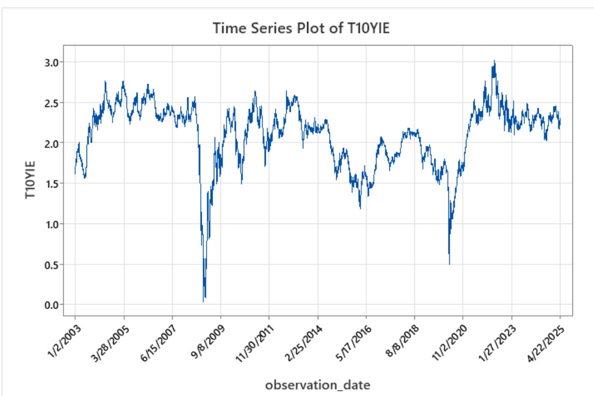
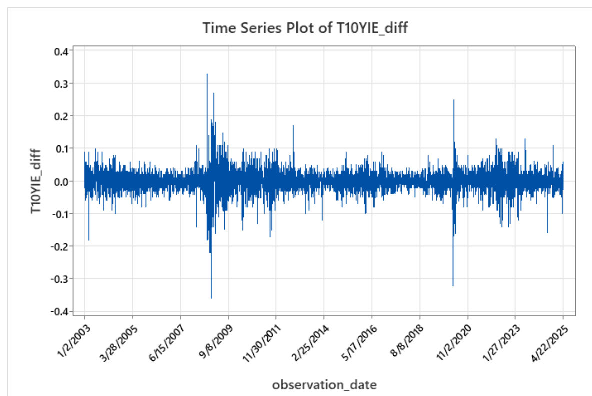
SARIMA - GARCH Model for 10 yr Breakeven Inflation Daily Time Series

In this project, we investigate the daily 10 Yr Breakeven Inflation T10YIE time series. The goal is to find the best (S)ARIMA-(G)ARCH model that will produce reasonable forecast points and forecast intervals, using ACF, PACF, AICc and other tools. The analysis reveals that an $ARIMA(2,1,0)(0,0,1)_{16}$ without a constant combined with $GARCH(1,1)$ for innovations/noise is the best choice. The source of data is Fred's website: https://fred.stlouisfed.org/series/T10YIE, with daily observations covering the time period from 01/02/2003 to 04/25/2025. We removed the last day 04/25/2025 from the data, as we used this observation to check the accuracy of the selected forecasting model. The times series T10YIE, its first difference and their ACF and PACF correlograms are plotted below:

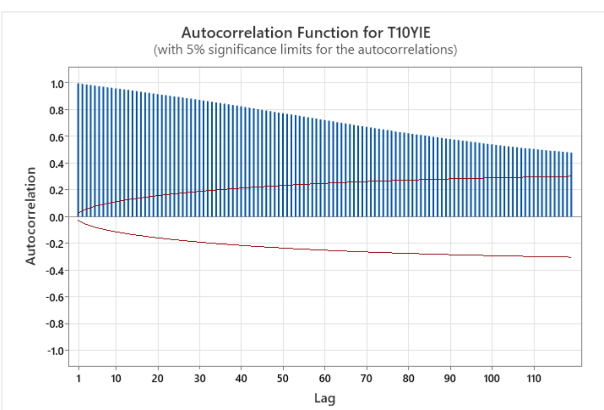
Plot of T10YIE:



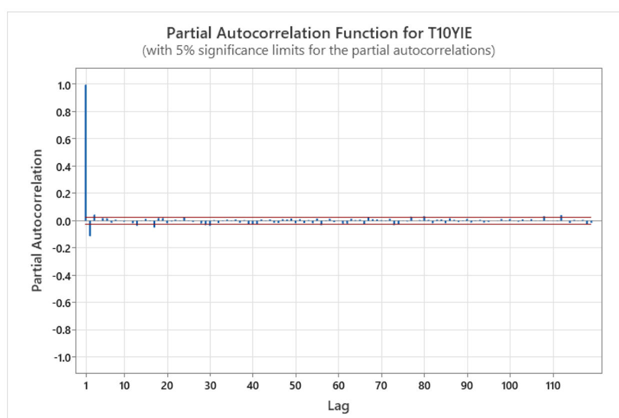
Plot of first difference of T10YIE:



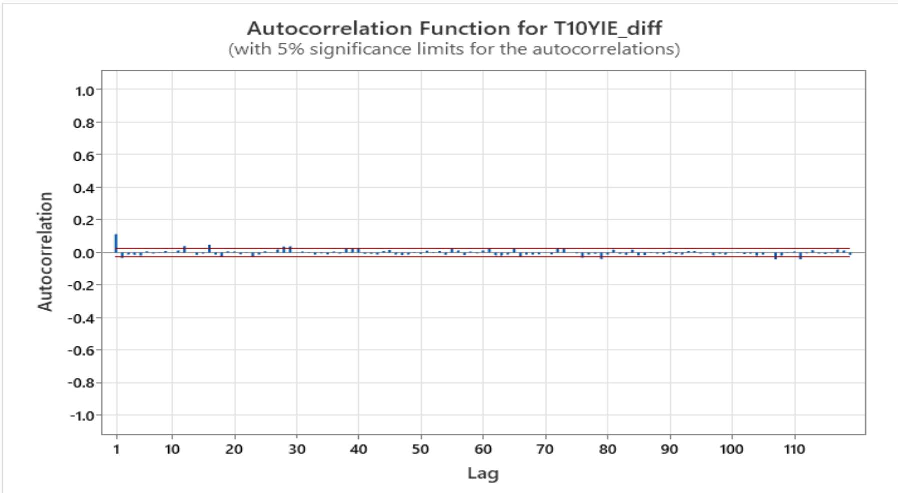
Plot of ACF for T10YIE:



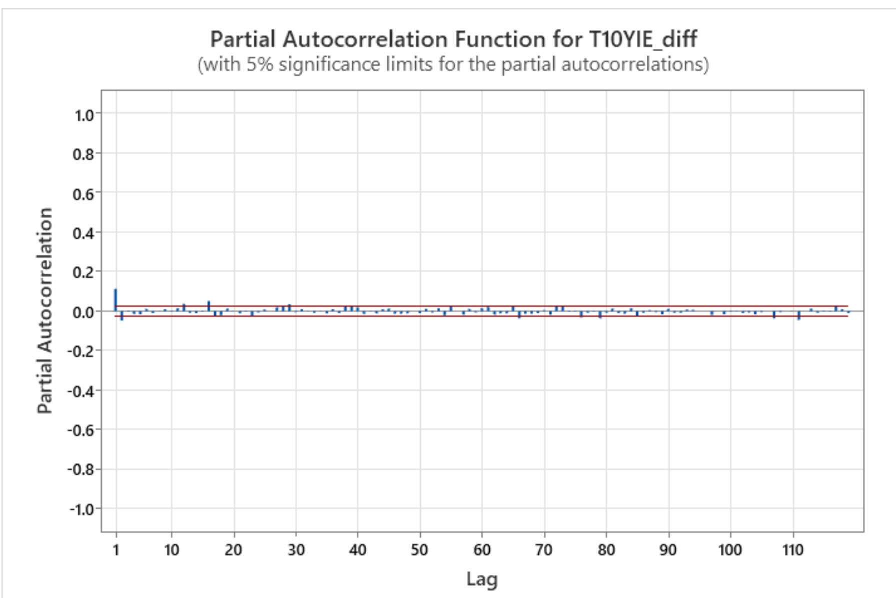
Plot of PACF for T10YIE:



Plot of ACF for first difference of T10YIE:



Plot of PACF for first difference of T10YIE



We did not use log transformation because for ARIMA type models for ln(T10YIE) the residuals diagnostics using ACF, PACF, Ljung-Box statistics still show statistically significant autocorrelation at many lags. The ACF for T10YIE is dying down extremely slowly, while the PACF for T10YIE cuts off at lag 2. However, the first lags of ACF and the lag 1 of PACF are very close to 1, indicating that T10YIE is potentially non-stationary time series. For this reason, we calculate the first difference for T10YIE, together with its ACF and PACF correlograms. We also obtained the second order difference of T10YIE, but ACF and PACF first lag are statistically significant and negative, which indicates overdifferencing. Based on the ACF and PACF correlograms of the first difference of T10YIE – neither the ACF nor the PACF die down even if ACF and PACF cut off at lag 2 first – we cannot decide on a specific ARIMA(p,1,q) or $ARIMA(p,1,q)(p1,1,q1)_s$ model just based on ACF and PACF. We notice that there is still a slightly statistically significant lag at 16 (and few other lags, but with lower magnitude than the one for lag 16) for the first difference of T10YIE. Therefore, we run both ARIMA(p,1,q) and $ARIMA(p,1,q)(p1,0,q1)_{16}$ with or without constant models and decided that models of type $ARIMA(p,1,q)(p1,0,q1)_{16}$ are better suited – the residuals diagnostic check using Ljung-Box statistics shows no autocorrelations at lags 12, 24, 36, 48; this does not happen for models of type ARIMA(p,1,q). The AICc results for $ARIMA(p,1,q)(p1,0,q1)_{16}$ are presented below:

d =1 with no constant

Model Selection

Model (d = 1, D = 0)	LogLikelihood	AICc	AIC	BIC
p = 2, q = 0, P = 0, Q = 1*	10961.1	-21914.2	-21914.2	-21887.7
p = 0, q = 2, P = 1, Q = 0	10960.9	-21913.7	-21913.7	-21887.2
p = 0, q = 2, P = 0, Q = 1	10960.6	-21913.3	-21913.3	-21886.8
p = 1, q = 1, P = 1, Q = 0	10960.4	-21912.8	-21912.8	-21886.3
p = 2, q = 0, P = 1, Q = 0	10960.1	-21912.2	-21912.2	-21885.7
p = 1, q = 1, P = 1, Q = 1	10961.0	-21912.0	-21912.0	-21878.9
p = 0, q = 2, P = 1, Q = 1	10960.6	-21911.2	-21911.2	-21878.1
p = 0, q = 1, P = 0, Q = 1	10958.6	-21911.2	-21911.2	-21891.3
p = 0, q = 1, P = 1, Q = 0	10958.3	-21910.7	-21910.7	-21890.8
p = 2, q = 0, P = 1, Q = 1	10959.2	-21908.4	-21908.4	-21875.3
p = 1, q = 1, P = 0, Q = 1	10958.2	-21908.3	-21908.3	-21881.8
p = 0, q = 1, P = 1, Q = 1	10958.0	-21908.0	-21908.0	-21881.5
p = 1, q = 0, P = 0, Q = 1	10955.3	-21904.7	-21904.7	-21884.8
p = 2, q = 0, P = 0, Q = 0	10954.3	-21902.6	-21902.6	-21882.7
p = 0, q = 2, P = 0, Q = 0	10954.0	-21902.0	-21902.0	-21882.1
p = 1, q = 2, P = 0, Q = 0	10954.9	-21901.8	-21901.8	-21875.3
p = 1, q = 0, P = 1, Q = 0	10953.9	-21901.7	-21901.7	-21881.8
p = 1, q = 1, P = 0, Q = 0	10953.5	-21900.9	-21900.9	-21881.1
p = 2, q = 1, P = 0, Q = 0	10954.4	-21900.8	-21900.8	-21874.3
p = 2, q = 2, P = 0, Q = 0	10955.0	-21899.9	-21899.9	-21866.8
p = 0, q = 1, P = 0, Q = 0	10951.1	-21898.3	-21898.3	-21885.0
p = 1, q = 0, P = 1, Q = 1	10952.7	-21897.4	-21897.4	-21870.9
p = 2, q = 1, P = 0, Q = 1	10952.0	-21894.0	-21894.0	-21860.9
p = 1, q = 0, P = 0, Q = 0	10948.1	-21892.2	-21892.2	-21879.0
p = 2, q = 1, P = 1, Q = 0	10947.1	-21884.3	-21884.3	-21851.1
p = 1, q = 2, P = 0, Q = 1	10945.0	-21880.1	-21880.1	-21846.9
p = 2, q = 2, P = 0, Q = 1	10944.1	-21876.2	-21876.2	-21836.4
p = 2, q = 1, P = 1, Q = 1	10943.4	-21874.8	-21874.8	-21835.1
p = 2, q = 2, P = 1, Q = 0	10942.6	-21873.3	-21873.3	-21833.5
p = 1, q = 2, P = 1, Q = 0	10940.0	-21870.0	-21870.0	-21836.9
p = 1, q = 2, P = 1, Q = 1	10935.7	-21859.4	-21859.5	-21819.7
p = 2, q = 2, P = 1, Q = 1	10934.5	-21854.9	-21854.9	-21808.5
p = 0, q = 0, P = 1, Q = 0	10919.3	-21834.7	-21834.7	-21821.4
p = 0, q = 0, P = 0, Q = 1	10919.3	-21834.6	-21834.6	-21821.4
p = 0, q = 0, P = 1, Q = 1	10919.4	-21832.7	-21832.7	-21812.8
p = 0, q = 0, P = 0, Q = 0	10913.0	-21823.9	-21823.9	-21817.3

** Best model with minimum AICc. Output for the best model follows.*

d =1 with constant

Model Selection

Model (d = 1, D = 0)	LogLikelihood	AICc	AIC	BIC
p = 2, q = 0, P = 0, Q = 1*	10961.1	-21912.2	-21912.3	-21879.1
p = 0, q = 2, P = 1, Q = 0	10960.9	-21911.8	-21911.8	-21878.7
p = 0, q = 2, P = 0, Q = 1	10960.7	-21911.3	-21911.3	-21878.2
p = 1, q = 1, P = 1, Q = 0	10960.4	-21910.9	-21910.9	-21877.7
p = 2, q = 0, P = 1, Q = 0	10960.1	-21910.3	-21910.3	-21877.1
p = 1, q = 1, P = 1, Q = 1	10961.0	-21910.1	-21910.1	-21870.3
p = 0, q = 2, P = 1, Q = 1	10960.6	-21909.2	-21909.3	-21869.5
p = 0, q = 1, P = 0, Q = 1	10958.6	-21909.2	-21909.2	-21882.7
p = 0, q = 1, P = 1, Q = 0	10958.4	-21908.7	-21908.7	-21882.2
p = 2, q = 0, P = 1, Q = 1	10959.2	-21906.4	-21906.5	-21866.7
p = 1, q = 1, P = 0, Q = 1	10958.2	-21906.4	-21906.4	-21873.3
p = 0, q = 1, P = 1, Q = 1	10957.8	-21905.7	-21905.7	-21872.5
p = 1, q = 0, P = 0, Q = 1	10955.4	-21902.7	-21902.7	-21876.2
p = 2, q = 0, P = 0, Q = 0	10954.3	-21900.6	-21900.6	-21874.1
p = 0, q = 2, P = 0, Q = 0	10954.0	-21900.0	-21900.1	-21873.5
p = 1, q = 2, P = 0, Q = 0	10954.9	-21899.9	-21899.9	-21866.8
p = 1, q = 0, P = 1, Q = 0	10953.9	-21899.8	-21899.8	-21873.3
p = 1, q = 1, P = 0, Q = 0	10953.5	-21899.0	-21899.0	-21872.5
p = 2, q = 1, P = 0, Q = 0	10954.4	-21898.9	-21898.9	-21865.8
p = 2, q = 2, P = 0, Q = 0	10955.0	-21898.0	-21898.0	-21858.2
p = 0, q = 1, P = 0, Q = 0	10951.2	-21896.3	-21896.3	-21876.4
p = 1, q = 0, P = 1, Q = 1	10952.7	-21895.5	-21895.5	-21862.4
p = 2, q = 1, P = 0, Q = 1	10952.0	-21892.0	-21892.1	-21852.3
p = 1, q = 0, P = 0, Q = 0	10948.1	-21890.3	-21890.3	-21870.4
p = 2, q = 1, P = 1, Q = 0	10947.1	-21882.3	-21882.3	-21842.5
p = 1, q = 2, P = 0, Q = 1	10945.1	-21878.1	-21878.1	-21838.3
p = 2, q = 1, P = 1, Q = 1	10943.9	-21873.7	-21873.7	-21827.3
p = 2, q = 2, P = 0, Q = 1	10943.4	-21872.8	-21872.8	-21826.4
p = 1, q = 2, P = 1, Q = 0	10940.0	-21868.0	-21868.0	-21828.3
p = 2, q = 2, P = 1, Q = 0	10938.6	-21863.2	-21863.2	-21816.8
p = 1, q = 2, P = 1, Q = 1	10935.7	-21857.5	-21857.5	-21811.1
p = 2, q = 2, P = 1, Q = 1	10934.5	-21852.9	-21853.0	-21799.9
p = 0, q = 0, P = 1, Q = 0	10919.4	-21832.8	-21832.8	-21812.9
p = 0, q = 0, P = 0, Q = 1	10919.3	-21832.7	-21832.7	-21812.8
p = 0, q = 0, P = 1, Q = 1	10919.4	-21830.8	-21830.8	-21804.3
p = 0, q = 0, P = 0, Q = 0	10913.0	-21822.0	-21822.0	-21808.8

** Best model with minimum AICc. Output for the best model follows.*

Based on the information above, the model with lowest AICc is $ARIMA(2, 1, 0)(0, 0, 1)_{16}$ without a constant. The estimated parameters are given below together with Ljung-Box statistics showing that all p-values for autocorrelations at lags 12, 24, 36, 48 are higher than 0.05, which indicates that residuals are uncorrelated at these lags.

Final Estimates of Parameters

Type	Coef	SE Coef	T-Value	P-Value
AR 1	0.1184	0.0134	8.85	0.000
AR 2	-0.0454	0.0134	-3.40	0.001
SMA 16	-0.0494	0.0134	-3.68	0.000

Differencing: 1 Regular
Number of observations after differencing: 5581

Modified Box-Pierce (Ljung-Box) Chi-Square Statistic

Lag	12	24	36	48
Chi-Square	13.51	24.68	41.42	56.34
DF	9	21	33	45
P-Value	0.141	0.261	0.149	0.120

We obtain the expression for $x_t = T10Y1E$ based on this model using backshift operator $Bx_t = x_{t-1}$ as follows: $(1 - 0.1184B + 0.0454B^2)(1 - B)x_t = (1 + 0.0494B^{16})\varepsilon_t$ or $(1 - B - 0.1184B + 0.1184B^2 + 0.0454B^2 - 0.0454B^3)x_t = \varepsilon_t + 0.0494\varepsilon_{t-16}$ or $(1 - 1.1184B + 0.1638B^2 - 0.0454B^3)\, x_t = \varepsilon_t + 0.0494\varepsilon_{t-16}$.

Therefore, the complete form for model $ARIMA(2, 1, 0)(0, 0, 1)_{16}$ is given by

$$x_t = 1.1184x_{t-1} - 0.1638x_{t-2} + 0.0454x_{t-3} + \varepsilon_t + 0.0494\varepsilon_{t-16},$$

where ε_t is zero-mean white noise.

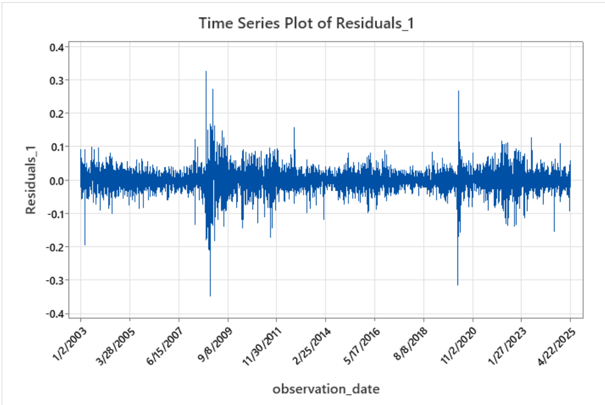
Forecasts from period 5582

Period	Forecast	95% Limits Lower	95% Limits Upper	Actual
5583	2.30009	2.23352	2.36666	

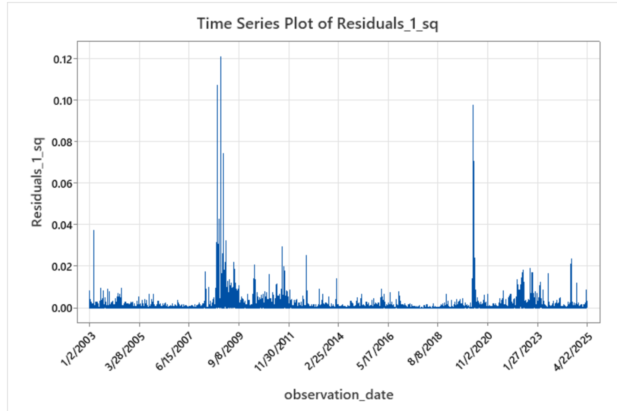
Based on the information above, at time $04/24/2025$ (corresponding to $n = 5582$ for T10YIE) for step $h = 1$, the one day ahead forecast point for T10YIE is $f_{5582,1} = 2.30009$ and the 95% one day ahead forecast interval for T10YIE is (2.23352, 2.36666), which contains the actual value 2.27 of T10YIE at $04/25/2025$ (corresponding to $n + 1 = 5583$).

Coming back to diagnostic check for residuals of the selected model, we plot the residuals ε_t and residuals squared ε_t^2, together with their corresponding ACF and PACF correlograms below:

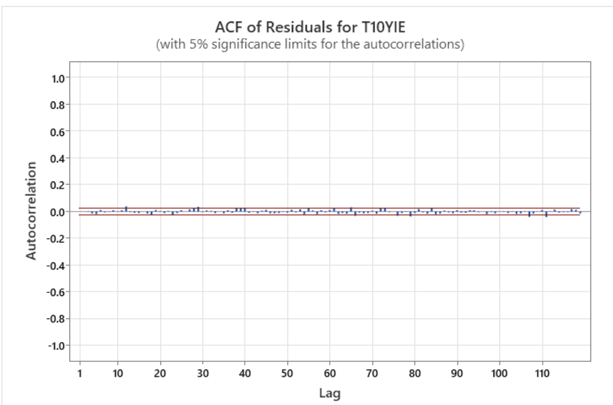
Plot of residuals for $ARIMA(2, 1, 0)(0, 0, 1)_{16}$:



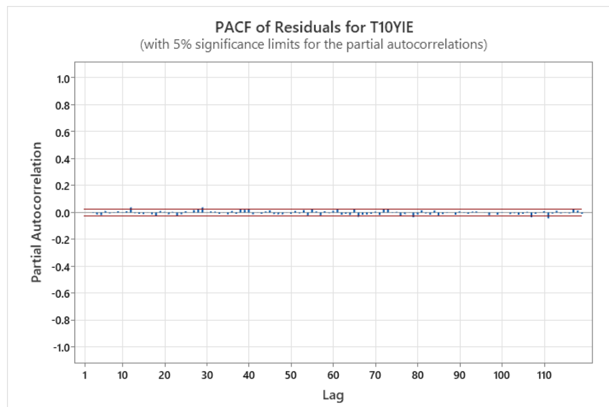
Plot of residuals squared for $ARIMA(2, 1, 0)(0, 0, 1)_{16}$:



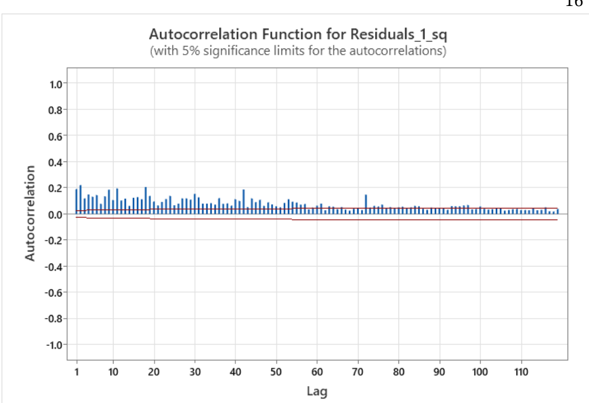
Plot of ACF for residuals for $ARIMA(2, 1, 0)(0, 0, 1)_{16}$:



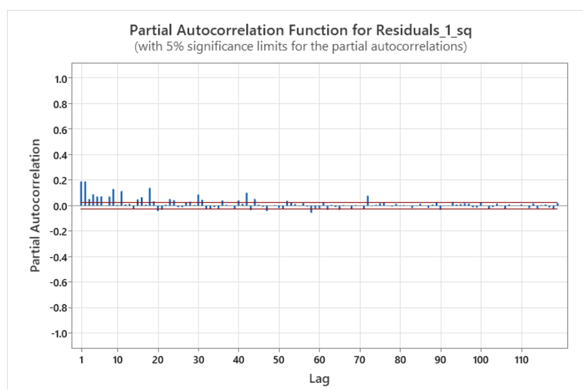
Plot of PACF for residuals for $ARIMA(2, 1, 0)(0, 0, 1)_{16}$:



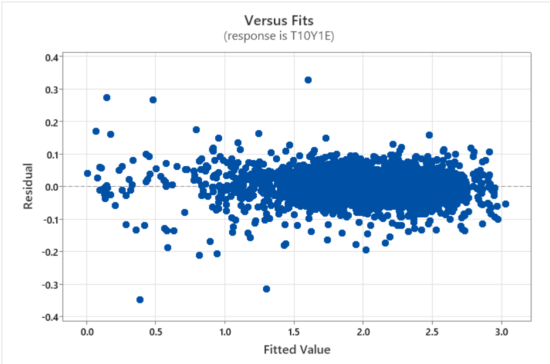
Plot ACF for residuals squared $ARIMA(2, 1, 0)(0, 0, 1)_{16}$:



Plot PACF for residuals squared $ARIMA(2, 1, 0)(0, 0, 1)_{16}$:



We also plot residuals for $ARIMA(2, 1, 0)(0, 0, 1)_{16}$ vs fitted values below:



Based on ACF and PACF plots for residuals together with Ljung-Box statistics, we observe that residuals have no statistically significant correlated or partially autocorrelated lags of any order (maybe at some further lags, but very slightly significance). Based on the ACF plot for squared residuals, we observe that first ACF seems to die down slowly, PACF plot seems to cut off at lag 7 first, but then there are several spikes at other statistically significant lags of order higher than 7. Based on all these, it seems that while residuals are zero mean white noise, the squared residuals are forecastable. Also, investigating the plot of residuals vs fitted values, we observe that there may be at least 2 types of different 'groups' - one less dense and the other more dense and it looks slightly like a cone, which may indicate some patterns within the variance (i.e.conditional heteroscedasticity). Based on these observations, we can infer that residuals are uncorrelated but not independent, and they show evidence of conditional heteroscedasticity.

We proceed further to find the best fitted model of type ARCH(q) or GARCH(1,1) for residuals ε_t. First, we run ARCH(q) models with $0 \leq q \leq 10$ in R, and we use logLik(model) to find log likelihood for each model, which we then plug into the AICc formula from class notes. For $q = 0$, we calculate the log likelihood of ARCH(0) by hand, using the formula given in class handout, namely: $Loglike0 = -0.5 * N * (1 + log(2 * pi * mean(resid^2))) = 10959.16$, (resid denotes residuals data for which $d = 1$, $N = 5581 - d = 5580$) which plugged in the formula for AICc gives AICc = -21916.31 for ARCH(0). All likelihood for ARCH(q) with $0 \leq q \leq 10$ and AICc for ARCH(q) with $0 \leq q \leq 10$ are given in the table below:

q	Log Likelihood	AICc	q	Log Likelihood	AICc
0	10959.16	-21916.31	6	11709.61	-23405.20
1	11273.93	-22543.86	7	11725.45	-23434.88
2	11437.41	-22868.81	8	11740.96	-23463.88
3	11550.27	-23092.52	9	11750.51	-23480.99
4	11635.63	-23261.26	10	11755.18	-23488.32
5	11680.64	-23349.26			

Therefore, amongst ARCH(q) models with $q = 1,\ldots, 10$, ARCH(10) has the lowest AICc.

Next, we run GARCH(1,1) model (denoted model_garch) in R, find the likelihood with logLik(model_garch) to be 11846.85 and plug this into the AICc formula from class notes with $q = 2$ to obtain AICc_garch = -23687.69. Also, the estimation of parameters for GARCH(1,1) gives model_garch$coef[1] = 0.00001172, model_garch$coef[2] =$0.07774$ and model_garch$coef[3] = 0.913. Since model_garch$coef[2] + model_garch$coef[3] = 0.990740 is close to 1 and model_garch$coef[3] = 0.913 is close to 1, GARCH(1,1) may be nearly stationary.

Upon taking the minimum of AICc for ARCH(q), q = 0 ,...,10 and GARCH(1,1), we find out that GARCH(1,1) has lowest AICc (i.e. $min(-23488.32, -23687.69) = -23687.69$). Therefore, we picked GARCH(1,1) as our best fitted model for residuals ε_t. The R printouts from summary(model_garch) and logLik(model_garch) are given below. From the printouts, after dividing p-values of the coefficients and constant by 2, we still have that all p-values/2 are less than 0.05, which supports the alternative hypothesis that all coefficients and the constant are statistically significant up to 5% level.

The complete form of GARCH(1,1) model is given by :

$$\varepsilon_t | \Psi_{t-1} \sim N(0, h_t) \quad \text{with} \quad h_t = 0.00001172 + 0.07774\,\varepsilon_t^{2} + 0.913 h_{t-1},$$

where Ψ_{t-1} is prior information.

The unconditional variance of ε_t is given by

var(ε_t) =model_garch$coef[1] /(1– model_garch$coef[2] – model_garch$coef[3])= 0.001272449.

The printout for logLik(model_garch) in R is given below:

```
> logLik(model_garch)
'log Lik.' 11846.85 (df=3)
```

The printout for summary(model_garch) in R is given below:

```
> #garch(1,1)
> summary(model_garch)

Call:
garch(x = resid, order = c(1, 1), trace = F)

Model:
GARCH(1,1)

Residuals:
      Min        1Q    Median        3Q       Max 
-7.077936 -0.596337  0.008785  0.623882  6.439964 

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)    
a0 1.172e-05   1.543e-06    7.597 3.02e-14 ***
a1 7.774e-02   3.214e-03   24.187  < 2e-16 ***
b1 9.130e-01   3.905e-03  233.830  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Diagnostic Tests:
	Jarque Bera Test

data:  Residuals
X-squared = 1288.1, df = 2, p-value < 2.2e-16


	Box-Ljung test

data:  Squared.Residuals
X-squared = 6.9896, df = 1, p-value = 0.008198
```

We have resid[5581] = 0.012433 (which actually corresponds to the original date of observation 5582). Using R, we find the conditional variances ht_garch_1_1←model_garch$fit[,1]^2 with ht_garch_1_1[5581] = 0.00151606. Also, we define:
omegahat ← model_garch$coef[1] = 0.00001172, alphahat ← model_garch$coef[2] = 0.07774 and betahat ← model_garch$coef[3] = 0.913. All these allow us to calculate ht_5583 (which actually corresponds to original observation 5583 which we removed from the data):

ht_ 5583 ← omegahat + alphahat*resid[5581]^2 + betahat*ht_garch[5581] =0.001407972

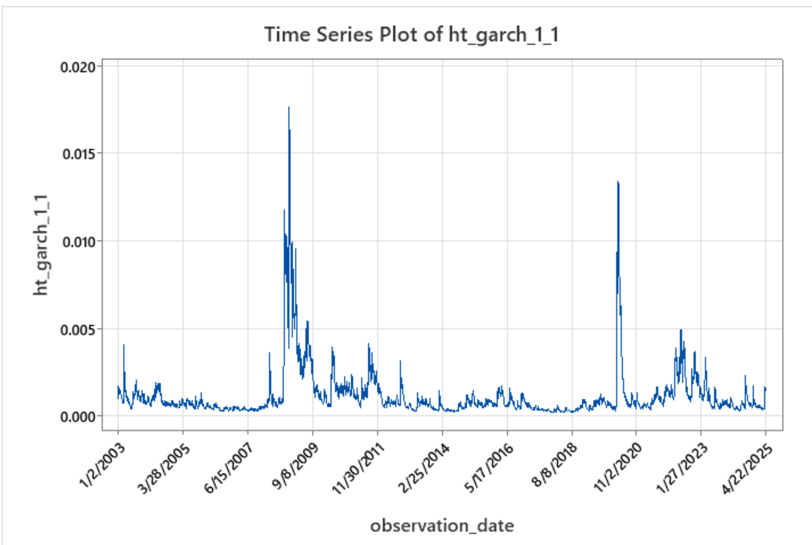
and based on the SARIMA – GARCH model, the 95% one step ahead interval for x_{5583} at $n = 5582$ for step $h = 1$ is given by :

$$(f_{5582,1} - 1.96\sqrt{ht_{5583}}, f_{5582,1} + 1.96\sqrt{ht_{5583}}) = (2.226545, 2.373635),$$

which contains actual value 2.27 of T10YIE at $05/25/2025$ (corresponding to $n + 1 = 5583$). Based on the ARIMA model, the 95% one step forecast interval for x_{5583} at $n = 5582$ for step $h = 1$ is given by $(2.23352, 2.36666)$. We can see that the 95% one day ahead forecast interval for SARIMA–GARCH model seems to contain the 95% one day ahead forecast interval for SARIMA model. Also, the 5th percentile of conditional distribution of the next period should be around $f_{5582,1} - 1.65\sqrt{ht_{5583}} = 2.238096$ by interpretation of 95% one sided prediction intervals. We performed simulations (100k) in R to get the conditional distribution of the next period of T10YIE and the 5% percentile of this conditional distribution equals 2.238548, which is close to 2.238096. Any mismatch is due to the residuals distribution conditional on previous information not following a normal distribution, and simulating conditional distribution of the next period gives a more precise value in this case.

Next we plot the conditional variances ht_garch_1_1 ← model_garch$fit[,1]^2 (indexed from 1 to 5581, but corresponding for original date observations 2 to 5582), plus also the forecast ht_5583 at 5583 — so, in total we have 5582 points:

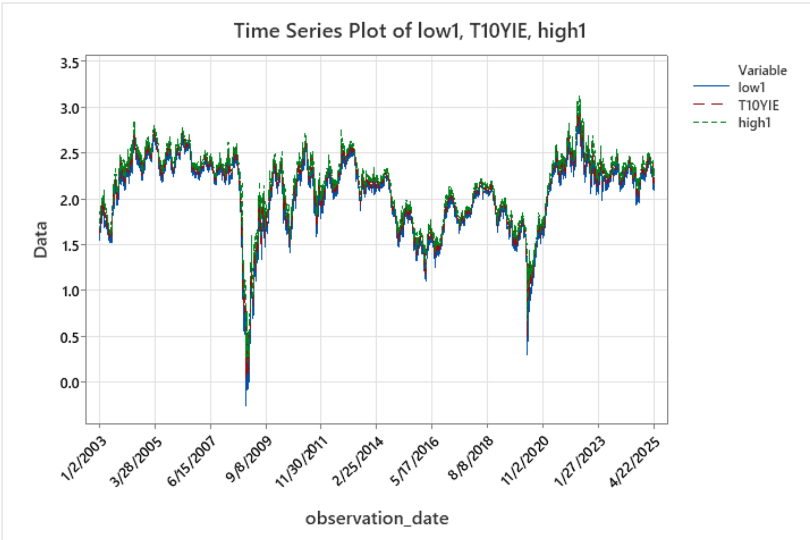
Plots of conditional variances ht_garches_1_1:



We identify bursts of higher volatility during periods 5/6/2009-10/8/2009, 8/1/2011-3/27/2012, 2/25/2022-9/28/2022, 9/19/2022-2/23/2023, some spikes around 3/14/2003, 6/7/2010, 3/24/2023 and 7/18/2024, and very high volatility during periods 9/12/2008-5/6/2009 and 2/24/20-8/18/2020. These volatile periods agree with those found from examination of the time series plot of T10YIE and its first difference, as we can observe by inspecting the corresponding plots for these time series (at the beginning of the report).

Plots of T10YIE and one step ahead 95% forecast intervals, based on information available up to are given below:

Plots of T10YIE and one step ahead 95% forecast interval:



It seems that the 95% one step ahead forecast intervals contains T10YIE actual values 5526 times out of 5582 (we have used Minitab to calculate sum(low < T10YIE < high) and obtained 5526), which represents $5526/5582 = 98.99\%$. This means the 95% one step ahead forecast intervals look too good and accurate, but the explanation is that these forecast intervals are based on all data, not just the observations up to the time the forecast is to be constructed. We would expect that a 95% one step ahead forecast interval should have a conditional coverage of 95%, given the previous only observed information.

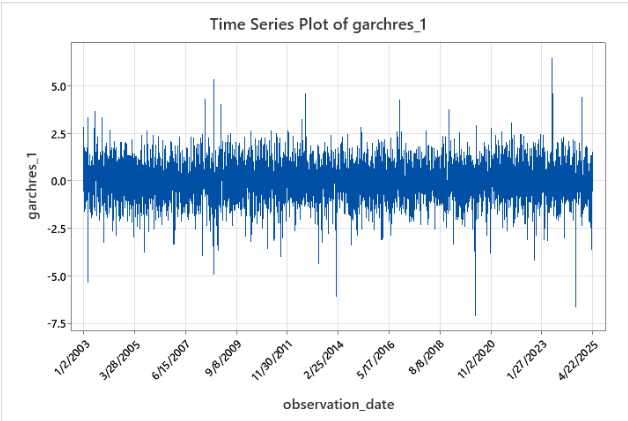
We refer to 'garchres_1' as being the residuals $e_t = \varepsilon_t/\sqrt{ht\_garch\_1\_1}$. We investigate if these residuals are normally distributed with mean 0 and variance 1. The descriptive statistics (see below) show that mean of e_t is 0.002647, i.e. very close to 0, variance of e_t is 1.00038, i.e. very close to 1, skewness is -0.22 (i.e. close to 0), but excess kurtosis is 2.32 (i.e. kurtosis is $3 + 2.32 = 5.32 > 3$). Also, normality test for e_t (see results below) shows that the 'garchres_1' are not normally distributed (p-value < 0.05) besides having kurtosis higher than 3 -- so, longer tails than normal distribution. Therefore, the SARIMA-GARCH does seem to adequately describe the long tailedness in the data for original residuals.

Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
garchres_1	5581	1	0.0026470	0.0133909	1.00038	-7.07794	-0.596602	0.0085517	0.624046

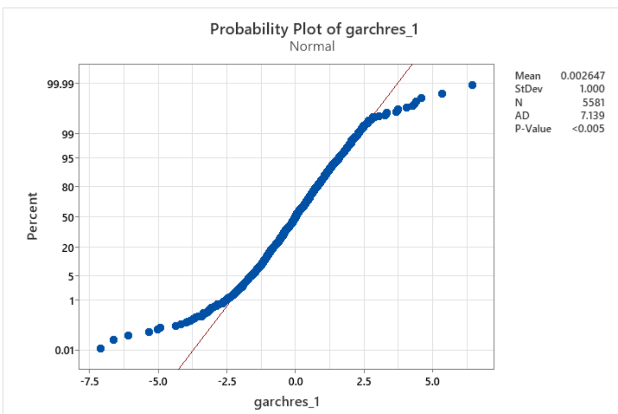
Variable	Maximum	Skewness	Kurtosis
garchres_1	6.43999	-0.22	2.32

Plot of garchres_1:

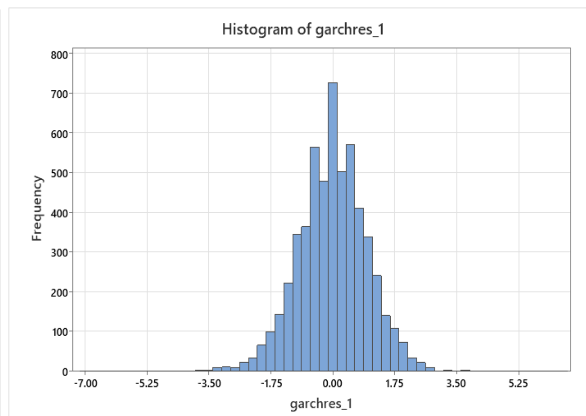


Normality test (left) and histogram (right) forn garchres_1 are illustrated below:

Normality test for garches_1:



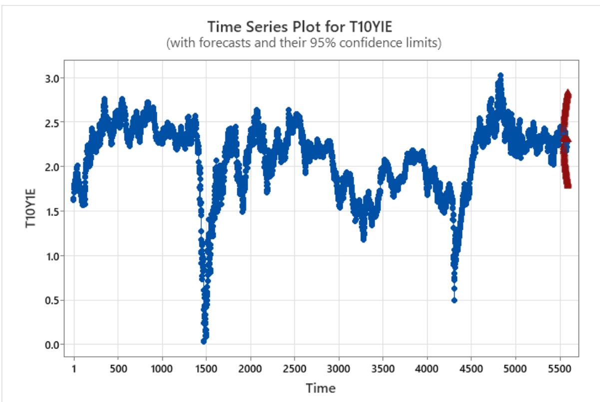
Histogram fro garches_1:



If we count sum(abs(garchres_1) > 1.96), we obtain that we have 268 failures. We have 5581 prediction intervals corresponding to 5581 garchres_1 values. Therefore, the percentage of the time when 95% prediction intervals fail is given by $268/5581 = 4.8\%$. We would expect around 5% failures, but since 'garchres_1' does not follow a normal distribution, it is acceptable to have the percentage of time when 95% prediction intervals fail to be slightly around 5% , but not exactly 5% .

Regarding the 95% one day ahead interval forecasted by SARIMA, given by $(2.23352,\ 2.36666)$ at $04/24/2025$ with step $h = 1$, and by SARIMA-GARCH, given by $(2.226545,\ 2.373635)$ at $04/24/2025$ with step $h = 1$, they both contain the actual value of 2.27 for T10YIE at time $04/24/2025$ (corresponding to $n + 1 = 5583$). The SARIMA-GARCH forecast interval contains (i.e. it is wider than) the SARIMA forecast interval, and both one day ahead forecast intervals at $04/24/2025$ seem to be just about right. Related to a task in project 1, we plot the data for T10YIE with forecasts at lead times 1-50 for $ARIMA(2, 1, 0)(0, 0, 1)_{16}$, together with one backtesting at origin point given by observation 5500. These forecast intervals seem to become wider as lead time increases. Compared with these, the forecast intervals for lead time 1-50 (not required to plot) for SARIMA-GARCH would have adaptive width length as lead time increases.

95% Forecast intervals for lead time 1-50 $ARIMA(2, 1, 0)(0, 0, 1)_{16}$



Backtesting for $ARIMA(2, 1, 0)(0, 0, 1)_{16}$ with origin at: observation 5500: