ECON20005 Assignment 3

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Question 1: Entry Deterrence

1.1

When the firms set quantities simultaneously it is an example of Cournot competition. Since firms A and B are the only ones in the market, $Q = q_A + q_B \implies P = 25 - (q_A + q_B)$.

Firm A has profit $\Pi_A = P \times q_A - c_A \times q_A = (25 - q_A - q_B)q_A - 5 \times q_A = 20q_A - q_A^2 - q_A q_B$.

Maximising with respect to q_A:

$$\begin{aligned}\frac{\partial \Pi_A}{\partial q_A} &= 20 - 2q_A - q_B = 0 \\ q_A &= \frac{20 - q_B}{2}\end{aligned}$$

Since the firms are subject to identical conditions:

$$q_B = \frac{20 - q_A}{2}$$

Substituting B's best response quantity into A's:

$$\begin{aligned}q_A &= \frac{20 - \frac{20 - q_A}{2}}{2} \\ &= \frac{\frac{20 + q_A}{2}}{2} \\ &= \frac{20 + q_A}{4} \\ 0 &= 20 - 3q_A \\ q_A &= \frac{20}{3}\end{aligned}$$

Because the firms face symmetric best response quantities, $q_A = q_B \implies q_B = \frac{20}{3}$. Substituting this back into the profit function:

$$\begin{aligned}\Pi_A = 20q_A - q_A^2 - q_A q_B &= 20 \cdot \frac{20}{3} - \left(\frac{20}{3}\right)^2 - \left(\frac{20}{3}\right)^2 \\ &= \frac{400}{9}\end{aligned}$$

So the equilibrium quantities are $q_A = q_B = \frac{20}{3}$ and the equilibrium profits are $\Pi_A = \Pi_B = \frac{400}{9}$.

1.2

Since B is the second mover, their response will depend on firm A, from Q1.1 we have

$$q_B = \frac{20 - q_A}{2}.$$

Doing backward induction with this, firm A's optimal choice can be found by taking the first order condition of the profit function and setting equal to zero:

$$\begin{aligned}
\Pi_A = 20q_A - q_A^2 - q_A q_B &= 20q_A - q_A^2 - q_A \frac{20 - q_A}{2} \\
&= 10q_A - \frac{q_A^2}{2} \\
\frac{\partial \Pi_A}{\partial q_A} &= 10 - q_A = 0 \\
q_A &= 10
\end{aligned}$$

Then to find B's quantity:

$$\begin{aligned}
q_B &= \frac{20 - q_A}{2} \\
&= \frac{20 - 10}{2} \\
q_B &= 5
\end{aligned}$$

So the total market quantity is $q_A + q_B = 15$, making the market price $25 - 15 = 10$. This means A's profit is:

$$\Pi_A = P \times q_A - c_A \times q_A = 10 \times 10 - (5 \times 10) = 50$$

and B's profits are:

$$10 \times 5 - (5 \times 5) = 25.$$

The equilibrium quantities and profits differ from Q1.1 because as the first mover A is able to commit to producing a larger quantity knowing that the best way for B to respond is by producing less. A partially pushes B out of the market.

1.3

Taking B's profit equation and subtracting the fixed entry cost: $\Pi_B = P \times q_B - c_B \times q_B - F = 20q_B - q_B^2 - q_A q_B - 9$. Importantly, since the derivative of the constant fixed cost is 0, B's best response doesn't change, so substituting in B's best response from Q1.1 in:

$$\begin{aligned}
\Pi_B &= 20q_B - q_B^2 - q_A q_B - 9 \\
&= 20 \times \frac{20 - q_A}{2} - \left(\frac{20 - q_A}{2}\right)^2 - q_A \times \frac{20 - q_A}{2} - 9 \\
&= 200 - 10q_A - (100 - 10q_A + q_A^2/4) - 10q_A + q_A^2/2 - 9 \\
&= 91 - 10q_A + q_A^2/4
\end{aligned}$$

Therefore for B to not enter the market, $91 - 10q_A + q_A^2/4$ must be less than or equal to 0.

$$\begin{aligned}
91 - 10q_A + q_A^2/4 &\leq 0 \\
q_A^2 - 40q_A + 364 &\leq 0 \\
(q_A - 14)(q_A - 26) &\leq 0
\end{aligned}$$

Since the equation for Π_B is a positive quadratic, B's profit is 0 or negative for $q_A \in [14, 26]$, so the minimum quantity A can choose to push B out of the market is $q_A = 14$.

1.4

A's profit when B enters the market in Q1.2 is 50. In Q1.3 where A pushes B out of the market, A sells a quantity $q_A = 14$. With A being the only firm in the market, the price is $P = 25 - 14 = 11$, so A's profit is $\Pi_A = 11 \times 14 - 5 \times 14 = 84$. $84 > 50$ so it is profit maximising for A to deter B from entering the market.

Question 2: Twists on the Linear City and Ice-Cream Salesman Models

2.1

Let the consumer indifferent between A and B be located at position $x_1 \in [0, 0.5]$, their travel cost per unit distance is $t = 3$ so the indifference condition is:

$$\begin{aligned} p_A + 3x_1 &= p_B + 3(0.5 - x_1) \\ 6x_1 &= p_B - p_A + 1.5 \\ x_1 &= \frac{p_B - p_A + 1.5}{6} \end{aligned}$$

Then if the consumer indifferent between B and C is located at $x_2 \in [0.5, 1]$, their indifference condition is:

$$\begin{aligned} p_B + 3(x_2 - 0.5) &= p_C + 3(1 - x_2) \\ 6x_2 &= p_C - p_B + 4.5 \\ x_2 &= \frac{p_C - p_B + 4.5}{6} \end{aligned}$$

If x_1 or x_2 are out of their defined boundaries then no indifferent consumer exists in that section of the city.

2.2

Starting with firm A, their profit is equal to the amount of consumers they sell to times the price, minus their cost:

$$\begin{aligned} \Pi_A &= 100 \times x_1 \times (p_A - c) \\ &= 100(p_A - 1) \times \frac{p_B - p_A + 1.5}{6} \end{aligned}$$

Maximising their profit by setting the first order condition equal to zero:

$$\begin{aligned} \frac{\partial \Pi_A}{\partial p_A} = 100(p_A - 1) \times \frac{-1}{6} + 100 \times \frac{p_B - p_A + 1.5}{6} &= 0 \\ \frac{100 - 100p_A}{6} + \frac{100p_B - 100p_A + 150}{6} &= 0 \\ \frac{250 - 200p_A + 100p_B}{6} &= 0 \\ 250 + 100p_B &= 200p_A \\ p_A &= \frac{5 + 2p_B}{4} \end{aligned}$$

Now the same for firm B but considering that they get customers from both ends of the street:

$$\begin{aligned} \Pi_B &= 100 \times (x_2 - x_1) \times (p_B - c) \\ &= 100(p_B - 1) \times \left(\frac{p_C - p_B + 4.5}{6} - \frac{p_B - p_A + 1.5}{6} \right) \\ &= 100(p_B - 1) \times \left(\frac{p_A - 2p_B + p_C + 3}{6} \right) \end{aligned}$$

$$\begin{aligned}\frac{\partial \Pi_B}{\partial p_B} = 100(p_B - 1) \times \frac{-1}{3} + 100 \times \frac{p_A - 2p_B + p_C + 3}{6} &= 0\\ \frac{200 - 200p_B}{6} + \frac{100p_A - 200p_B + 100p_C + 300}{6} &= 0\\ \frac{500 + 100p_A - 400p_B + 100p_C}{6} &= 0\\ 500 + 100p_A + 100p_C &= 400p_B\\ p_B &= \frac{5 + p_A + p_C}{4}\end{aligned}$$

Since the location of consumers and firms on the street is symmetric about the centre, the firms face identical costs, and because typesetting LaTeX takes longer than I'd like to admit, we will use the symmetry to conclude that firm C's best response price is equal to firm A's, therefore

$$p_C = \frac{5 + 2p_B}{4}.$$

Next, solving for the equilibrium prices, start with B:

$$\begin{aligned}p_B &= \frac{5 + p_A + p_C}{4}\\ &= \frac{5 + \frac{5+2p_B}{4} + \frac{5+2p_B}{4}}{4}\\ &= \frac{30 + 4p_B}{16}\\ 16p_B &= 30 + 4p_B\\ 12p_B &= 30\\ p_B &= 2.5\end{aligned}$$

Then

$$p_A = p_C = \frac{5 + 2p_B}{4} = \frac{10}{4} = 2.5.$$

So all firms charge a price equal to 2.5 at equilibrium.

Finally to find the profits:

$$\begin{aligned}100(p_A - 1) \times \frac{p_B - p_A + 1.5}{6} &= 100(2.5 - 1) \times \frac{2.5 - 2.5 + 1.5}{6}\\ &= 150 \times \frac{1.5}{6}\\ &= 37.5\end{aligned}$$

So by symmetry Π_C also equals 37.5, and:

$$\begin{aligned}\Pi_B &= 100(p_B - 1) \times \left(\frac{p_A - 2p_B + p_C + 3}{6}\right)\\ &= 150 \times \frac{3}{6}\\ &= 75\end{aligned}$$

So although all firms charge identical prices, B's location in the middle of the street allows them to attract twice as many customers as the other firms, giving them twice the profits.

2.3

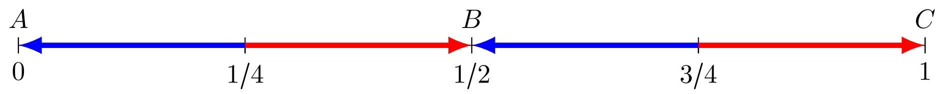
Firstly, since $p = 20$ is greater than their cost $c = 1$, all firms maximise their profit by attracting the most consumers. Since all prices are equal and the cost of travel for a consumer is constant along the whole street, consumers simply purchase from the closest firm.

With firms located at 0, 0.5, and 1, the indifferent consumers are located at $\frac{0+0.5}{2} = 0.25$ and $\frac{0.5+1}{2} = 0.75$, meaning that firm A captures the consumers in $[0, 0.25)$, firm B gets $(0.25, 0.75)$ and the consumers in

$(0.75, 1]$ purchase from C. In this situation if A moves to position 0.1, then the midpoint and therefore indifferent consumer between A and B is located at $\frac{0.1+0.5}{2} = 0.3$, so A captures the region $[0, 0.3)$ by unilaterally deviating to position 0.1 (since there is no firm to A's "left" that they lose space to by moving). By doing this A increases their market share from 25% to 30%, and since sales are profitable A therefore increases their profit by unilaterally deviating, so setting up at 0, 0.5 and 1 cannot be a Nash equilibrium when $p = 20$.

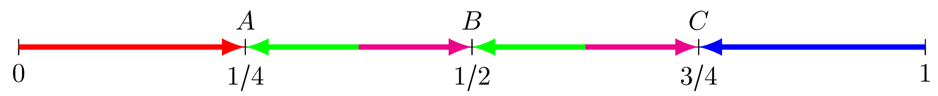
2.4

In the scenario with firms placed at 0, 1/2 and 1, the firms divide the market into these intervals, where the arrows point to a consumer's nearest firm:



The four intervals are of equal length, so the average travel distance for the entire linear city is the same as the average travel distance for one segment; $\frac{1}{4} \div 2 = \frac{1}{8}$.

In the other scenario with firms at 1/4, 1/2 and 3/4, the market is divided like this:



The average travel distance for a consumer on the larger end arrows is the same as above, $\frac{1}{8}$, and the average distance on the shorter inner arrows is $\frac{1}{8} \div 2 = \frac{1}{16}$. Since half the market is covered with short arrows and half with long arrows, the overall average travel distance is the mean of the two, $\frac{3}{32} = 0.9375$. Therefore this is the socially desirable configuration.

Question 3: Rent Seeking for the National Broadband Network

3.1

Telstra's expected profit $\mathbb{E}[\pi_T(r_T, r_O)]$ is equal to their probability of winning times their payoff for winning, minus the rent seeking effort.

$$\mathbb{E}[\pi_T(r_T, r_O)] = p_T \times V^T - r_T = \frac{750 r_T}{r_T + r_O} - r_T$$

And doing the same for Optus:

$$\mathbb{E}[\pi_O(r_T, r_O)] = p_O \times V^O - r_O = \frac{500 r_O}{r_T + r_O} - r_T$$

3.2

To maximise the expected profits, set the first partial derivative with respect to the rent seeking effort equal to zero. For Telstra:

$$\begin{aligned}
\frac{\partial \mathbb{E}[\pi_T(r_T, r_O)]}{\partial r_T} = \frac{750(r_T + r_O) - 750 r_T}{(r_T + r_O)^2} - 1 &= 0 \\
\frac{750 r_O}{(r_T + r_O)^2} - 1 &= 0 \\
750 r_O &= r_T^2 + 2 r_T r_O + r_O^2 \\
r_T^2 + 2 r_O r_T + (r_O^2 - 750 r_O) &= 0 \\
r_T &= \frac{-2 r_O \pm \sqrt{4 r_O^2 - 4(r_O^2 - 750 r_O)}}{2} \\
r_T &= -r_O \pm 5\sqrt{30 r_O}
\end{aligned}$$

And for Optus:

$$\begin{aligned}
\frac{\partial \mathbb{E}[\pi_O(r_T, r_O)]}{\partial r_O} = \frac{500(r_T + r_O) - 500r_O}{(r_T + r_O)^2} - 1 &= 0 \\
\frac{500r_T}{(r_T + r_O)^2} - 1 &= 0 \\
500r_T &= r_T^2 + 2r_Tr_O + r_O^2 \\
r_O^2 + 2r_Tr_O + (r_T^2 - 500r_T) &= 0 \\
r_O &= \frac{-2r_T \pm \sqrt{4r_T^2 - 4(r_T^2 - 500r_T)}}{2} \\
r_O &= -r_T \pm 10\sqrt{5r_T}
\end{aligned}$$

And since both rent seeking efforts are non-negative the negative solutions can be excluded, giving

$$r_T = -r_O + 5\sqrt{30r_O}; \qquad r_O = -r_T + 10\sqrt{5r_T}.$$

3.3

Solve simultaneously to find the equilibrium rent seeking efforts:

$$r_T = -r_O + 5\sqrt{30r_O}; \qquad r_O = -r_T + 10\sqrt{5r_T}$$

$$\begin{aligned}
\implies r_T &= -(-r_T + 10\sqrt{5r_T}) + 5\sqrt{30(-r_T + 10\sqrt{5r_T})} \\
&= r_T + 10\sqrt{5r_T} + 5\sqrt{30(-r_T + 10\sqrt{5r_T})} \\
100(5r_T) &= 25 \times 30(-r_T + 10\sqrt{5r_T}) \\
5r_T &= 30\sqrt{5r_T} \\
25r_T^2 &= 30^2 \times 5r_T \\
r_T^2 - 180r_T &= 0 \\
r_T(r_T - 180) &= 0
\end{aligned}$$

So $r_T = 0$ and $r_T = 180$. Substituting these into Optus's optimal effort:

$$r_O = -r_T + 10\sqrt{5r_T} = 0 + 0 = 0$$

And:

$$r_O = -r_T + 10\sqrt{5r_T} = -180 + 10\sqrt{900} = 120$$

From this it appears that $(r_T, r_O) = (0, 0)$ and $(r_T, r_O) = (180, 120)$ are both Nash equilibria, but this is not true. Since $(r_T, r_O) = (0, 0) \implies p_T = p_O = \frac{0}{0}$, $(r_T, r_O) = (0, 0)$ *cannot* be a Nash equilibrium, as $\frac{0}{0}$ is *undefined*.

The only true Nash equilibrium is therefore $(r_T, r_O) = (180, 120)$, where Telstra is willing to make a greater rent seeking investment than Optus due to their higher valuation of the contract.

Question 4: The Impact of Omar on Drug Dealing

4.1

4.2

4.3

4.4