Electron interaction with the self-emitted field in the bending magnet

..., Advanced Photon Source, Argonne National Laboratory, Lemont, IL, USA

A brief description of calculations

PACS numbers:

MAGNETIC FIELD

Maofei Qian and Joe Xu designed a 2° bending magnet with the aperture of 8 mm and the magnetic field shown in Figure 1

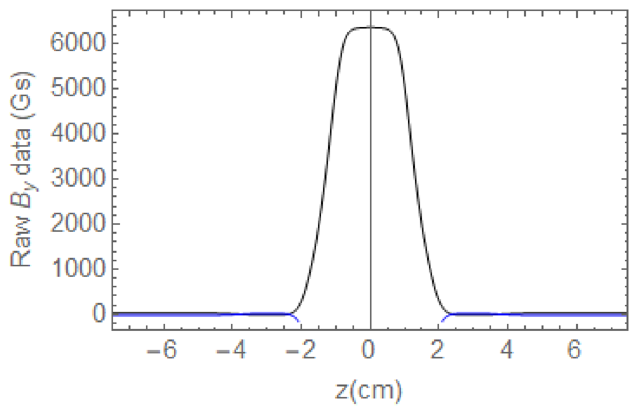

FIG. 1: Magnetic field on the axis of the bending magnet.

The electron with the energy of 147 MeV passes this magnet following the trajectory shown in Figure 2. Here

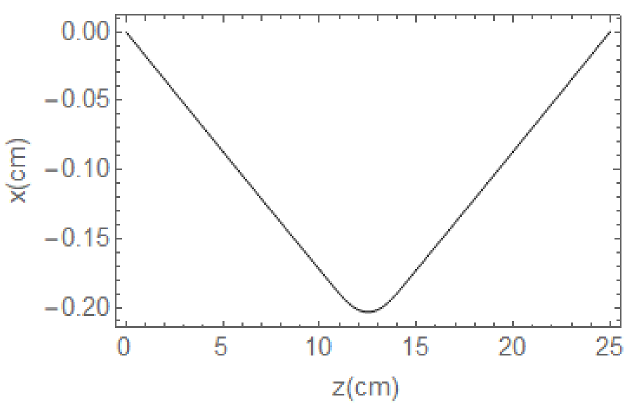

FIG. 2: Electron trajectory in the two-pole magnet. The beginning of the z-coordinate is shifted by 6.25 cm with respect to what is used in Figure 1.

we use a Cartesian coordinate system where the z-coordinate unit vector is tangential to the electron trajectory at the location with the peak magnetic field, the y coordinate unit vector is directed along the magnetic field and the x-coordinate unit vector is orthogonal to z and y unit vectors.

ELECTRIC FIELD

A. Numerical calculation

When electron with the charge e propagates the magnet, it oscillates and radiates the electromagnetic field. Eq. 1 taken from Ref. [1–3] defines this field at the observer's location on the axes z at a far distance R_0 from the magnet center in the MKS system of units:

$$E_{x_1}(t) = -\frac{e}{4\pi\epsilon_0 R_0}\frac{d^2x'}{c^2dt^2}. \quad (1)$$

Here ϵ_0 is the vacuum permittivity, c is the speed of light, x' is the *apparent* trajectory of the electron and t is the time in the observer's frame as defined in [2, 3].

In the optical stochastic cooling (OSC) method this radiation is transported to the identical downstream magnet as shown in Figure 3. The light optics inverts the image of the electron at the source magnet when the light reaches the second magnet. The electron optics guides the electron to the second magnet and reverses the sign of the electron's offset and angle with respect to z axes in the first magnet when the electron reaches the second magnet. The light and electron paths between the

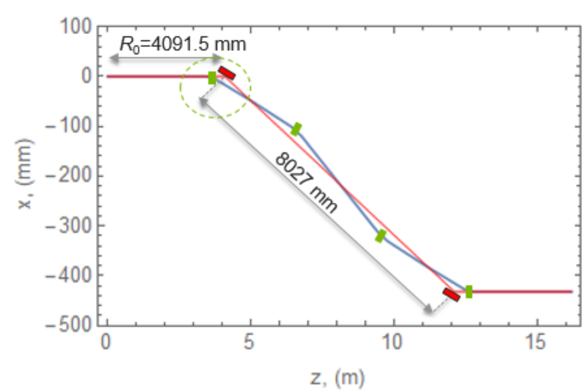

FIG. 3: A schematic of the OSC system. The red line shows the light path and two small red boxes are the paraboloid mirrors. The green line shows the electron path and four green boxes are bending magnets. The quadrupole lenses used in the electron beamline are not shown.

magnets are arranged such as the electron appears in the second magnet in front of the light pulse.

Defining the angular acceptance of the optical system as θ and using in (1) the anzats $r_e = \frac{e^2}{4\pi\epsilon_0 mc^2}$, where r_e is the classical electron radius, we write for the electric field in the second magnet:

$$E_{x_2}(t) = -\frac{mc^2}{e}\theta^2 \frac{r_e}{\lambda_c}\frac{d^2x'}{d(ct)^2}. \tag{2}$$

Here $\lambda_c = \frac{4\pi}{3}\frac{\rho}{\gamma^3}$ is the critical wavelength of the synchrotron radiation, γ is the relativistic factor, and ρ is the radius of curvature of the electron trajectory. If $\theta > 1/\gamma$, then $1/\gamma$ should be used in (1) instead of θ. Using 147 MeV for the electron energy and 6374 Gs for the peak magnetic field, we obtain ρ=0.769 m, λ_c= 135 nm and calculate for corresponding photon energy $\hbar\omega$=9.16 eV.

The main difficulty in calculating E_{x_2} is a calculation of $\frac{d^2x'}{d(ct)^2}$. We followed the instruction given in [1–3] and obtained the field plotted in Figure 4. FWHM of the pulse as shown on the plot is 0.016 mum or 53.4 attoseconds and the maximum field E_x=46 V/cm is located at ct=17.923 μm.

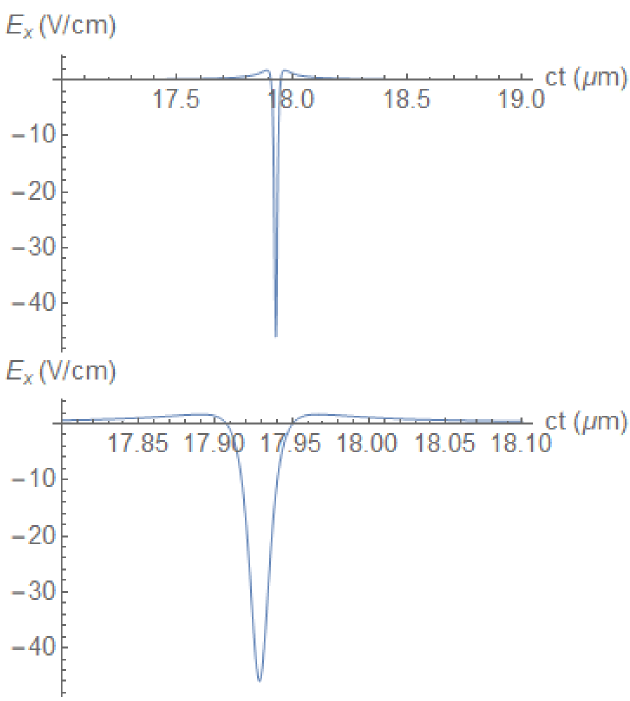

FIG. 4: Electric field as it is seen by the observer located in the second magnet. The bottom panel is used to show more details of the plot shown in the top pane.

B. Analytical calculation

Here we present the analytical calculation of the electric field based on the equation derived in the paragraph 4.4.1 in [4].

$$E_x = -E_{max}\frac{1-4\sinh^2\left(\frac{1}{3}\sinh^{-1}(\tau)\right)}{(1+u^2)^2\left(1+4\sinh^2\left(\frac{1}{3}\sinh^{-1}(\tau)\right)\right)^3} \tag{3}$$

Here

$$E_{max} = \frac{mc^2}{e}\frac{r_e\gamma^2}{\rho\lambda_c},$$
$$\tau = \frac{4\pi(ct)}{\lambda_c\,(1+u^2)^{3/2}},$$
$$u = \gamma\psi,$$

where ψ is the angle from the source to the observer locate with the offset to the plane of the electron motion.

Figure 5 shows the analitical result in comparison with the numercical result.

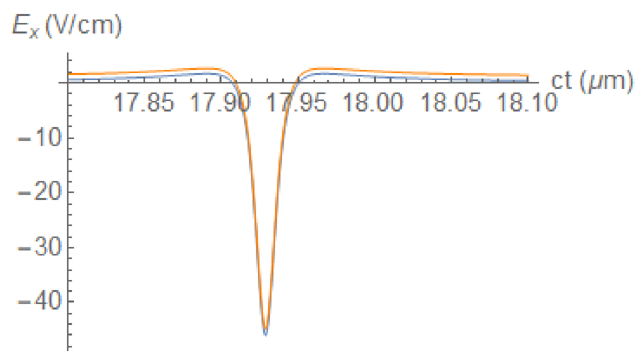

FIG. 5: Comparison of the analytical result (orange curve) with the numerical result shown in Figure 4 (blue curve). The analytical result is deliberately shifted by 1 V/cm.

C. Spectrum

To obtain the spectrum of the electron radiation from the magnet described at the beginning, we applied Fourier transform to the field given by Eq. 3. The result is shown in Figure 6. We note that the half of the power is radiated in the spectrum range above 4.52 eV and that the spectrum range from 1.5 eV to 25 eV contains 89% of radiated energy.

ENERGY KICK

In the second magnet, the electron interacts with the field that it radiated in the first magnet. In result it gains

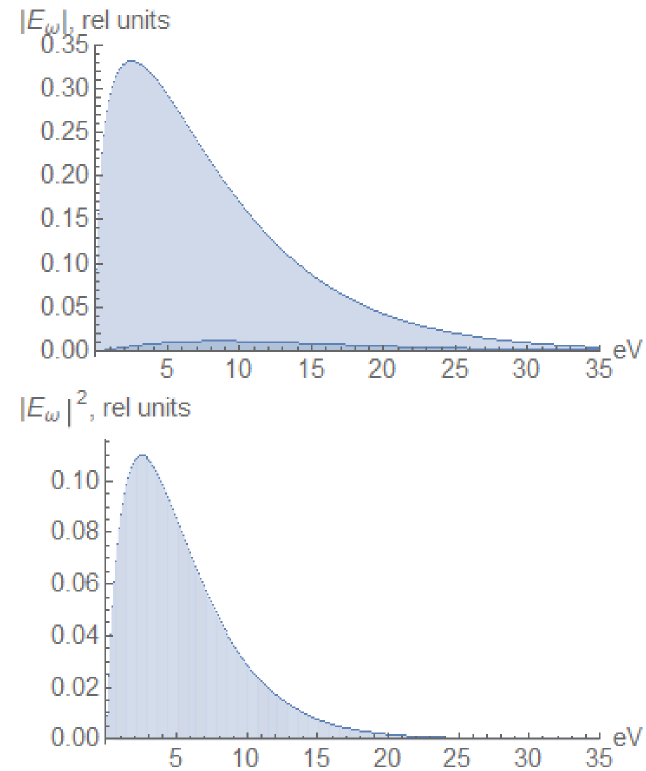

FIG. 6: Frequency spectrum of the electric filed (top plot) and radiation power (bottom plot) in relative units as a function of the photon energy.

or lose energy δE according to the following equation:

$$\delta E(\tau) = ec \int E_{x_2}(t-\tau)\beta_x dt. \tag{4}$$

The exact amount of the energy gain or loss depends on the difference τ between arrival times of the light pulse and the electron in the second magnet. We varied this difference and plotted the result in Figure 7.

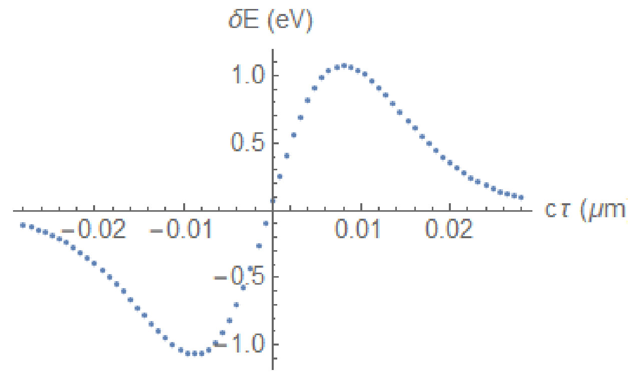

FIG. 7: The energy gain or loss experienced by the electron after interaction in the second magnet with the light that it radiated in the first magnet . Here τ is the difference in the arrival times of the light pulse and the electron in the second magnet.

[1] R. P. Feynman, R. B. Leighton, and M. Sands, The Feynman Lectures on Physics (Addison-Wesley, New York, 1963), Vol. 1, Chap. 28.
[2] 12ibid, Chap. 34.
[3] B. D. Patterson, "A simplified approach to synchrotron radiation", American Journal of Physics 79, 1046 (2011); doi: 10.1119/1.3614033.
[4] A. Hofmann, "The Physics of Synchrotron Radiation", Cambridge University Press, 2004.