Modal Analysis – Prof. Arcanjo Lenzi

Lucas Schroeder – Mar. 8th, 2021

Exercise List 2.3

# SDOF – Hysteretic Damping

The objective of this work is investigate how FRF signals measured from real continuous systems can be used to construct a MDOF vibration model using modal analysis.

The system used in this particular work is a simply supported beam, illustrated in Figure 1. This beam is 1 m long, 30 mm wide and 5 mm thick, with a density of , totaling . The structural damping assumed is .

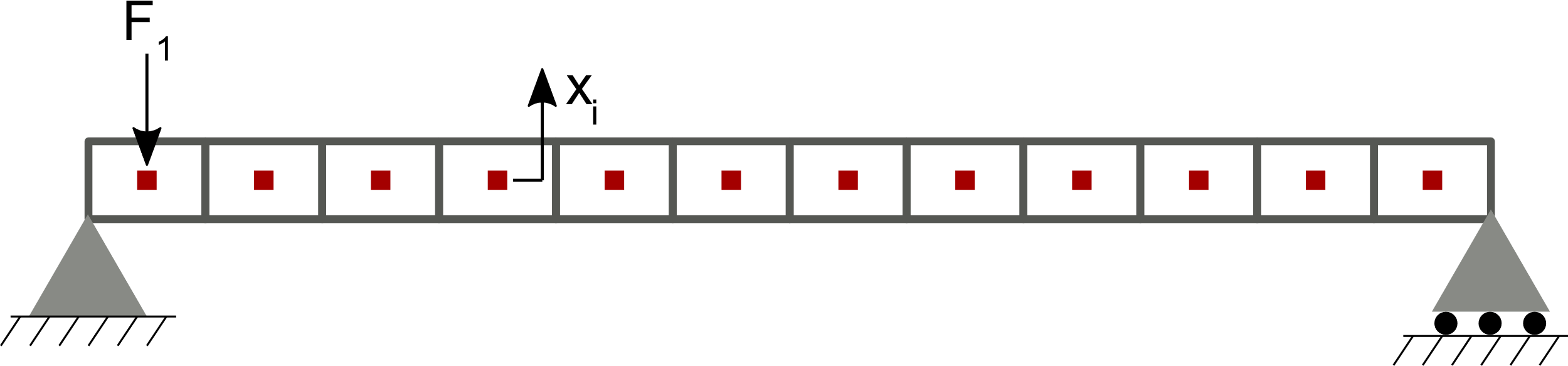


Figure 1 - System sketch.

To simulate the acquired data, an analytical model is used to find the natural frequencies and mode shapes. The structure is instrumented with 12 accelerometers, evenly spaced, as illustrated in Figure 1, resulting in an experimental mesh with spatial resolution of . The analytical model gives the receptance by

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 1 |

where

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 2 |

is the element of the mass-normalized eigenvector associated to the eigenvalue , is the mode number, and is the position of the experimental DoF. The natural frequency is given by

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 3 |

The analytical model requires that we consider all modes, which, for a continuous systema is infinite. However, we shall consider only the contribution of the first four modes to obtain the FRFs, this give us the four natural frequencies to be considered:

Objective: Plot the analytical receptance for each DoF in relation to an excitation in . Use only the first for modes ( in Eq. 1), with a frequency resolution of .





Objective: Plot the Nyquist circle for . Do the modes interfere? Are the shapes circular?

Figure 2 - Nyquist plot for the receptance for and

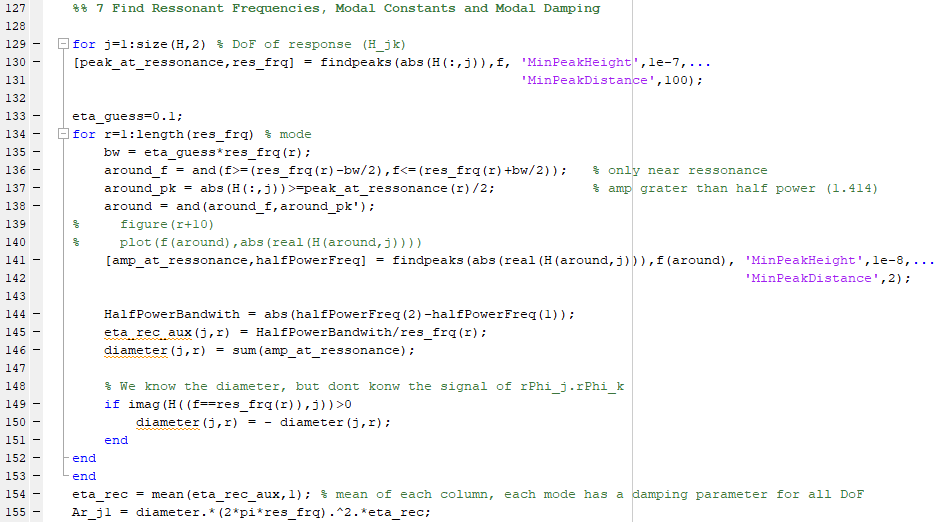
The resonant frequencies are sufficiently spaced so that the shapes are very close to a perfect circle.

Objective: find the resonant frequencies , modal damping , and modal constants from the properties of Nyquist circles.

First, we found the peaks and their respective frequencies . Then, we found the diameter of the Nyquist plot using the maximum and minimum of the real part of receptance around . We also note the frequencies the maximum and minimum occurs, so we can calculate the half-power bandwidth and the damping parameter associated to that mode . Knowing this, we can finally calculate the modal constant using

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 4 |

The following code was developed to find the peaks, resonant frequencies , half-power bandwidth, modal damping and modal constants.



The results recuperated from the FRFs (index ) are:

Now we are able to reconstruct the FRF from the parameter measured using

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 1 |

The results are shown in Figure 3.



Figure 3 - Reconstructed FRFs

Objective: From modal constants , extract mode shapes .

By definition, modal constants are the product of two mass-normalized components of the mode shapes

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 4 |

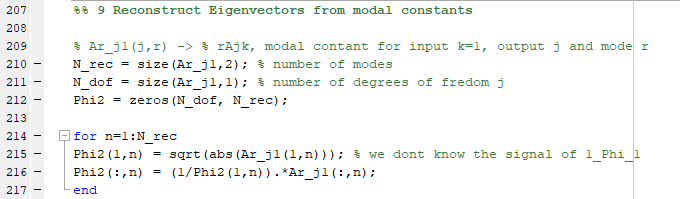
So, for the point response of DoF 1, the first component of the mode shape is

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 5 |

From Eq. 4, we can write the components of the mode shape, for an input force at DoF 1 and response at DoF 1 through

|  |  |  |
| --- | --- | --- |
|  |  | Eq. 6 |

The following piece of code was used to recuperate the mode shape vectors .



The recuperated mode shape vectors are plotted against the original ones in Figure 4.



Figure 4 - Mode shapes.

## General Observations

The method implemented is known as Peak-Find, and it worked really well. This is mainly because the FRF used as input were analytically calculates, that means there were no noise in the signal. Furthermore, the resonant frequencies were very far apart considering the damping parameter and the frequency resolution. This scenario, almost ideal and far from real cases, made the results obtained very close to the target.

There are other FRF processing methods to extract the modal parameters , and , such as circle fitting, that may present better results for less than perfect data.