1. Resolver: $\int x^2(4-x^2)^3dx$

   Solucion:

   $\int x^2(4-x^2)^3dx = \int x^8+12x^6-48x^4+64x^2 = \frac{x^9}{9}+\frac{12x^7}{7}-\frac{48x^5}{5a}+\frac{64x^3}{3}+C$

2. Resolver: $\int \frac{(x+1)dx}{\sqrt{x^2+2x-4}}$

   Solucion:

   $seau = x^2+2x-4$

   $du = 2x+2dx = \frac{du}{2}(x+1)dx$

   $\int \frac{(x+1)dx}{\sqrt{x^2+2x-4}} = \int \frac{du}{2\sqrt{u}} = \frac{1}{2}\int \frac{du}{\sqrt{u}}$

   $\int \frac{(x+1)dx}{\sqrt{x^2+2x-4}} = 2.\frac{1}{2}u^{1/2}+c = (x^2+2x-4)^{1/2}+C$

3. Resolver: $\int \frac{(1+\sqrt{x})^2}{\sqrt{x}}dx$

   Solucion:

   $u = 1+\sqrt{x}$

   $du = \frac{1}{2\sqrt{x}}dx$

   $dx = 2\sqrt{x}du$

   $\int \frac{(1+\sqrt{x})^2}{\sqrt{x}}dx = \int \frac{u^2 2}{2\sqrt{x}}dx = 2\int u^2du = \frac{2}{3}u^3+C = \frac{2}{3}(1+\sqrt{x})^3+C$

4. Resolver: $\int \frac{(x^2+2x)dx}{\sqrt{x^3+3x^2+1}}$

   Solucion:

   $u = x^3+3x^2+1$

   $du = (3x^2+6x)dx = 3(x^2+2x)dx$

   $\int \frac{(x^2+2x)dx}{\sqrt{x^3+3x^2+1}} = \frac{1}{3}\int \frac{3(x^2+2x)dx}{\sqrt{x^3+3x^2+1}} = \frac{1}{3}\int \frac{du}{\sqrt{u}} = \frac{2}{3}u^{1/2}+C = \frac{2}{3}(x^3+3x^2+1)^{1/2}+C$

5. Resolver: $\int \frac{x^3-4x^2+5x-1}{x^2-2x+1}dx$

   Solucion:

   $\frac{x^3-4x^2+5x-1}{x^2-2x+1} = (x-2)+\frac{1}{x^2-2x+1}$

$$\int \frac{x^3-4x^2+5x-1}{x^2-2x+1}dx = \int \left((x-2)+\frac{1}{x^2-2x+1}\right)dx$$

$$\int \frac{x^3-4x^2+5x-1}{x^2-2x+1}dx = \int \left((x-2)+\frac{1}{x^2-2x+1}\right)dx = \int (x-2)dx + \int \frac{1}{(x-1)^2}$$

$$\int \frac{x^3-4x^2+5x-1}{x^2-2x+1} = \int xdx - \int 2dx + \int (x-1)^{-2}dx$$

$$\int \frac{x^3-4x^2+5x-1}{x^2-2x+1} = \frac{x^2}{2} - 2x - (x-1)^{-1} + C$$

$$\int \frac{x^3-4x^2+5x-1}{x^2-2x+1} = \frac{x^2}{2} - 2x - \frac{1}{(x-1)} + C$$

6. Resolver: $\int \frac{xdx}{(x-5)^6}$

   Solucion:

   combio de variable

   $u = x-5 \Rightarrow x = u+5$

   $dx = du$, sustituyendo se tiene:

   $$\int \frac{xdx}{(x-5)^6} = \int \frac{(u+5)}{u^6}du = \int \frac{u}{u^6}du = \int u^{-2}du + 5\int u^{-6}du = \frac{u^{-4}}{-4} + 5\frac{u^-5}{-5} + C$$

   $$\int \frac{xdx}{(x-5)^6} = \frac{-u^{-4}}{4} - u^{-5} + C = -\frac{1}{4(x-5)^4} - \frac{1}{(x-5)^5} + C$$

7. Resolver: $\int (x+1)(x-2)^9 dx$

   Solucion:

   sea $u = x-2 \Rightarrow x = u+2, du = dx$, luego se tiene:

   $$\int (x+1)(x-2)^9 dx = \int (u+2+1)(u+2-2)^9 dx = \int (u+3)u^9 dx$$

   $$\int (x+1)(x-2)^9 dx = \int (u^10 + 3u^9)du = \int u^10du + \int 3u^9 du = \frac{u^11}{11} + 3\frac{u^10}{10} + C$$

   $$\int (x+1)(x-2)^9 dx = \frac{(x-2)^11}{11} + 3\frac{(x-2)^10}{10} + C$$

8. Resolver: $\int \left(x+\frac{1}{x}\right)^{3/4}\left(1-\frac{1}{x^2}\right)dx$

   Solucion:

sea $u = x + \dfrac{1}{x}, u = x + x^{-1} \Rightarrow du = 1 - x^{-2}dx$ luego $du = 1 - \dfrac{1}{x^2}dx$

entonces se tendra:

$\displaystyle\int \left(x + \frac{1}{x}\right)^{3/4}\left(1 - \frac{1}{x^2}\right)dx = \int u^{3/4}du = \frac{u^{7/4}}{7/4} + C = \frac{4}{7}u^{7/4} + C\frac{4}{7}\left(x + \frac{1}{x}\right)^{7/4} +$

C

9. Resolver: $\displaystyle\int \frac{x+3}{(3-x)^{2/3}}dx$

   Solucion:

   sea $u = 3 - x, \Rightarrow x = 3 - u, du = -dx$, luego se tiene:

   $\displaystyle\int \frac{x+3}{(3-x)^{2/3}}dx = -\int \frac{3-u+3}{u^{2/3}}du = -\int \frac{6-u}{u^{2/3}}du = -\left[6\int u^{-2/3}du - \int u^{1/3}du\right]$

   $\displaystyle\int \frac{x+3}{(3-x)^{2/3}}dx = -\left[6\frac{u^{1/3}}{1/3} - \frac{u^{4/3}}{4/3}\right] + C = -\left[6\frac{(3-x)^{1/3}}{1/3} - \frac{(3-x)^{4/3}}{4/3}\right] +$

   C

10. Resolver: $\displaystyle\int \sqrt{1 + \frac{1}{2x}}.x^{-3}$

    Solucion:

    sea $u = 1 + \dfrac{1}{2x} \Rightarrow x = \dfrac{1}{2(u-1)}, du = -\dfrac{1}{2x^2}dx \Rightarrow dx = -2x^2du$

    Reemplazando se tiene:

    $\displaystyle\int \sqrt{1 + \frac{1}{2x}}.x^{-3} = \int u^{1/2}\left(\frac{1}{2(u-1)}\right)^{-3}(-2)\left(\frac{1}{2(u-1)}\right)^{2}du = (-2)\int u^{1/2}\left(\frac{1}{2(u-1)}\right)^{-1}du$

    $\displaystyle\int \sqrt{1 + \frac{1}{2x}}.x^{-3} = (-4)\int[u^{1/2}(u-1)]du = (-4)\int(u^{3/2} - u^{1/2})du =$

    $\displaystyle(-4)\left(\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} + C\right)$

    $\displaystyle\int \sqrt{1 + \frac{1}{2x}}.x^{-3} = \left(\frac{8\left(1 + \frac{1}{2x}\right)^{3/2}}{3} - \frac{8\left(1 + \frac{1}{2x}\right)^{5/2}}{5}\right) + C$

11. Resolver: $\displaystyle\int \frac{2x}{(1-x)^{2/3}}dx$

    Solucion:

    sea $u = 1 - x$, entonces $x = 1 - u, dx = -du$, luego se tiene:

    $\displaystyle\int \frac{2x}{(1-x)^{2/3}}dx = -\int \frac{2(1-u)}{u^{2/3}}du = -\int (2-2u)u^{-2/3}du$

    $\displaystyle\int \frac{2x}{(1-x)^{2/3}}dx = -\int 2u^{-2/3}du + \int 2u^{1/3}du = -6u^{1/3} + \frac{3}{2}u^{4/3} + C$

    $\displaystyle\int \frac{2x}{(1-x)^{2/3}}dx = -6(1-x)^{4/3} + C$

12. Resolver: $\int x^2\sqrt[3]{x+2}dx$

 Solucion:

 sea $u = x+2$ entonces $x = u-2, dx = du$,luego se tiene:

 $\int x^2\sqrt[3]{x+2}dx = \int (u-2)^2 u^{1/3}du = \int (u^2-4u+4)u^{1/3}du$

 $\int x^2\sqrt[3]{x+2}dx = \int \left(u^{7/3}-4u^{4/3}+4u^{1/3}\right)du = \frac{3}{10}u^{10/3} - \frac{12}{7}u^{10/3} - \frac{12}{7}u^{7/3}+3u^{4/3}+C$

 $\int x^2\sqrt[3]{x+2}dx = \frac{3}{10}(x+2)^{10/3} - \frac{12}{7}(x+2)^{7/3} + 3(x+2)^{4/2} + C$

13. Resolver: $\int \frac{xdx}{\sqrt{x+3}}$

 Solucion:

 sea $u = x+3$ entonces $x = u-2, dx = du$, luego se tiene:

 $\int \frac{xdx}{\sqrt{x+3}} = \int x(x+3)^{-1/2}du = \int (u-3)(u)^{-1/2}du = \int u^{1/2}du - 3\int u^{-1/2}du$

 $\int \frac{xdx}{\sqrt{x+3}} = \frac{u^{3/2}}{3/2} - 3\frac{u^{1/2}}{1/2} + C = \frac{2(x+3)^{3/2}}{3} - 6(x+3)^{1/2} + C$

14. Resolver: $\int \frac{2dx}{(1-x)^{2/3}}$

 Solucion:

 sea $u = 1-x$, entonces $x = 1-u, dx = -du$,luego se tiene:

 $\int \frac{2dx}{(1-x)^{2/3}} = -\int \frac{2(1-u)du}{(u)^{2/3}}$

 $\int \frac{2dx}{(1-x)^{2/3}} = -\left[\int \frac{2du}{(u)^{2/3}} - \frac{2udu}{(u)^{2/3}}\right]$

 $\int \frac{2dx}{(1-x)^{2/3}} = -\left[6u^{1/3} - \frac{3u^{4/3}}{2}\right] + C$

 $\int \frac{2dx}{(1-x)^{2/3}} = \frac{3(1-x)^{4/3}}{2} - 6(1-x)^{1/3} + C$

15. Resolver: $\int \sqrt{3+x}(x+1)^2dx$

 Solucion:

 sea $u = 3+x$ entonces $x = u-3, dx = du$, luego se tiene:

 $\int \sqrt{3+x}(x+1)^2dx = \int \sqrt{u}(u-3+1)^2du$

$$\int \sqrt{3+x}(x+1)^2dx = \int u^{1/2}(u^2-4u+4)du$$
$$\int \sqrt{3+x}(x+1)^2dx = \int (u^{5/2}-4u^{3/2}+4u^{1/2})du$$
$$\int \sqrt{3+x}(x+1)^2dx = \int u^{5/2}du - \int 4u^{3/2}du + \int 4u^{1/2}du$$
$$\int \sqrt{3+x}(x+1)^2dx = \frac{u^7/2}{7/2} - 4\frac{u^{5/2}}{5/2} + 4\frac{u^{3/2}}{3/2}$$
$$\int \sqrt{3+x}(x+1)^2dx = \frac{2}{7}(3+x)^{7/2} - \frac{8}{5}(3+x)^{5/2} + \frac{8}{3}(3+x)^{3/2} + C$$

16. Resolver: $\int \frac{3dx}{1-2x}$

    Solucion:

    sea $u = 1+2x$ entonces $x = (u-1)/2, 2dx = du$ luego se tiene:

    $$\int \frac{3dx}{1-2x} = \int \frac{3}{u}\left(\frac{du}{2}\right) = \frac{3}{2}ln|u| + C = \frac{3}{2}ln|1+2x| + C$$

17. Resolver: $\int \frac{x^2dx}{1-x^3}$

    Solucion:

    sea $u = 1-x^3$ entonces $du = -3x^2dx$ luego se tiene:

    $$\int \frac{x^2dx}{1-x^3} = -\frac{1}{3}\int \frac{-3x^2dx}{1-x^3} = -\frac{1}{3}\int \frac{du}{u} = -\frac{1}{3}ln|u| + C = -\frac{1}{3}ln|1-x^3| + C$$

18. Resolver: $\int \frac{(x+1)dx}{x^2+2x+3}$

    Solucion:

    sea $u = x^2+2x+3$ entonces $du = (2x+2)dx$, luego se tiene:

    $$\int \frac{(x+1)dx}{x^2+2x+3} = \frac{1}{2}\int \frac{2(x+1)}{x^2+2x+3}dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}ln|u| + C = \frac{1}{2}ln|x^2+2x+3| + C$$

19. Resolver: $\int \frac{x-1}{x+1}dx$

    Solucion:

    sea $u = x+1$ entonces $x = u-1, du = dx$, luego se tiene:

    $$\int \frac{x-1}{x+1}dx = \int \frac{(u-1)-1}{u}du = \int \frac{(u-2)du}{u} = \int \frac{udu}{u} - 2\int \frac{du}{u}$$
    $$\int \frac{x-1}{x+1}dx = u - 2ln|u| + C = x - 2ln|x+1| + C$$

20. Resolver: $\displaystyle\int \frac{ln(lnx)+1}{xlnx}dx$

    Solucion:

    se puede ver que la dirivada del numerador es $\dfrac{1}{xlnx}$,luego la Integral es directa, es decir,

    $$\int \frac{ln(lnx)+1}{xlnx}dx = \frac{[ln(lnx)+1]^2}{2} + C = \frac{1}{2}(ln^2(lnx) + 2ln(lnx) + 1^2) + C$$

    $$\int \frac{ln(lnx)+1}{xlnx}dx = \frac{1}{2}ln^2(lnx) + ln(lnx) + C$$

21. Resolver: $\displaystyle\int \frac{e^{2x}}{e^{2x}+1}dx$

    Solucion:

    sea $u = e^2x$ entonces $du = e^{2x}dx$, luego se tiene:

    $$\int \frac{e^{2x}}{e^{2x}+1}dx = \frac{1}{2}\int \frac{2e^{2x}}{e^{2x}+1}dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}ln|u| + C = \frac{1}{2}ln|e^{2x}+1| + C$$

22. Resolver: $\displaystyle\int \frac{e^x-1}{e^x+1}dx$

    Solucion: