Modal Analysis – Prof. Arcanjo Lenzi

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Exercise List I

# SDOF – Hysteretic Damping

, N/m, , , Frequencies from 100 to 500 Hz.

Graph: Bode Plot, Coincidence-Quadrature Plot, Nyquist Plot



Figure - Bode Plot



Figure - Coincidence-Quadrature Plot



Figure - Nyquist plot

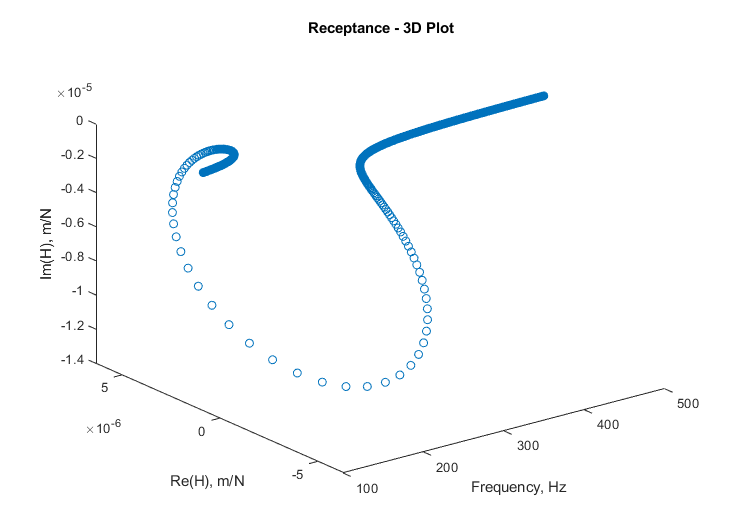


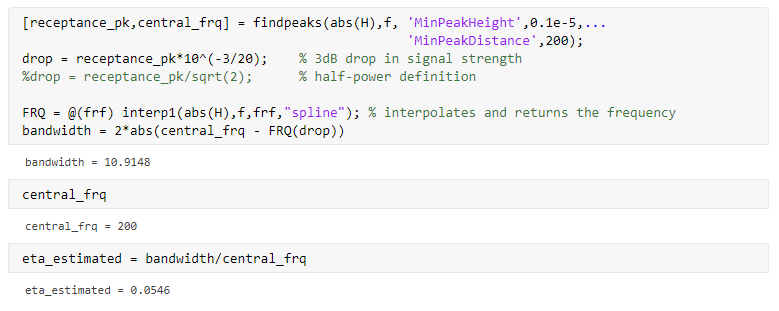
Figure - 3D Nyquist plot

Calculate damping from half-power using peak-picking method.

In this method, we must find the frequency and amplitude at the resonance and then find the frequencies that corresponds to amplitudes times the peak. This process resulted in a central frequency , with a bandwidth of . The hysteretic damping coefficient in found using the approximation

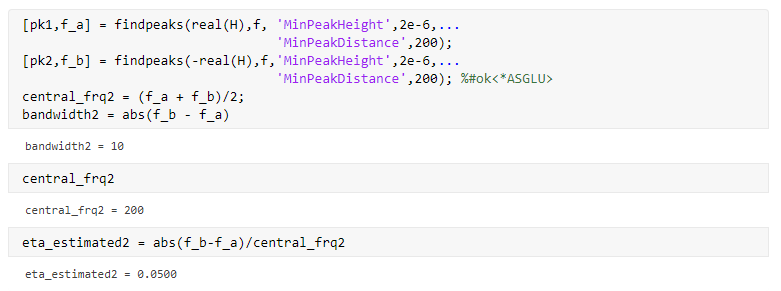
|  |  |  |
| --- | --- | --- |
|  |  | Eq. |

which results in . This value is deviates from the original by , and is due, probably, to the frequency discretization.



Find half-power frequency bandwidth from the imaginary part of the FRF.

Locating the peaks in Figure 2 results in central frequency , with a bandwidth of . Plugging them in Eq. 1 results in .



Find half-power frequency bandwidth from the Nyquist Plot

The process is identical to the item 1.2.

Calculate the damping using the peak in the imaginary part of receptance.

The definition of receptance with hysteretic damping is

|  |  |  |
| --- | --- | --- |
|  |  | Eq. |

which can be decomposed into real and imaginary parts, leading to

|  |  |  |
| --- | --- | --- |
|  |  | Eq. |

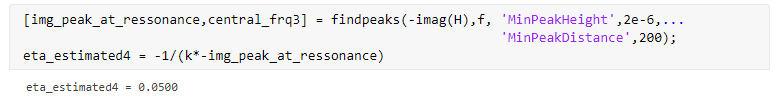
Solving this equation for gives

|  |  |  |
| --- | --- | --- |
|  |  | Eq. |

which is a function of the frequency and the imaginary value of the receptance at that frequency. In the case where , it become much simpler:

|  |  |  |
| --- | --- | --- |
|  |  | Eq. |

A simple routine finds the negative peak of the imaginary part of the receptance, and uses Eq. 5 to calculate the damping coefficient, resulting in .



Find the energy dissipated in each vibration cycle at the resonant frequency. Find the dumping using the energy dissipated.

The damping coefficient is defined as the ratio of damping energy loss per radian,

|  |  |  |
| --- | --- | --- |
|  |  | Eq. |

The vibration energy is defined as

|  |  |  |
| --- | --- | --- |
|  |  | Eq. |

For a fixed frequency and damping factor, the amplitude of vibration remains constant. This implies that the energy input to the system by the excitation force is equal to the dissipated energy bay the damping effects

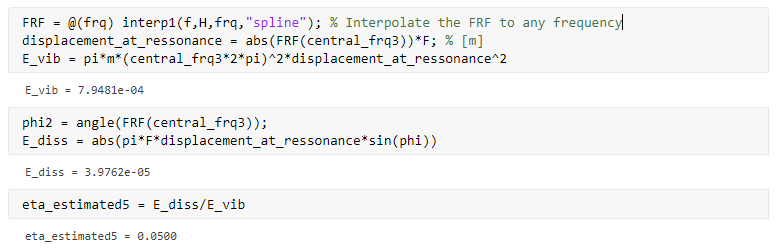
|  |  |  |
| --- | --- | --- |
|  |  | Eq. |

Solving the integral for a harmonic excitation gives

|  |  |  |
| --- | --- | --- |
|  |  | Eq. |

where is the phase difference between excitation and response, and is the response amplitude. Now we can calculate and using Eq. 7 and Eq. 9, respectively, and then use Eq. 6 to calculate the damping factor.

At the resonant frequency, found in Item 1.4, the vibration energy is and the dissipated energy is , resulting in .



Plot the receptance and find the modal stiffness

The receptance plots can be seen in Figure 1, Figure 2, Figure 3, and Figure 4. It can be shown from Eq. 2 that, for small damping

|  |  |  |
| --- | --- | --- |
|  |  | Eq. |

Taking the values for , we obtain , which is very close to the actual value.

Plot the inertance and find the modal mass.

Inertance is defined as .



Figure - Coincidence-Quadrature plot of inertance

It can be shown from Eq. 2 that, for small damping

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 10 |

Taking the values for , we obtain , which is very close to the actual value.

# SDOF – Viscous Damping

, N/m, , , Frequencies from 100 to 500 Hz.

Compare FRF (receptance) plots between different damping models; Compare resonant frequencies.



Figure – Bode plot of receptance for viscous and hysteretic damping





The resonance frequencies for both models were calculated using the peak in the imaginary part, ad both resulted in , exactly.

# Inverse of Receptance

Plot the inverse of the receptance for both damping models; Calculate damping using the imaginary part.

From Eq. 2, we see that for hysteretic damping

|  |  |  |
| --- | --- | --- |
|  |  | Eq. |

which is constant, since and are constants. In item 1.6 we calculated , now from the value of we obtain .

