MACROECNOMICS PROJECT

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1 Introduction

When talk about macroeconomics variables we should understand how they are created, what do they represent and why they are not linear as we could expect. All these questions can be, in part, answered by the so called "Business Cycles".

The business cycle is the natural rise and fall of economic growth that occurs over time. A series of positive deviations from trend (culminating in a peak) represents a boom, while a series of negative deviations from trend (culminating in a trough) represents a recession.
This is in brief what I am going to study in this short educational project: a real world example of how can we study a macroeconomic aggregate time series.

2 State of Art

Before to start with the real project, it is important to introduce the theoretical concept that stand behind the Business Cycle.
First of all, to measure business cycle we have to break down a time series into a growth or **trend** component and a **cyclical** component:

$$x_t = \tau_t^x + \nu_t^x$$

The idea is the following: the trend in a macroeconomic time series is determined by factors that influence the speed of long-run growth but not cyclical fluctuations.
Due to the fact that the trend isn't always a simple linear trend that could be find with a linear regression of the output variable on the time variable, there will be presented some filtering methods aiming to detrend data.

2.1 Hodrick - Prescott (HP) filter

The HP filter constructs the trend that minimizes loss function:

$$\min_{\tau_t^x} \sum_{t=1}^{T} \underbrace{(x_t - \tau_t^x)^2}_{\text{size of the cycle}} + \lambda \sum_{t=2}^{T-1} \underbrace{[(\tau_{t+1}^x - \tau_t^x) - (\tau_t^x - \tau_{t-1}^x)]^2}_{\text{smothness of the trend}}$$

The first term puts a penalty on the distance of τ_t^x from x_t, while the second term on the changes in the growth rates of τ_t^x.

- if $\lambda = 0$ then $\tau_t^x = x_t \Rightarrow \nu_t^x = 0$
- if $\lambda \to \infty$ then $\tau_t^x \to \delta_0 + \delta_1 t$ (linear trend)

2.2 Baxter-King Filter (BK)

The Baxter-King band pass filter is a method of smoothing a time series, that is a modification of the Hodrick-Prescott filter that provides wider opportunities for removing cycle component from a time series.

The filter procedure consists of singling out the repeated component of a time series by setting the width for oscillations of periodic component. The Baxter-King filter is a band pass filter that removes the cycle component S from the time series Y based on weighted moving average with specified weights. To calculate weights, the user should set cutoff frequencies that describe permissible non-seasonal oscillations of the smoothed series.

Let w_u, w_l are the upper and the lower limits of the cutoff frequency. Then the B_j weights for the specified lag/lead K are calculated by the following formulas:

$$w_1 = \frac{2\pi}{w_u};\ w_2 = \frac{2\pi}{w_l}$$

$$b_0 = \frac{w_2 - w_1}{\pi};\ b_j = \frac{\sin(w_2 j) - \sin(w_1 j)}{j\pi}, j \geq 1$$

$$\theta = b_0 + \frac{\sum_{j=1}^{K} b_j}{2K+1}$$

$$B_j = b_j + \theta,\ j = 0, ..., K$$

Seasonal (cycle) component of the source series is calculated by the formula:

$$S_t = Y_t B_0 + \sum_{j=1}^{K} Y_{t-j} B_j + \sum_{j=1}^{K} Y_{t+j} + B_j$$

where:

- B_j: weight value that corresponds to value of the source series Y, positioned at the distance j from the current element.
- K: sets lead or lag value, with which the moving average is calculated.

The result of smoothing is the source series with removed seasonal (cycle) component.

2.3 Christiano-Fitzgerald Filter (CF)

The Christiano-Fitzgerald random walk filter is a band pass filter that was built on the same principles as the Baxter and King (BK) filter.It uses the whole time series for the calculation of each filtered data point. The advantage of the CF filter is that it is designed to work well on a larger class of time series than the BK filter, converges in the long run to the optimal filter, and in real time applications outperforms the BK filter.

The CF filter has a steep frequency response function at the boundaries of the filter band (i.e. low leakage); it is an asymmetric filter that converges in the long run to the optimal filter. It can be calculated as follows:

$$c_t = B_0 y_t + B_1 y_{t+1} + \cdots + B_{T-1-t} y_{T-1} + \tilde{B}_{T-t} y_T + B_1 y_{t-1} + \cdots + B_{t-2} y_2 + \tilde{B}_{t-1} y_1$$

where

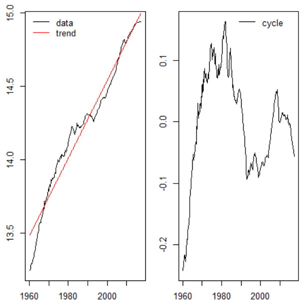
$$B_j = \frac{\sin(jb) - \sin(ja)}{\pi j},\ j \geq 1,\ \text{and}\ B_0 = \frac{b-1}{\pi},\ a = \frac{2\pi}{p_u},\ b = \frac{2\pi}{p_l}$$

$$\tilde{B}_k = -\frac{1}{2} B_0 - \sum_{j=1}^{k-1} B_j$$

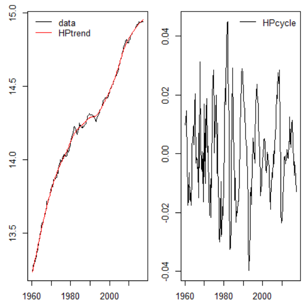
The parameters p_u and p_l are the cut-off cycle length in month. Cycles longer than p_l and shorter than p_u are preserved in the cyclical term c_t.

3 Application

The time series regards the South African GDP from 1960 to 2017.
I used quarterly logarithmic data taken from a library open source in GitHub. My project follow the State of Art of previous chapter with the aim of comparing different type of cyclical component extracted from the "complete" time series (Trend + Cycle) with different filtering methods.
I start with the most common linear regression and I continue with the filtering methods mentioned before. Finally, I compared all the cycle result obtained with the different 'detrend' methods.



(a) Linear regression



(b) HP filtering method

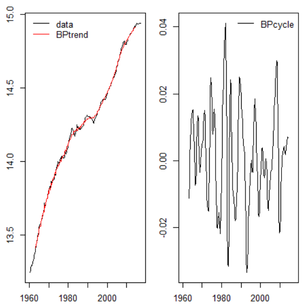


Figura 2: BK Filtering method

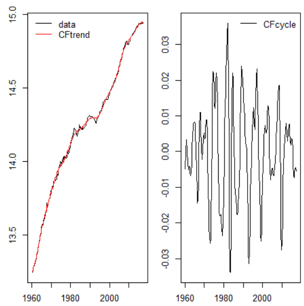


Figura 3: CF filtering method

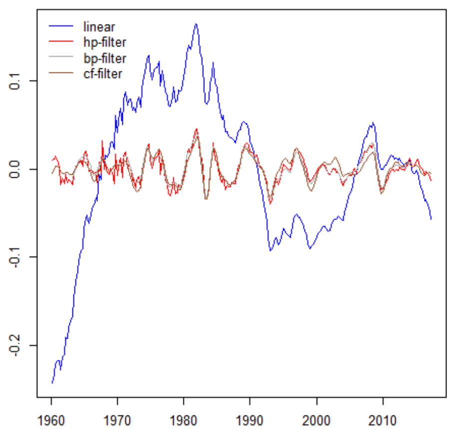


Figura 4: Comparison of all filtering methods

As we can see from the last plot, the linear regression method is the worst one and also the most simple. Whether in the linear regression we can not specify many parameters in order have an accurate result, in the 3 others filtering methods the choose of the parameters is a key factor if we want to break down correctly the series in trend and cyclical components.