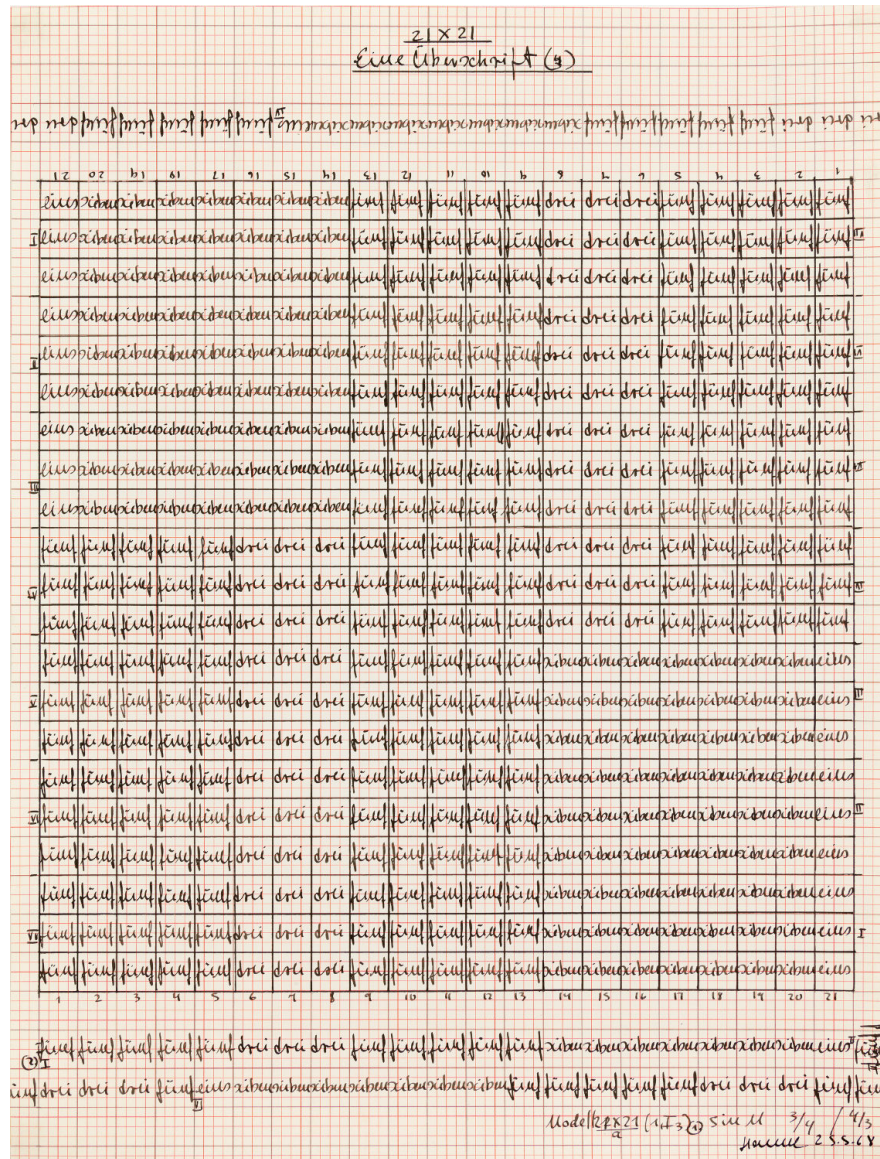# Six Functor Formalisms

Kaif Hilman[*], Lucas Piessevaux[†], Qi Zhu[‡]

WiSe 25/26

Hanne Darboven, *Eine Überschrift*, 1968

[*]kaif@mpim-bonn.mpg.de
[†]lucas@math.uni-bonn.de
[‡]qzhu@mpim-bonn.mpg.de

Schedule

The seminar meets at TBD on TBD in Room TBD. The lecture notes from Juan Esteban Rodríguez Camargo [Cam25, Part II] might be useful for the Heyer-Mann talks.

Date		Topic	Speaker
TBD	1	Correspondences and Six-Functor Formalisms	TBD
TBD	2	Category of Kernels I	TBD
TBD	3	Category of Kernels II	TBD
TBD	4	Six-Functor Formalism on Condensed Anima	TBD
TBD	5	Applications to Representation Theory	TBD
TBD	6	Sheaves and Cosheaves	TBD
TBD	7	Verdier Duality	TBD
TBD	8	Six-Functor Formalism on Topological Spaces	TBD
TBD	9	Analytic Manifolds	TBD
TBD	10	Étale sheaves on Light Condensed Anima	TBD
TBD	11	Six-Functor Formalism for Étale sheaves	TBD
TBD	12	Duality for p-adic Lie Groups	TBD
TBD	13	Chromatic Applications	TBD

Schedule of talks

Syllabus

Talk 1: Correspondences and Six-Functor Formalisms

Motivate and define six-functor formalisms. Discuss how to construct six-functor formalisms from a suitable decomposition (I, P) of the allowed forward maps E and discuss how to extend six functor formalisms to a stacky setup. You should omit the example on condensed anima which will be discussed in Talk 4 but feel free to mention other examples.
References: [HM24, Sections 1–3]

Talk 2: Category of Kernels I

Define the category of kernels and introduce suave and prim maps and objects. Discuss the relation to duality.
References: [HM24, Sections 4.1–4.5]

Talk 3: Category of Kernels II

Discuss the notion of étale and proper maps in a six-functor formalism as a continuation of the discussion on suave and prim maps. Discuss exceptional descent, taking time to cover the notion of descendability following Mathew and illustrate this notion with some classical examples.
References: [HM24, Sections 4.6–4.8], [Mat16].

Talk 4: Six-Functor Formalism on Condensed Anima

Give a crash course introduction to condensed mathematics and then construct a six functor formalism on condensed anima. Discuss this gadget using the material from the previous talks and give applications to topological spaces.
References: [HM24, Sections 3.5, 4.8]

Talk 5: Applications to Representation Theory

Discuss how the six functor formalisms (particularly the one on condensed anima) can be applied to smooth representation theory.
References: [HM24, Section 5]

Talks 6: Sheaves and Cosheaves

Set up the background material on tensor products of cocomplete ∞-categories and the notion of (co)sheaves. Discuss relative shape theory for topoi and how this can be used to state and prove a smooth projection formula for shape submersions.
References: [Vol21, Sections 2–3]

Talks 7: Verdier Duality

Discuss the localisation sequences of categories of sheaves associated to open-closed decompositions, and how these give rise to recollements. Proceed to discuss Verdier duality for sheaves by introducing the notion of $\mathcal{K}$-sheaves
References: [Vol21, Sections 4–5]

Talks 8: Six-Functor Formalism on Topological Spaces

Bring together the previous results by setting up the full six-functor-formalism of sheaves on topological spaces and discuss the usual formulæ: base change, projection, and Künneth. Use this to set up a J-homomorphism and prove relative Atiyah duality.
References: [Vol21, Sections 6–7]

Talk 9: Analytic Manifolds

Discuss some preliminaries on the theory of analytic manifolds, culminating in the construction of the deformation to the tangent bundle.
References: [Cla25, Sections 2–3]

Talks 10: Étale Sheaves on Light Condensed Anima

Discuss the theory of étale sheaves on light condensed anima, and relate it to the theory of sheaves on analytic manifolds. Discuss (relative) uniform pro-p-groups and their classifying stacks.
References: [Cla25, Sections 4–5]

Talks 11: Six Functors for Étale Sheaves

Set up the six functor formalism for étale sheaves on light condensed anima as an application of [HM24]. Proceed to characterise smooth morphisms of interest to us as well as the theory of *paths*.
References: [Cla25, Sections 6–9]

Talks 12: Duality for p-adic Lie Groups

Reap the fruits of the previous talks by setting up the theory of dualising objects for p-adic Lie groups. Discuss the more general framework for setting up J-homomorphisms in general six functor formalisms.
References: [Cla25, Sections 10–11]

Talks 13: Chromatic Applications

Discuss the applications of the previously established duality and linearisation results in chromatic homotopy theory.
References: [Cla25, Sections 12–14]

References

[Cam25]   Juan Esteban Rodríguez Camargo. *Notes on D-modules via derived algebraic stacks*. 2025. URL: https://drive.google.com/file/d/1HyZ3u7_Zq_nzrcn56BNZSqrdb8FJjzh7/view.

[Cla25]   Dustin Clausen. "Duality and linearization for p-adic lie groups". *arXiv preprint arXiv:2506.18174* (2025).

[HM24]   Claudius Heyer and Lucas Mann. "6-Functor Formalisms and Smooth Representations". *arXiv preprint arXiv:2410.13038* (2024).

[Mat16]   Akhil Mathew. "The Galois group of a stable homotopy theory". *Advances in Mathematics* 291 (2016), pp. 403–541.

[Vol21]   Marco Volpe. "The six operations in topology". *arXiv preprint arXiv:2110.10212* (2021).