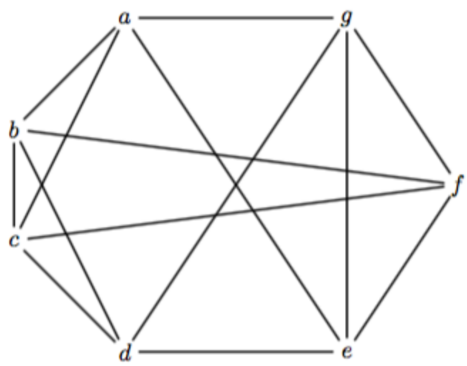
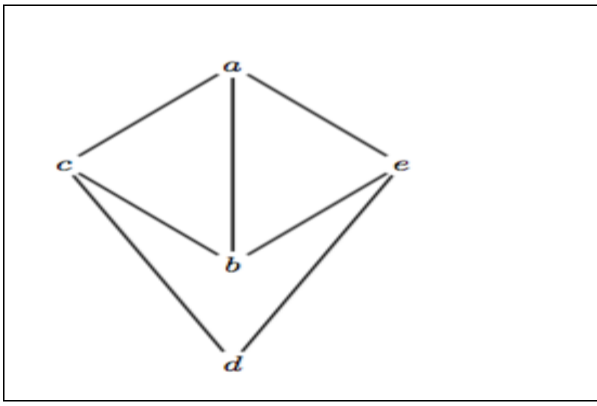
3805ICT Advanced Algorithms
Graph Problems

1. Eulerian Graphs
   a. Explain what is meant by an Eulerian circuit in a finite undirected graph.
   b. What simple property of the graph corresponds exactly to the existence of Eulerian circuits?
   c. Explain clearly how to compute an Eulerian circuit in such graphs (you need not write a program, but whatever algorithm you choose must be clearly explained).

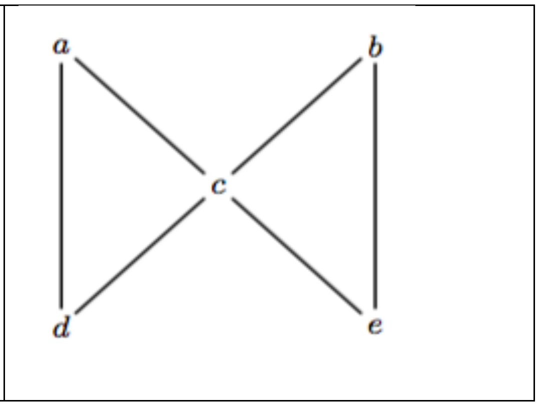
2. Explain why the following graph is Eulerian and find an Eulerian circuit.



3. Determine if both the following graphs are Hamiltonian or not. Justify your answers.

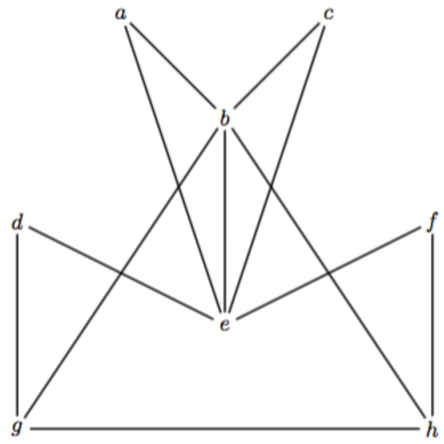


A

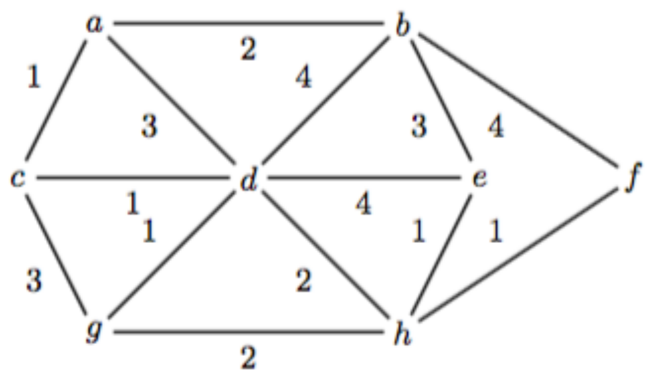


B

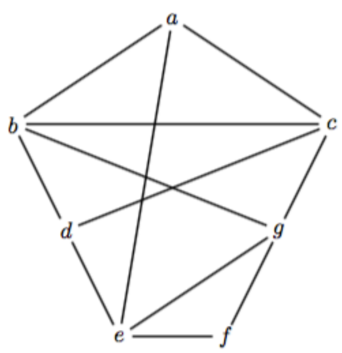
4. Is the graph Hamiltonian and/or Eulerian? Justify your answers.



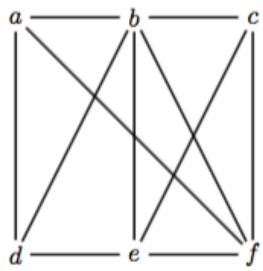
5. Solve the Chinese Postman Problem for the following graph.



6. Is the following graph isomorphic to $K_{3,4}$?



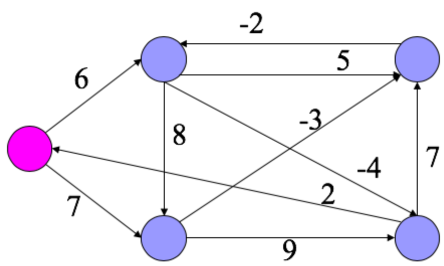
7. Show that the following graph is planar by drawing an isomorphic plane graph with straight edges.



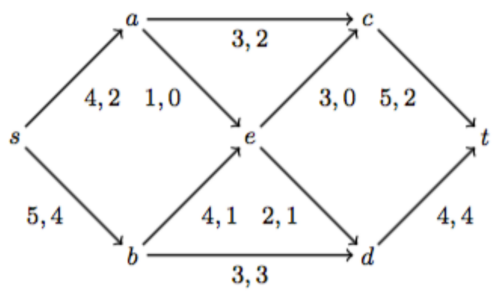
8. Define the following terms, stating their defining properties and making use of equations where appropriate.

   a. flow network (a graph G = (V, E))
   b. flow (a function $f : V \times V \rightarrow \mathbb{R}$)
   c. value of a flow (a real number)
   d. residual network (a graph $G_f = (V, E_f)$)
   e. residual capacity (a function $c_f : V \times V \rightarrow \mathbb{R}$)
   f. augmenting path (a sequence of edges).

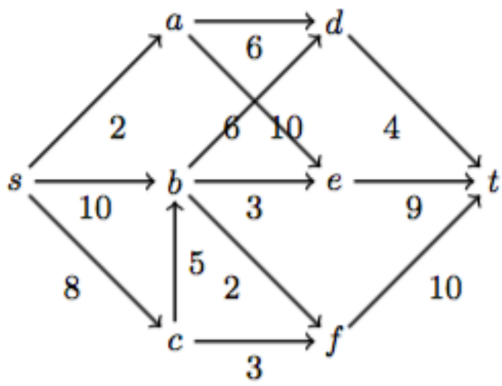
9. Write a working C++ program that uses the Bellman-Ford algorithm to find the shortest paths from the pink node to all other nodes:



10. For the following network flow:
    a. Verify the law of conservation at *a*, *e* and *d*.
    b. Find the value of the indicated flow.
    c. Find the capacity of the (*s*,*t*)-cut defined by $S = \{s, a, b\}$ and $T = \{c, d, e, t\}$.
    d. Can the flow be increased along the path *sbedt*? If so, by how much?
    e. Is the given flow maximum? Explain.



11. Find a maximum flow in the following directed network and prove that it is a maximum (capacities are clockwise from flow arrow).



12. Run the Floyd-Warshall algorithm on the following graph showing the $D^{(k)}$ matrix at each step.