### **Distributions**

| Distribution                                   | Probability Density Function  | $\mathbb{E}(X)$                                    | $\mathbf{Var}(X)$   |
|--|---|--|---|
| $X \sim \mathrm{Bi}(n,p)$                      | $p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$   | np   | np(1-p)   |
| $N \sim G(p)$                                  | $p_N(n) = (1-p)^n p$  | $\frac{1-p}{p}$                                    | $\frac{1-p}{p^2}$   |
| $X \sim \operatorname{Pn}(\alpha t)$           | $p_X(x) = \frac{e^{-\alpha}(\alpha)^x}{x!}$   | $\alpha$   | $\alpha$  |
| $X \sim \mathrm{U}(m,n)$                       | $p_X(x) = \frac{1}{n - m + 1}$  | $\frac{m+n}{2}$                                    | $\frac{1}{12}((n-m+1)^2-1)$   |
| $X \sim \mathrm{R}(a,b)$                       | $f_X(x) = \frac{1}{b-a}$  | $\frac{a+b}{2}$                                    | $\frac{1}{12}(b-a)^2$   |
| $T \sim \exp(\alpha)$                          | $f_T(t) = \alpha e^{-\alpha t}$ , for $t \ge 0$   | $\frac{1}{\alpha}$                                 | $\frac{1}{\alpha^2}$  |
| $T \sim \gamma(r, \alpha)$                     | $f_T(t) = \frac{\alpha^r t^{r-1} e^{-\alpha t}}{\Gamma(r)}, \ t > 0$                                    | $\frac{r}{\alpha}$                                 | $\frac{r}{\alpha^2}$  |
| $X \sim \operatorname{Pareto}(\alpha, \gamma)$ | $f_X(x) = \frac{\gamma \alpha^{\gamma}}{x^{\gamma+1}}$ , for $\alpha \le x < \infty$                    | $\frac{\gamma\alpha}{\gamma-1}$ , for $\gamma > 1$ | $\frac{\gamma \alpha^2}{(\gamma-1)^2(\gamma-2)}$ , for $\gamma > 2$ |
| $X \sim N(\mu, \sigma^2)$                      | $f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, \text{ for } x \in \mathbb{R}$ | $\mu$  | $\sigma^2$  |
| $X \sim \mathrm{C}(m,a)$                       | $f_X(x) = \frac{1}{\pi} \frac{a}{a^2 + (x - m)^2}$ , for $-\infty < x < \infty$                         | Undefined  | Undefined   |
| $Y \sim \text{LN}(\mu, \sigma^2)$              | $f_Y(y) = \frac{1}{\sqrt{2\pi}\sigma y} e^{-\frac{(\ln y - \mu)^2}{2\sigma^2}}, \text{ for } y > 0$     | $e^{r\mu + \frac{1}{2}r^2\sigma^2}, \ r \ge 0$     | $e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$                                 |

## Confidence Intervals

### **Estimating Means**

Normal, Single Mean, Known  $\sigma$ 

$$\left(\bar{x} \pm c \frac{\sigma}{\sqrt{n}}\right)$$
; with  $c = F^{-1}(1 - \frac{\alpha}{2})$  from pivot N(0, 1). Normal, Single Mean, Unknown  $\sigma$ 

$$\left(\bar{x} \pm c \frac{s}{\sqrt{n}}\right)$$
; with  $c = F^{-1}(1 - \frac{\alpha}{2})$  from pivot  $t_{n-1}$ .

$$\left((\bar{x}-\bar{y})\pm c\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}}\right)$$
; with  $c$  from pivot N(0,1).

Normal, Two Means, Two Known 
$$\sigma$$
's  $\left((\bar{x} - \bar{y}) \pm c\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}\right)$ ; with  $c$  from pivot N(0,1).  
Normal, Two Means, Unknown  $\sigma$ 's, Many Samples  $\left((\bar{x} - \bar{y}) \pm c\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}\right)$ ; with  $c$  from pivot N(0,1).

Normal, Two Means, Unknown  $\sigma$ 's, Common Variance

Normal, Two Means, Unknown 
$$\sigma$$
 s, Common val
$$\left((\bar{x} - \bar{y}) \pm c \cdot s_P \sqrt{\frac{1}{n} + \frac{1}{m}}\right); \text{ with } c \text{ from pivot } t_{n+m-2}.$$

$$s_P = \sqrt{\frac{(n-1)s_X^2 + (m-1)s_Y^2}{n+m-2}}$$
Normal, Two Means, Unknown  $\sigma$ 's,  $\neq$  Variances

Normal, Two Means, Unknown  $\sigma$ 's,  $\neq$  Variances

$$\left((\bar{x} - \bar{y}) \pm c\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}\right); \text{ with } c \text{ from pivot } t_r.$$

$$r = \left(\frac{s_X^2}{n} + \frac{s_Y^2}{m}\right)^2 / \left(\frac{s_X^4}{n^2(n-1)} + \frac{s_Y^4}{m^2(m-1)}\right)$$
**Normal, Paired Samples**
For pairs  $(X_i, Y_i)$ , let  $D_i = X_i - Y_i$ .  $D_i \sim \mathrm{N}(\mu_D, \sigma_D^2)$ .

For pairs 
$$(X_i, Y_i)$$
, let  $D_i = X_i - Y_i$ .  $D_i \sim N(\mu_D, \sigma_D^2)$   
 $(\bar{d} + c \frac{s_d}{2})$ ; with c from pivot  $t$ 

# $\left(\bar{d} \pm c \frac{s_d}{\sqrt{n}}\right)$ ; with c from pivot $t_{n-1}$ .

### **Estimating Variance**

Normal, Single Variance

$$\left(\frac{(n-1)s^2}{b}, \frac{(n-1)s^2}{a}\right)$$
; with  $a, b$  from pivot  $\chi^2_{n-1}$ .

Normal, Two Variances

A confidence interval for the ratio of the variances  $\sigma_X^2/\sigma_Y^2$  is  $\left(a \cdot \frac{s_x^2}{s_u^2}, b \cdot \frac{s_x^2}{s_u^2}\right)$ ; with a, b from pivot  $\mathbf{F}_{m-1, n-1}$ .

### **Estimating Proportions**

Single Proportion

$$\approx \left(\hat{p} \pm c\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$$
; with  $c$  from pivot N(0,1).

Two Proportions

$$\approx \left(\hat{p_1} - \hat{p_2} \pm c\sqrt{\frac{\hat{p_1}(1-\hat{p_1})}{n} + \frac{\hat{p_2}(1-\hat{p_2})}{n}}\right); c \text{ from pivot } N(0,1).$$

## Other Formulae

Sample Variance

$$s^{2} = \frac{1}{n-1} \sum_{i=1}^{n} (x_{i} - \bar{x})^{2} = \frac{1}{n-1} \left( \left( \sum_{i=1}^{n} x_{i}^{2} \right) - n\bar{x}^{2} \right)$$