Distributions

Distribution	Probability Density Function	$\mathbb{E}(X)$	Var(X)
$X \sim \mathrm{Bi}(n,p)$	$p_X(x) = \binom{n}{x}p^x(1-p)^{n-x}$	np	$np(1-p)$
$N \sim \mathrm{G}(p)$	$p_N(n) = (1-p)^n p$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$
$X \sim \mathrm{Pn}(\alpha t)$	$p_X(x) = \frac{e^{-\alpha}(\alpha)^x}{x!}$	α	α
$X \sim \mathrm{U}(m,n)$	$p_X(x) = \frac{1}{n-m+1}$	$\frac{m+n}{2}$	$\frac{1}{12}((n-m+1)^2-1)$
$X \sim \mathrm{R}(a,b)$	$f_X(x) = \frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{1}{12}(b-a)^2$
$T \sim \exp(\alpha)$	$f_T(t) = \alpha e^{-\alpha t}$, for $t \geq 0$	$\frac{1}{\alpha}$	$\frac{1}{\alpha^2}$
$T \sim \gamma(r,\alpha)$	$f_T(t) = \frac{\alpha^r t^{r-1} e^{-\alpha t}}{\Gamma(r)}$, $t > 0$	$\frac{r}{\alpha}$	$\frac{r}{\alpha^2}$
$X \sim \mathrm{Pareto}(\alpha,\gamma)$	$f_X(x) = \frac{\gamma\alpha^\gamma}{x^{\gamma+1}}$, for $\alpha \leq x < \infty$	$\frac{\gamma\alpha}{\gamma-1}$, for $\gamma > 1$	$\frac{\gamma\alpha^2}{(\gamma-1)^2(\gamma-2)}$, for $\gamma > 2$
$X \sim \mathrm{N}(\mu,\sigma^2)$	$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$, for $x \in \mathbb{R}$	μ	σ^2
$X \sim \mathrm{C}(m,a)$	$f_X(x) = \frac{1}{\pi}\frac{a}{a^2+(x-m)^2}$, for $-\infty < x < \infty$	Undefined	Undefined
$Y \sim \mathrm{LN}(\mu,\sigma^2)$	$f_Y(y) = \frac{1}{\sqrt{2\pi}\sigma y}e^{-\frac{(\ln y-\mu)^2}{2\sigma^2}}$, for $y > 0$	$e^{r\mu+\frac{1}{2}r^2\sigma^2}$, $r \geq 0$	$e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$

Confidence Intervals

Estimating Means

For large r, $t_r \longrightarrow \mathrm{N}(0,1)$ is a good approximation.

Normal, Single Mean, Known σ

$\left(\bar{x} \pm c\frac{\sigma}{\sqrt{n}}\right)$; with $c = \mathrm{F}^{-1}(1-\frac{\alpha}{2})$ from pivot $\mathrm{N}(0,1)$.

Normal, Single Mean, Unknown σ

$\left(\bar{x} \pm c\frac{s}{\sqrt{n}}\right)$; with $c = \mathrm{F}^{-1}(1-\frac{\alpha}{2})$ from pivot t_{n-1}.

Normal, Two Means, Two Known σ's

$\left((\bar{x}-\bar{y}) \pm c\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}}\right)$; with c from pivot $\mathrm{N}(0,1)$.

Normal, Two Means, Unknown σ's, Many Samples

$\left((\bar{x}-\bar{y}) \pm c\sqrt{\frac{s_X^2}{n}+\frac{s_Y^2}{m}}\right)$; with c from pivot $\mathrm{N}(0,1)$.

Normal, Two Means, Unknown σ's, Common Variance

$\left((\bar{x}-\bar{y}) \pm c \cdot s_P\sqrt{\frac{1}{n}+\frac{1}{m}}\right)$; with c from pivot t_{n+m-2}.

$$s_P = \sqrt{\frac{(n-1)s_X^2+(m-1)s_Y^2}{n+m-2}}$$

Normal, Two Means, Unknown σ's, $\neq$ Variances

$\left((\bar{x}-\bar{y}) \pm c\sqrt{\frac{s_X^2}{n}+\frac{s_Y^2}{m}}\right)$; with c from pivot t_r.

$$r = \left(\frac{s_X^2}{n}+\frac{s_Y^2}{m}\right)^2 \Big/ \left(\frac{s_X^4}{n^2(n-1)}+\frac{s_Y^4}{m^2(m-1)}\right)$$

Normal, Paired Samples

For pairs (X_i, Y_i), let $D_i = X_i - Y_i$. $D_i \sim \mathrm{N}(\mu_D, \sigma_D^2)$.

$\left(\bar{d} \pm c\frac{s_d}{\sqrt{n}}\right)$; with c from pivot t_{n-1}.

Estimating Variance

Normal, Single Variance - estimate of σ^2 not σ!

$\left(\frac{(n-1)s^2}{b}, \frac{(n-1)s^2}{a}\right)$; with a, b from pivot χ^2_{n-1}.

Normal, Two Variances

A confidence interval for the ratio of the variances σ_X^2/σ_Y^2 is $\left(a \cdot \frac{s_x^2}{s_y^2}, b \cdot \frac{s_x^2}{s_y^2}\right)$; with a, b from pivot $\mathrm{F}_{m-1,n-1}$.

Estimating Proportions

Single Proportion

$\approx \left(\hat{p} \pm c\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$; with c from pivot $\mathrm{N}(0,1)$.

Two Proportions

$\approx \left(\hat{p}_1 - \hat{p}_2 \pm c\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n}+\frac{\hat{p}_2(1-\hat{p}_2)}{n}}\right)$; c from pivot $\mathrm{N}(0,1)$.

Prediction Intervals

Let X^* be a future realisation of X.

If $X \sim \mathrm{N}(\mu,\sigma^2), \bar{X} \sim \mathrm{N}(\mu, \frac{\sigma^2}{n})$, and $(\bar{X}-X^*) \sim \mathrm{N}(\mu, \sigma^2+\frac{\sigma^2}{n})$.

$\left(\bar{x} \pm c\sqrt{s^2+\frac{s^2}{n}}\right)$ is a prediction interval with c from t_{n-1}.

If σ is known, use pivot $\mathrm{N}(0,1)$. If μ is known use χ^2_{n-1}.

Other Formulae

Sample Variance

$s^2 = \frac{1}{n-1}\sum_{i=1}^n (x_i-\bar{x})^2 = \frac{1}{n-1}\left(\left(\sum_{i=1}^n x_i^2\right) - n\bar{x}^2\right)$