1a) $P((A\cup C)^c) = 1 - P(A\cup C)$
$= 1 - 0.4$
$= 0.6$

b) For any 2 events A and C;

$P(A\cup C) = P(A) + P(C) - P(A\cap C)$
$\Rightarrow P(A) = P(A\cup C) - P(C) + P(A\cap C)$

$\therefore P(A) = 0.4 - 0.2 + 0.1$
$P(A) = 0.3$

c) $P(C|A\cup B\cup C) = \dfrac{P(C\cap(A\cup B\cup C))}{P(A\cup B\cup C)}$

$C \subseteq (A\cup B\cup C) \Rightarrow C\cap(A\cup B\cup C) = C$

$\therefore P(C|A\cup B\cup C) = \dfrac{P(C)}{P(A\cup B\cup C)}$

By DeMorgan's theorem;
$A\cup B\cup C = (A^c \cap (B\cup C)^c)^c$
$= (A^c\cap((B^c\cap C^c)^c)^c)^c$
$= (A^c\cap B^c\cap C^c)^c$

$\therefore P(A\cup B\cup C) = 1 - P(A^c\cap B^c\cap C^c)$
$= 0.6$

and $P(C) = 0.2$

$\therefore P(C|A\cup B\cup C) = \dfrac{0.2}{0.6} = \dfrac{1}{3}$

d) $P(B|C^c) = \dfrac{P(B\cap C^c)}{P(C^c)}$

$P(C^c) = 1 - P(C)$
$= 0.8$

$P(B\cap C^c) = P(B) - P(B\cap C)$

$P(B\cap C) = P(B) + P(C) - P(B\cup C)$
$= 0.2$
$\therefore P(B\cap C^c) = 0.4 - 0.2 = 0.2$

and $P(B|C^c) = \dfrac{0.2}{0.8} = \dfrac{1}{4}$

2a) $P(A) = P(A\cap B^c) + P(A\cap B)$
$\Rightarrow P(A\cap B) = P(A) - P(A\cap B^c)$

$P(B) = P(A^c\cap B) + P(A\cap B)$
$\Rightarrow P(A\cap B) = P(B) - P(A^c\cap B)$

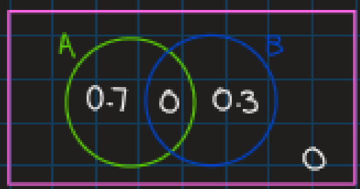
$\therefore P(A) - P(A\cap B^c) = P(B) - P(A^c\cap B)$

$P(A) - P(B) = P(A\cap B^c) - P(A^c\cap B)$

$P(A) - P(B) = 0$ iff $P(A\cap B^c) - P(A^c\cap B) = 0$

$\therefore P(A) = P(B)$ iff $P(A\cap B^c) = P(A^c\cap B)$

b) $P(A\cap B^c) = 0.7$
$P(A^c\cap B) = 0.3$
can be represented by the following venn diagram, where the numbers represent probabilities.



This clearly shows a valid sample space with sum of probabilities equal to 1 that satisfies:

- $P(A) = 0.7$ • $P(B) = 0.3$
- $P(A^c\cup B^c) = 1$

thus it is possible.

c) $P(A|B) = \dfrac{P(A\cap B)}{P(B)} = 0.7$

$\Rightarrow P(A\cap B) = 0.7\cdot P(B)$
$= 0.245$

$P(B\cap A^c) = P(B) - P(A\cap B)$
$= 0.105$

$P(A\cup B^c) = P(A) + P(B^c) - P(A\cap B^c)$
$= P(B^c) + P(A\cap B)$
$= 1 - P(B) + P(A\cap B)$
and since $P(B\cap A^c) = 0.105$
$\Rightarrow P(B) > P(A\cap B)$ so
$P(A\cup B^c) < 1$; it is not possible.

3a) Let A be the event that jar A is selected, and R be that a red ball is picked.

$P(R) = P(R|A)\cdot P(A) + P(R|A^c)\cdot P(A^c)$
since A and A^c partition Ω.

$\therefore P(R) = \frac{2}{7}\cdot\frac{1}{3} + \frac{4}{10}\cdot\frac{2}{3}$

$= \frac{38}{105} \approx 0.36$

b) The coin showing a tail is equivalent to A^c, so we want;

$P(A^c|R) = \frac{P(A^c \cap R)}{P(R)}$

$= \frac{P(R|A^c)P(A^c)}{P(R)}$

$= \frac{4/10 \cdot 2/3}{38/105}$ (using P(R) from 3a)

$= \frac{14}{19} \approx 0.74$

c) "a tail is showing" is again equivalent to event A^c.

$P(R) = \frac{38}{105}$

$P(R|A^c) = \frac{4}{10}$

$P(R) < P(R|A^c)$; so the events are positively related.

4) $P(B_1) = 1 - P(B_1^c)$
$= 1 - (\frac{1}{2})^3$
$= \frac{7}{8}$

$P(B_2) = P(\text{"0 odd"}) + P(\text{"1 odd"})$
$= \frac{1}{8} + \binom{3}{1}\cdot\frac{1}{8}$
$= \frac{1}{2}$

$P(B_3) = P(\text{"All even"}) + P(\text{"All odd"})$
$= \frac{1}{8} + \frac{1}{8}$
$= \frac{1}{4}$

4a) B_1 and B_2:

$P(B_1 \cap B_2) = P(\text{"2 or 3 even numbers"})$
$= \binom{3}{2}\frac{1}{8} + \frac{1}{8}$
$= \frac{1}{2}$

$P(B_1)\cdot P(B_2) = \frac{7}{16} \neq \frac{1}{2}$ so B_1 and B_2 are dependant.

B_1 and B_3:
$P(B_1 \cap B_3) = P(\text{"All even"})$
$= \frac{1}{8}$
$P(B_1)\cdot P(B_3) = \frac{7}{8}\cdot\frac{1}{4}$
$= \frac{7}{32} \neq \frac{1}{8}$ so B_1 and B_3 are dependant

B_2 and B_3:
$P(B_2 \cap B_3) = P(\text{"All even"})$
$= \frac{1}{8}$
$P(B_2)\cdot P(B_3) = \frac{1}{2}\cdot\frac{1}{4}$
$= \frac{1}{8}$ $\therefore$ B_2 and B_3 are independant.

b) $P(B_2) = P(\text{"0 odd"}) + P(\text{"1 odd"})$
$= \frac{1}{16} + \binom{4}{1}\frac{1}{16}$
$= \frac{5}{16}$

$P(B_3) = \frac{1}{16} + \frac{1}{16}$
$= \frac{1}{8}$

$P(B_2 \cap B_3) = P(\text{"All even"})$
$= \frac{1}{16}$

$P(B_2)\cdot P(B_3) = \frac{5}{16}\cdot\frac{1}{8}$
$= \frac{5}{128} \neq \frac{1}{16}$

$\therefore$ B_2 and B_3 are now dependant and this implies B_1, B_2 and B_3 cannot be mutually independant.

5a) let the sum of the numbers be X

$f_X(x) =$

x	2	3	4	5	6	7	8
$P(X=x)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

$$F_X(x) = \begin{cases} 0, & x < 2 \\ \frac{1}{16}, & 2 \leq x < 3 \\ \frac{3}{16}, & 3 \leq x < 4 \\ \frac{6}{16}, & 4 \leq x < 5 \\ \frac{10}{16}, & 5 \leq x < 6 \\ \frac{13}{16}, & 6 \leq x < 7 \\ \frac{15}{16}, & 7 \leq x < 8 \\ 1, & 8 \leq x \end{cases}$$

b) Let the absolute value of the difference between the numbers be Y

$f_Y(y) =$

y	0	1	2	3
$P(Y=y)$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{2}{16}$

$$F_Y(y) = \begin{cases} 0, & x < 0 \\ \frac{4}{16}, & 0 \leq x < 1 \\ \frac{10}{16}, & 1 \leq x < 2 \\ \frac{14}{16}, & 2 \leq x < 3 \\ 1, & 3 \leq x \end{cases}$$