Chiral Quantum Walks for Network Tasks

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May 2024

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1 Introduction

[1, 2] has been introduced as the quantum analog of classical random walks (CRW). In general QW are a powerful tools to describe the dynamics of a quantum excitation over a network, however, due to historical reasons, for many years only dynamics governed by real Hamiltonians have been considered.
It is only recently that the possibility of introducing complex phases have been explored, leading to the so called chiral quantum walks (CQW) [3, 4, 5]. Chiral Hamiltonians have revealed of wide interest for network tasks both because time reversal is broken and because they add more degree of freedom to the dynamics, as will be extensively shown in the following sections. Other than CQW the idea of complex weighted networks is arousing interest in many other fields [6].

In this work we explored the potential of chirality in relation to the problem of routing of information. Indeed, as will be extensively shown in Section 2, phases can break spatial symmetries allowing directional transport [7, 8]. In a recent work, Bottarelli et al. [9] developed a 3 outputs quantum router, able to transfer a quantum excitation to a designated node only changing the phase applied to the only internal cycle. It is worth it to mention that many works about chiral directional transport appeared recently. Particularly, Cavazzoni et al. [10], developed a topology which allows perfect routing of both excitations and superpositions only modifiyng the chiral phases. However the topology described in [10] could be difficult to implement, consequently we retained useful to attack the more general problem of a random topology. Indeed, in this work we developed a method to optimize the phases necessary to route excitations and superpositions between nodes of a network with any given topology.

This paper is structured as follow. In Section 2 we will provide a brief description of the mathematical tools commonly employed to describe the dynamics of QW and CQW. In Section 3 we will address a strategy to exploit chirality for the problem of directional state transfer. We will show that in most network topologies the routing of both localized excitations and quantum superpositions is possible only optimizing the chiral phases. Finally, in Section 4, we conclude the paper with some observations and remarks.

2 Quantum Walks

In this section we will provide a short description of the QW dynamics considering also the possibility of using generalized complex Hamiltonians. For a more extensive review we refer to [1] for QW and [3] for CQW.

Let us consider a graph $\mathcal{G}(V, E)$ where V is a set of vertexes and E is a set of edges, whose elements are pairs of elements of V. We assume the graph to be undirected, meaning that each element of E is unordered. Then we define the so called adjacency matrix matrix A as :

$$A = \begin{cases} 1 \text{ if } (i,j) \in E \\ 0 \text{ if } (i,j) \notin E \end{cases} \tag{1}$$

In general the entries of A can be different from only 0 or 1 but for our purposes we will consider unweighted simple graph. Defining then the degree matrix D as:

$$D = \begin{cases} k_i \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases} \tag{2}$$

where k_i is the degree of the vertex i, i.e. the number of elements of E containing i. It is now possible to define the Laplacian matrix $L = D - A$. As anticipated before QW are based on CRW, and the equation governing the dynamics of a CRW on a graph is commonly described as:

$$\frac{d\vec{p}_j(t)}{dt} = L\vec{p}_j(t) \tag{3}$$

where the i-th element $p^i_j(t)$ of the vector $\vec{p}_j(t)$ represent the probability of finding the particle at the time t in the vertex i, starting the dynamics at $t = 0$ with the particle in the vertex j. In the same way we can define :

$$\frac{d}{dt}|\psi\rangle = H|\psi\rangle \tag{4}$$

where $|\psi\rangle$ is a quantum state and H is considered to be either the Adjacency or the Laplacian matrix. For motivations that will be explained later in this work we will focus on the dynamics described by the Adjacency matrix. Assuming to start in a localized state $|j\rangle$, through Born rule, it is possible to assign a probability to the possibility of measuring the state in the site i at the time t:

$$p^i_j(t) = |\langle i|e^{-iHt}|j\rangle|^2 \tag{5}$$

the dynamics is clearly different from whose described in in Eq. (3). The main differences are two: the first one is that the whole evolution of a CRW is clearly a stochastic process while, regarding the QW, the dynamics is deterministic and stochasticity is only introducted by the measurement process. The second is that clearly the QW opens the way to quantum interference in the sense that the state gather a complex phase during the evolution which can lead to interference between different paths, as can be seen from Eq. (5).

2.1 Chiral Quantum Walks

As anticipated before, due to historical reasons, most of the works regarding quantum walks explored only the dynamic governed by real Hamiltonians. Clearly this is not the most general dynamics in the QW scenario, indeed it has been recently introduced the possibility of adding phases to the edges, namely, assuming $H_0 = A$:

$$H_{ij} = A_{ij}e^{i\phi_{ij}} \tag{6}$$

with $\phi_{ji} = -\phi_{ij}$ to preserve hermiticity. It is trivial to notice that some gauge symmetries are present which will reduce the number of physical phases [8, 11]. In particular the total number of non trivial phases will be found to be equal to the number of independent cycles. Regarding planar graphs this is easily identified as:

$$F = E - N + 1 \tag{7}$$

which is the Euler's characteristic of the graph. This generalization is particularly interesting, indeed for a real Hamiltonian we have:

$$p^i_j(t) = p^j_i(t) \tag{8}$$

which forbids directional transport. However this symmetry is broken by chiral Hamiltonians, opening the possibility to new applications.

3 Routing of information over a network via chiral quantum walks

A crucial task for both classical and quantum networks is the routing of information [12, 13, 14, 15]. However, the standard approach to quantum walks does not allow directional transport depending on any properties of the system because of the symmetries induced from the fact that the Hamiltonian is real, as can be seen from Eq. (8). These symmetries can be broken considering all possible complex Hamiltonians, that, in principle, can be simulated over a given network. Indeed, the problem has been attacked exploiting CQW [9, 10]. Recent works focus on the development of optimal quantum routers [10], which are the primary object for directional transport, particularly the 3-outputs router has been charcterized by Bottarelli et al [9], its topology is shown in Fig. 1, from which can be seen that there is only one free phase.

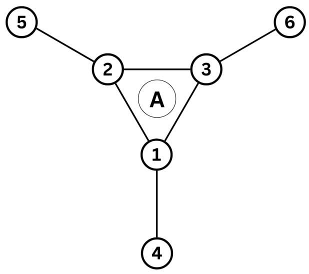


Figure 1: 3-outputs router topology, the free phase is identified with A

Bottarelli et al. showed that high fidelity routing of a single excitation between two external sites is possible setting the phase parameter $\theta = \pm\frac{\pi}{2}$ depending on the target site.

In this work we will focus on the development of optimal many-outputs quantum routers in order to perform optimal directional transport over a random network. Recent works have pointed out that, in general, cycles formed by an even number of vertexes do not provide quantum advantage when coupled with a phase [16], due to symmetry reasons. Moreover K_3 has revealed to be the more versatile cycle for chiral directional transport, this, combined with the hypothesis of unweighted networks, brought us to focus on composition of equilateral triangles in order to develop optimal many-outputs routers.

The complexity of the task relies in the fact that parameter space becomes rapidly huge, growing as the number of phases. To address this issue we chose to employ a Genetic Algorithm(GA). GAs are a class of optimization algorithms based on biological evolution where a population of sets of parameters, called individuals, is randomly generated and then evoveld creating fitter generations through genetic functions each iteration, the output will be the optimal individual

[17]. We focused on this method other than others optimization tecniques because GAs are known to be quite robust i.e. do not get easily stucked in local minima, which is basically is the reason why humanity will extinct quite soon. Indeed, the signature of quantum chaos have been individuated in phase dependence [16].

3.1 Genetic Algorithm

GAs have been already employed in various quantum tasks such as identification of a network topology [18], design of quantum experiments[19] and adaptive phase estimation [20], both in the continuous- and discrete-valued parameters scenarios. The main common steps of all GAs is to generate a random population which elements, called individuals, are sets of parameters to be estimated; the goodness of an individual is then determined through a fitness score. The initial population is evolved through genetic functions in order to obtain a new generation, performing the procedure many times GAs will converge to optimal solution.

3.1.1 Design of optimal phases

As anticipated before, our problem is the design of optimal phases, consequently, individuals will be constituted of set of phases whose cardinality will be the number of independent loops in the graph. An individual can be thought as a vector $\varphi \in (-\pi, \pi]^N$ where N is the number of free phases plus one time parameter which needs optimization too. The time taken by the transport to occur is an important parameter because of decoherence, which is our worst enemy in this framework. In order to address this problem we considered a characteristic time τ, defined as:

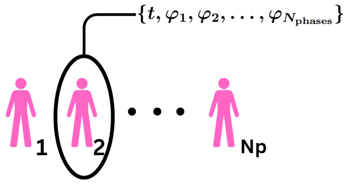
$$\tau = \frac{1}{E_1 - E_0}\,, \tag{9}$$

where E_1 is the first excited state and E_0 is the ground state. The choice of Eq. (9) is meaningful since τ governs the timescale of decoherence.

3.1.2 Genetic functions

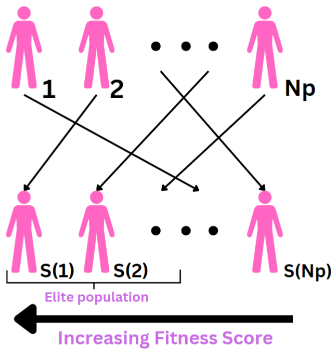
The population described in the previous section is refined through generations applying genetic functions, which will be described in this section.

Random generation
Initial population is chosen from a uniform distribution over an interval. Regarding phases, they are picked in the interval $(-\pi, \pi]$, while for time, usually we chose it in $[0, 2d\tau]$ where d is the length of the shortest path between the start and target vertexes. We briefly mention that, if a solution is needed around a specific time $\bar{t}$, it is possible to initialize the time parameter picking it on a normal distribution peaked on $\bar{t}$



Fitness evaluation

During this step, firstly, the fitness score of the individuals is evaluated.Then the parent population are sorted by its fitness score and the $P_e \times N_p$ fittest will be automatically selected to survive the next generation that we will call child population.



The choice of the fitness score depended on the task. Indeed regarding the routing of single excitations e.g. $|\mathbf{I}\rangle \to |\mathbf{F}\rangle$ was chosen to be:

$$\mathcal{F} = P_{|\mathbf{I}\rangle \to |\mathbf{F}\rangle} + \alpha \sum_{j \in \textbf{int nodes}} P_{|\mathbf{I}\rangle \to |j\rangle} \,, \tag{10}$$

where the first part is the probability of routing correctly the excitation. The second part of Eq (10), optimized with a weight α, reflects the fact that we prefer the excitation to be stucked inside the graph than being routed to another external node seen as a potential output.

Regarding the problem of routing superpositions of the form $|\mathbf{I}_1\rangle + e^{i\chi}|\mathbf{I}_2\rangle \to |\mathbf{F}_1\rangle + e^{i\chi}|\mathbf{F}_2\rangle$, a similar fitness score has been chosen as:

$$\mathcal{F} = \sum_{i=0}^{N_{ph}} \left[P^{\chi_i}_{|\mathbf{I}_1\rangle|\mathbf{I}_2\rangle \to |\mathbf{F}_1\rangle|\mathbf{F}_2\rangle} + \alpha \sum_{j \in \textbf{int nodes}} P^{\chi_i}_{|\mathbf{I}_1\rangle \to |j\rangle} \right] , \tag{11}$$

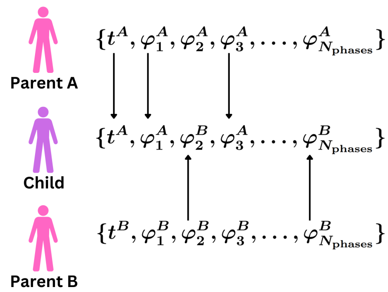
where $\chi_j = -\pi + j\frac{2\pi}{N_{ph}}$ and N_{ph} is the number of superposition phases using which the optimization is performed.

The reason for the choice of this fitness score Eq (11) is to maximize the integral over the superposition phase χ, since we want the process to be maximally independent on χ. In other words we want to be able to encode some information in the state and being able to route it independently on the information itself.

Tournament and crossover

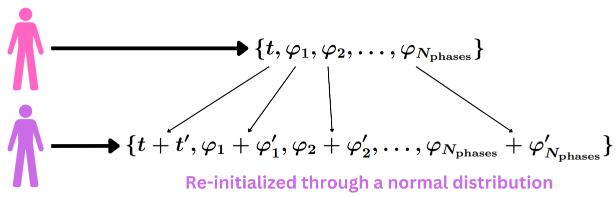
The remaining $(1 - P_e) \times N_p$ child individuals will be generated through crossover selection. In practice we randomly select two groups k individuals from the parent population . Each group compete in a tournament, meaning that they are sorted by fitness score. Then the two fittest individuals resulting from the two different tournament will be crossed over to generate an individual of the child population.

After trying various possibilities we chose to apply the uniform crossover. Namely, we pick a random number between 0 and 1 N times and each time if the result is 0 we set the child chromosome with the first parent, if it is 1 we set with the second.



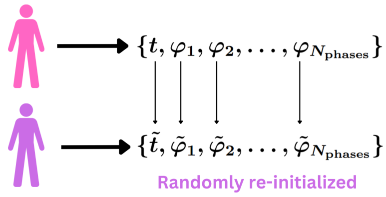
Mutation

This step is necessary to preserve genetic diversity and will only interest the last $(1 - P_e) \times N_p$ individuals of child population. Each of these individuals will mutate with probability $(1 - \mathcal{F}) \times P_m$. The new value of each chromosome will be chosen from a gaussian distribution centred in the old value with variance $\sigma = (1 - \mathcal{F}) \times \sigma_0$.



Infection

The last step is the infection and will only interest the last $N_p \times Pc_i$ individuals of the child population. Each individual will be infected with probability P_i. Meaning that its chromosomes will be reinitialized randomly. To do that we choose the method of initialization picked in the first step. Then the child population will become the parent population of a new generation an the algorithm will start again from the fitness evaluation. The maximum number of generation correspond to the maximum number of iterations of the algorithm is M_{it}



N_p	2000-5000
k	20
P_e	0.05
P_m	0.5
σ_0	$\frac{\pi}{5}, \frac{d\tau}{10}$
α	0.05-0.1
Pc_i	0.1
P_i	0.3
M_{it}	30

Table 1: Parameters employed in the GA.

3.2 Routing of Classical Information

In this section we focus on the task of routing classical information. Namely routing of excitations of the form: $|i\rangle \to |j\rangle$. This transmission is addressed as classical since the information is encoded in a discrete set and not in a continuum. In this framework of routing single excitations we set a threshold of 0.5 for the transition amplitude to define a transport event. Moreover we decide to normalize the evolution over the characteristic time τ Eq. (9).

3.2.1 Design of Optimal Chiral Quantum Routers

Regarding the 4-outputs switch two topologies have been considered Fig. 2.

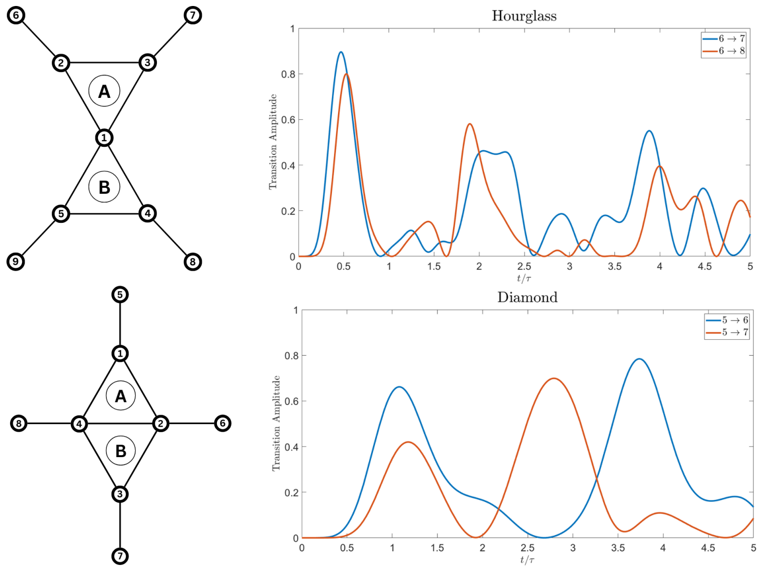


Figure 2: On the left we have the two considered topologies to design the optimal 4 outputs router, the hourglass on top and the diamond on the bottom. On the right it is plotted the probability of routing excitatations between different pairs of vertexes. The plot of transition amplitude is made with respect to time t normalized with the characteristic time τ.

Both have two free phases, and the Hourglass topology has been found to be optimal in all conditions as can be easily seen from Fig. 2 providing transport between external vertices with fidelity always higher than 0.8.

However it is important to point out that optimal phases are not always those one would expect from the fact that basically the topology is a composition of two 3-switches. Indeed often they are different from the values of $\pm\frac{\pi}{2}$.

For the 5-outputs scenario, only one topology Fig. 3 was considered with 3 free phases, providing good results for every couple of external vertexes and obtaining maximums before 2τ.

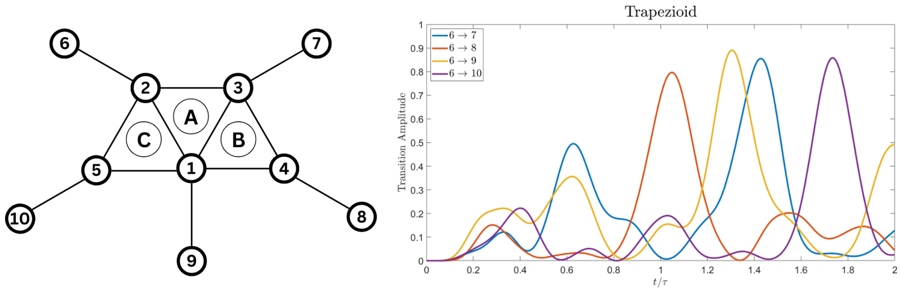


Figure 3: On the left we have the only considered topology to design the optimal 5 outputs router, the trapezoid. On the right it is plotted the probability of routing excitatations between different pairs of vertexes. The plot of transition amplitude is made with respect to time t normalized with the characteristic time τ.

Regarding the 6-outputs router, two topology have been considered Fig. 4. Respectively with four and six free phase parameters.

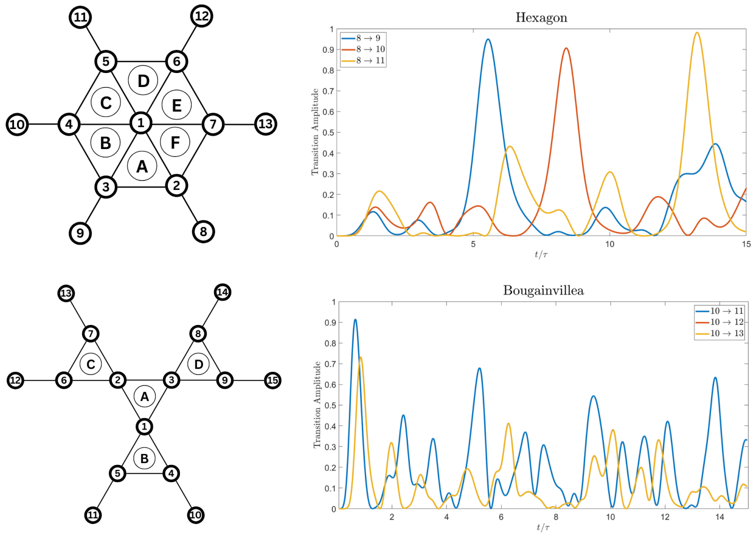


Figure 4: On the left we have the two considered topologies to design the optimal 6 outputs router, the hexagon on top and the bougainvillea on the bottom. On the right it is plotted the probability of routing excitatations between different pairs of vertexes. The plot of transition amplitude is made with respect to time t normalized with the characteristic time τ.

A and B topologies offer different performances the first is optimal in term of fidelity since propagating an excitation with the phases found to be optimal, measuring an external vertex one will find the excitation with probability around 0.9 at a given time as can be observed from Fig. 4. On the other hand topology B is not performing as the first in terms of fidelity but the propagation time is significantly reduced.

The last router we purpose is the triangular lattice topology(TLT) (with and without handles) which can be thought as a N-outputs quantum router. As can be seen from Fig. 5 genetic technique provide pretty good results in the non handled scenario. However the transition amplitude drops when handles are added.

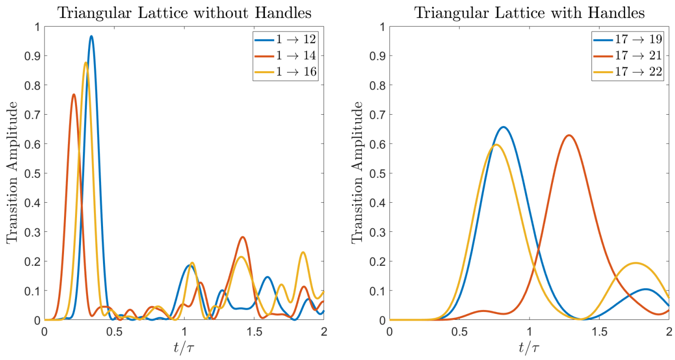


Figure 5: Plot of the probability of routing excitatations between different pairs of vertexes for a triangular lattice of 18 cells. On the left we considered the topology with no handles while on the right we considered the same topology but adding handles to the external vertexes. The plot of transition amplitude is made with respect to time t normalized with the characteristic time τ.

It could be considered a topology where only necessary handles are added to design N-routers.

3.2.2 Routing of Information Over a Random Network

Now that we have characterized a considerable amount of router it is the moment to build a network with those! Firstly, we considered a random network constituted by a fungus mycelium [21], showed in Fig. 6 and adapted to our purpose, substituting routers to the nodes with degree higher than 2 and trying to keep a sort of geometric distance segmenting long paths. As can be seen from Fig. 6 GA provides pretty good results exceeding in every scenario the threshold of 0.5.

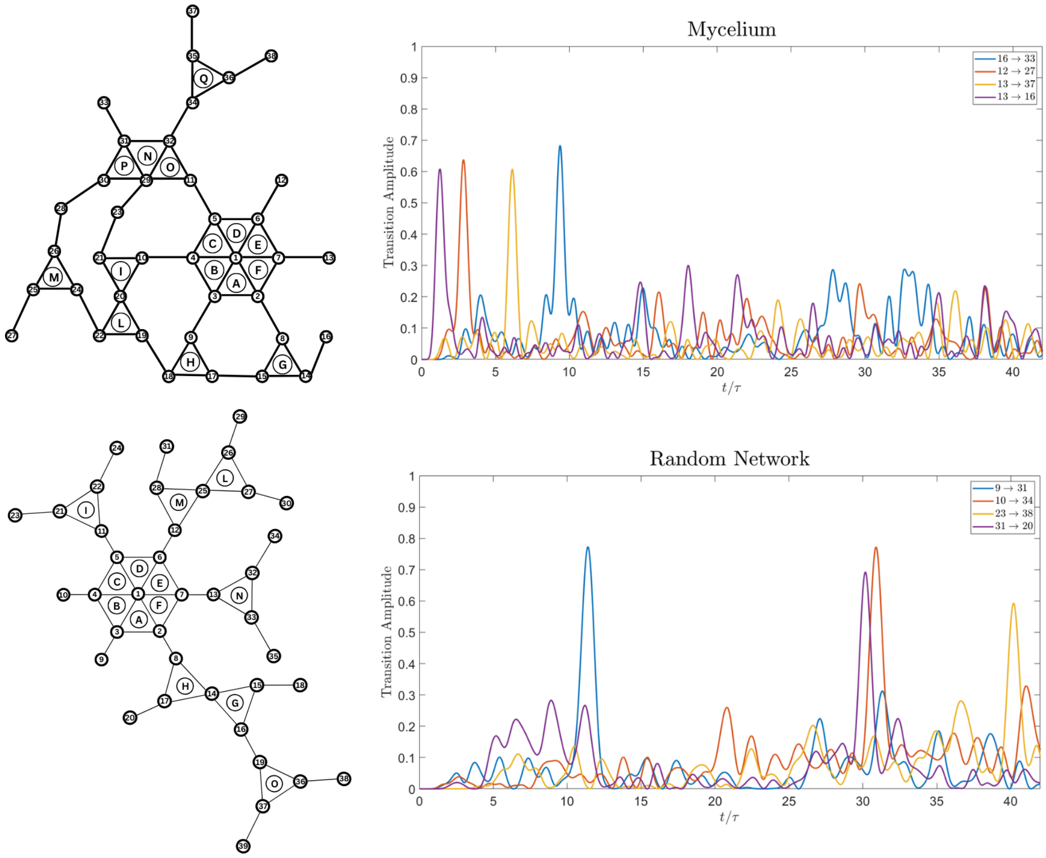


Figure 6: On the left we have the two considered topologies, the mycelium (top) and the random network (bottom). On the right it is plotted the probability of routing excitatations between different pairs of vertexes. The plot of transition amplitude is made with respect to time t normalized with the characteristic time τ.

Secondly we considered a more controlled structure avoiding loops outside the routers, always based on a mycelium Fig. 6. As can be observed from Fig. 6 fidelity is always around 0.7 except for the most distant nodes.

3.2.3 Further Analysis

As anticipated before, when routing a single excitation, is desirable to have the excitation stucked internally in the graph rather than routed to a different output. This is the reason behind the form of the fitness score Eq. (10).

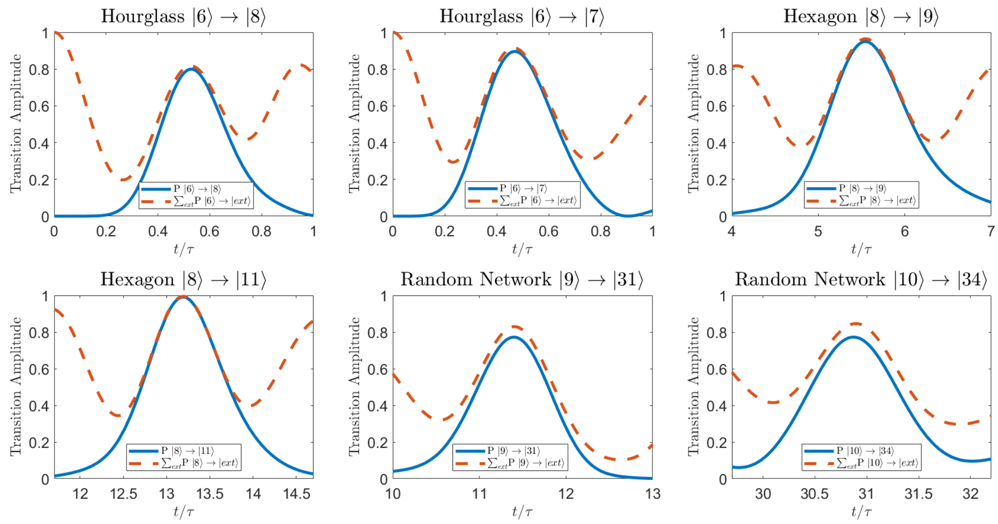


Figure 7: Plot of the transition amplitudes of routing excitations between different pairs of nodes (solid) and the sum transition amplitudes of routing from the initial node to all the other external nodes (dashed). The plot of transition amplitude is made with respect to time t normalized with the characteristic time τ for different topologies among those considered before.

It can be easily seen from Fig. 7 that even in the cases where the the transition amplitude is lower it is highly improbable to measure the excitation in a different output.

3.3 Routing of Quantum Superpositions

We focus in this section to the task of routing quantum superpositions of the form: $|i\rangle + e^{i\chi}|j\rangle \rightarrow |m\rangle + e^{i\chi}|n\rangle$. As anticipated before the fitness score has been chosen of the form Eq. (11) in order to make the process the most independent on the encoded information. To spot a transport event we set a threshold of 2/3 coming from the maximum fidelity reached from no-entanglement teleportation. The protocol is showed in Fig. 8 and consists in Alice measuring a pure state $|\psi\rangle$ multiple times and then communicating the results classically to Bob. The resulting state ρ will have average fidelity 2/3 with the initial state $|\psi\rangle$. In other words, if the fidelity of the routed state is below 2/3 I could have made a phone call.

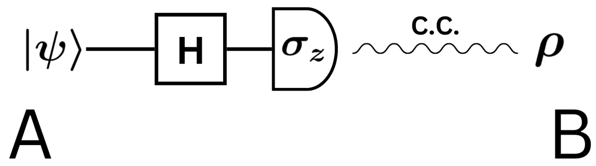


Figure 8: Pictorial representation of the no-entanglement communication protocol.

As can be seen from Fig. 9 the algorithm is performing quite good, exceeding the threshold for any value of the superposition phase χ. Moreover the transition amplitude is almost independent on χ.
Finally a non trivial feature is that the maximums are large in time, not requiring a high time precision in measurements.

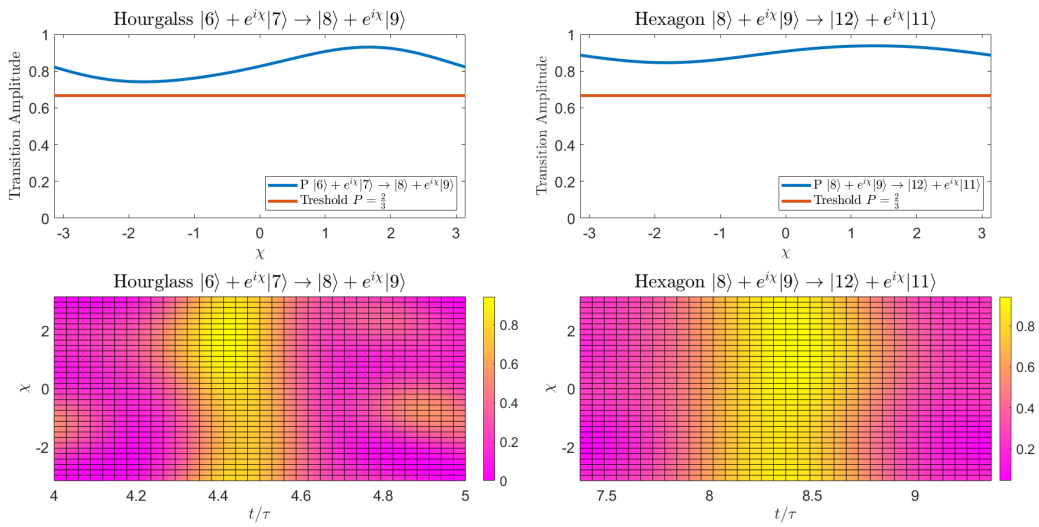


Figure 9: On the top we plotted the transition amplitude of routing a quantum superposition between two pairs of nodes (blue line) and the threshold (red line) in relation with the superposition phase χ. On the bottom we plotted the same transition amplitudes but with respect to both the superposition phase χ and the normalized time t/τ

4 Conclusions and Outlooks

In this work we used CQW as a tool to address the problem of routing information over a network. In order to do that we developed a GA to optimize free phases in fixed topologies. Firstly we optimized the shape of 4,5 and 6 outputs routers, that are the primary object for directional transport. We showed that the considered topologies offer very good performance regarding the task of routing single excitations. Next, we took into account a mycelium and a random network and also in this scenario we were able to perform the routing with very high fidelity. We also fixed the problem of routing the excitation to an undesired node through an appropriated choice of the fitness score. Indeed, we showed that, also in the cases where the transition amplitude is lower, the probability of finding the excitation in another vertex is almost null. Finally we focused on the problem of routing equally weighed superpositions, where can be encoded actual quantum information. The main problem we faced was that the transition amplitude depended on the superposition phase and it not possible optimize the chiral phases differently for different states, since we want the routing to be independent on the information. We fixed this issue with an oculate choice of the fitness score. We considered also a threshold which was overcame in all the scenarios we took into account.

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