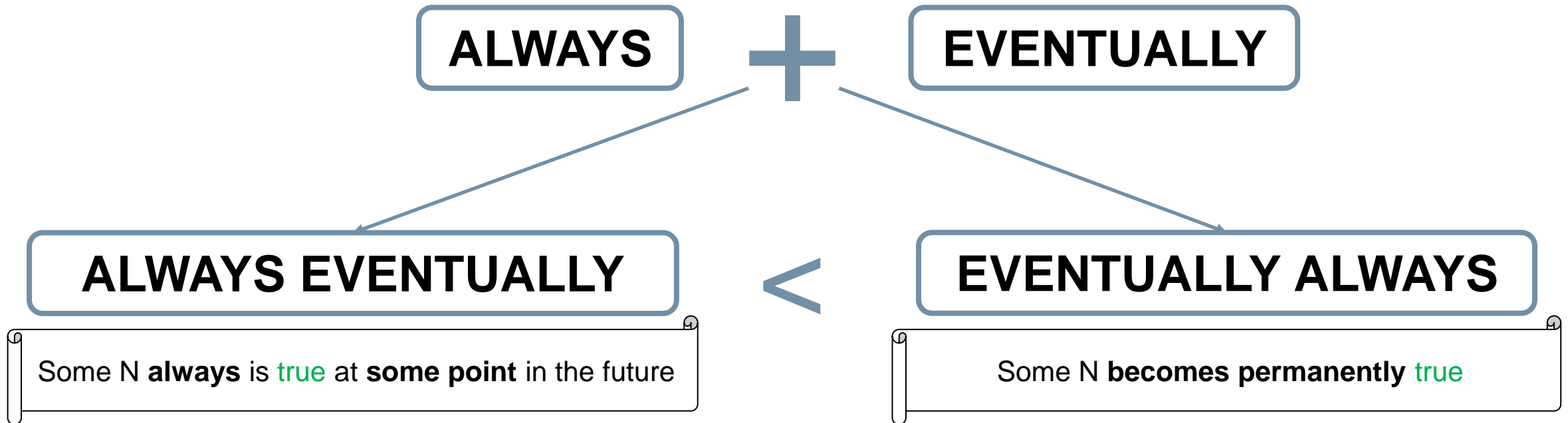
LIVENESS PROPERTY

EVENTUALLY F



EVENTUALLY

```
fact noImmortality {  
  always (all p:Person |  
    p.liveness = Alive implies  
    after (eventually  
      p.liveness = Dead))}  
}
```



ALWAYS
+
EVENTUALLY

```
pred Mortality1 {  
  all p:Person |  
  always eventually  
  p.liveness = Dead  
}  
  
pred Mortality2 {  
  all p:Person |  
  eventually always  
  p.liveness = Dead  
}
```


Historically

Syntax: $\text{Exp} ::= \underline{\text{un0p}} \text{ expr} \mid \text{expr} \underline{\text{bin0p}} \text{ expr}$

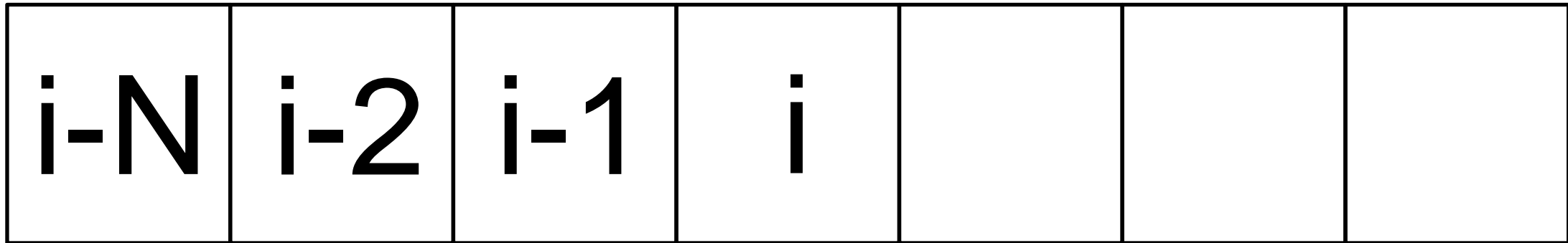
● F IS TRUE

❖ $\text{Un0p} ::= \text{historically} \mid \text{once} \mid \text{before}$

❖ $\text{Bin0p} ::= \text{since} \mid \text{triggered}$

HISTORICALLY F = true in i iff $F = \text{true}$ in $k \leq i$
for each state k

HISTORICALLY F



HISTORICALLY

```
fact NoDeadThenAlive {  
  always (all p:Person |  
    p.liveness = Alive  
    implies historically  
    p.liveness = Alive)}
```

Once

Syntax: $\text{Exp} ::= \underline{\text{un0p}} \text{ expr} \mid \text{expr} \underline{\text{bin0p}} \text{ expr}$

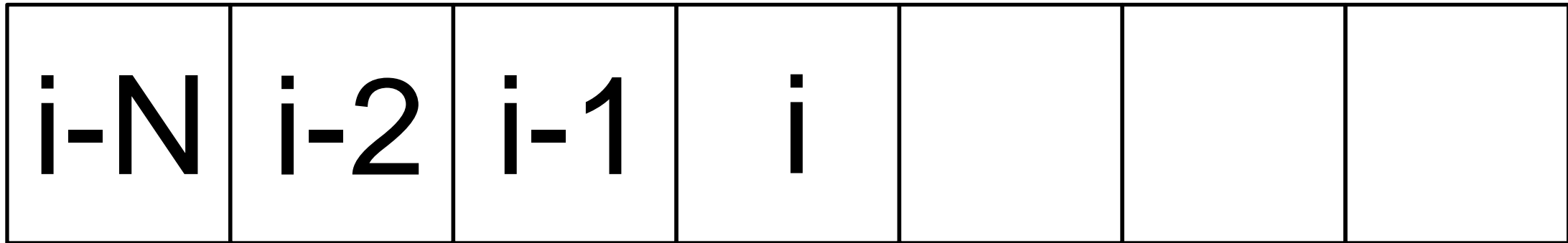
● F IS TRUE

❖ $\text{Un0p} ::= \text{historically} \mid \text{once} \mid \text{before}$

❖ $\text{Bin0p} ::= \text{since} \mid \text{triggered}$

ONCE $F = \text{true}$ in i iff $F = \text{true}$ in $k \leq i$
for some state k

ONCE F



ONCE

```
fact DeadSinceDeath {  
  always (all p:Person |  
    p.liveness = Dead implies  
    once Die [p]))}
```

Before

Syntax: $\text{Exp} ::= \underline{\text{un0p}} \text{ expr} \mid \text{expr} \underline{\text{bin0p}} \text{ expr}$

● F IS TRUE

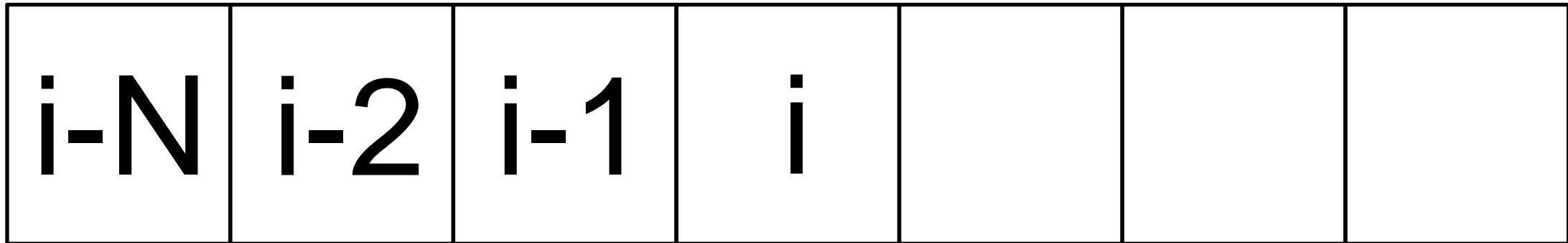
❖ $\text{Un0p} ::= \text{historically} \mid \text{once} \mid \text{before}$

❖ $\text{Bin0p} ::= \text{since} \mid \text{triggered}$

BEFORE $F = \text{true}$ in i iff $F = \text{true}$ in $i-1$
for $i > 0$

BEFORE F

FALSE IN STATE 0



BEFORE

```
assert IfAliveBeforeAlive {  
  after  
  (always (all p:Person |  
    p.liveness = Alive implies  
    before p.liveness = Alive)  
  )}
```


Until

Syntax: $\text{Exp} ::= \underline{\text{un0p}} \text{ expr} \mid \text{expr} \underline{\text{bin0p}} \text{ expr}$

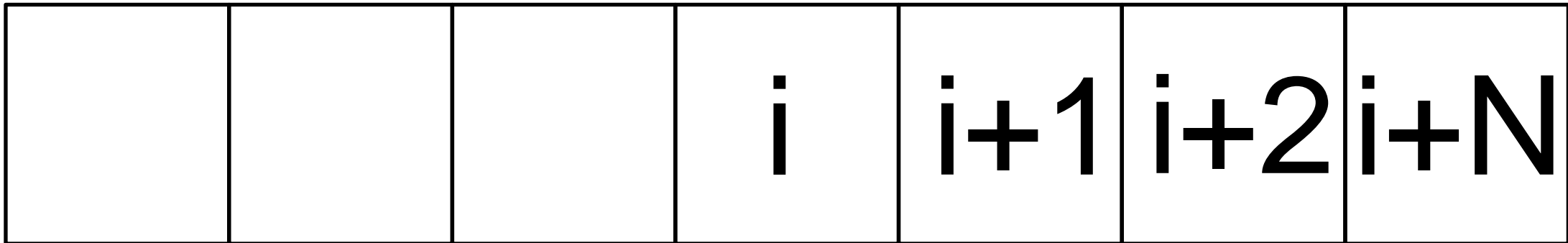
● F IS TRUE
● G IS TRUE

❖ $\text{Un0p} ::= \text{always} \mid \text{after} \mid \text{eventually}$

❖ $\text{Bin0p} ::= \text{until} \mid \text{releases} \mid ;$

F UNTIL G = true in i iff $G = \text{true}$ in $j \geq i$ && $F = \text{true}$ in $k: i \leq k < j$
for some state j , for each state k

F UNTIL G



UNTIL

```
fact AliveUntilDeath {  
  always (all p:Person |  
    p.liveness = Alive implies  
    (p.liveness=Alive until  
    p.liveness = Dead))}
```

TEMPORAL CONNECTIVES

Releases

22

Syntax: $\text{Exp} ::= \underline{\text{un0p}} \text{ expr} \mid \text{expr} \underline{\text{bin0p}} \text{ expr}$

❖ $\text{Un0p} ::= \text{always} \mid \text{after} \mid \text{eventually}$

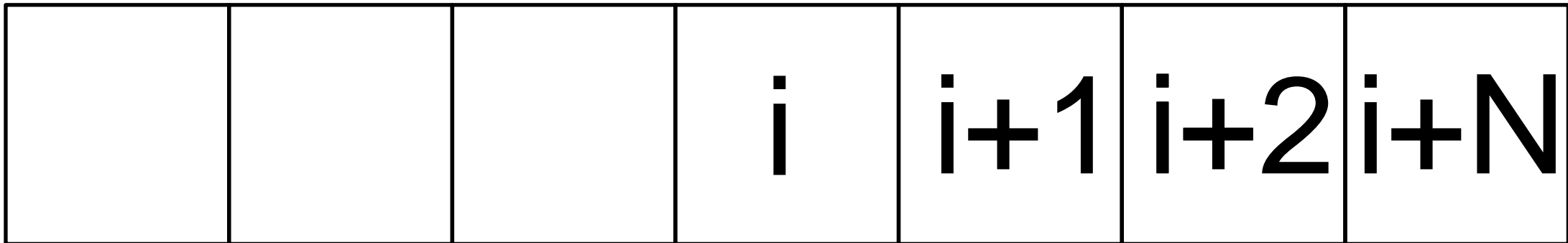
❖ $\text{Bin0p} ::= \text{until} \mid \text{releases} \mid ;$

● F IS TRUE

● G IS TRUE

F RELEASES G = true in i iff $F = \text{true}$ in k && $G = \text{true}$ in j : $i \leq j \leq k$
for some state k , for each state j
OR
no such k && $G = \text{true}$ in $j \geq i$ for each j

F RELEASES G



RELEASES

```
assert AliveUntilDeath2 {  
  always (all p:Person |  
    p.liveness = Alive implies  
    (Die[p] releases  
    p.liveness = Alive))}
```

Triggered

Syntax: $\text{Exp} ::= \underline{\text{un0p}} \text{ expr} \mid \text{expr} \underline{\text{bin0p}} \text{ expr}$

❖ $\text{Un0p} ::= \text{historically} \mid \text{once} \mid \text{before}$

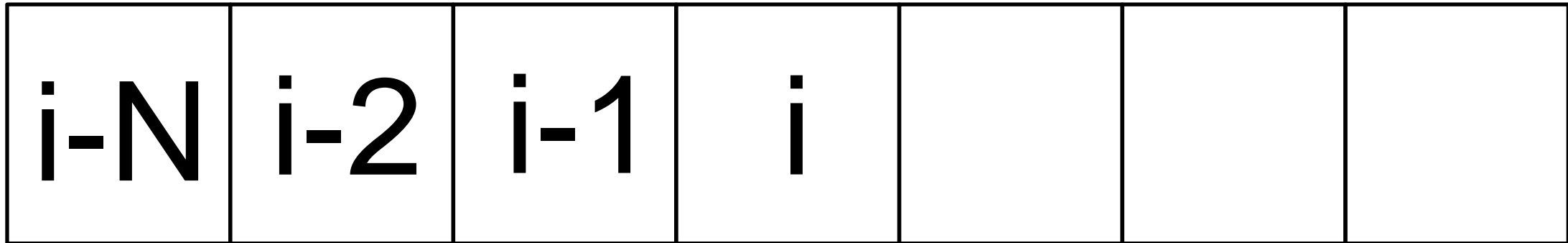
❖ $\text{Bin0p} ::= \text{since} \mid \text{triggered}$

● F IS TRUE

● G IS TRUE

F TRIGGERED G = true in i iff $F = \text{true}$ in $j \leq i$ && $G = \text{true}$ in $k: j < k \leq i$
for some state j , for each state k
OR
 $F = \text{false}$ in $j \leq i$ && $G = \text{true}$ in $k \leq j$
for each j , for each k

F TRIGGERED G



TRIGGERED

```
assert DeadSinceDeath2 {  
  always (all p: Person |  
    p.liveness = Dead implies  
    (Die[p] triggered  
    p.liveness = Dead))}
```


Since

Syntax: $\text{Exp} ::= \underline{\text{un0p}} \text{ expr} \mid \text{expr} \underline{\text{bin0p}} \text{ expr}$

❖ $\text{Un0p} ::= \text{historically} \mid \text{once} \mid \text{before}$

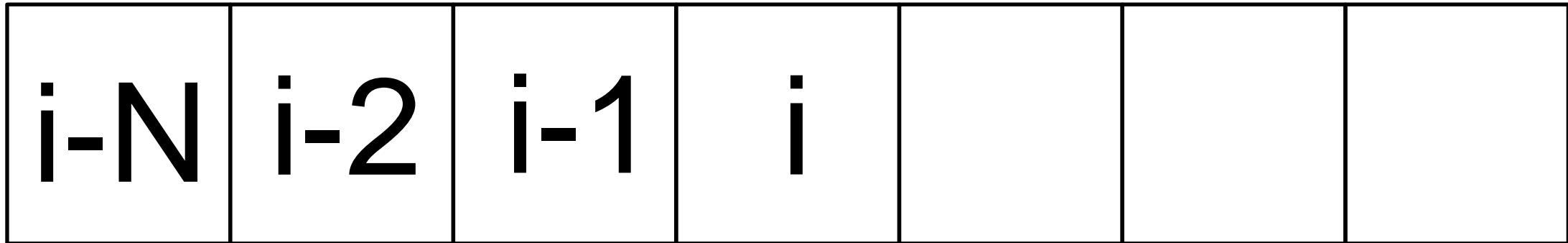
❖ $\text{Bin0p} ::= \text{since} \mid \text{triggered}$

● F IS TRUE

● G IS TRUE

$\underline{\text{F ONCE G}}$ = true in i iff $G = \text{true}$ in $j \leq i$ && $F = \text{true}$ in $k: j < k \leq i$
for some state j , for each state k

F SINCE G



SINCE

```
assert DeadSinceDeath2 {  
  always (all p: Person |  
    p.liveness = Dead implies  
    (p.liveness=Dead since  
    Die[p]))}
```

Syntax: $\text{Exp} ::= \underline{\text{un0p}} \text{ expr} \mid \text{expr} \underline{\text{bin0p}} \text{ expr}$

❖ $\text{Un0p} ::= \text{always} \mid \text{after} \mid \text{eventually}$

❖ $\text{Bin0p} ::= \text{until} \mid \text{releases} \mid ;$

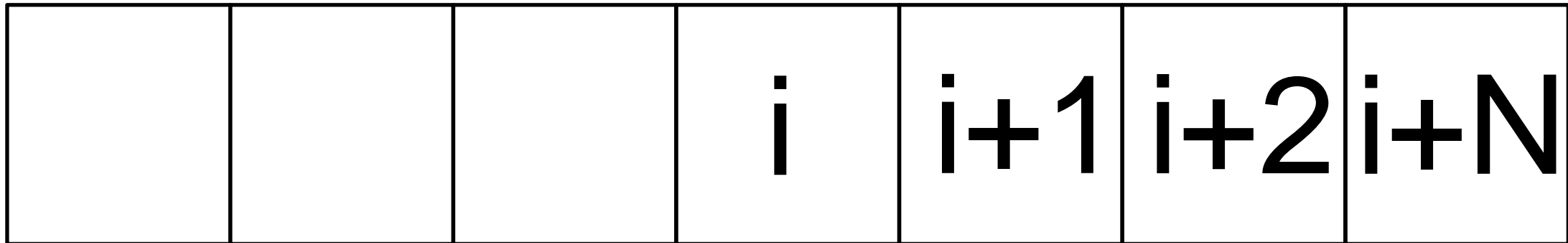
● F IS TRUE

● G IS TRUE

$\underline{F ; G} = \text{true in } i \text{ iff } F \text{ true in } i \ \&\& \ G = \text{true in } i+1$

$;$ = AND AFTER

F ; G



;

```
run{#Person = 4 and  
some p: Person |  
(p.liveness = Alive ;  
p.liveness = Alive ;  
p.liveness = Dead)}  
for 5
```

-
;