Watch it! – dowel design to *TR34*

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Abstract
This Note compares the design approach for dowels in concrete given in Concrete Society *Technical report 34*[1] (*TR34*) with an expression for their strength taken from CEB-FIP Model Code 1990[2] (*MC90*) and an alternative version of this derived here. Although *TR34*'s approach gives similar results for normal strength concretes with little eccentricity, it is unsafe for dowels in higher-strength concretes and where the eccentricity exceeds 10% of the dowel diameter.

Theory to *MC90* and alternative expressions
An expression for the design strength of a dowel in concrete, taken from *MC90*, was given in 1995NST_11; an expression of similar form is derived below. The bearing strength of the concrete is taken as the basic compressive design strength, f_{cd}, (for example $0.85f_c$' in *ACI318*[3], $0.85f'_c$ in *AS3600*[4], $0.45f_{cu}$ in *BS8110*[5] and f_{cd} in *EC2*[6]) factored by a constant k. The minimum edge distances specified on the diagram in 1995NST_11 ensure that failure occurs by crushing of the concrete and yielding of the steel, rather than by punching or splitting of the concrete.

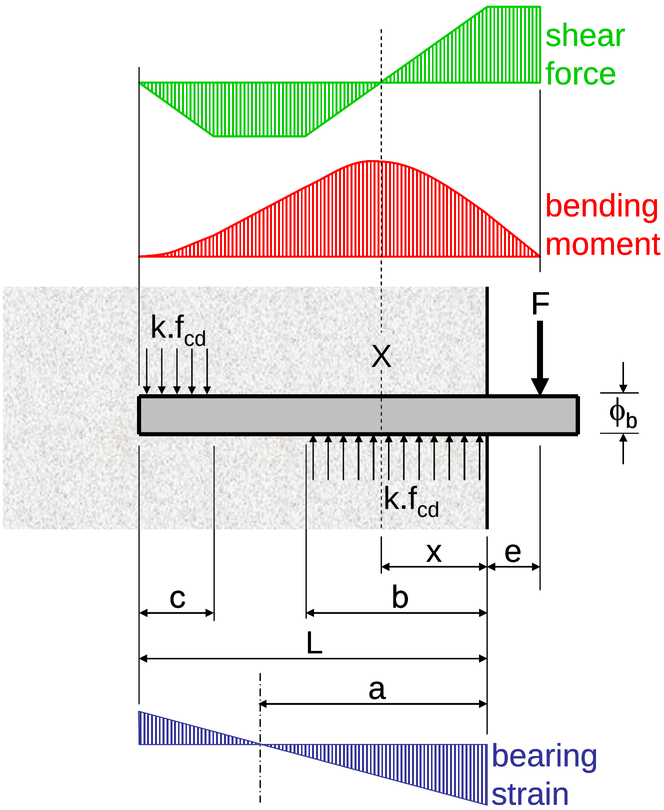

Fig 1. Geometry of dowel with external and internal forces.

Strength
The maximum moment is at X where shear = 0 $\therefore$ $x = F/(k.f_{cd}.\phi_b)$ (1)

Considering loads and reactions to the right of X, moment $M = F.(e + x/2)$ (2)

Maximum strength of bar in bending, assuming a plastic distribution = $\phi_b^3.f_{yd}/6$ (3)

Substituting (1) into (2) and equating with (3) gives: $F_{max}.(e + [F_{max}/(k.f_{cd}.\phi_b)]/2) = \phi_b^3.f_{yd}/6$
$\therefore F_{max}^2 + F_{max}.2kf_{cd}.\phi_b e - k.f_{cd}.f_{yd}.\phi_b^4/3 = 0$

Substituting ε for $3(e/\phi_b).\sqrt{[f_{cd}/f_{yd}]}$ gives $F_{max}^2 + F_{max}.2k.\sqrt{[f_{cd}.f_{yd}]}.\phi_b^2.\varepsilon/3 - k.f_{cd}.f_{yd}.\phi_b^4/3 = 0$
giving:

$$F_{max} = \sqrt{[k.f_{cd}.f_{yd}.\ \phi_b^4/3 + k^2.f_{cd}.f_{yd}.\ \phi_b^4.\varepsilon^2/9} - k.\sqrt{[f_{cd}.f_{yd}]}.\phi_b^2.\varepsilon$$
$$= \phi_b^2.\sqrt{[f_{cd}.f_{yd}]}\ .(\sqrt{[\ k/3 + (k/3)^2.\varepsilon^2]} - k\varepsilon/3) \qquad (4)$$

Expression (4) gives the *MC90* expression (5) if $\sqrt{[k/3]} = 1.3$, that is k = 5.07, and a model partial factor, γ_{Rd}, with a recommended value of 1.3, is introduced:

$$F_{max} = 1.3\phi_b^2.\sqrt{[f_{cd}.f_{yd}]}\ .(\sqrt{[1 + (1.3\varepsilon)^2]} - 1.3\varepsilon)\ /\ \gamma_{Rd} \qquad (5)$$

Although the concrete on which the dowel bears is well confined, *EC2* limits the bearing stress in these situations to three times the normal compressive design strength compared with which *MC90*'s implicit value of 5.07 seems rather high. Restricting the factor k to 3.00 probably makes an additional model partial factor unnecessary. An alternative expression for the dowel strength (referred to later as *MC90a*) is therefore:

$$F_{max} = \phi_b^2.\sqrt{[f_{cd}.f_{yd}]}\ .(\sqrt{[1 + \varepsilon^2]} - \varepsilon) \qquad (6)$$

Naturally, F_{max} must not exceed the shear strength of the dowel itself.

Minimum length
Let L be the minimum length of dowel embedment.

For a beam in bending, a rectangular stress block, which acts over a given proportion of the depth from the compression face to the neutral axis, is often used. In a similar way, the bearing stress will be assumed to act over a proportion, λ, of the neutral axis distance, a. This results in:

$$a = (L+x)/2$$
$$b = \lambda a \quad = \lambda(L+x)/2$$

At x, shear = 0 so there can be no net load to the left of X

$$\therefore \quad c = b\text{-}x \quad = \lambda(L+x)/2 - x$$

M can be expressed, considering reactions to the left of X, as $c.(L - x - c).k.f_{cd}.\phi_b$

$$\therefore\ M = [\lambda(L+x)/2 - x].[L - \lambda(L+x)/2].\ k.f_{cd}.\phi_b \qquad (7)$$

Equating expressions (2) and (7) gives:

$$ex + x^2/2 = L^2.(-\lambda^2/4 + \lambda/2) + L.x.(-\lambda^2/2 + \lambda - 1) + x^2.(-\lambda^2/4 + \lambda/2)$$
$$\therefore\ L/x = \{2-2\lambda+\lambda^2 + \text{sqrt}[2.(2-2\lambda+\lambda^2 + 2.(2\lambda-\lambda^2).e/x]\}\ /\ (2\lambda-\lambda^2)$$

$$\therefore\ L = \{F/(k.f_{cd}.\phi_b)\}\ .\ \{2-2\lambda+\lambda^2 + \text{sqrt}[2.(2-2\lambda+\lambda^2 + 2.(2\lambda-\lambda^2).e.k.f_{cd}.\phi_b/F]\}\ /\ \{2\lambda-\lambda^2\}$$

EC2 gives λ a value of 0.8 for class 50/60 concrete and below, for which value:

$$L = \{F/(k.f_{cd}.\phi_b)\}\ .\ \{1.083 + \text{sqrt}[2.257 + 4.167.e.k.f_{cd}.\phi_b/F]\}$$

Substituting k as 3.00 gives:

$$L = \{F/(f_{cd}.\phi_b)\}\ .\ \{0.361 + \text{sqrt}[0.251 + 1.389.e.f_{cd}.\phi_b/F]\} \qquad (8)$$

Dowel strength to *TR34*
In addition to checking the shear strength of the dowel, *TR34* considers independently the bearing capacity of the concrete and the bending capacity of the dowels.

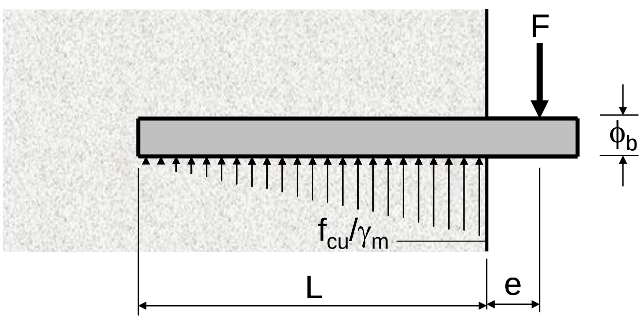

Fig 2. Geometry of dowel with reaction assumed by *TR34.*

The bearing stress is assumed to have a triangular distribution, as shown in Fig 2, with a peak value of f_{cu}/γ_m, where γ_m has a value of 1.5. The effective bearing length, L, is not to be taken as greater than $8\phi_b$. While this bearing stress can be in vertical equilibrium with the applied load, **it clearly cannot provide rotational equilibrium**.

The bending strength of the dowel is checked at the face of the concrete, assuming a plastic stress distribution across the section. It is plain that the bending moment continues to increase within the concrete.

Comparison

The design strengths of a dowel, of steel with a characteristic strength of 500MPa, calculated using the expressions in *TR34*, *MC90* and *MC90a*, are compared in Fig 3 (for concrete class 25/30) and Fig 4 (for concrete class 50/60). It will be seen that there is little difference between the values for *MC90* and *MC90a*; they are identical for no eccentricity, with *MC90* becoming more slightly conservative for high eccentricities. In comparison, although *TR34* gives only slightly higher values for zero eccentricity and normal strength concrete, the ratio approaches a value of two for eccentricities of about the bar diameter. For eccentricities exceeding three dowel diameters, the strengths are similar, being almost the same as *MC90a*.

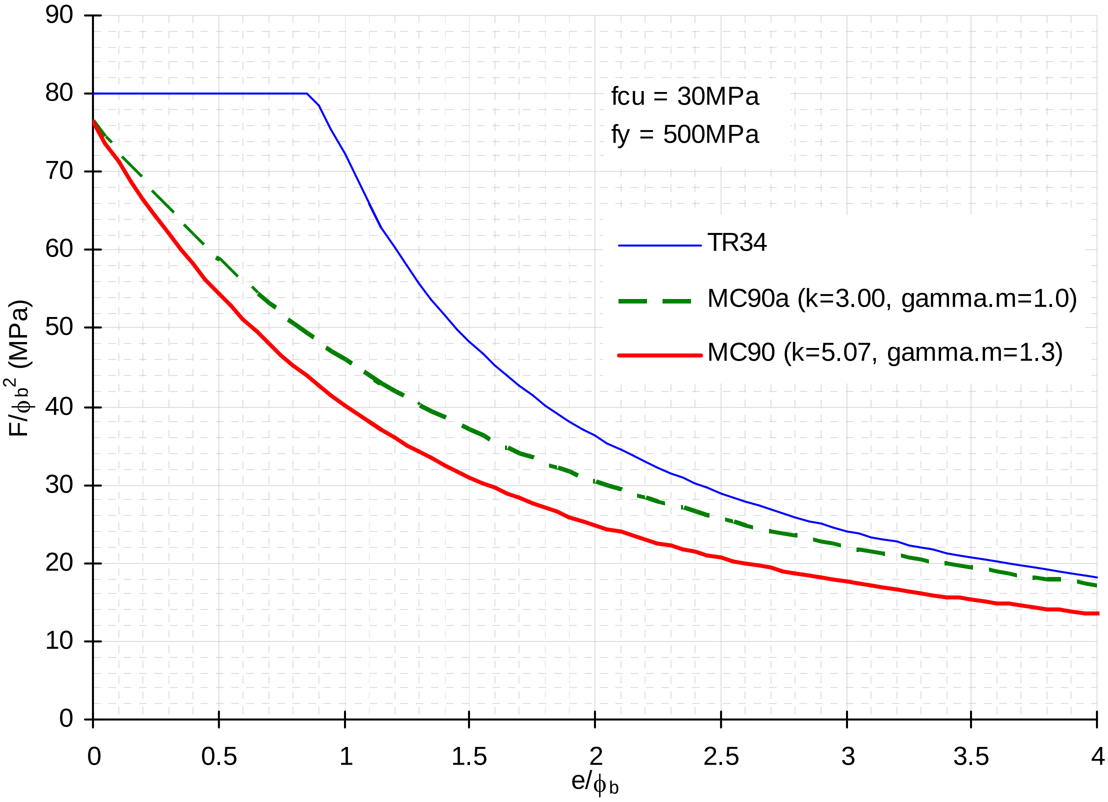

Fig 3. Comparison of dowel design strengths as given by *TR34*, *MC90a* and *MC90* – class 25/30 concrete.

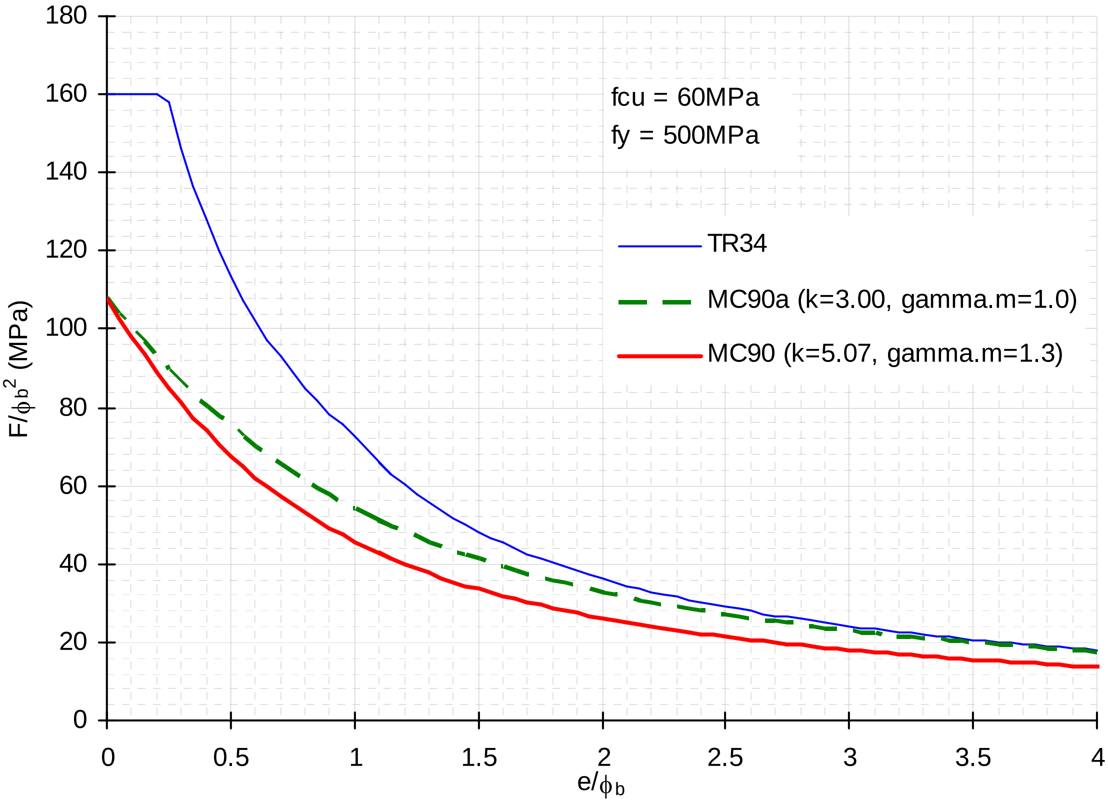

Fig 4. Comparison of dowel design strengths as given by *TR34*, *MC90a* and *MC90* – class 50/60 concrete

Conclusions

- The *MC90* dowel expression can be obtained considering the internal and external forces in Fig 1 and taking appropriate values for the bearing strength and model partial factor.
- Expression (6) (*MC90a*) gives similar strengths to *MC90* and can be obtained adopting the limiting bearing strength from EC2, which seem more credible.
- The *TR34* approach does not have a consistent model. However, it gives similar strengths for dowels with no eccentricity in low strength concretes. In other situations it may give strength of up to double those given by *MC90*.
- The minimum embedment length of the dowel is given by expression (8). Note that *MC90* requires a minimum embedment of eight diameters; in some situations it may be appropriate to use expression (8) to justify a reduced length.

References

(1) CONCRETE SOCIETY. *Technical report 34*. Concrete industrial ground floors: a guide to design and construction (third edition). Concrete society, 2003.
(2) COMITÉ EURO-INTERNATIONAL DU BÉTON. CEB-FIP *Model code 1990*. Design code. Thomas Telford 1993.
(3) AMERICAN CONCRETE INSTITUTE. *ACI318-02*. Building code requirements for structural concrete. ACI, 2002.
(4) STANDARDS ASSOCIATION OF AUSTRALIA. *AS3600-2001*. Concrete structures. SAA, 2001.
(5) BRITISH STANDARDS INSTITUTION. *BS8110: Part 1: 1997*. Structural use of concrete. Part 1. Code of practice for design and construction. BSI, 1997.
(6) BRITISH STANDARDS INSTITUTION. *BSEN1992-1-1:2004*. Eurocode 2. Design of concrete structures. Part 1-1: General rules and rules for buildings. BSI, 2004.

Related Note

'Design of dowels and fastenings in concrete', *Notes on Structures* 1995NST_11

First published September 2006
Revised (a) October 2006 - numbering of expressions 7 and 8 corrected; expression for k = 3 added
Revised (b) January 2007 – incorrect bracket in expression 8 removed