

1. **Resolver:**  $\int x^2(4-x^2)^3 dx$

**Solucion:**

$$\int x^2(4-x^2)^3 dx = \int x^8 + 12x^6 - 48x^4 + 64x^2 = \frac{x^9}{9} + \frac{12x^7}{7} - \frac{48x^5}{5} + \frac{64x^3}{3} + C$$

2. **Resolver:**  $\int \frac{(x+1)dx}{\sqrt{x^2+2x-4}}$

**Solucion:**

**Sea**  $u = x^2 + 2x - 4$

$$du = 2x + 2dx = \frac{du}{2}(x+1)dx$$

$$\int \frac{(x+1)dx}{\sqrt{x^2+2x-4}} = \int \frac{du}{2\sqrt{u}} = \frac{1}{2} \int \frac{du}{\sqrt{u}}$$

$$\int \frac{(x+1)dx}{\sqrt{x^2+2x-4}} = 2 \cdot \frac{1}{2} u^{1/2} + c = (x^2 + 2x - 4)^{1/2} + C$$

3. **Resolver:**  $\int \frac{(1+\sqrt{x})^2}{\sqrt{x}} dx$

**Solucion:**

$$u = 1 + \sqrt{x}$$

$$du = \frac{1}{2\sqrt{x}} dx$$

$$dx = 2\sqrt{x} du$$

$$\int \frac{(1+\sqrt{x})^2}{\sqrt{x}} dx = \int \frac{u^2 \cdot 2}{2\sqrt{x}} dx = 2 \int u^2 du = \frac{2}{3} u^3 + C = \frac{2}{3} (1+\sqrt{x})^3 + C$$

4. **Resolver:**  $\int \frac{(x^2+2x)dx}{\sqrt{x^3+3x^2+1}}$

**Solucion:**

$$u = x^3 + 3x^2 + 1$$

$$du = (3x^2 + 6x)dx = 3(x^2 + 2x)dx$$

$$\int \frac{(x^2+2x)dx}{\sqrt{x^3+3x^2+1}} = \frac{1}{3} \int \frac{3(x^2+2x)dx}{\sqrt{x^3+3x^2+1}} = \frac{1}{3} \int \frac{du}{\sqrt{u}} = \frac{2}{3} u^{1/2} + C = \frac{2}{3} (x^3 + 3x^2 + 1)^{1/2} + C$$

5. **Resolver:**  $\int \frac{x^3 - 4x^2 + 5x - 1}{x^2 - 2x + 1} dx$

**Solucion:**

$$\frac{x^3 - 4x^2 + 5x - 1}{x^2 - 2x + 1} = (x - 2) + \frac{1}{x^2 - 2x + 1}$$

$$\begin{aligned}
\int \frac{x^3 - 4x^2 + 5x - 1}{x^2 - 2x + 1} dx &= \int \left( (x - 2) + \frac{1}{x^2 - 2x + 1} \right) dx \\
\int \frac{x^3 - 4x^2 + 5x - 1}{x^2 - 2x + 1} dx &= \int \left( (x - 2) + \frac{1}{x^2 - 2x + 1} \right) dx = \int (x - 2) dx + \\
&\int \frac{1}{(x - 1)^2} \\
\int \frac{x^3 - 4x^2 + 5x - 1}{x^2 - 2x + 1} &= \int x dx - \int 2 dx + \int (x - 1)^{-2} dx \\
\int \frac{x^3 - 4x^2 + 5x - 1}{x^2 - 2x + 1} &= \frac{x^2}{2} - 2x - (x - 1)^{-1} + C \\
\int \frac{x^3 - 4x^2 + 5x - 1}{x^2 - 2x + 1} &= \frac{x^2}{2} - 2x - \frac{1}{(x - 1)} + C
\end{aligned}$$

6. **Resolver:**  $\int \frac{x dx}{(x - 5)^6}$

**Solucion:**

**Cambio de variable**

$$u = x - 5 \Rightarrow x = u + 5$$

$dx = du$ , **sustituyendo se tiene:**

$$\begin{aligned}
\int \frac{x dx}{(x - 5)^6} &= \int \frac{(u + 5)}{u^6} du = \int \frac{u}{u^6} du = \int u^{-2} du + 5 \int u^{-6} du = \frac{u^{-4}}{-4} + \\
&5 \frac{u^{-5}}{-5} + C \\
\int \frac{x dx}{(x - 5)^6} &= \frac{-u^{-4}}{4} - u^{-5} + C = -\frac{1}{4(x - 5)^4} - \frac{1}{(x - 5)^5} + C
\end{aligned}$$

7. **Resolver:**  $\int (x + 1)(x - 2)^9 dx$

**Solucion:**

**sea**  $u = x - 2 \Rightarrow x = u + 2, du = dx$ , **luego se tiene:**

$$\begin{aligned}
\int (x + 1)(x - 2)^9 dx &= \int (u + 2 + 1)(u + 2 - 2)^9 dx = \int (u + 3)u^9 dx \\
\int (x + 1)(x - 2)^9 dx &= \int (u^{10} + 3u^9) du = \int u^{10} du + \int 3u^9 du = \frac{u^{11}}{11} + \\
&3 \frac{u^{10}}{10} + C \\
\int (x + 1)(x - 2)^9 dx &= \frac{(x - 2)^{11}}{11} + 3 \frac{(x - 2)^{10}}{10} + C
\end{aligned}$$

8. **Resolver:**  $\int \left( x + \frac{1}{x} \right)^{3/4} \left( 1 - \frac{1}{x^2} \right) dx$

**Solucion:**

**Sea**  $u = x + \frac{1}{x}, u = x + x^{-1} \Rightarrow du = 1 - x^{-2}dx$  **luego**  $du = 1 - \frac{1}{x^2}dx$

**Entonces se tendrá:**

$$\int_C \left(x + \frac{1}{x}\right)^{3/4} \left(1 - \frac{1}{x^2}\right) dx = \int u^{3/4} du = \frac{u^{7/4}}{7/4} + C = \frac{4}{7} u^{7/4} + C \frac{4}{7} \left(x + \frac{1}{x}\right)^{7/4} +$$

9. **Resolver:**  $\int \frac{x+3}{(3-x)^{2/3}} dx$

**Solucion:**

**Sea**  $u = 3 - x, \Rightarrow x = 3 - u, du = -dx$ , **luego se tiene:**

$$\begin{aligned} \int \frac{x+3}{(3-x)^{2/3}} dx &= - \int \frac{3-u+3}{u^{2/3}} du = - \int \frac{6-u}{u^{2/3}} du = - \left[ 6 \int u^{-2/3} du - \int u^{1/3} du \right] \\ \int_C \frac{x+3}{(3-x)^{2/3}} dx &= - \left[ 6 \frac{u^{1/3}}{1/3} - \frac{u^{4/3}}{4/3} \right] + C = - \left[ 6 \frac{(3-x)^{1/3}}{1/3} - \frac{(3-x)^{4/3}}{4/3} \right] + \end{aligned}$$

10. **Resolver:**  $\int \sqrt{1 + \frac{1}{2x}} \cdot x^{-3} dx$

**Solucion:**

**Sea**  $u = 1 + \frac{1}{2x} \Rightarrow x = \frac{1}{2(u-1)}, du = -\frac{1}{2x^2} dx \Rightarrow dx = -2x^2 du$

**Reemplazando se tiene:**

$$\begin{aligned} \int \sqrt{1 + \frac{1}{2x}} \cdot x^{-3} dx &= \int u^{1/2} \left( \frac{1}{2(u-1)} \right)^{-3} (-2) \left( \frac{1}{2(u-1)} \right)^2 du = (-2) \int u^{1/2} \left( \frac{1}{2(u-1)} \right)^{-1} du \\ \int \sqrt{1 + \frac{1}{2x}} \cdot x^{-3} dx &= (-4) \int [u^{1/2}(u-1)] du = (-4) \int (u^{3/2} - u^{1/2}) du = \\ &= (-4) \left( \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} + C \right) \\ \int \sqrt{1 + \frac{1}{2x}} \cdot x^{-3} dx &= \left( \frac{8(1 + \frac{1}{2x})^{3/2}}{3} - \frac{8(1 + \frac{1}{2x})^{5/2}}{5} \right) + C \end{aligned}$$

11. **Resolver:**  $\int \frac{2x}{(1-x)^{2/3}} dx$

**Solucion:**

**Sea**  $u = 1 - x$ , **entonces**  $x = 1 - u, dx = -du$ , **luego se tiene:**

$$\begin{aligned} \int \frac{2x}{(1-x)^{2/3}} dx &= - \int \frac{2(1-u)}{u^{2/3}} du = - \int (2-2u)u^{-2/3} du \\ \int \frac{2x}{(1-x)^{2/3}} dx &= - \int 2u^{-2/3} du + \int 2u^{1/3} du = -6u^{1/3} + \frac{3}{2}u^{4/3} + C \\ \int \frac{2x}{(1-x)^{2/3}} dx &= -6(1-x)^{4/3} + C \end{aligned}$$

12. Resolver:  $\int x^2 \sqrt[3]{x+2} dx$

**Solucion:**

**Sea**  $u = x + 2$  **entonces**  $x = u - 2, dx = du$ , **luego se tiene:**

$$\begin{aligned} \int x^2 \sqrt[3]{x+2} dx &= \int (u-2)^2 u^{1/3} du = \int (u^2 - 4u + 4) u^{1/3} du \\ \int x^2 \sqrt[3]{x+2} dx &= \int (u^{7/3} - 4u^{4/3} + 4u^{1/3}) du = \frac{3}{10} u^{10/3} - \frac{12}{7} u^{7/3} - \\ &\quad \frac{12}{7} u^{7/3} + 3u^{4/3} + C \\ \int x^2 \sqrt[3]{x+2} dx &= \frac{3}{10} (x+2)^{10/3} - \frac{12}{7} (x+2)^{7/3} + 3(x+2)^{4/3} + C \end{aligned}$$

13. Resolver:  $\int \frac{x dx}{\sqrt{x+3}}$

**Solucion:**

**Sea**  $u = x + 3$  **entonces**  $x = u - 3, dx = du$ , **luego se tiene:**

$$\begin{aligned} \int \frac{x dx}{\sqrt{x+3}} &= \int x(x+3)^{-1/2} du = \int (u-3)(u)^{-1/2} du = \int u^{1/2} du - \\ &\quad 3 \int u^{-1/2} du \\ \int \frac{x dx}{\sqrt{x+3}} &= \frac{u^{3/2}}{3/2} - 3 \frac{u^{1/2}}{1/2} + C = \frac{2(x+3)^{3/2}}{3} - 6(x+3)^{1/2} + C \end{aligned}$$

14. Resolver:  $\int \frac{2dx}{(1-x)^{2/3}}$

**Solucion:**

**Sea**  $u = 1 - x$ , **entonces**  $x = 1 - u, dx = -du$ , **luego se tiene:**

$$\begin{aligned} \int \frac{2dx}{(1-x)^{2/3}} &= - \int \frac{2(1-u)du}{(u)^{2/3}} \\ \int \frac{2dx}{(1-x)^{2/3}} &= - \left[ \int \frac{2du}{(u)^{2/3}} - \frac{2udu}{(u)^{2/3}} \right] \\ \int \frac{2dx}{(1-x)^{2/3}} &= - \left[ 6u^{1/3} - \frac{3u^{4/3}}{2} \right] + C \\ \int \frac{2dx}{(1-x)^{2/3}} &= \frac{3(1-x)^{4/3}}{2} - 6(1-x)^{1/3} + C \end{aligned}$$

15. Resolver:  $\int \sqrt{3+x}(x+1)^2 dx$

**Solucion:**

**Sea**  $u = 3 + x$  **entonces**  $x = u - 3, dx = du$ , **luego se tiene:**

$$\int \sqrt{3+x}(x+1)^2 dx = \int \sqrt{u}(u-3+1)^2 du$$

$$\begin{aligned}
\int \sqrt{3+x}(x+1)^2 dx &= \int u^{1/2}(u^2 - 4u + 4) du \\
\int \sqrt{3+x}(x+1)^2 dx &= \int (u^{5/2} - 4u^{3/2} + 4u^{1/2}) du \\
\int \sqrt{3+x}(x+1)^2 dx &= \int u^{5/2} du - \int 4u^{3/2} du + \int 4u^{1/2} du \\
\int \sqrt{3+x}(x+1)^2 dx &= \frac{u^{7/2}}{7/2} - 4 \frac{u^{5/2}}{5/2} + 4 \frac{u^{3/2}}{3/2} \\
\int \sqrt{3+x}(x+1)^2 dx &= \frac{2}{7}(3+x)^{7/2} - \frac{8}{5}(3+x)^{5/2} + \frac{8}{3}(3+x)^{3/2} + C
\end{aligned}$$

16. **Resolver:**  $\int \frac{3dx}{1-2x}$

**Solucion:**

**Sea**  $u = 1 + 2x$  **entonces**  $x = (u - 1)/2, 2dx = du$  **luego se tiene:**

$$\int \frac{3dx}{1-2x} = \int \frac{3}{u} \left( \frac{du}{2} \right) = \frac{3}{2} \ln|u| + C = \frac{3}{2} \ln|1 + 2x| + C$$

17. **Resolver:**  $\int \frac{x^2 dx}{1-x^3}$

**Solucion:**

**Sea**  $u = 1 - x^3$  **entonces**  $du = -3x^2 dx$  **luego se tiene:**

$$\int \frac{x^2 dx}{1-x^3} = -\frac{1}{3} \int \frac{-3x^2 dx}{1-x^3} = -\frac{1}{3} \int \frac{du}{u} = -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|1-x^3| + C$$

18. **Resolver:**  $\int \frac{(x+1)dx}{x^2+2x+3}$

**Solucion:**

**Sea**  $u = x^2 + 2x + 3$  **entonces**  $du = (2x + 2)dx$ , **luego se tiene:**

$$\int \frac{(x+1)dx}{x^2+2x+3} = \frac{1}{2} \int \frac{2(x+1)dx}{x^2+2x+3} = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|x^2 + 2x + 3| + C$$

19. **Resolver:**  $\int \frac{x-1}{x+1} dx$

**Solucion:**

**Sea**  $u = x + 1$  **entonces**  $x = u - 1, du = dx$ , **luego se tiene:**

$$\begin{aligned}
\int \frac{x-1}{x+1} dx &= \int \frac{(u-1)-1}{u} du = \int \frac{(u-2)du}{u} = \int \frac{udu}{u} - 2 \int \frac{du}{u} \\
\int \frac{x-1}{x+1} dx &= u - 2 \ln|u| + C = x - 2 \ln|x+1| + C
\end{aligned}$$

20. **Resolver:**  $\int \frac{\ln(\ln x) + 1}{x \ln x} dx$

**Solucion:**

Se puede ver que la derivada del numerador es  $\frac{1}{x \ln x}$ , luego la Integral es directa, es decir,

$$\int \frac{\ln(\ln x) + 1}{x \ln x} dx = \frac{[\ln(\ln x) + 1]^2}{2} + C = \frac{1}{2}(\ln^2(\ln x) + 2\ln(\ln x) + 1^2) + C$$

$$\int \frac{\ln(\ln x) + 1}{x \ln x} dx = \frac{1}{2}\ln^2(\ln x) + \ln(\ln x) + C$$

21. **Resolver:**  $\int \frac{e^{2x}}{e^{2x} + 1} dx$

**Solucion:**

sea  $u = e^{2x}$  entonces  $du = e^{2x} dx$ , luego se tiene:

$$\int \frac{e^{2x}}{e^{2x} + 1} dx = \frac{1}{2} \int \frac{2e^{2x}}{e^{2x} + 1} dx = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|e^{2x} + 1| + C$$

22. **Resolver:**  $\int \frac{e^x - 1}{e^x + 1} dx$

**Solucion:**