Graphical Model for State Estimation in Electric Power Systems

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I. Abstract

This paper is motivated by major needs for fast and accurate on-line state estimation (SE) in the emerging electric energy systems, due to recent penetration of distributed green energy, distributed intelligence, and plug-in electric vehicles. Different from the traditional deterministic approach, this paper uses a probabilistic graphical model to account for these new uncertainties by efficient distributed state estimation. The proposed graphical model is able to discover and analyze unstructured information and it has been successfully deployed in statistical physics, computer vision, error control coding, and artificial intelligence. Specifically, this paper shows how to model the traditional power system state estimation problem in a probabilistic manner. Mature graphical model inference tools, such as belief propagation and variational belief propagation, are subsequently applied. Simulation results demonstrate better performance of SE over the traditional deterministic approach in terms of accuracy and computational time. Notably, the near-linear computational time of the proposed approach enables the scalability of state estimation which is crucial in the operation of future large-scale smart grid.

II. Introduction

Regarded as a seminal national infrastructure, the electric power grid provides clean, convenient, and relatively easy way to transmit electricity to both urban and suburban areas. However, the grid often exhibits fragileness, such as blackouts. To monitor potential problems, hidden state information is extracted from measurements using state estimation (SE) [1]; the results of SE are used for on-line assessment of reliability problems.

Currently, there exists a significant gap in performance between the state-of-the-art methods and the desired informative data exploration. This problem may grow as new technologies are being deployed. [2]. For instance, according to utility records, for the past ten years, numerous distributed power plants (wind, solar, etc.), each serving primarily a local area, have been connected to the existing grid and their presence has raised great concerns about possible reliability problems in the future electric power grids. Further, President Obama's goal of putting one million electric vehicles on the road by 2015 will also contribute to the grid architecture shift; robustness of these new architectures will have to be studied. Because of their unconventional characteristics, new modeling of these technologies must be done for accurate SE in smart grid.

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Scalability is as important as proper modeling for the future large-scale smart grid. The state estimators used by the industry today are hard to scale up and are computationally complex [3]. To avoid excessive computational complexity, only Extra High Voltage (EHV), High Voltage (HV) and occasionally Medium Voltage (MV) representation of the complex multi-voltage level power grids are included. The low voltage (LV) distribution networks are not modeled nor supported by the on-line SE today. This, in turn, makes it difficult to estimate the status and states of many new diverse resources and users connected to the LV level distribution systems. More generally, the power system operators of traditional power grids face inherent difficulties in managing the effects of small scale generations and loads including but not limited to renewable energy generators, such as wind and solar generators; responsive small electricity users; and electricity users which can offer storage to utility, such as electric cars. While having potential to reduce the impact on the environment, increase fuel diversity, and bring in economic benefits, these new components also raise tremendous concerns regarding secure and reliable operation of the backbone EHV/HV power grids; in particular, their state needs to be estimated to account for their effects on the state of the backbone power grid. This need to estimate the on-line state in the entire electric power grid makes it even more difficult to manage all data in a centralized way than in the past. A multi-layered, distributed implementation of state estimators for future electric energy systems is likely to become the preferred approach; this requires systematic design of distributed algorithms whose performance does not worsen relative to the centralized methods.

The above challenges are enormous as the industry paradigm shifts from the traditionally deterministic model based centralized monitoring architecture to probabilistic model based highly distributed interactive data and resource management. Therefore, to properly model the system stochastic properties and to conduct efficient distributed state estimation, we propose a graphical model description [4] of the electric power grid; this is inspired by the exciting results made available by applying graphical models to compact uncertainty representations and computationally-tractable inferences [5], [6]. Specifically, we consider a graph representing the electrical power system as a graphical model, and model power grid states (bus voltages) as random variables on the graph vertices; the edges of the graph determine the interaction of state variables' according to physical laws (i.e. Kirchoff laws). Viewed together, the graphical model is specified by the joint density of random variables in the network for state estimation, subject to the constraints imposed by the physical laws.

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Unlike the traditional state estimation process which aims at minimizing mean square error (MMSE), this paper aims at maximizing a posteriori probability (MAP) popular in the graphical model. To achieve MAP, exact inference on distributed SE for trees is conducted via belief propagation (BP) [4], [7]–[9]. This is one way of organizing the "global" computation of marginal beliefs in terms of smaller local computations in graphical model enabled by distributed computation capability (such as a small embedded system) and communication capability (a wireless, telephony or Internet link) of components in the future smart grid (such as a small generator). We adopt variational belief propagation (VBP) algorithm [10], [11] to deal with meshed networks, such as transmission systems, by breaking a meshed structure into multiple spanning trees. Finally, we use uninformative prior to avoid bias.

Such BP-based algorithms can form the basis for smart grids by enabling many small system users to participate in enhancing system operation in predictable ways. For example, the state (power consumed, voltage) of smart meters will not have to be estimated by the operator of the backbone system based on the measurement provided at this location. Instead, the state of small users can be estimated in a distributed, message-passing manner with the neighboring system users where smart meters are located. Then the aggregated information is communicated in a bottom-up way to the backbone system operator. Therefore, the BP method is a unique way to break the current centralized monitoring architecture that requires both large communication/computation overhead and tight data synchronization.

We show using simulations of several IEEE Test Systems up to 300 buses that this graphical model approach can result in higher accuracy than the weighted least square (WLS) SE. Further, the proposed approach features an approximately linear computational time unavailable in the past. Based on these preliminary results, we present that the proposed method can offer a major promise for scalable SE with high accuracy for the future smart grid.

This paper is organized as follows: in Section III we introduce a graphical model for state estimation; in Section IV we review the basics of the BP algorithm for the tree networks; we then extend BP to variational BP in the meshed networks (transmission systems) for SE; in Section V, we illustrate the application of the proposed approach on a small example; in Section VI we simulate results for IEEE test systems. Finally, in Section VII, we conclude.

III. Graphical Model for State Estimation

A power grid is defined as a physical graph $G(V,E)$ with vertices V that represent the buses (generators and loads), and edges E that represent transmission lines and transformers. The graph of the physical network can be visualized as the physical layer in Fig.1. The probabilistic measurement model of AC power system state estimation is expressed as

$$z_i = h_i(\boldsymbol{v}) + u_i \tag{1}$$

where the vector $\boldsymbol{v} = [|v_1|e^{j\delta_1}, |v_2|e^{j\delta_2}, \cdots, |v_n|e^{j\delta_n}]^T$ represents the probabilistic power system states, instead of the conventionally used deterministic states. u_i is the i^{th} additive

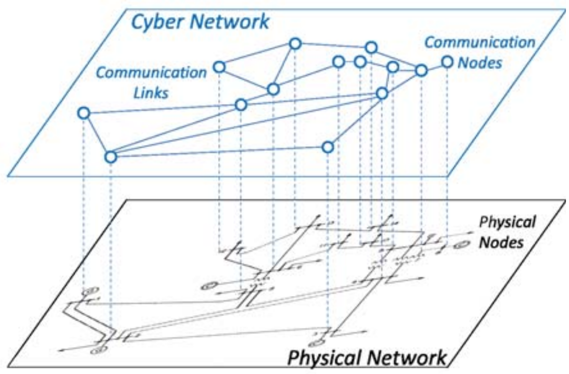


Fig. 1. Physical Network and Cyber Network (14 Bus System).

measurement noise assumed to be independent Gaussian random variable with zero mean, i.e., $\boldsymbol{u} \sim \mathcal{N}(\boldsymbol{0}, \Sigma)$, where Σ is a diagonal matrix, with the i^{th} diagonal element σ_i^2. z_i is the i^{th} telemetered measurement, such as power flow and voltage magnitude. $h_i(\cdot)$ is the nonlinear function associated with the i^{th} measurement.

The probabilistic power system state estimator aims to find an estimate ($\hat{\boldsymbol{v}}$) of the true states ($\boldsymbol{v}$) that achieves the maximum a posteriori probability (MAP), given the measurement set $\boldsymbol{z}$ and the priori information on the state $\boldsymbol{v}$ according to the measurement model in (1). It is mathematically expressed as

$$\max_{\boldsymbol{v}} p(\boldsymbol{v}|\boldsymbol{z}) = \frac{p(\boldsymbol{v})p(\boldsymbol{z}|\boldsymbol{v})}{p(\boldsymbol{z})} \tag{2}$$

where $p(\cdot)$ represents the probability density function. Such a process is achieved via the cyber network layer as in Fig.1. In this work, the cyber network topology is the same as the physical network topology.

IV. Graphical-Model-based State Estimation

To obtain the MAP estimate in (2), we need to 1) obtain proper formulations for $p(\boldsymbol{v})$, $p(\boldsymbol{z}|\boldsymbol{v})$, and $p(\boldsymbol{z})$; and 2) employ efficient algorithms to conduct marginalization over $p(\boldsymbol{v}|\boldsymbol{z})$ with respect to $\boldsymbol{v}$. While the first problem will be addressed in subsection IV-C, we start by introducing efficient marginal computations for trees, which will be extended to account for meshed networks such as the IEEE 14 bus system.

A. Belief Propagation for Trees

Similar to other efficient algorithms in electric power system analysis, BP explores network sparsity to compute marginal probabilities, in a time that grows only linearly with the number of nodes in the systems. The underlying principle of this BP process is that of divide and conquer process like in the usual serial dynamic programming (DP): we solve a large problem by breaking it down into a sequence of simpler problems. However, as a form of non-serial dynamic programming, BP generalizes the serial form of deterministic dynamic programming to arbitrary tree-structured graphs, where each subgraph is again a tree disjoint from other subgraphs (trees).

Here we provide a simple illustration of four binary variables in a chain, defined by the joint probability $p(v_1, v_2, v_3, v_4)$. When the graph sparsity is disregarded, $2^3 = 8$ summations (grows exponentially) are needed for obtaining marginal distribution of v_1 via $p(v_1) = \sum_{v_2,v_3,v_4} p(v_1, v_2, v_3, v_4)$. Instead, BP needs only $2 \cdot 3 = 6$ summations (grows linearly) when exploring the system structure via $p(v_1) = \sum_{v_2} p(v_1|v_2) \sum_{v_3} p(v_2|v_3) \sum_{v_4} p(v_4)p(v_3|v_4)$. In such an operation, $M_{4\to3} \triangleq \sum_{v_4} p(v_4)p(v_3|v_4)$, a function of v_3, is interpreted as a message passed from node 4 to node 3 [7]. It represents how node 4 believes node 3's probability mass function (pmf) is, based on node 4's own pmf and the joint pmf between them.

As a generalization [4], BP can be conducted on an arbitrary tree-structured graph $G(V, E)$ with a pairwise Markov Random Field factorization

$$p(v_1, v_2, \cdots, v_n) = \alpha \prod_{s\in V} \phi_s(v_s) \prod_{(s,t)\in E} \phi_{st}(v_s, v_t) \tag{3}$$

where ϕ_s and ϕ_{st} are compatibility functions for the joint distribution $p(v_1, v_2, \cdots, v_m)$; α denotes a positive constant chosen to ensure the distribution normalization. Finally, BP conducts message updates according to

$$M_{s\to t}(v_t) \leftarrow \sum_{v_s} \phi_s(v_s)\phi_{st}(v_s, v_t) \prod_{k\in\mathcal{N}(\mathrm{s}),\ \mathrm{k}\neq\mathrm{t}} M_{k\to s}(v_s) \tag{4}$$

where $\mathcal{N}(s)$ is the set of neighboring buses of the bus s.

In such a message-passing calculation, product is taken over all messages going into node s except for the one coming from node t. In practice, we can start with the nodes on the graph edge, and computes a message only when all necessary messages are received. Therefore, each message needs to be computed only once for a tree structured graph.

B. Variational Belief Propagation for Mesh Networks

As observed above, the key assumptions of the BP algorithm are: 1) each subgraph remains a tree after graph division, and 2) the subgraphs are disjoint. Such assumptions often do not hold for most electric power transmission networks, such as the test case of IEEE 14 bus system. To overcome this problem, variational BP approach [10] is used by randomly generating spanning trees of the meshed network. The key is to assign probability to the edges based on its appearance probability ρ_{st} in the spanning trees according to

$$\rho_{st} = \frac{\text{No. of spanning trees with edge (s,t)}}{\text{No. of all spanning trees}}. \tag{5}$$

Subsequently, convex combination methods are adopted to approximate the inference on meshed networks with the BP algorithm on the trees [11]. Mathematically, the new message-passing algorithm below is run with ρ_{st} until convergence of the states,

$$M_{t\to s}^{n+1}(v_s) = \alpha \sum_{v_t} \Bigg\{ \exp\left(\frac{\theta_{st}(v_s, v_t)}{\rho_{st}} + \theta_t(v_t)\right) \frac{\prod_{k\in\mathcal{N}(\mathrm{t})\backslash \mathrm{s}} [M_{k\to t}^n(v_t)]^{\rho_{kt}}}{[M_{s\to t}^n(v_t)]^{(1-\rho_{ts})}} \Bigg\} \tag{6}$$

where θ_{st} and θ_t are the exponential parameters associated with compatible functions such as ϕ_{st}, and ϕ_s in (3). Note that, if $\rho_{st} = 1$, $\forall(s, t) \in E$, the VBP in (6) degrades to the BP form in (4) due to the tree structure implication. Further, VBP approach relies on the special exponential family that includes many common distributions, such as the normal, exponential, gamma, and chi-square distributions, instead of the more general arbitrary distribution in (3) and (4). However, the exponential family is sufficient for the purpose of this paper.

C. Variational Belief Propagation for AC Power System State Estimation

In this subsection, we explain how to apply the VBP algorithm in (6) to the posterior probability distribution in (2) in an AC power system state estimation setting. Specifically, we use uniform prior probability distribution for voltage magnitude, i.e. $|v_i| \in [0, 10]$ and phase angle, i.e. $\delta_i \in [0, 2\pi]$, instead of historical data [6], [12], [13] to avoid bias. This is because maximum a posteriori probability (MAP) estimation can be viewed as a regularization of currently used maximum likelihood estimation (MLE). By relaxing the regularization conditions to non-informative prior probability distributions, we can prevent bias in the Bayesian framework. With $p(\boldsymbol{v})$ defined above and $p(\boldsymbol{z})$ as a constant, we show next the forms of $p(\boldsymbol{z}|\boldsymbol{v})$ needed to calculate the posterior probability distribution $p(\boldsymbol{v}|\boldsymbol{z})$ in (2).

Since VBP is built on the exponential family, we use the additive Gaussian noise in (1) to represent

$$p(\boldsymbol{z}|\boldsymbol{v}) \sim \exp\Big\{ -\sum_i (z_i - h_i(\boldsymbol{v}))^2/\sigma_i^2 \Big\}. \tag{7}$$

Without loss of generality, we omit the variance σ_i^2 in the rest of the paper for simplicity. In the following, we specify each measurement type.

1) The complex valued power flow (pf) measurement on the branch (edge) $s - t$ near bus s:

$$p(z_i^{pf}|\boldsymbol{v}) \sim \exp\Big\{ -\sum_i \Big| z_i - (v_s - v_t)Y_{st}^* v_s^* \Big|^2 \Big\} \tag{8}$$

This form can be easily extended into real power measurements and reactive power measurements.

2) The voltage magnitude (vm) on bus s:

$$p(z_i^{\mathrm{vm}}|\boldsymbol{v}) \sim \exp\Big\{ -\sum_i \Big(z_i - (v_s v_s^*)^{\frac{1}{2}} \Big)^2 \Big\} \tag{9}$$

3) The voltage phase angle (va) on bus s:

$$p(z_i^{\mathrm{va}}|\boldsymbol{v}) \sim \exp\Big\{ -\sum_i \Big(z_i - \tan^{-1}\frac{Im(v_s)}{Re(v_s)} \Big)^2 \Big\} \tag{10}$$

Partition functions associated with the measurement types above satisfy the pairwise Markov Random Field representation requirement of the SE problem in (3) and (6). Now we discuss how to deal with the power injection measurement that violates the pairwise requirement.

4) The power injection into bus s:

$$p(z_i^{\text{pinj}}|\boldsymbol{v}) \sim \exp\Big\{-\sum_i \Big|z_i - \sum_{t\in\mathcal{N}(s)} (v_s - v_t)Y_{st}^* v_s^*\Big|^2\Big\}$$
$$= \exp\{-T\} \quad (11)$$

where T equals to

$$\sum_i \Big|z_i - \sum_{t\in\mathcal{N}(s)} (v_s - v_t)Y_{st}^* v_s^*\Big|^2$$
$$= \sum_i \Big\{z_i - \sum_{t\in\mathcal{N}(s)} (v_s - v_t)Y_{st}^* v_s^*\Big\}\Big\{z_i - \sum_{k\in\mathcal{N}(s)} (v_s - v_k)Y_{sk}^* v_s^*\Big\}^*$$
$$= \sum_i \Big\{|z_i|^2 - z_i \sum_{k\in\mathcal{N}(s)} (v_s^* - v_k^*)Y_{sk} v_s - z_i^* \sum_{t\in\mathcal{N}(s)} (v_s - v_t)Y_{st}^* v_s^*$$
$$+ \sum_{t\in\mathcal{N}(s)} \sum_{k\in N_s} |v_s|^2 Y_{st}^* Y_{sk}(v_s - v_t)(v_s^* - v_k^*)\Big\} \quad (12)$$

which can be abstracted as

$$\sum_s \Big\{\theta_s(v_s) + \sum_{t\in\mathcal{N}(s)} \theta(v_s, v_t) + \sum_{t\in\mathcal{N}(s)} \sum_{k\in\mathcal{N}(s)} |v_s|^2 Y_{st}^* Y_{sk} v_t v_k^*\Big\} \quad (13)$$

By including a multiplication of three different state variables, $|v_s|^2 Y_{st}^* Y_{sk} v_t v_k^*$ violates the pairwise Markov random assumption required by VBP algorithm. To resolve the problem, dummy variable vector $\boldsymbol{w}_{stk} \triangleq [w_{stk}^{(1)}, w_{stk}^{(2)}, w_{stk}^{(3)}]^T$ and the corresponding $\phi(\boldsymbol{w}_{stk}) \triangleq |v_s|^2 Y_{st}^* Y_{sk} v_t v_k^*$ are defined for regularization. In this way, the original problem of either maximizing (11) or equivalently minimizing (13) can be regarded as the problem of minimizing the following regularized problem

$$\sum_s \Big\{\theta_s(v_s) + \sum_{t\in\mathcal{N}(s)} \theta_{st}(v_s, v_t) + \sum_{t\in\mathcal{N}(s), k\in\mathcal{N}(s)} \phi_{stk}(w_{stk}) \quad (14)$$
$$+ 10|w_{stk}^{(1)} - v_s|^2 + 10|w_{stk}^{(2)} - v_t|^2 + 10|w_{stk}^{(3)} - v_k|^2\Big\}$$

where 10 is adopted as the penalty coefficient. $|w_{stk}^{(1)} - v_s|^2 = |w_{stk}^{(1)}|^2 - w_{stk}^{(1)} v_s^* - w_{stk}^{(1)*} v_s + |v_s|^2$ results in a pairwise expression, with similar extension to regularization on v_t and v_k in (14).

Now all measurement types can be written in a pairwise Markov form as in (3). Further, since the prior state distribution is uniform, which is uninformative, maximizing the posterior distribution $p(\boldsymbol{v}|\boldsymbol{z})$ is equivalent to maximizing the conditional distribution $p(\boldsymbol{z}|\boldsymbol{v})$ according to the Bayes' rule in (2), given $\boldsymbol{z}$. We can apply the VBP algorithm (6) to (7) (a compact summary of different measurements types (8), (9), (10), and (11)), to obtain the marginal distribution of $p(\boldsymbol{v}|\boldsymbol{z})$ or the probabilistic state estimate for all buses.

D. Algorithm Summary

We provide here a summary of conducting the proposed graphical model-based SE in Fig.2 with the following steps:

- Step 1: randomly generate spanning trees of the graph based on physical power system topology according to [14], then calculate the appearance probability ρ_{st} of each edge with them according to (5);
- Step 2: use the measurement values of $\boldsymbol{z}$ in (8), (9), (10), and (11) to obtain the joint probability function (7) over state $\boldsymbol{v}$;
- Step 3: initialize the state variables; (i.e., ones for voltage magnitudes, and zeros for voltage phase angles.)
- Step 4: with the regularization (14) of power injection measurements, apply the result obtained in from Step 1 to Step 3 to the VBP algorithm in (6) for state variable updates;
- Step 5: repeat Step 4 until state variables converge.

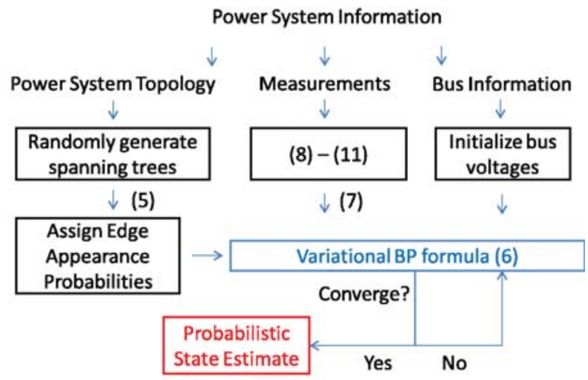
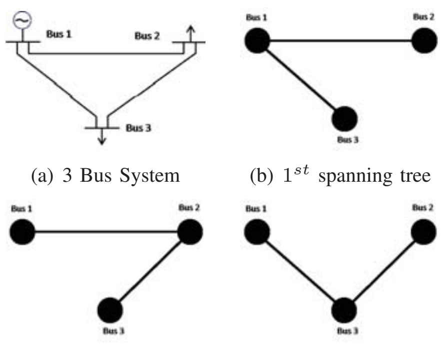


Fig. 2. Flow Chart of the proposed approach.

V. Illustration on a Small Example

Fig.2(a) represents a three bus system, to which the proposed VBP is applied. In this example, we assume that we have a voltage measurement z_1^{vm} on bus 1, a voltage phase angle measurement z_2^{va} on bus 2, a complex power flow measurement z_3^{pf} on the branch 2−3 near bus 2, and a complex power injection measurement z_4^{pinj} on bus 3.



(a) 3 Bus System (b) 1^{st} spanning tree (c) 2^{nd} spanning tree (d) 3^{rd} spanning tree

Fig. 3. Generation of spanning tree to obtain the value of ρ_{ij}

- Step 1: because there are three possible spanning trees shown in Fig.2(b)(c)(d) with equal edge probability, $\rho_{12} = \rho_{23} = \rho_{13} = 2/3$;

- Step 2: write $p(\boldsymbol{v}|\boldsymbol{z})$ as

$$\begin{aligned}p(v_1, v_2, v_3|\boldsymbol{z}) \sim \exp\Bigg\{ &- \Big(z_1^{\mathrm{vm}} - (v_1 v_1^*)^{\frac{1}{2}}\Big)^2 \\ &- \Big(z_2^{\mathrm{va}} - \tan^{-1}\frac{Im(x_2)}{Re(v_2)}\Big)^2 - \Big|z_3^{\mathrm{pf}} - (v_2 - v_3)Y_{ij}^* v_2^*\Big|^2 \\ &- \Big|z_4^{\mathrm{pinj}} - \big[(v_3 - v_1)Y_{31} + (v_3 - v_2)Y_{32}\big]v_3^*\Big|^2 \Bigg\}\end{aligned} \tag{15}$$

and use the measurement value z_i and the admittance value Y_{ij}, leading to

$$\begin{aligned}p(v_1, v_2, v_3) \sim \exp\Big\{&\theta_{v_1}(v_1) + \theta_{v_2}(v_2) + \theta_{v_1,v_2}(v_1, v_2) \\ &+ \theta_{v_2,v_3}(v_2, v_3) + \theta_{v_1,v_3}(v_1, v_3) \\ &+ \phi_{v_1,v_2,v_3}(v_1, v_2, v_3)\Big\};\end{aligned} \tag{16}$$

- Step 3: initialize the voltage belief on each bus with magnitude one and phase angle zero;
- Step 4: with regularization (14) on $\phi_{v_1,v_2,v_3}(v_1, v_2, v_3)$, apply step 1 to step 3 to VBP algorithm in (6) for message (M_{ij}) passing;
- Step 5: repeat Step 4 until state variables converge.

VI. Numerical Results

The simulations are implemented on the IEEE standard test systems for IEEE 9, 14, 30, 39, 57, 118, and 300 bus systems. Similar performance improvements are observed. Due to page limit, only 14 bus simulation results are presented for state domain and error domain comparisons. The data has been preprocessed by the MATLAB Power System Simulation Package (MATPOWER) [15], [16]. To obtain the measurements, we first run a power flow to generate the true states of the power system, followed by adding Gaussian noise to the corresponding measurements. The measurements include (1) the power injection on each bus; (2) the transmission line power flow '*from*' or '*to*' each connected bus; (3) the direct voltage magnitude of each bus and (4) the voltage phase angle of each bus. Further, the measurements are randomly chosen with system observability check, and the measurement number is selected as three times the bus number. Finally, the convergence criteria to stop the calculation is defined as 0.01 in per unit value.

A. Simulation Results

We demonstrate the performance of the graphical model-based approach by conducting SE with Gaussian noise.

1) Accuracy: Fig.4 shows 30 simulations for the weighted residual sum of squares (WRSS) error in the 14 bus system

$$\mathrm{WRSS} = \sum_{\mathrm{i}=1}^{\mathrm{m}} \left(\frac{\mathrm{z_i} - \mathrm{h_i}(\hat{\mathrm{v}})}{\sigma_{\mathrm{i}}}\right)^2 \tag{17}$$

where m is the total measurement number.

In this illustration, the x axis is the simulation test number. The y axis is the metric WRSS (normalized with respect to Newton's method). It can be seen that the graphical model-based approach can reduce the error by 20% on average when compared to Newton's method with flat start. The reason for simulating a flat start is to mimic the smart grid environment with strong state variation, making typical initial guess method (from the last static state estimate) less informative for the new estimation process. More than 50% improvement is achieved at simulation number 8, 12, 20, 24, and 29,. These facts lead to a natural interpretation for the proposed SE procedure: the possibility for the graphical model approach to come closer to global optimum is greatly increased since the results do not rely heavily on the initial guess. Flat start seems to perform better only in test case 27, because it successfully helps Newton's method reach the global optimum, over which VBP cannot improve. VBP fails because the objective of VBP is MAP, instead of MMSE. Further, such a case rarely occurs in practice in a true smart grid scenario, where flat start performs poorly. Note that, the proposed distributed method is superior to the distributed implementation of Newton's method. This is because the centralized method is equal or better to its distributed realization.

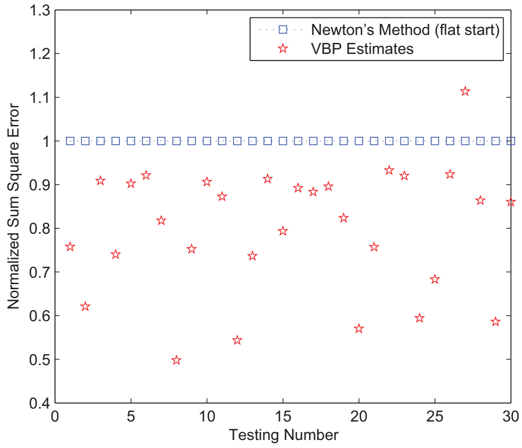
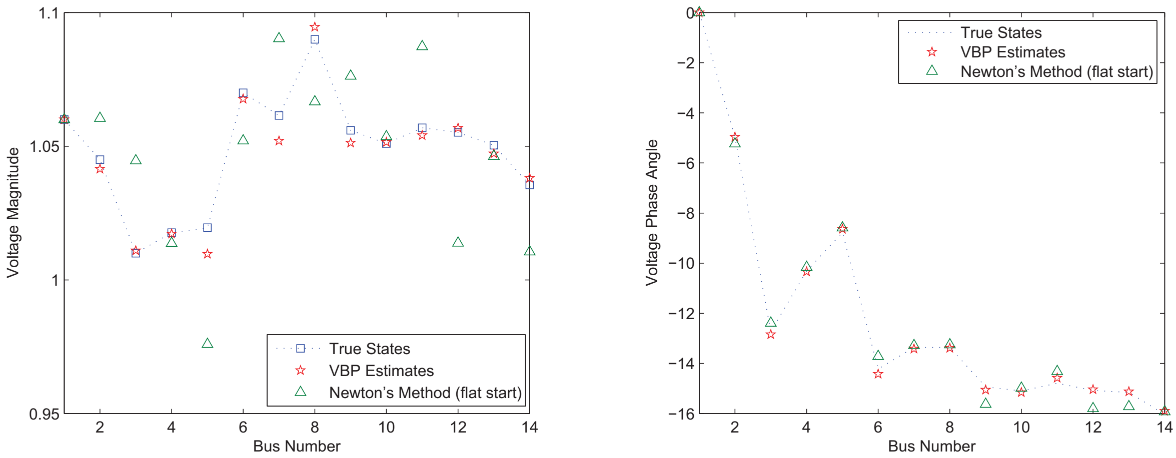


Fig. 4. Normalized Sum Square Errors (14 Bus System).

Fig.5(a) and Fig.5(b) show estimate of voltage magnitudes and phase angles of the proposed approach. Likewise, it can be seen from the two plots, that the graphical model-based approach (red star) is superior to the flat start method (green triangle) as it provides an estimate much closer to the true state.

2) Computational Cost: On the other hand, a plot of CPU time is provided in Fig.6 for the computational time comparison of the regular WLS and the graphical model-based method. The x axis is the test case bus number. The y axis stands for the $\log(CPU\ time)$. All simulations are obtained using MATLAB on an Intel Core 2 CPU with 3GB RAM. The computational time achieved by the graphical model-based method grows linearly, and is much lower than in the regular WLS method from MATPOWER where matrix inversion has a computational complexity of $O(n^2 \log n)$, growing exponentially. This confirms VBP's scalable feature, which is the key to the design of the future Wide Area Monitoring, Control and Protection (WAMPAC) systems.



(a) Voltage magnitudes

(b) Voltage phase angles

Fig. 5. Results obtained from the IEEE 14 bus.

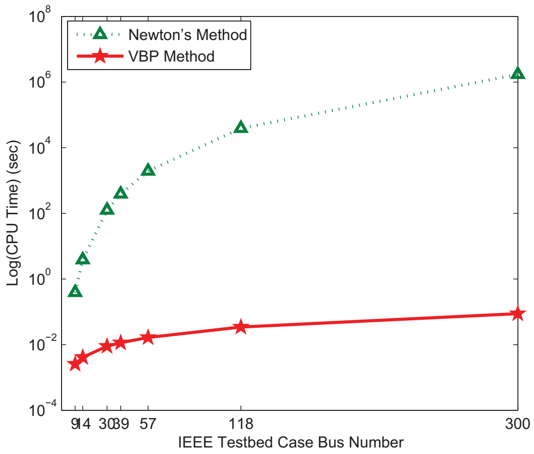


Fig. 6. CPU time comparison.

VII. Conclusions

In this paper, we propose for the first time a probabilistic model for the whole power grid. Unlike many cyber models that do not account for physical constraints, and unlike deterministic engineering modeling defined solely by physical laws, this paper combines the two into a single cyber-physical graphical model. This paper introduces a distributed graphical model approach for AC power system state estimation; as such, it is sufficiently scalable to account for LV distributed technologies. The proposed approach is able to relate state variables and their interactions through measurements in a graphical model according to physical laws such as KCL and KVL. Besides, a state estimate can be efficiently located via local computation (belief propagation), and achieved by the increasingly available embedded cyber (i.e., computational and communication) intelligence. We demonstrate that the proposed approach can significantly reduce the error in SE. Further, its linear computational time is attractive due to the network scalability needs, which is vital for the future large-scale smart grid.

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