Lab 4 – Beam Deflection and Analysis

Section 5, Group 5
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09/29/2023

Introduction

In this lab, we investigated the mechanical behavior of aluminum beams with varying cross-sectional shapes and loading configurations. The primary objective was to determine the deflection of these beams under different loading conditions and compare the experimental results to theoretical predictions. To achieve this, we made use of aluminum beams with square and rectangular cross-sections, a digital displacement gauge, a wood base board, pasco weight blocks, and a support. We conducted five tests, including both cantilever and statically indeterminate setups, to assess the beam's performance. This report presents the methodology, results, and analysis of our findings, to determine on the accuracy of theoretical predictions in real-world scenarios.

Methods and procedure

For this lab, the following materials and instruments were used:

- An aluminum beam with length = 90 cm and a square cross-section (12.8 x 12.8 mm)
- An aluminum beam with length = 90 cm and a rectangular cross-section (6.4 x 12.8 mm)
- A digital displacement gauge
- A wood base board and roller stand
- A vise
- Weights of different values
- A level gauge and a metal stand
- A ruler
- A wrench

The Young's Modulus of the aluminum beams is 68.9 GPa (Aluminum 6061-T6511 alloy).

Our group conducted 5 tests, for which 1-3 had the same cantilever beam setup as seen in Figure 1. The 4^{th} and 5^{th} tests had a statically indeterminate setup.

To measure deflection y, we used the needle digital displacement gauge. The beam was fixed to the support structure using clamps, simulating a situation that the beam would be completely fixed to a wall. A level was used to make sure the beam was leveled.

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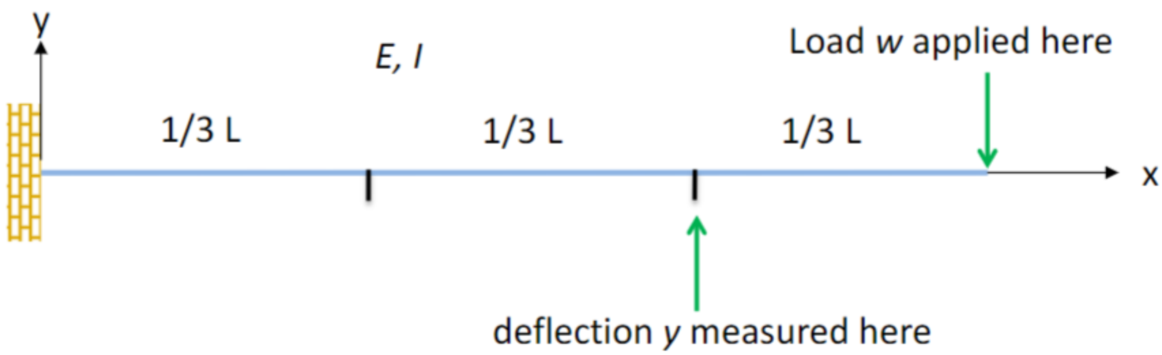


Figure 1. Cantilever beam setup for tests 1-3

For test 1, we used the beam with rectangular cross-section which the base was the 12.8 mm side and the height was the 6.4 mm side. A 100g load was applied to the end of the beam (as seen in Figure 1).

For test 2, we used the beam with rectangular cross-section which the base was the 6.4 mm side and the height was the 12.8 mm side. A 200g load was applied to the end of the beam.

For test 3, we used the beam with square cross-section. A 500g load was applied to the end of the beam.

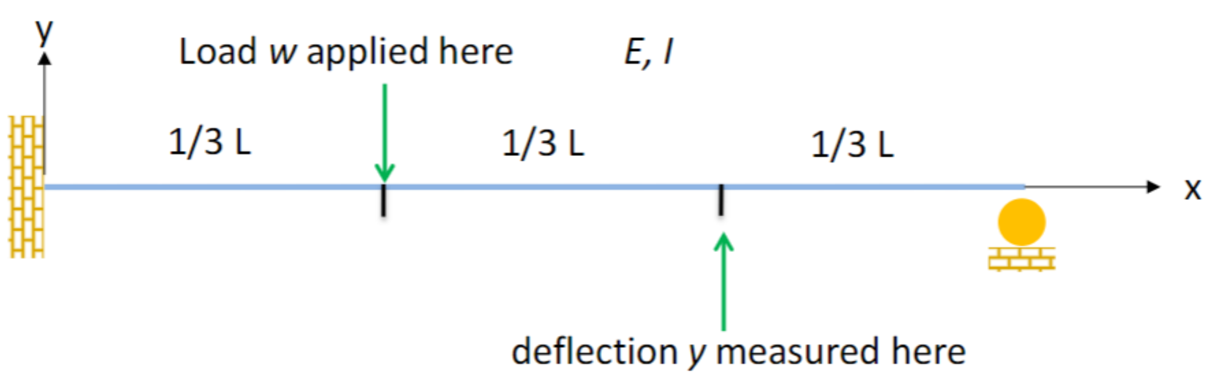


Figure 2. Beam setup for test 4

Figure 3. Figure of beam setup for test 4

For test 4, a roller support at the end of the beam was added. Now, the beam structure is statically indeterminate. We used the beam with rectangular cross-section which the base was the 12.8 mm side and the height was the 6.4 mm side. The 1 kg load was applied at 1/3 of the length, as seen in figures 2 and 3.

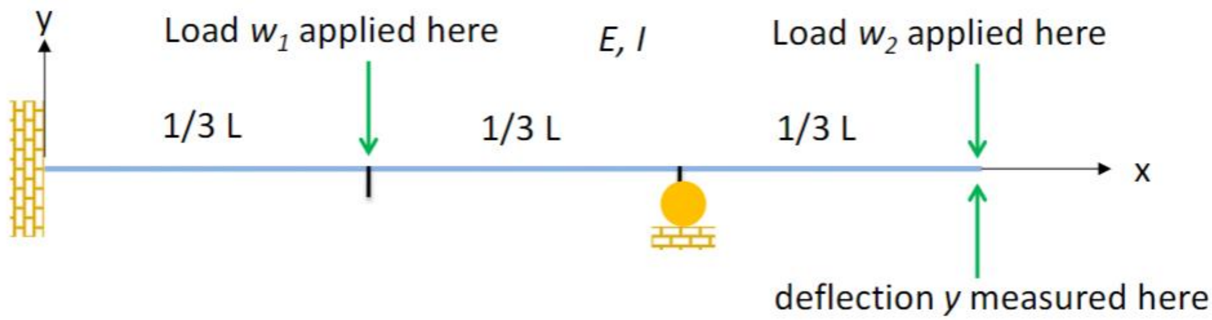


Figure 4. Beam setup for test 5

The beam structure is still statically indeterminate for test 5. Now, the roller support is positioned at 2/3 of the length and the needle digital displacement gauge at the end of the beam. The beam used was the one with rectangular cross-section which the base was the 12.8 mm side and the height was the 6.4 mm side. We applied two loads to the system. The first load was 2.5 kg applied at 1/3 of the length, the second was 200 g applied at the end of the beam, as seen in Figure 4.

Work Assignments

The setup that was done for the lab was done by Ethan, Joseph, and Lucas. The data collection was done by Ethan and changing the apparatus for different variations of the lab was done by Lucas and Joseph.

The work for the report was done by everyone in the group. Lucas did the methodology and procedure section of the report. Ethan Did the results section and contributed to the analysis of the report. Joseph did the work assignments part of the analysis. Isaac wrote the introduction and conclusion of the lab report.

Results

Test Configuration	Theoretical Deflection (mm)
1	6.42
2	3.21
3	4.01
4	1.95
5	2.005

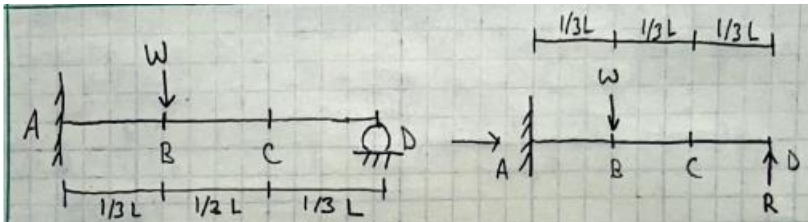
Table 1. Theoretical beam deflection results

Test Configuration	Deflection (mm)
1	5.73
2	3.53
3	4.83
4	2.14
5	1.33

Table 2. Experimental beam deflection results

Analysis

1) Derivation of deflection formula and deflection calculation for test configuration (4):



Deflection at D=0 $\rightarrow Y_D = (Y_D)_w + (Y_D)_R$

First find $(Y_D)_w$: $X = L$ $L_1 = L/3$

$$(Y_w)_D = -\frac{(W)(L_1^2)}{6EI}(3X - L_1) \rightarrow = -\frac{W(L^2/9)}{6EI}\left(3L - L/3\right)$$

$$= \frac{(W)(8L^3)}{(6EI)(9)(3)} \rightarrow = \frac{-4WL^3}{81EI}$$

Now find $(Y_D)_R$: $X = L$

$$(Y_D)_R = \frac{R(X^2)}{6EI}(3L - X) \rightarrow = \frac{RL^2}{6EI}(2L) \rightarrow = \frac{RL^3}{3EI}$$

$Y_D = (Y_D)_w + (Y_D)_R = 0$:

$$-\frac{4WL^3}{81EI} + \frac{RL^3}{3EI} = 0 \rightarrow \frac{RL^3}{EI} = \frac{4WL^3}{27EI} \rightarrow R = \frac{4}{27}W$$

$Y_C = (Y_C)_w + (Y_C)_R$

First Solve $(Y_C)_w$: $L_1 = \frac{L}{3}$ $X = 2L/3$

$$(Y_C)_w = \frac{-WL_1^2}{6EI}(3X - L_1) \rightarrow = \frac{-W(L^2/9)}{6EI}\left(3(2L/3) - L/3\right)$$

$$= \frac{-5WL^3}{162EI}$$

Figure 5. Beam deflection derivation for test configuration (4)

Now $(Y_c)_R$: $\quad x = 2L/3 \quad L_i = L$

$$(Y_c)_R = \frac{Rx^2}{6EI}(3L - x) \longrightarrow = \frac{(4/27)(2L/3)^2}{6EI}\left(3L - 2L/3\right)$$

$$= \frac{56WL^3}{2187EI}$$

$Y_c = (Y_c)_w + (Y_c)_R$:

$$Y_c = -\frac{5WL^3}{162EI} + \frac{56WL^3}{2187EI} \longrightarrow \boxed{Y_c = \frac{-23WL^3}{4374EI}}$$

Beam deflection for test Configuration 4:

$$Y_c = \frac{-23(9.81)(900)^3}{4374(68.9\times10^3)(279.62)} = \boxed{1.952 \text{ mm} \downarrow}$$

Figure 6. Beam deflection derivation for test configuration (4) and calcullated beam deflection

2) Derivation of deflection formula and deflection calculation for test configuration (5):

$$Y_{w_1 c} = \frac{w_1\left(\frac{L}{3}\right)^2}{6EI}\cdot\left(3\cdot\frac{2L}{3} - \frac{L}{3}\right) = \frac{w_1 L^2}{9(6EI)}\cdot\left(\frac{5L}{3}\right) \quad (1)$$

$$Y_{w_2 c} = \frac{w_2\left(\frac{2L}{3}\right)^2}{6EI}\cdot\left(3L - \frac{2L}{3}\right) = \frac{4 w_2 L^2}{9\,6EI}\cdot\left(\frac{7L}{3}\right) \quad (2)$$

$$Y_{R_1 c} = \frac{R_1\cdot\left(\frac{2L}{3}\right)^2}{6EI}\cdot\left(3\cdot\frac{2L}{3} - \frac{2L}{3}\right) = \frac{4 R_1 L^2}{9(6EI)}\left(\frac{4L}{3}\right) \quad (3)$$

deflection at C = 0

$$\frac{w_1 L^2}{9(6EI)}\cdot\left(\frac{5L}{3}\right) + \frac{4 w_2 L^2}{9(6EI)}\cdot\frac{7L}{3} = \frac{4 R_1 L^2}{9(6EI)}\cdot\frac{4L}{3}$$

$$\Rightarrow R_1 = \frac{5w_1 + 28w_2}{16}$$

$$Y_{w_1 D} = \frac{w_1}{6EI}\left(\frac{L}{3}\right)^2\left(3L - \frac{L}{3}\right) = \frac{w_1 L^2}{9(6EI)}\cdot\left(\frac{8L}{3}\right)$$

$$Y_{w_2 D} = \frac{w_2 L^2}{6EI}\cdot(3L - L) = \frac{w_2 L^2}{6EI}\cdot 2L$$

$$Y_{R_1 D} = \frac{R_1\left(\frac{2L}{3}\right)^2}{6EI}\left(3L - \frac{2L}{3}\right) = \frac{4 R_1 L^2}{9(6EI)}\cdot\left(\frac{7L}{3}\right)$$

$$y = \frac{L^3}{18EI}\left[\frac{2}{9}w_1 + 6w_2 - \frac{16}{9}\left(\frac{5w_1 + 28w_2}{16}\right)\right]$$

$$\boxed{y = \frac{L^3}{EI}\left[\frac{5w_2}{162} - \frac{w_1}{216}\right]}$$

$$I = \frac{12.8\,(6.4)^3}{12} = 279.62$$

$$Y_D = \frac{900^3}{(68.4\times10^3)(279.62)}\left[\frac{5(0.2\cdot9.81)}{162} - \frac{(2.5\cdot9.81)}{216}\right] = \boxed{2.005\ \text{mm} \downarrow}$$

Figure 7. Beam deflection derivation for test configuration (5) and calcullated beam deflection

3) Comparison between model predictions and experimental results:

Looking at our data in the results section above, we can compare our theoretical deflection values to the deflection values found in the experiment. For test 1, the theoretical deflection value came out to 6.42 mm, which was calculated in the prelab. Doing the experiment, the deflection value came out to be 5.73 mm. The experimental data shows about 11 percent error from the expected value.

Analyzing the results from test 2, we see that the deflection calculated in the experiment was 3.53 mm. The expected value calculated in the prelab came out to 3.21 mm. The experimental deflection value has about 10 percent error from the expected value.

The data from test 3 shows that the deflection calculated in the experiment was 4.83mm. The expected value from the prelab was 4.01. Comparing these two values, we can see that the experimental value had about 20 percent error from the expected value.

The calculations from the expected value of test 4 was 1.95 mm. The value that was experimentally found in the lab was 2.14mm. This means that there is about a 9 percent error in our lab findings.

The data from the experiment shows that the deflection of the beam at point D was 1.33 mm. While the expected value from the calculated formulas was 2.005 mm. Meaning that our error for this test is about 33 percent, which is extremely high.

Conclusions

Our experiments provided insight into the deflection behavior of aluminum beams under different loading conditions. We found that the theoretical predictions closely aligned with the experimental results for some test configurations, with minimal percentage error. However, in other cases, there was a noticeable deviation between the expected and observed deflection values, indicating potential discrepancy in the theoretical models used. This can be interpreted as indicating the importance of practical experimentation in verifying theoretical assumptions. Overall, this study contributes to our knowledge of beam deflection mechanics and offers opportunities for further investigation and refinement of analytical models.