S T Q Q S S D ___/___

5) Continuação 2:

→ Logo, temos que:

$$\iint_D F = 180\left[\underbrace{\left(\frac{-\cos(\theta)\sin^3(\theta)}{4}\right)_0^{\pi}}_{=0} + \frac{3}{4}\cdot\frac{\pi}{2}\right]$$

$$\Rightarrow 180\cdot\frac{3\pi}{8} \Rightarrow \frac{135}{2}$$

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5) Continuação

$$\Rightarrow 3\int_0^{\pi}\left[\int_{2\sin(\theta)}^{4\sin(\theta)} r^3\,dr\right]d\theta \Rightarrow 3\int_0^{\pi}\left[\frac{r^4}{4}\right]\Bigg|_{2\sin(\theta)}^{4\sin(\theta)}$$

$$\Rightarrow 3\int_0^{\pi}\frac{(4\sin(\theta))^4}{4}-\frac{(2\sin(\theta))^4}{4}\,d\theta$$

$$\Rightarrow \frac{3}{4}\int_0^{\pi}240\sin^4(\theta)-16\sin^4(\theta)\,d\theta$$

$$\Rightarrow 180\int_0^{\pi}\sin^4(\theta)\,d\theta \Rightarrow 180\left[\left(\frac{-\cos(\theta)\sin^3(\theta)}{4}+\frac{3}{4}\right)_0^{\pi}\cdot\int_0^{\pi}\sin^2(\theta)\,d\theta\right]$$

$$\Rightarrow \int_0^{\pi}\sin^2(\theta)\,d\theta \Rightarrow \int_0^{\pi}\frac{1-\cos(2\theta)}{2}\,d\theta \Rightarrow \frac{1}{2}\int_0^{\pi}1-\cos(2\theta)\,d\theta$$

$$\Rightarrow \frac{1}{2}\left[\int_0^{\pi}1\,d\theta-\int_0^{\pi}\cos(2\theta)\,d\theta\right] \Rightarrow \frac{1}{2}\left[\pi-0\right] \Rightarrow \frac{\pi}{2}$$

π

$u=2x$, $dx=\frac{1}{2}du$, $\frac{du}{dx}=2$

$du=2dx$

Então: $\frac{1}{2}\int_0^{2\pi}\cos(u)\,du=\frac{1}{2}\left[\sin(u)\right]\Big|_0^{2\pi}=0$

S T Q Q S S D

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NOME: LUCAS LOBATO DA SILVA AMORIM

1) Letra E

2) Letra C

3) Letra B

4) Parametrizando a curva temos que:

$$\begin{cases} x = r\cos(t) \\ y = r\sin(t) \end{cases}, \ r = 1, \quad \begin{cases} x = \cos(t) \\ y = \sin(t) \end{cases}, \ t \in [0, 2\pi]$$

-> Calculando a interseção da curva:

$$\begin{cases} x^2 - y^2 - z = 0 \\ z = 2 - x^2 - y^2 \end{cases}, \quad \begin{aligned} x^2 + y^2 - (2 - x^2 - y^2) &= 0 \\ 2x^2 + 2y^2 - 2 &= 0 \\ x^2 + y^2 &= 1 \end{aligned}$$

-> Logo $\begin{cases} x^2 + y^2 = 1 \\ z = 2 - x^2 - y^2 \end{cases}$

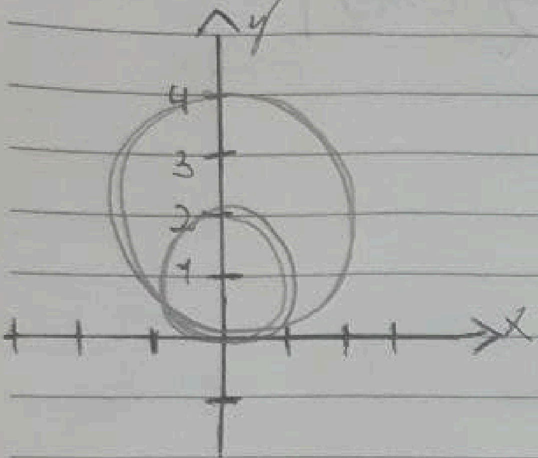
*Com isso temos que:

$$\gamma(t) = (\cos(t), \sin(t), 2 - (\cos^2(t) + \sin^2(t)))$$

$$\gamma'(t) = (-\sin(t), \cos(t), 0)$$

$$F(\gamma) = (2\cos(t)\sin(t), 2\cos^2(t) + 2\sin(t), 1)(-\sin(t), \cos(t), 0)$$

5) $C = \gamma_1 \cup \gamma_2$, completando quadrados, temos: $(x^2+(y-1)^2)=1$ e $x^2+(y-2)^2=4$, que são círculos orientados positivamente.



-> Sejam $F_1 = e^{x^2} - y^3$ e $F_2 = e^{y^2} + x^3$ os componentes do campo, temos que:

$$\frac{\partial F_1}{\partial y} = -3y^2 \quad e \quad \frac{\partial F_2}{\partial x} = 3x^2$$

$$\Rightarrow \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 3(x^2+y^2)$$

(i) Aplicando o Teorema de Green temos:

$$\int_C F = \int_{\gamma_1} F + \int_{\gamma_2} F \Rightarrow \iint_D \left[\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right] dx\,dy$$

$$\Rightarrow \iint_D 3\left[x^2+y^2\right] dx\,dy, \quad D = \{(x,y) \,/\, 2y \leq x^2+y^2 \leq 4y\}$$

(ii) Aplicando coordenadas polares temos:

$2\sin(\theta) \leq r \leq 4\sin(\theta)$ e $0 \leq \theta \leq \pi$, então:

$$\iint_D 3\left[x^2+y^2\right] dx\,dy = 3\int_0^{\pi} \left[\int_{2\sin(\theta)}^{4\sin(\theta)} r^2 - r \, dx\right] d(\theta)$$

S T Q Q S S D

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4) Continuação

$$\int_{c} \vec{F}\, d\vec{r} \Rightarrow \int_{0}^{2\pi} F(\gamma)\, dt \Rightarrow \int_{0}^{2\pi} -2\cos(t)\sin^2(t) - 2\cos(t)\sin(t) + \cos^3(t) + 2\sin(t)\cos(t)\, dt$$

$$\Rightarrow \int_{0}^{2\pi} -2\cos(t)\sin^2(t) + \cos^3 t\, dt$$

$$\Rightarrow \int_{0}^{2\pi} -\cos(t)\, dt + \int_{0}^{2\pi} 3\cos^3(t)\, dt$$

$$\Rightarrow -\sin(t)\Big|_{0}^{2\pi} + \frac{9\sin(t)}{4}\Big|_{0}^{2\pi} + \frac{1}{4}\sin(3t)\Big|_{0}^{2\pi}$$

$$\Rightarrow 0 + 0 + 0$$

$$\Rightarrow 0 \;//$$