Modal Analysis – Prof. Arcanjo Lenzi

Lucas Schroeder – Feb 25th, 2021

Exercise List I

1 SDOF – Hysteretic Damping

$m = 1\ kg$, $k = 1{,}58 \cdot 10^6$N/m, $\eta_{hist} = 0.05$, $F = 1\ N$, Frequencies from 100 to 500 Hz.

Graph: Bode Plot, Coincidence-Quadrature Plot, Nyquist Plot

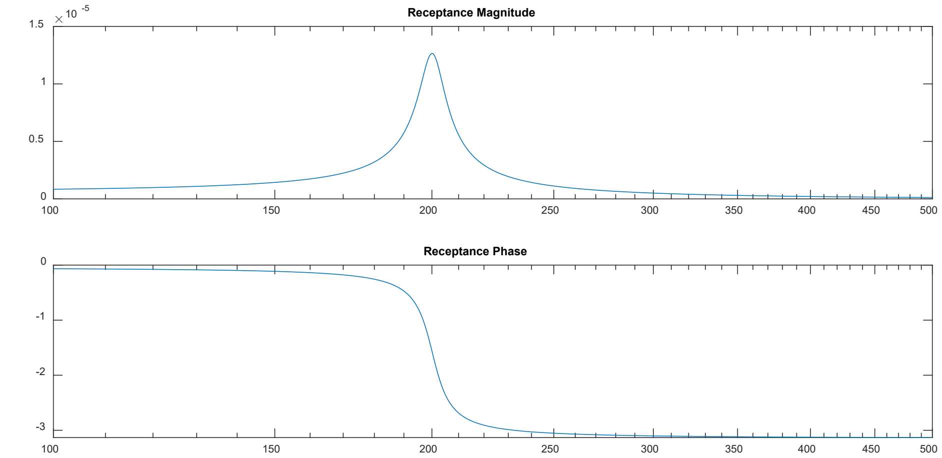


Figure 1 - Bode Plot

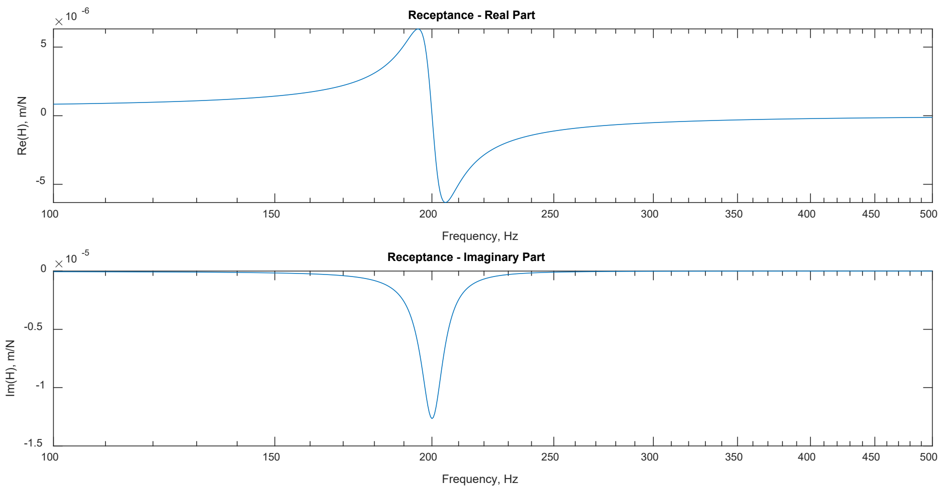


Figure 2 - Coincidence-Quadrature Plot

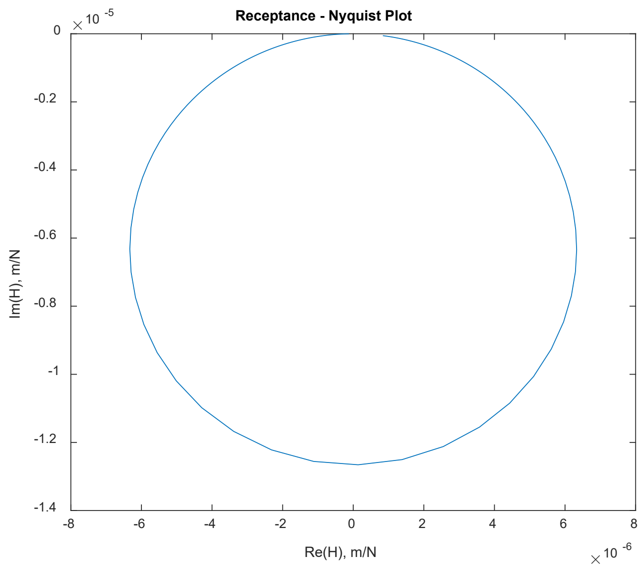


Figure 3 - Nyquist plot

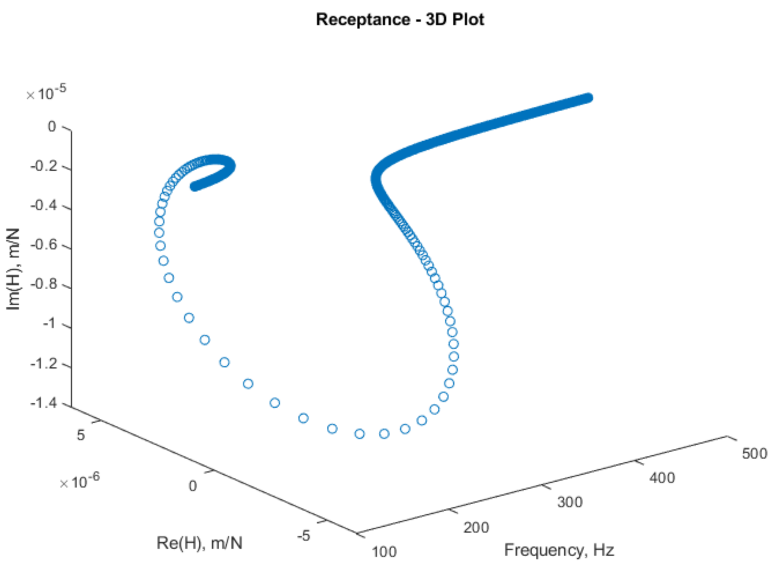


Figure 4 - 3D Nyquist plot

1.1

Calculate damping from half-power using peak-picking method.

In this method, we must find the frequency and amplitude at the resonance and then find the frequencies that corresponds to amplitudes $1/\sqrt{2}$ times the peak. This process resulted in a central frequency $f_0 = 200\ Hz$, with a bandwidth of $\Delta f = 10.9\ Hz$. The hysteretic damping coefficient in found using the approximation

$$\eta_{hp} = \frac{\Delta f}{f_0}, \qquad \text{Eq. 1}$$

which results in $\eta = 0.0546$. This value is deviates from the original 0.05 by 9.2%, and is due, probably, to the frequency discretization.

```
[receptance_pk,central_frq] = findpeaks(abs(H),f, 'MinPeakHeight',0.1e-5,...
                                         'MinPeakDistance',200);
drop = receptance_pk*10^(-3/20);     % 3dB drop in signal strength
%drop = receptance_pk/sqrt(2);       % half-power definition

FRQ = @(frf) interp1(abs(H),f,frf,"spline"); % interpolates and returns the frequency
bandwidth = 2*abs(central_frq - FRQ(drop))
```

```
bandwidth = 10.9148
```

```
central_frq
```

```
central_frq = 200
```

```
eta_estimated = bandwidth/central_frq
```

```
eta_estimated = 0.0546
```

1.2

Find half-power frequency bandwidth from the imaginary part of the FRF.

Locating the peaks in Figure 2 results in central frequency $f_0 = 200\,Hz$, with a bandwidth of $\Delta f = 10\,Hz$. Plugging them in Eq. 1 results in $\eta = 0.0500$.

```
[pk1,f_a] = findpeaks(real(H),f, 'MinPeakHeight',2e-6,...
                                 'MinPeakDistance',200);
[pk2,f_b] = findpeaks(-real(H),f,'MinPeakHeight',2e-6,...
                                 'MinPeakDistance',200); %#ok<*ASGLU>
central_frq2 = (f_a + f_b)/2;
bandwidth2 = abs(f_b - f_a)
```

```
bandwidth2 = 10
```

```
central_frq2
```

```
central_frq2 = 200
```

```
eta_estimated2 = abs(f_b-f_a)/central_frq2
```

```
eta_estimated2 = 0.0500
```

1.3

Find half-power frequency bandwidth from the Nyquist Plot

The process is identical to the item 1.2.

1.4

Calculate the damping using the peak in the imaginary part of receptance.

The definition of receptance with hysteretic damping is

$$\alpha(\omega) = \frac{1}{k - m\omega^2 + jk\eta}, \qquad \text{Eq. 2}$$

which can be decomposed into real and imaginary parts, leading to

$$Im\{\alpha(\omega)\} = \frac{-k\eta}{(k - m\omega^2)^2 + (k\eta)^2}. \qquad \text{Eq. 3}$$

Solving this equation for η gives

$$\eta = \frac{1}{2}\left[\frac{1}{Im\{\alpha(\omega)\}k} \pm \sqrt{\frac{1}{Im^2\{\alpha(\omega)\}k^2} - 4\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2}\,\right], \qquad \text{Eq. 4}$$

which is a function of the frequency and the imaginary value of the receptance at that frequency. In the case where $\omega = \omega_n$, it become much simpler:

$$\eta = \frac{-1}{Im\{\alpha(\omega_n)\}k}. \qquad \text{Eq. 5}$$

A simple routine finds the negative peak of the imaginary part of the receptance, and uses Eq. 5 to calculate the damping coefficient, resulting in $\eta = 0.050$.

```
[img_peak_at_ressonance,central_frq3] = findpeaks(-imag(H),f, 'MinPeakHeight',2e-6,...
                                                    'MinPeakDistance',200);
eta_estimated4 = -1/(k*-img_peak_at_ressonance)
```

```
eta_estimated4 = 0.0500
```

1.5

Find the energy dissipated in each vibration cycle at the resonant frequency. Find the dumping using the energy dissipated.

The damping coefficient is defined as the ratio of damping energy loss per radian,

$$\eta = \frac{E_{diss}/radian}{E_{vib}}. \qquad \text{Eq. 6}$$

The vibration energy is defined as

$$E_{vib} = m\dot{x}^2{}_{rms} = m\left(\frac{\omega X}{\sqrt{2}}\right)^2 = \frac{1}{2}m\omega^2 X^2. \qquad \text{Eq. 7}$$

For a fixed frequency and damping factor, the amplitude of vibration remains constant. This implies that the energy input to the system by the excitation force is equal to the dissipated energy bay the damping effects

$$E_{in} = \int_0^{\frac{2\pi}{\omega}} F(x)\dot{x}dt = E_{diss}. \qquad \text{Eq. 8}$$

Solving the integral for a harmonic excitation gives

$$E_{diss} = \pi F X \sin\Phi, \qquad \text{Eq. 9}$$

where $\Phi = \tan^{-1}\frac{Im\{\alpha(\omega)\}}{Re\{\alpha(\omega)\}}$ is the phase difference between excitation and response, and X is the response amplitude. Now we can calculate E_{vib} and E_{diss} using Eq. 7 and Eq. 9, respectively, and then use Eq. 6 to calculate the damping factor.

At the resonant frequency $f_0 = 200\ Hz$, found in Item 1.4, the vibration energy is $E_{vib} = 7.95 \cdot 10^{-4}\ J$ and the dissipated energy is $E_{diss} = 3.97 \cdot 10^{-5}\ J/cycle$, resulting in $\eta = 0.050$.

```
FRF = @(frq) interp1(f,H,frq,"spline"); % Interpolate the FRF to any frequency
displacement_at_ressonance = abs(FRF(central_frq3))*F; % [m]
E_vib = pi*m*(central_frq3*2*pi)^2*displacement_at_ressonance^2
```

```
E_vib = 7.9481e-04
```

```
phi2 = angle(FRF(central_frq3));
E_diss = abs(pi*F*displacement_at_ressonance*sin(phi))
```

```
E_diss = 3.9762e-05
```

```
eta_estimated5 = E_diss/E_vib
```

```
eta_estimated5 = 0.0500
```

1.6

Plot the receptance and find the modal stiffness

The receptance plots can be seen in Figure 1, Figure 2, Figure 3, and Figure 4. It can be shown from Eq. 2 that, for small damping

$$\alpha(\omega)|_{\omega \ll \omega_n} \approx \frac{1}{k(1+j\eta)} \approx \frac{1}{k}. \qquad \text{Eq. } 10$$

Taking the values for $f = 10\,Hz \ll f_0$, we obtain $k_{modal} = 1.578 \cdot 10^6\ N/m$, which is very close to the actual value.

1.7

Plot the inertance and find the modal mass.

Inertance is defined as $A(\omega) = -\omega\alpha(\omega)$.

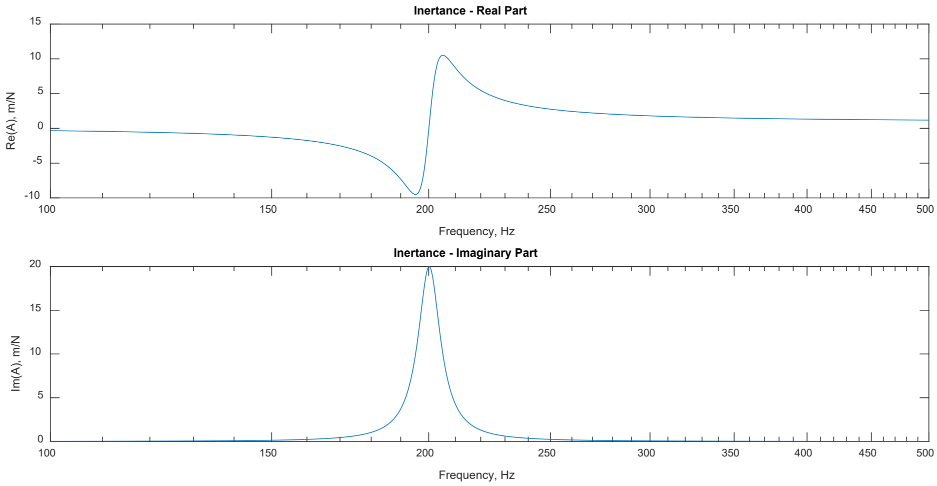


Figure 5 - Coincidence-Quadrature plot of inertance

It can be shown from Eq. 2 that, for small damping

$$\alpha(\omega)|_{\omega \gg \omega_n} \approx \frac{1}{m\omega^2}. \qquad \text{Eq. } 11$$

Taking the values for $f = 2000\,Hz \gg f_0$, we obtain $m_{modal} = 0.990\ kg$, which is very close to the actual value.

2 SDOF – Viscous Damping

$m = 1\,kg$, $k = 1{,}58 \cdot 10^6$N/m, $c_{visc} = 62.83\ kg/s$, $F = 1\,N$, Frequencies from 100 to 500 Hz.

Compare FRF (receptance) plots between different damping models; Compare resonant frequencies.

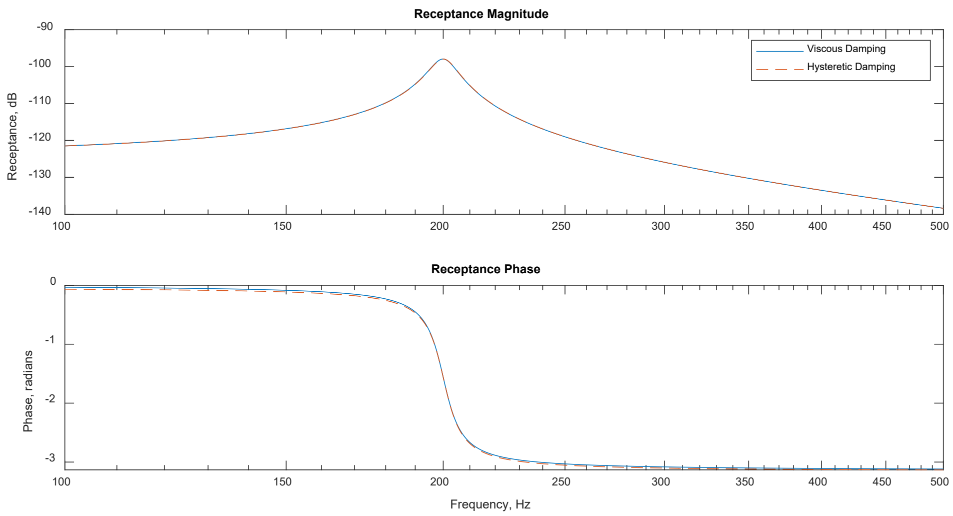
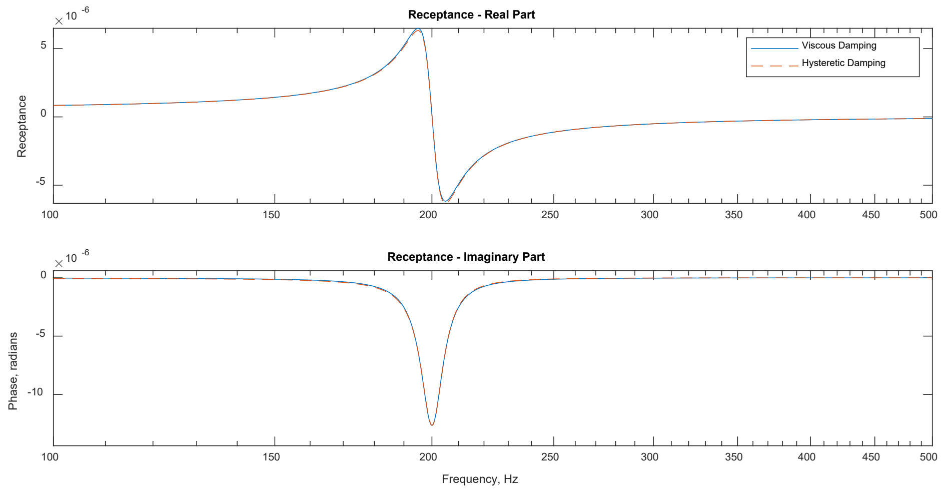
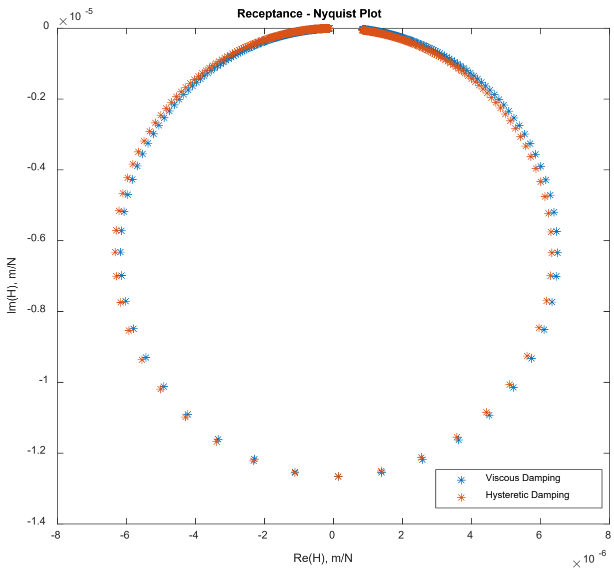


Figure 6 – Bode plot of receptance for viscous and hysteretic damping





The resonance frequencies for both models were calculated using the peak in the imaginary part, ad both resulted in $f_0 = 200\ Hz$, exactly.

3 INVERSE OF RECEPTANCE

Plot the inverse of the receptance for both damping models; Calculate damping using the imaginary part.

From Eq. 2, we see that for hysteretic damping

$$Im\left\{\frac{1}{\alpha(\omega)}\right\} = \eta k, \qquad \text{Eq. 12}$$

which is constant, since η and k are constants. In item 1.6 we calculated $k_{modal} = 1.578 \cdot 10^6\ N/m$, now from the value of $Im\left\{\frac{1}{\alpha(\omega)}\right\} = 79000$ we obtain $\eta = 0.0501$.