

The advanced heuristic is like the original Manhattan distance heuristic, however, the advanced doubled the value from calculating the Manhattan distance except when the 2x2 block is only one step from the solution. In other words, the advanced heuristic function calculates the Manhattan distance and checks whether the distance is 1 or not, if distance is 1 then the advanced heuristic function returns just 1, otherwise it returns $2 \times \text{Manhattan distance}$.

It is admissible because for every state, the heuristic value is greater than or equal to 0 and is less than or equal to the cost of the cheapest path. For the common configuration, the heuristic value of the initial state is 6, which is less than the cost of the cheapest path. Consider some edge cases: it never exceeds the cheapest cost because when the current state is only one step from the solution it's just 1 and the real cost would be 1 as well. Also, let's say the current state is not one step from the solution but the Manhattan distance is 1, in this case the cost of the cheapest path from current state to the goal state is greater than the advance heuristic value. Moreover, heuristic value for the goal state is 0 since the Manhattan distance is 0.

It dominates the original Manhattan distance heuristic since it doubles the heuristic value of the original Manhattan distance heuristic. So,

- $h_{\text{advanced}}(n) \geq h_{\text{original}}(n)$, for every state n
- $h_{\text{advanced}}(n) > h_{\text{original}}(n)$, for at least one state n

In addition, I've tested for all 32 configurations, the A* search using the advance heuristic function expands lesser states than using the original Manhattan distance heuristic.