

ECE 6554 Adaptive Control Project Ducted Fan

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I. Introduction

The planar ducted fan is an example of a flying machine employing vectored thrust. A fan or other thrust generator sends air out of an orifice past to flaps. These flaps, moving identically, redirect the air. This system has been study using the equation of the center of the mass q, and of θ the orientation of the ducted fan. The aim of the study is to control this planar ducted fan and to compare performances to track a point (or a trajectory) between a linear and nonlinear controller but also between the static and the adaptive controller. This report is organized as follows: In section II we describe the equations of motion of the ducted fan. These equations will be linearized in section III in order to implement a static and adaptive linear controller. In section IV we rearrange the equation to implement a static nonlinear controller. This controller will be augmented to create an adaptive nonlinear controller in the section V. Throughout these sections, we will compare the performance of linear and nonlinear controllers, particularly in cases of model mismatch. Finally, we conclude this report in section VI.

II. Equations of motion

The model of the ducted fan is represented in Figure 1 with $f_1 = \tau * cos(\psi)$ and $f_2 = \tau * sin(\psi)$. This system is represented with $q = (x \ y)^T$ the center of the mass and θ the orientation. This gives the equation of motions:

$$m\ddot{q} = -d\dot{q} + R(\theta)e_2(-\psi)\tau - m\ddot{g}$$

$$J\ddot{\theta} = -r\sin(\psi)\tau$$

Where $R(\theta) = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$ is a planar rotation matrix.

 $e_2(.)$ is the second column of the rotation matrix.

And τ and ψ are the thrust and the flap angle respectively which control the system.

The constants are defined as:

- $m = 4.25 \, kg$
- $d = 0.1 \, kg/sec$
- r = 0.26 m
- $J = 0.0475 \, kg \, m^2$
- $\vec{g} = (0 \ g)^T$ and $g = 9.8 \ m/sec$

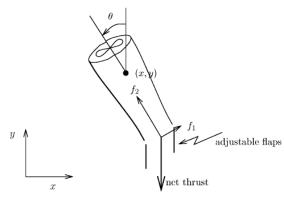


Figure 1: Planar ducted fan model

The state will be composed of

$$X = [x \quad y \quad \theta \quad \dot{x} \quad \dot{y} \quad \dot{\theta}]^T$$

III. Linear Controller

1) Linear Feedback Controller

For implementing this type of controller as shown in Figure 2, we need to linearize the



system about hover at $\theta = 0$ in order the avoid the nonlinear part.

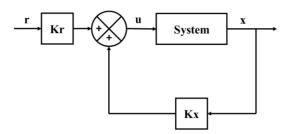


Figure 2: Block Diagram Linear Feedback Controller

The new equations of motion are $(\cos(\theta) = 1$ and $\sin(\theta) = \theta$):

$$\ddot{q} = -\frac{d}{m}\dot{q} + \frac{\tau}{m} \left[\frac{\sin(\psi) - \theta\cos(\psi)}{\theta\sin(\psi) + \cos(\psi)} \right]$$
$$\ddot{\theta} = -\frac{r}{J}\sin(\psi)\tau$$

We define f equal to the right part of these equations.

With the control $u = (\tau sin(\psi) \quad \tau cos(\psi))^T$, we get the state space equation:

$$\dot{X} = AX + Bu$$

With

$$A_{lin} = \frac{\partial f}{\partial X}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{\tau_0}{m}cos(\psi_0) & -\frac{d}{m} & 0 & 0 \\ 0 & 0 & \frac{\tau_0}{m}sin(\psi_0) & 0 & -\frac{d}{m} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_{lin} = \frac{\partial f}{\partial u} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{m} & \frac{-\theta_0}{m} \\ \frac{-\theta_0}{m} & \frac{1}{m} \\ \frac{-r}{J} & 0 \end{bmatrix}$$

The reference model is:

$$\dot{X}_m = A_m X_m + B_m r$$

With
$$A_m = A_{lin} - B_{lin} * K_{LOR}$$
 and $B_m = B_{lin}$

With K_{LQR} which come from the care() function of Matlab, this assure that A_m is Hurwitz and that the reference model is stable.

The controller is defined as
$$u = K_X^T X + K_r^T r$$
. With $K_X^T = B_{lin}^{-1} (A_m - A_{lin})$ And $K_r^T = B_{lin}^{-1} B_m$

Throughout this report we will study and compare the response of the controller for a reference signal equal to [3; 4] and to a trajectory equal to $[5 * cos(\frac{t}{10}); 5 * sin(\frac{t}{10})]$.

For the r = [3; 4], the result of the simulation is given below, the system succeeds to track the reference model and reach the goal of [3; 4] (the error in Figure 6 is scale at 10^{-8}).

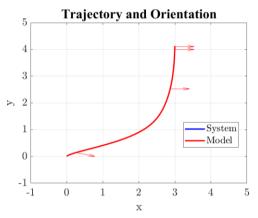


Figure 3: Trajectory and orientation of the center of mass for a Linear Feedback Controller with r=[3;4]

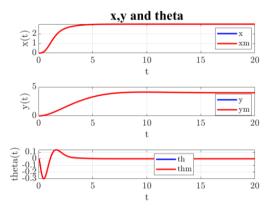


Figure 4: State for a Linear Feedback Controller with r=[3;4]



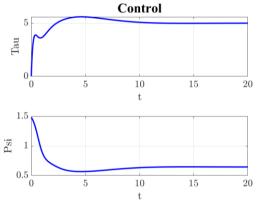


Figure 5: Control for a Linear Feeback Controller with r=[3;4]

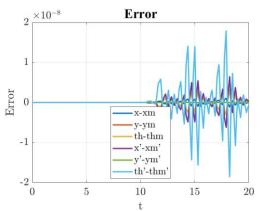


Figure 6: Error for a Linear Feedback Controller with r=[3:4]

With $r = [5 * cos(\frac{t}{10}); 5 * sin(\frac{t}{10})]$, the ducted fan must follow an ellipse. It still perfectly follows the model as shown in the Figure 7 (again the error is scale at 10^{-15}).

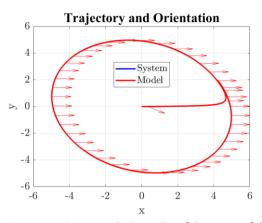


Figure 7: Trajectory and orientation of the center of the mass for a Linear Feedback Controller with $r=[5\cos(t/10);5\sin(t/10)]$

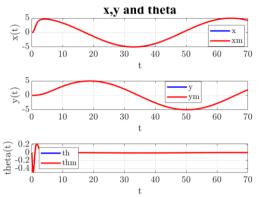


Figure 8: State for a Linear Feedback Controller with $r=[5\cos(t/10);5\sin(t/10)]$

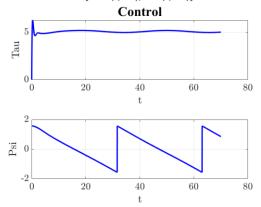


Figure 9: Control for a Linear Feedback Controller with $r=[5\cos(t/10);5\sin(t/10)]$

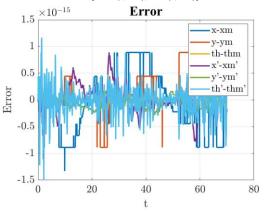


Figure 10: Error for a Linear Feedback Controller with $r=[5\cos(t/10);5\sin(t/10)]$

2) Linear Adaptive Controller

The implementation of this controller is shown in the figure below.



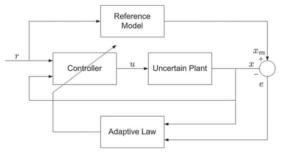


Figure 11: Block Diagram Linear Adaptive Controller

To implement this linear adaptive controller, we use the same reference model and controller, but we add the adaptive laws below.

$$\dot{K}_X = -\Gamma_X X(t) e^T(t) P B_{lin}$$
$$\dot{K}_r = -\Gamma_r r(t) e^T(t) P B_{lin}$$

With Γ_X , Γ_r some adaptive gains which will be fixed at 1 for now, $e = X - X_m$ the error and Pthe matrix which come from

$$A_m^T P + P A_m = -Q \text{ with } Q = I_6.$$

 $A_m^T P + P A_m = -Q$ with $Q = I_6$. Note that this is possible because A_m is Hurwitz.

With the adaptive controller, the system succeeds to reach the goal and to track the reference model. However, in the Figure 12 and Figure 17 the system seems to not follow better than the static controller. In Figure 13 and Figure 18, the adaptive controller creates some oscillations on y which result in less performance for the adaptive controller.

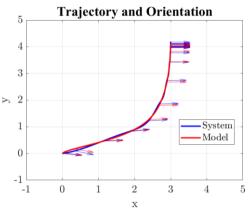


Figure 12: Trajectory and orientation of the center of mass for a Linear Adaptive Controller with r=[3;4]

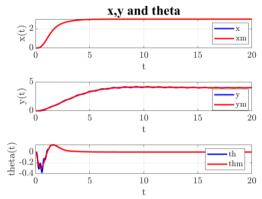


Figure 13: State for a Linear Adaptive Controller with r=[3;4]

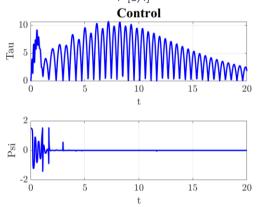


Figure 14: Control for a Linear Adaptive Controller with r=[3;4]

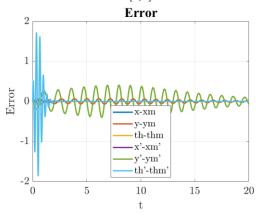


Figure 15: Error for a Linear Adaptive Controller with r=[3;4]

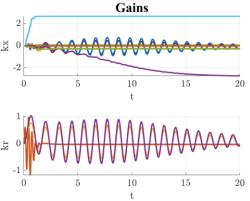


Figure 16: Gains for a Linear Adaptive Controller with r=[3;4]



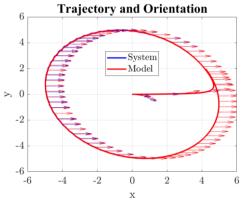


Figure 17: Trajectory and orientation of the center of the mass for a Linear Adaptive Controller with $r=[5\cos(t/10);5\sin(t/10)]$

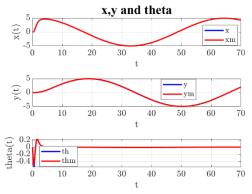


Figure 18: State for a Linear Adaptive Controller with r=[5cos(t/10);5sin(t/10)]

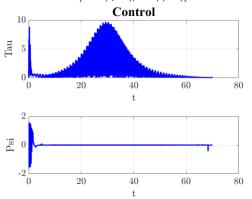


Figure 19: Control for a Linear Adaptive Controller with $r=[5\cos(t/10);5\sin(t/10)]$

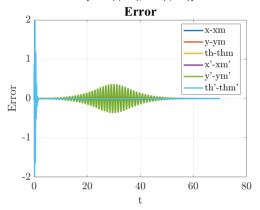


Figure 20: Error for a Linear Adaptive Controller with $r=[5\cos(t/10);5\sin(t/10)]$

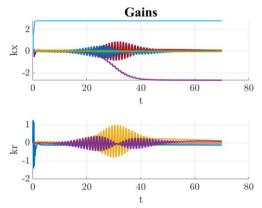


Figure 21: Gains for a Linear Adaptive Controller with $r=[5\cos(t/10);5\sin(t/10)]$

3) Use of Post-Transient Gains

In order to have a system which follows better the reference model, we re-simulate the system starting with the post-transient adaptive gains of the first simulation. With this technique we get much better results. Note that for r = [3; 4] the system still has a little error which is corrected by tuning the adaptive gains Γ_r . Indeed, with $\Gamma_r = 5$ the system tracks well the reference model. Therefore, we keep this value of Γ_r for r = [3; 4]. However, in Figure 23 and Figure 28 this post-transient technique indeed gives less oscillation in y.

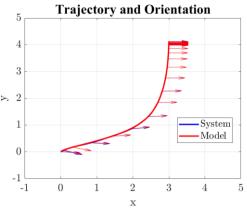


Figure 22: Trajectory and orientation of the center of mass using post-transient gains with r=[3;4]



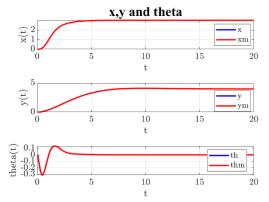


Figure 23: State using post-transient gains with r=[3;4]

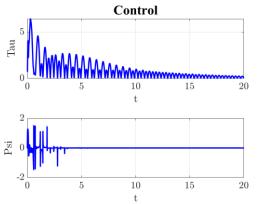


Figure 24: Control using post-transient gains with r=[3;4]

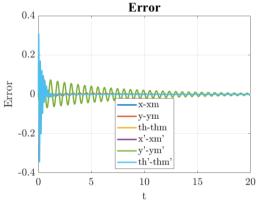


Figure 25: Error using post-transient gains with r=[3;4]

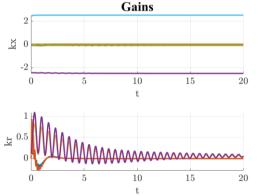


Figure 26:Gains using post-transient gains with r=[3;4]

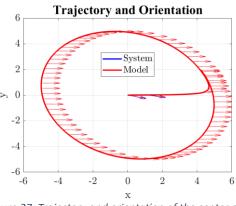


Figure 27: Trajectory and orientation of the center of the mass using post-transient gains with $r=[5\cos(t/10);5\sin(t/10)]$

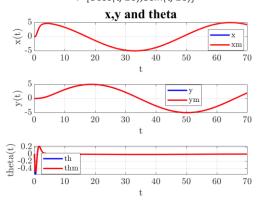


Figure 28: State using post-transient gains with $r=[5\cos(t/10);5\sin(t/10)]$

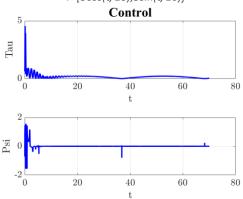


Figure 29: Control using post-transient gains with $r=[5\cos(t/10);5\sin(t/10)]$

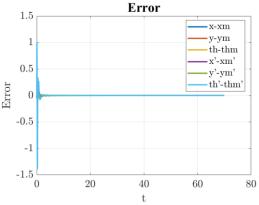


Figure 30: Error using post-transient gains with r=[5cos(t/10);5sin(t/10)]



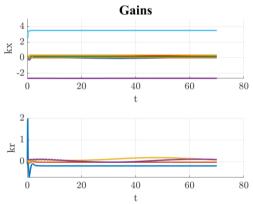


Figure 31: Gains using post-transient gains with $r=[5\cos(t/10);5\sin(t/10)]$

As expected, the gains start directly at the right value, which result in less oscillation in the gains and in the state. Now, the adaptive system follows the reference system as well as the static system.

Hence, a question could be why doing an adaptive controller? This question will be responded in the next subsection.

4) Model Mismatched

If we add some model mismatched (i.e. modify by 10-20% J, r and m), this will created unmatched condition for K_X and K_T .

For the next figure, we change the value of J, r and m to be 0.03, 0.2 and 3.6 in A_{lin} but we keep the same K_X and K_r computed before. This means that we suppose that our model on which we implement our controller was false.

In <u>belowFigure 32</u> and <u>Figure 36</u>, it is shown that now the static linear feedback controller do not succeed to track perfectly the reference model. This is confirm by the <u>Figure 33</u> and <u>Figure 37</u> on the state of the system and on <u>Figure 35</u> and <u>Figure 39</u> that the error is different of zero for both reference signal. Meanwhile, on the linear adaptive controller, it succeeds to correct this error by adapting its gains and track perfectly the reference model as well as before without the model mismatched (Note that for the adaptive controller the second simulation was used).

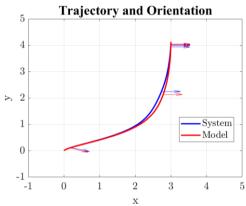


Figure 32: Trajectory and orientation of the center of mass with model mismatched for a static linear feedback controller with r=[3;4]

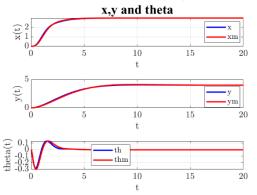


Figure 33: State with model mismatched for a static linear feedback controller with r=[3;4]

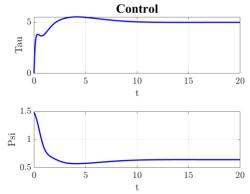


Figure 34: Control with model mismatched for a static linear feedback controller with r=[3;4]

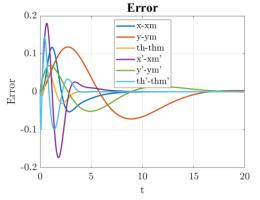


Figure 35: Error with model mismatched for a static linear feedback controller with r=[3;4]



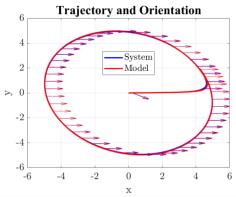


Figure 36: Trajectory and orientation of the center of the mass with model mismatched for a static linear feedback controller with $r=[5\cos(t/10);5\sin(t/10)]$

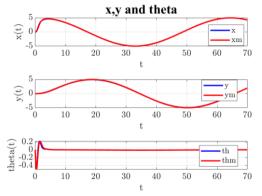


Figure 37: State with model mismatched for a static linear feedback controller with $r=[5\cos(t/10);5\sin(t/10)]$

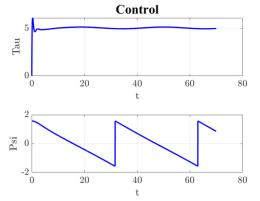


Figure 38: Control with model mismatched for a static linear feedback controller with $r=[5\cos(t/10);5\sin(t/10)]$

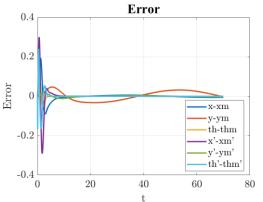


Figure 39: Error with model mismatched for a static linear feedback controller with $r=[5\cos(t/10);5\sin(t/10)]$

The figures below are for the linear adaptive controller.

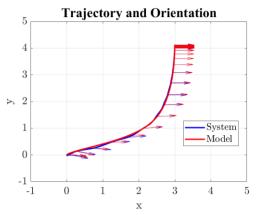


Figure 40: Trajectory and orientation of the center of mass with model mismatched for a linear adaptive controller with r=[3;4]

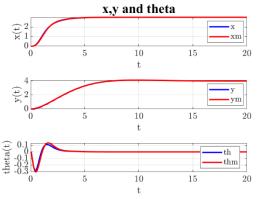


Figure 41: State with model mismatched for a linear adaptive controller with r=[3;4]

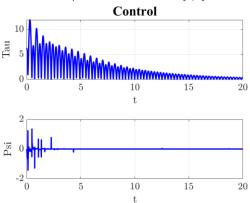


Figure 42: Control with model mismatched for a linear adaptive controller with r=[3;4]



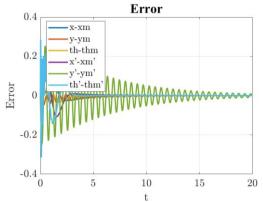


Figure 43: Error with model mismatched for a linear adaptive controller with r=[3;4]

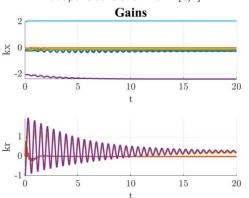


Figure 44: Gains with model mismatched for a linear adaptive controller with r=[3;4]

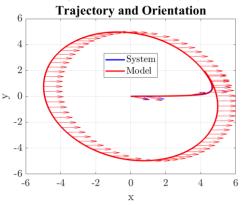


Figure 45: Trajectory and orientation of the center of the mass with model mismatched for a linear adaptive controller with r=[5cos(t/10);5sin(t/10)]

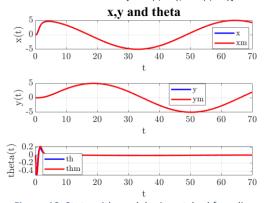


Figure 46: State with model mismatched for a linear adaptive controller with r=[5cos(t/10);5sin(t/10)]

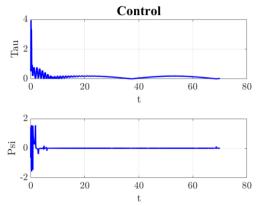


Figure 47: Control with model mismatched for a linear adaptive controller with r=[5cos(t/10);5sin(t/10)]

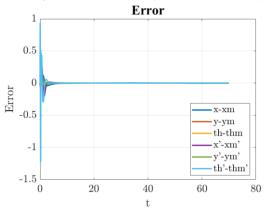


Figure 48: Error with model mismatched for a linear adaptive controller with r=[5cos(t/10);5sin(t/10)]

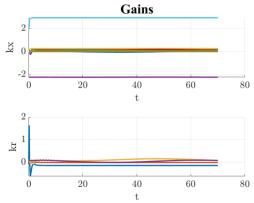


Figure 49: Gains with model mismatched for a linear adaptive controller with r=[5cos(t/10);5sin(t/10)]

With this result we can conclude for the linear part that the adaptive controller is better since it assures a good tracking even if the model of the system is not correct.



IV. Nonlinear Controller

We rearrange the equation, to take care of the nonlinear term. We have now this:

$$\dot{X} = Ax + B\left(u + \alpha^{\mathrm{T}}\Phi(x)\right)$$

With

In addition, we have now a new control:

$$u = \begin{bmatrix} \tau \sin(\psi) \\ \tau \cos(\psi) \\ 0 \end{bmatrix}$$

The nonlinear term we want to cancel is the gravitational constant g, and we have also some cosines and sinus, but they are link to the control, so we can't cancel them directly. For the reference model, we have taken the same as in the first part: $A_m = A_{lin} - B_{lin} * K_{LQR}$ and $B_m = B_{lin}$

We have now
$$u = K_X^T X + K_r^T r - \alpha^T \Phi$$
.
With $K_X^T = pinv(B) * (A_m - A_{lin})$
And $K_r^T = pinv(B) * B_m$

With a reference signal equal to [3; 4] (Figure 50, Figure 51, Figure 52), the model is tracking the reference with a very small error (about

 10^{-9}), and the ducted fan is going to the good coordinates (3, 4).

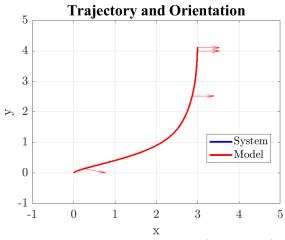


Figure 50: Trajectory and orientation of the center of mass for a Nonlinear Feedback Controller with r=[3;4]

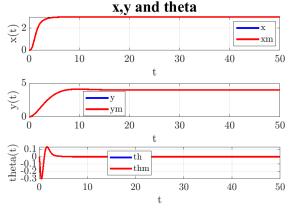


Figure 51: State for a Nonlinear Feedback Controller with r=[3;4]

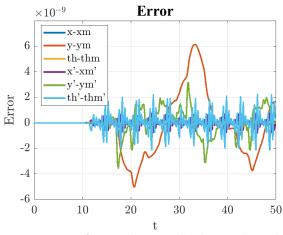


Figure 52: Error for a Nonlinear Feedback Controller with r=[3;4]

With a reference signal equal to $r = [5 * cos(\frac{t}{10}); 5 * sin(\frac{t}{10})]$, (Figure 53, Figure 54, Figure 55), we have the same pattern, it's a good



tracking, and the error is again very small (about 10^{-12}).

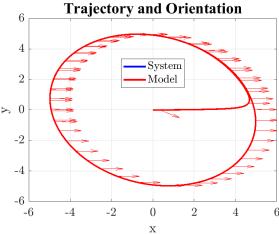


Figure 53: Trajectory and orientation of the center of mass for a Nonlinear Feedback Controller with r=[5cos(t/10);5sin(t/10)]

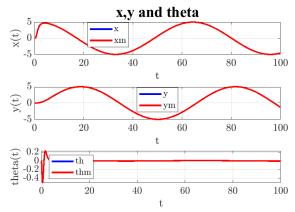


Figure 54: State for a Nonlinear Feedback Controller with $r=[5\cos(t/10);5\sin(t/10)]$

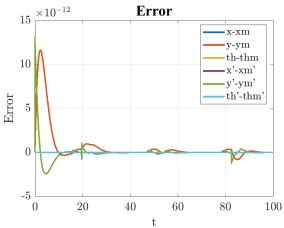


Figure 55: Error for a Nonlinear Feedback Controller with $r=[5\cos(t/10);5\sin(t/10)]$

In comparison with the linear feedback controller, the errors are globally the same, we cannot see a lot of difference and in both cases, the tracking is good.

V. NonLinear Adaptive Controller

1) New Equations

The implementation of the controller is about the same as in <u>Figure 11</u>, with just the addition of the nonlinear term. To implement adaptive controller, we use the same reference model and controller, but we add the adaptive laws below.

$$\dot{K}_X = -\Gamma_X X(t) e^T(t) PB$$

$$\dot{K}_r = -\Gamma_r r(t) e^T(t) PB$$

$$\dot{\hat{\alpha}} = -\Gamma_\alpha \Phi e^T(t) PB$$

With Γ_X , Γ_r , Γ_α some adaptive gains which will be fixed at 1 for now, $e = X - X_m$ the error and P the matrix which come from

$$A_m^T P + P A_m = -Q$$

With $Q = diag(30,1,1,30,1,1)$

Note that this Q allows to force the model to move along the x-axis, by putting more weight on x transition.

With a reference signal equal to [3; 4] (Figure 56), the reference model goes to the good coordinates, but the tracking of X with Xm is not as perfect as before. We can see in Figure 57, that the tracking of y is perfect, but there is a little error on x and theta, it means that the tracking has some little difficulties with horizontal transition. The control and gains do not diverge to infinity.

If we change the adaptive gain, we can have a better tracking such as in Figure 61, Figure 62 with $\Gamma_{X=1}$, $\Gamma_r = 10$, $\Gamma_{\alpha=10}$



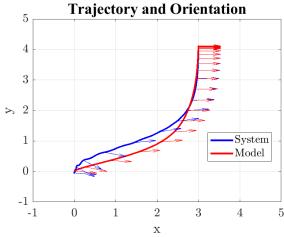


Figure 56: Trajectory and orientation of the center of mass for a Nonlinear Adaptive Controller with r=[3;4]

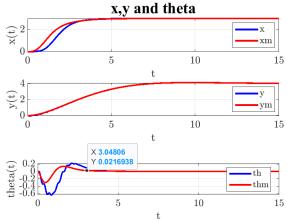


Figure 57: State for a Nonlinear Adaptive Controller with r=[3;4]

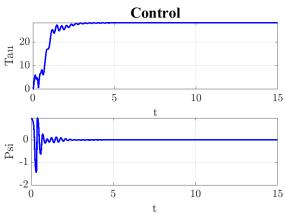


Figure 58: Control for a Nonlinear Adaptive Controller with r=[3;4]

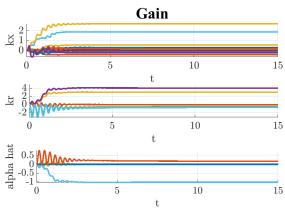


Figure 59: Gains for a Nonlinear adaptive controller with r=[3;4]

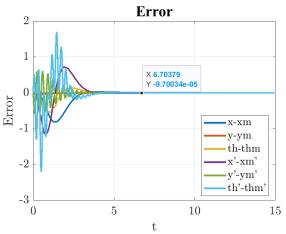


Figure 60: Error for a Nonlinear Adaptive Controller with r=[3;4]

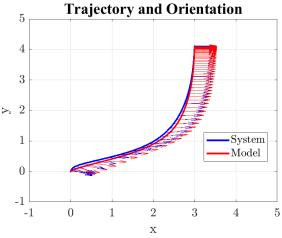


Figure 61: Trajectory and orientation of the center of mass for a Nonlinear Adaptive Controller with r=[3;4] with new adaptive gains



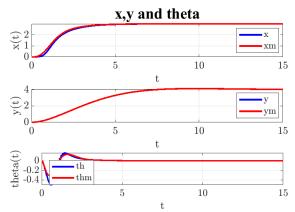


Figure 62: State for a Nonlinear Adaptive Controller with r=[3;4] with new adaptive gains

With a $r = [5 * cos(\frac{t}{10}); 5 * sin(\frac{t}{10})]$ (Figure 63), we have the same pattern, there is a little error on x and theta, and the tracking of y is perfect. Moreover, we can see that the gain on alpha diverge o infinity, it would be interesting to avoid the gain to drift.

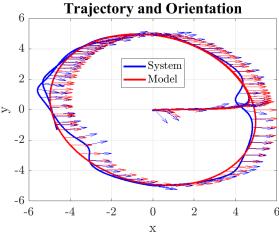


Figure 63: Trajectory and orientation of the center of mass for a Nonlinear Adaptive Controller with r=[5cos(t/10);5sin(t/10)]

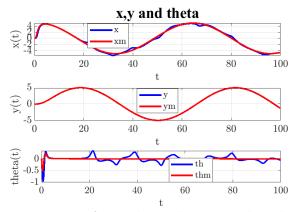


Figure 64: State for a Nonlinear Adaptive Controller with $r=[5\cos(t/10);5\sin(t/10)]$

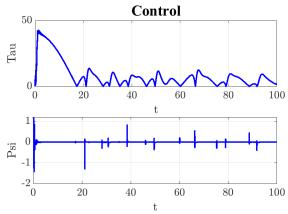


Figure 65: Control for a Nonlinear Adaptive Controller with r=[5cos(t/10);5sin(t/10)]

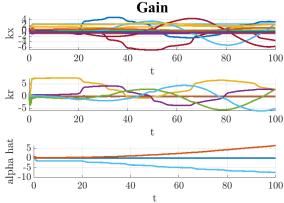


Figure 66: Gains for a Nonlinear Adaptive controller with $r=[5\cos(t/10);5\sin(t/10)]$

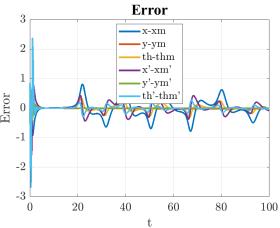


Figure 67: Error for a Nonlinear Adaptive Controller with $r=[5\cos(t/10);5\sin(t/10)]$

2) Use of Post-Transient Gains

As explained in the previous step, we can start with a better initial gain, for example, the gain at the end of the previous simulation, which can be considered as better. Let's have a look at the result.



With a reference signal equal to [3; 4], the results are much better! It can be explained by the fact that in the adaptive way, it depends a lot on the initial states. The error is really small on Figure 71 (about 0.1), in comparison with the initial gain equal to zero.

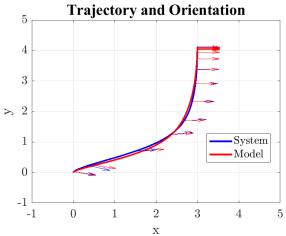


Figure 68: Trajectory and orientation of the center of mass for a Nonlinear Adaptive Controller with r=[3;4] using post-transient gains

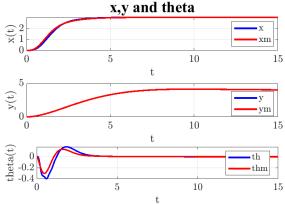


Figure 69: State for a Nonlinear Adaptive Controller with r=[3;4] using post-transient gains

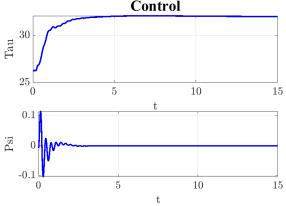


Figure 70 : Control for a Nonlinear Adaptive Controller with r=[3;4] using post-transient gains

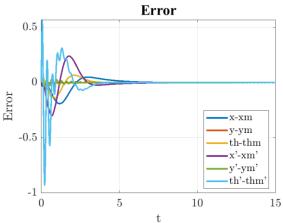


Figure 71: Error for a Nonlinear Adaptive Controller with r=[3;4] using post-transient gains

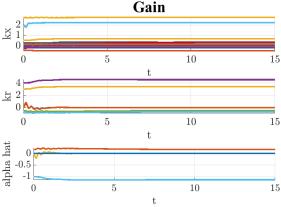


Figure 72: Gain for a Nonlinear Adaptive Controller with r=[3;4] using post-transient gains

We can observe the same pattern with a reference signal equal to $r = [5 * cos(\frac{t}{10}); 5 * sin(\frac{t}{10})]$, the tracking is much better, the error was about 0.5 Figure 67, and is now about 0.1

Moreover, the gain is not diverging to infinity (<u>Figure 76</u>), so it is better than the other experiment. We can see that the nonlinear feedback controller is better than the nonlinear adaptive controller in term of error (about

 10^{-12} in Figure 55 versus 10^{-1} in Figure 77).



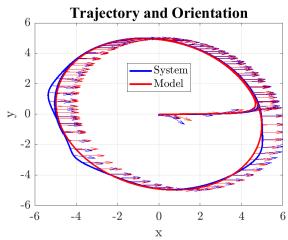


Figure 73: Trajectory and orientation of the center of mass for a Nonlinear Adaptive Controller with r=[5cos(t/10);5sin(t/10)] using post-transient gains

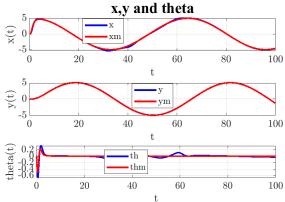


Figure 74: State for a Nonlinear Adaptive Controller with r=[5cos(t/10);5sin(t/10)] using post-transient gains

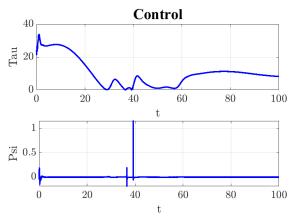


Figure 75 : Control for a Nonlinear Adaptive Controller with r=[5cos(t/10);5sin(t/10)] using post-transient gains

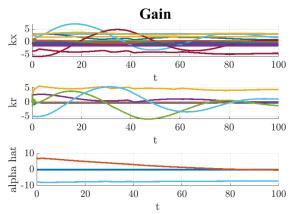


Figure 76: Gain for a Nonlinear Adaptive Controller with r=[5cos(t/10);5sin(t/10)] using post-transient gains

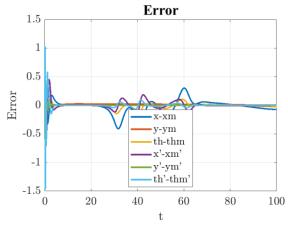


Figure 77: Error for a Nonlinear Adaptive Controller with r=[5cos(t/10);5sin(t/10)] using post-transient gains

3) Model Mismatched

As we have seen in the previous part, in reality we can have some mismatched value over some parameters such as m, J and r. Let's see an example with a reference signal equal to $r = [5*cos(\frac{t}{10}); 5*sin(\frac{t}{10})]$, m=4.4, r=0.3, J=0.04 (We change them by 10-20%).

With the static controller, we can see that the system did not track the reference as it should be (<u>Figure 78</u>). In fact, if we change r and J, it has a little influence on theta, but it's a really small influence. But when we change m, even a little, it creates big error on y (Figure 12)



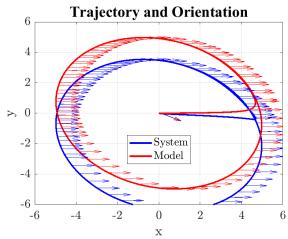


Figure 78: Trajectory and orientation of the center of mass for a Nonlinear Static Controller with r=[5cos(t/10);5sin(t/10)] using parameters mismatched

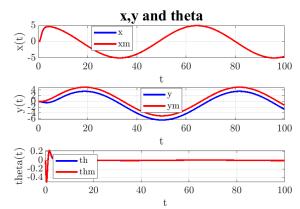


Figure 79: State for a Nonlinear Static Controller with r=[5cos(t/10);5sin(t/10)] using parameters mismatched

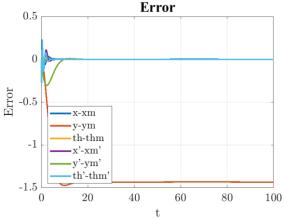


Figure 80: Error for a Nonlinear Static Controller with r=[5cos(t/10);5sin(t/10)] using parameters mismatched

Then we have done the same experiment with the adaptive controller, and as expected, the results are quite different, the tracking is better now. In fact, the error is about the same as before the parameter mismatched! It can be explained by the fact that, it is the benefit of the adaptive controller, it can adapt pretty well in case of error. This experiment uses the Post-Transient gains explained in the previous part. Moreover, the system is good, as the gain and the control are not drifting to infinity, which could be a problem in real life.

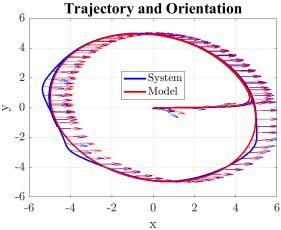


Figure 81: Trajectory and orientation of the center of mass for a Nonlinear Adaptive Controller with r=[5cos(t/10);5sin(t/10)] using parameters mismatched

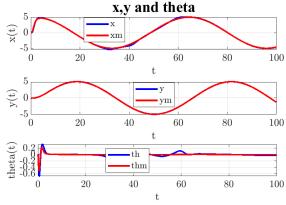


Figure 82: State for a Nonlinear Adaptive Controller with $r=[5\cos(t/10);5\sin(t/10)]$ using parameters mismatched

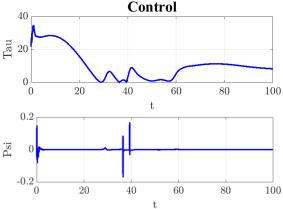


Figure 83: Control for a Nonlinear Adaptive Controller with r=[5cos(t/10);5sin(t/10)] using parameters mismatched



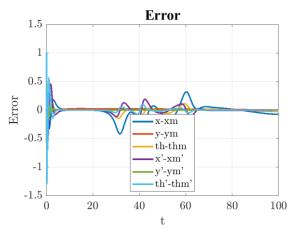


Figure 84: Error for a Nonlinear Adaptive Controller with $r=[5\cos(t/10);5\sin(t/10)]$ using parameters mismatched

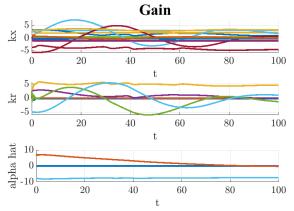


Figure 85: Gains for a Nonlinear Adaptive Controller with $r=[5\cos(t/10);5\sin(t/10)]$ using parameters mismatched

VI. Conclusion

What should we conclude about all these experiments? Firstly, as we have seen during our simulation of a scalar ducted fan, is that the linear approach allows tracking well the reference. Moreover, the static and the adaptive controller are about the same for the linear approach. In addition, with parameter mismatched, which can arrive in real life, it did not change a lot of things, the tracking is again good. Then if we add the nonlinear term, the static controller is really much better than the adaptive with no doubts! Even if we add the Post-Transient gains, to start with better gain for the adaptive way, which improve the tracking, the static way is again better. But this times, parameter mismatched, the controller has a lot of problem to follow the reference, in opposition with the adaptive controller, which continue to have the same tracking as before (it adapts itself to continue the tracking, in comparison with the static, who can't compensate the error of mismatched). The static way is having a lot of difficulties to track with the error on the weight of the ducted fan, but the error on r and J have not a big influence.

We have seen the interest of using an adaptive controller, in comparison with a static controller.