Iowa State University

Measurements of the Boundary Layer over a Flat Plate Laboratory

AER E 344 - Lab 08 - Measurements of the Boundary Layer over a Flat Plate

Section 3 Group 3

Matthew Mehrtens
Jack Mendoza
Kyle Ostendorf
Gabriel Pederson
Lucas Tavares Vasconcellos
Drew Taylor

Professor

Hui Hu, PhD

College of Engineering
Aerospace Engineering
Aerodynamics and Propulsion Laboratory

Ames, March 2024

Abstract

Using the high-speed wind tunnel at Iowa State University, we measured the velocity profile downstream of a flat plate to analyze the boundary layer profile. To measure the downstream velocity profile, we used a pitot rake to measure pressure at various locations. The coefficient of drag, coefficient of local shear stress, and the boundary layer thickness can be determined from this data.

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Glossary

A	Surface area. *(p. 4)*
C_D	drag coefficient. *(p. 2)*
C_f	Local shear stress coefficient. *(p. 2)*
D	Drag force. *(p. 4)*
L	Length of the plate. *(p. 4)*
δ	Boundary Layer thickness. *(p. 2)*
x	Characteristic linear dimension in x direction. *(p. 4)*

1

Introduction

A pitot rake is an instrument composed of a series of pitot tubes used to measure the velocity profile of a body. In this experiment, the pitot rake is used to track the pressure at various distances between 0 and 70 inches in the flow behind a the leading edge of a flat plate.

Using pressure measurements from the pitot rake, we will estimate the local shear stress coefficient, C_f, and find the coefficient of drag, C_D. We will also estimate the boundary layer thickness, δ, and compare it the theoretical values.

2

Methodology

2.1 Apparatus

A flat plate is positioned in the wind tunnel test chamber as seen in Figure 2.1. Downstream of the flat plate is a pressure rake, seen in Figure A.1. The data was collected using 3 DSA units for the pressure measurements. The computer configuration of this can be seen in Figure A.4.

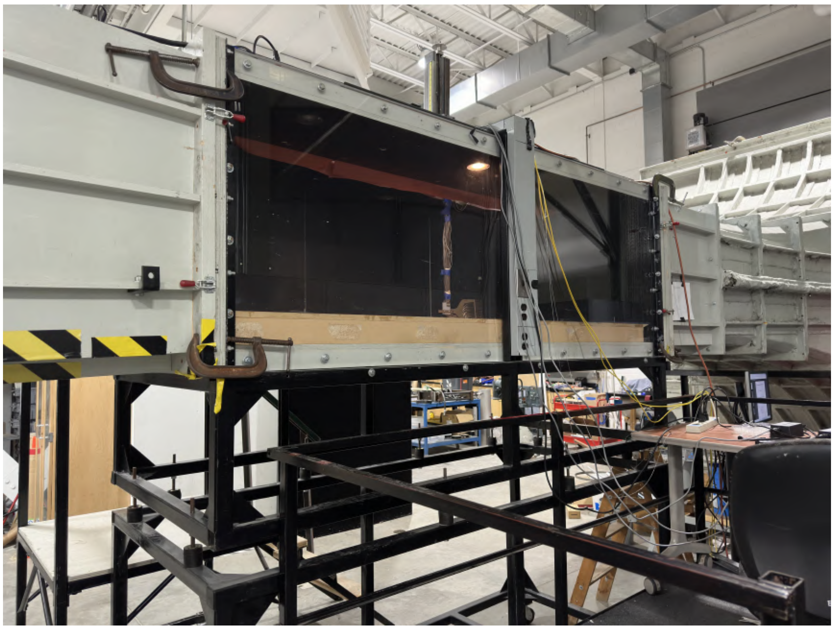

Figure 2.1: *Picture of the Test Section*

2.2 Procedures

1. Set the wind tunnel speed to 10 m/s
2. Take pressure measurements at 10 streamwise positions specified by the TA
3. After each adjustment, acquire and save the data to a data file

2.3 Derivations

The voltage outputs obtained by computer software can be converted to dynamic pressures, which can then be related to velocities. The local shear stress coefficient is defined in Equation 2.1. The momentum thickness is defined in Equation 2.2. Using Equation 2.2 and the data we collected, the momentum thickness can be calculated for each of the distances (Hu, 2024).

$$C_f = \frac{\tau_w}{1/2\rho U_e^2} \tag{2.1}$$

$$\theta = \int_0^Y \frac{u}{U_e}\left(1 - \frac{u}{U_e}\right) dy \tag{2.2}$$

The integral in Equation 2.2 was determined using the midpoint Riemann sum between the top and bottom pressure rake taps. The distance between each pressure tap is 4mm. To calculate the local shear stress coefficient, we used its relationship to the momentum thickness in Equation 2.3. In MATLAB, the *gradient* command is used to estimate $\frac{d\theta}{dx}$.

$$C_f = 2\frac{d\theta}{dx} \tag{2.3}$$

However, for the purpose of comparison, Equation 2.3 can be empirically related to Equation 2.4.

$$C_f = \frac{0.0583}{Re_x^{0.2}} \tag{2.4}$$

The coefficient of drag is also able to be calculated and related to the momentum thickness. Equation 2.5 defines the coefficient of drag, where D is drag force and A is the surface area. The coefficient of drag can be related to momentum thickness, as shown in Equation 2.6, where L is the length of the plate upstream of the measurement point.

$$C_d = \frac{D}{1/2\rho U_e^2 A} \tag{2.5}$$

$$C_d = \frac{2\theta}{L} \tag{2.6}$$

The coefficient of drag can also be estimated, as seen in Equation 2.7.

$$C_d = \frac{.074}{Re_L^{0.2}} \tag{2.7}$$

The following equations (Equation 2.8 and Equation 2.9) are theoretical estimations of the boundary layer thickness at a distance x over a flat plate.

$$\frac{\delta}{x} = \frac{5.0}{\sqrt{\mathrm{Re}_x}} \tag{2.8}$$

$$\frac{\delta}{x} = \frac{0.37}{\mathrm{Re}_x^{\frac{1}{5}}} \tag{2.9}$$

Assuming transition occurs at $Re_x = 10^5$, and using Equation 2.8 and Equation 2.9, we can estimate the thickness of the boundary layer at various locations.

3 Results

Figure 3.1, Figure 3.2, and Figure 3.3 show the Y/δ ratio vs. U/U_{inf} ratio at different distances from the leading edge. More plots of Y/δ vs. U/U_{inf} are located in Section A.2. The 10 locations chosen for analysis are at [1, 3, 5, 7, 9, 15, 25, 35, 45, 55] inches. The x-axis increases as the vertical distance from the flat plate increases.

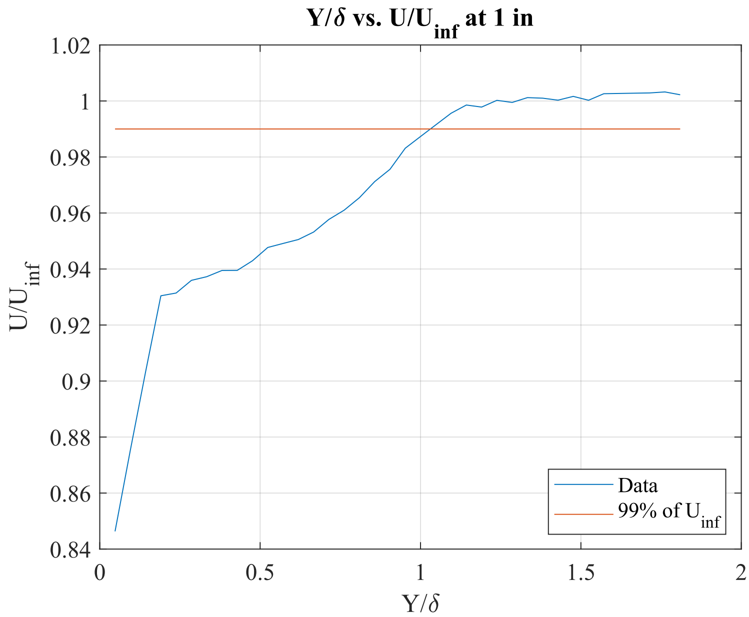

Figure 3.1: *Ydelta vs. UUinf at 1 in*

Figure 3.4 contains the graphs of the theoretical boundary layer at the laminar region and turbulent regions and the calculated thickness from the lab data. The momentum thickness, θ, at the chosen stream-wise distances are also plotted with their values on the right y-axis.

The coefficient of drag from the theoretical calculations and experimental data distances are located in Figure 3.5.

The coefficient of local shear stress from the theoretical calculations and experimental data distances are located in Figure 3.5.

Using Equation 3.1 and the subsequent parameters, we find the Reynolds number at the

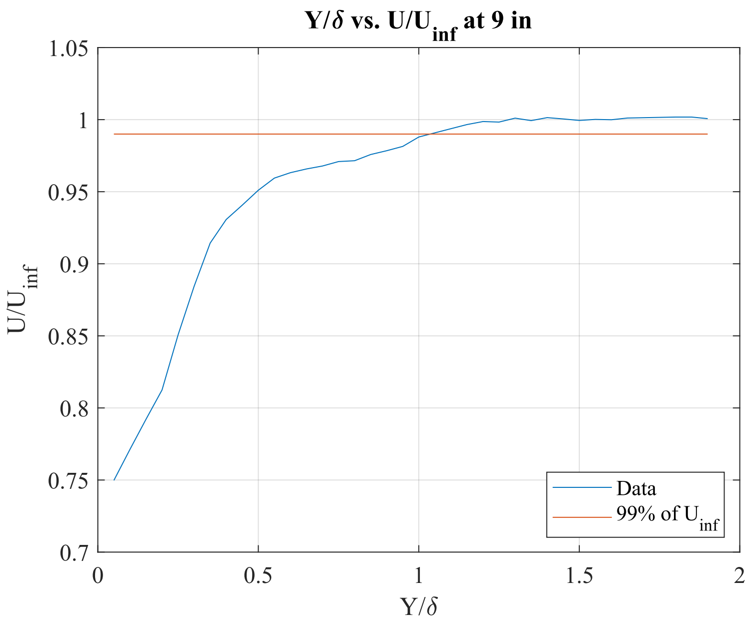

Figure 3.2: *Ydelta vs. UUinf at 9 in*

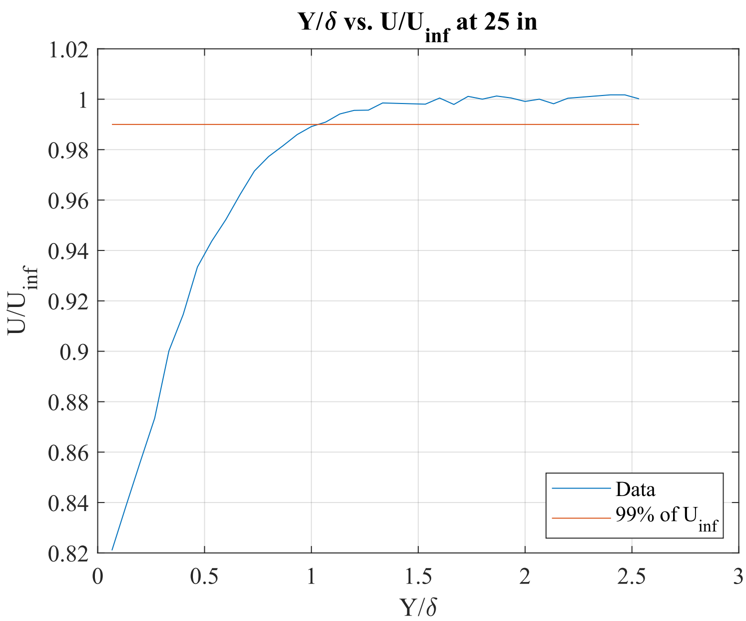

Figure 3.3: *Ydelta vs. UUinf at 25 in*

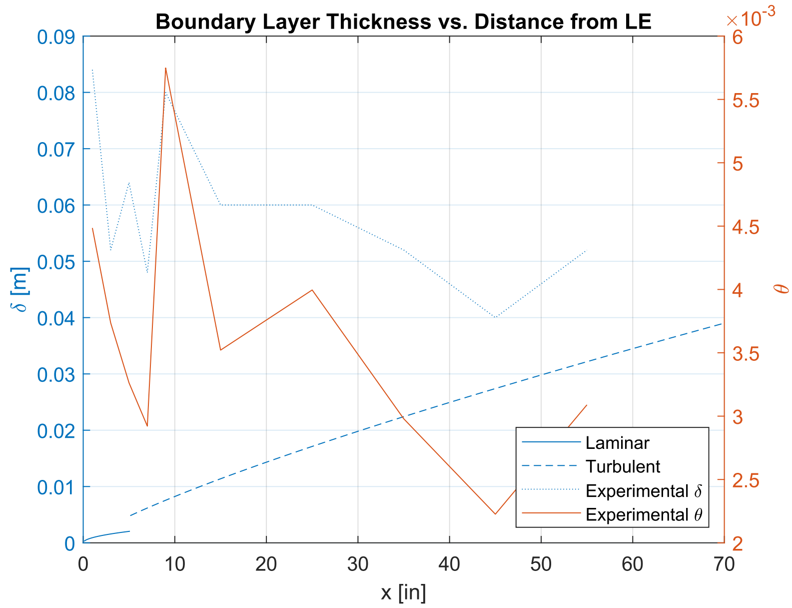

Figure 3.4: *Boundary Layer Thickness vs Distance from LE*

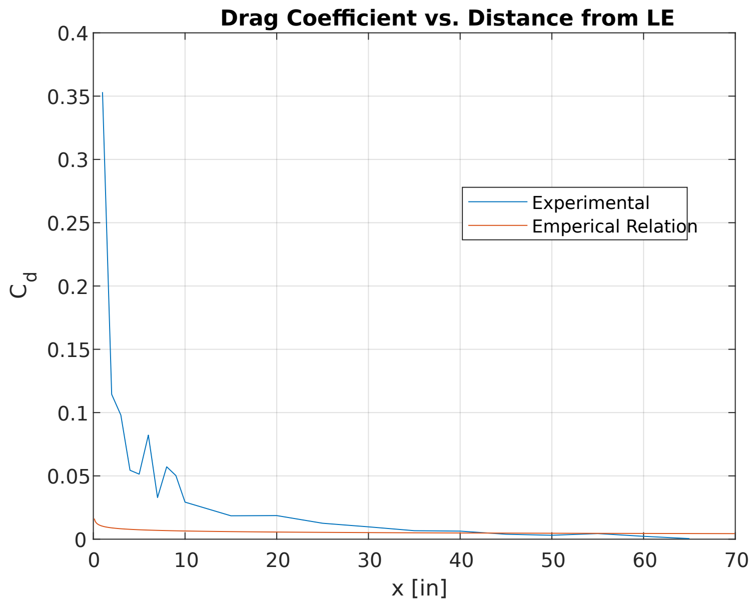

Figure 3.5: *Drag Coefficient vs Distance from Le*

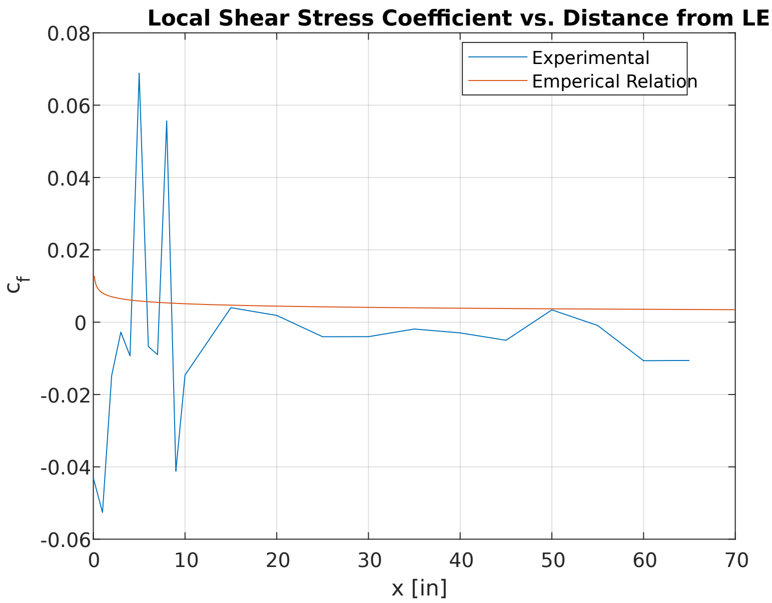

Figure 3.6: *Local Shear Stress Coefficient vs Distance from Le*

range from 0 to 65 inches in Figure 3.7. The length values are converted into meters to make the Reynold's Number unit-less.

$$Re = \frac{\rho V L}{\mu} \tag{3.1}$$

$\rho = 1.225\,\mathrm{kg/m^3}$
$\mu = 18.18 \times 10^{-6}\,\mathrm{Pa\,s}$
$L = 0\,\mathrm{in}$ to $65\,\mathrm{in}$
$V = 11.25\,\mathrm{m/s}$

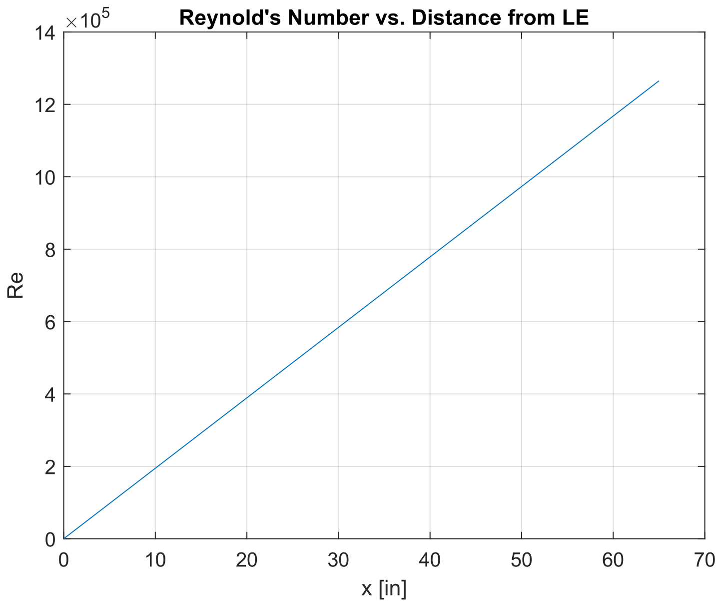

Figure 3.7: *Reynold's Number vs. Distance from LE.*

4

Discussion

4.1 Boundary Layer Analysis

Given the random nature of the boundary layer graph as seen in Figure 3.4, there is clearly an issue in our lab. The sources of this error are discussed in more detail in Section 4.3. Next, the Y/delta vs. $U/U\infty$ at distances of 10 inches or less, we noticed the boundary layer was much further from the plate surface than expected. The $U/U\infty$ tend to level off a few inches after the leading edge before increasing again, pushing the boundary layer higher (see Figure 3.1 or Figure 3.2). The boundary layer was estimated at the distance where the velocity is 99% of the free-stream velocity; however, if we estimate it where $U/U\infty$ begins to level off, the boundary layer may match the theoretical values. At distances that were greater than 10 inches, the 99% estimation seems more valid (Figure 3.3).

4.2 Coefficient of Drag and Local Shear Stress

The calculated coefficient of drag aligns with the theoretical relation, at distances further downstream from the leading edge. The given empirical relation (Equation 2.7) and calculated coefficient decrease to similar values of about 0.004 nearing a distance of 70in . The theoretical coefficient of local shear stress resembles the theoretical coefficient of drag; however, the calculate coefficient of local shear stress from the experimental data does not.

4.3 Sources of Error

Our data deviates from the theoretical predictions, indicating there might have been considerable sources of error either in the experimental setup or the data analysis process. Although we cannot assert the reliability of our analysis with full confidence, further validation with additional data sets over more time would be required to validate our process.

We suspect the primary discrepancies likely stem from faulty pressure taps. For example, in this lab, we know that pressure taps 2 and 22 were not working because the TAs told us to get rid of that data. Another discrepancy we could have easily had was one of the plastic tubes got pinched so it would not collect the data. Data was interpolated at pressure taps with known issues.

Another discrepancy that we had was with the 99% rule and that it may not be the best prediction of the boundary layer for the measurements closer to the leading edge. Due to the strange behavior of $U/U\infty$ at distances near the leading edge, an boundary layer estimation at 95% may have been a more accurate measurement compared to the theoretical calculations. For the other graphs that compared the Y/delta vs $U/U\infty$, the data looked more like how we would expect it to look. Where it would climb almost linearly and then eventually level off, indicating the boundary layer.

4.4 Future Work

Due to the errors in the data, we weren't able to accurately show the boundary layers. We cannot prove definitively the source of our error, but we think it is most likely due to how we collected the data. To improve our results, we would take the following corrective measures:

1. Test our analysis script on reliable data and compare the results to the expected values—fixing bugs and making changes as necessary.
2. Re-calibrate and verify the functionality and accuracy of the Pressure rake and the wind tunnel apparatuses.
3. Confirm the settings in the data acquisition software and repeat the experimental data collection.

5 Conclusion

To analyze the boundary layer profile over a flat plate, we used a pitot rake to collect a velocity profile at various locations downstream of the flat plate. Due to erraneous data collected from the pitot rake or a misconfiguration of the pitot rake, we were unable to generate a conclusive analysis and profile of the boundary layer. To rectify this and generate proper results, we would thoroughly review the Matlab analysis script and, if necessary, validate the functionality and accuracy of the pitot rake before repeating the experiment.

Bibliography

Hu, Hui (2024). *Measurements of the Boundary Layer over a Flat Plate*. Iowa State University. URL: https://www.aere.iastate.edu/~huhui/teaching/2024-01S/AerE344/lab-instruction/AerE344L-Lab-08-Instruction-BL-Flat-Plat.pdf.

A

Appendix A

A.1 Additional Apparatus Pictures

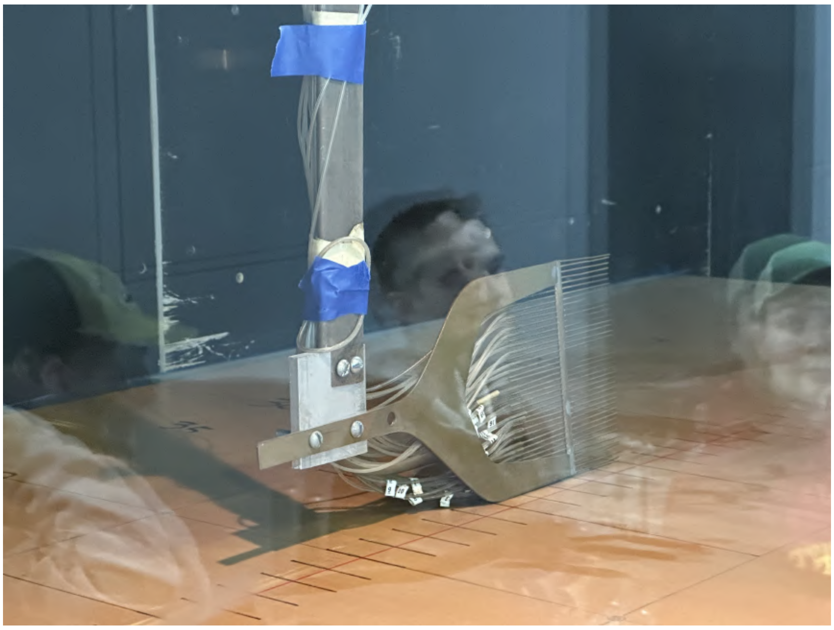

Figure A.1: *Pressure rake in open circuit wind tunnel*

Figure A.2: *Picture of the flat plate in test section*

Figure A.3: *Picture of the control panel for wind tunnel*

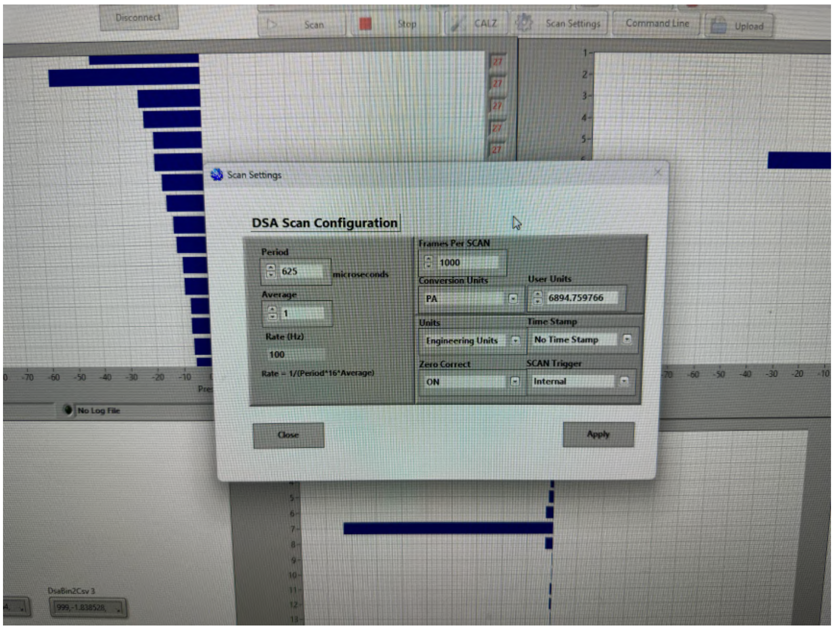

Figure A.4: *Picture of the Computer program that we used for collecting the data.*

A.2 Additional Figures

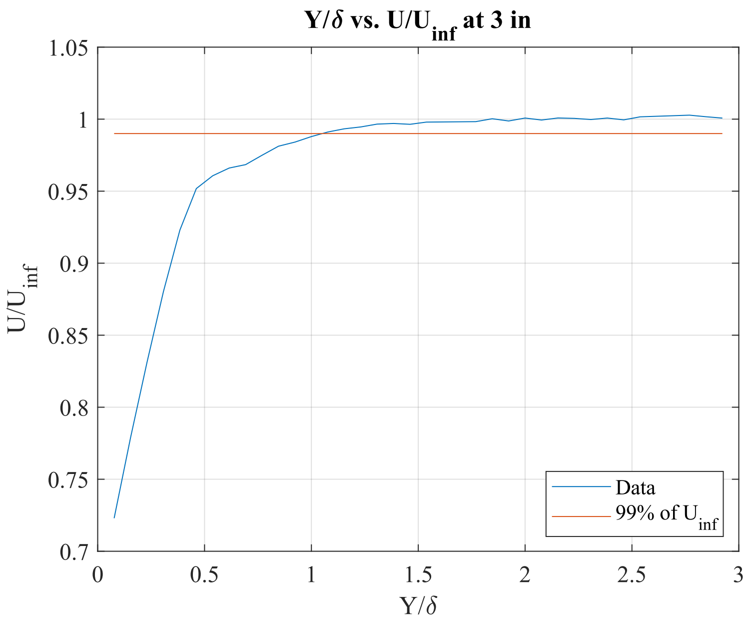

Figure A.5: *Ydelta vs. UUinf at 3 in*

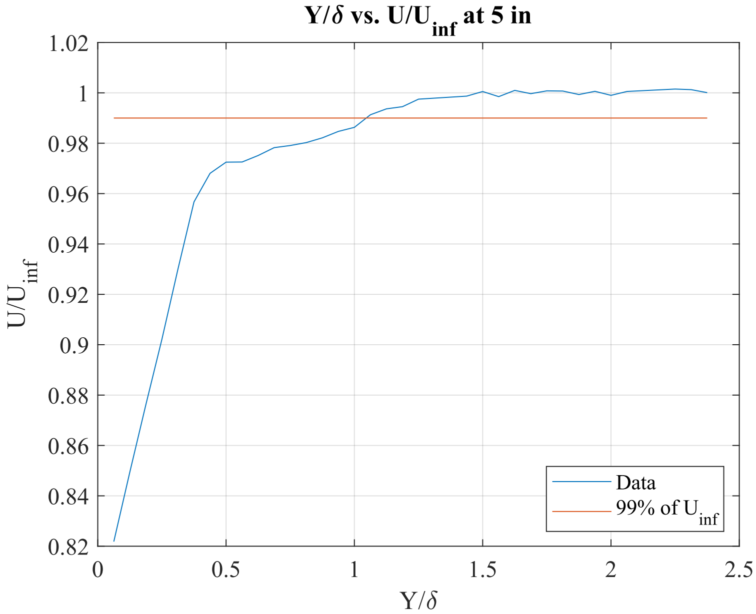

Figure A.6: *Ydelta vs. UUinf at 5 in*

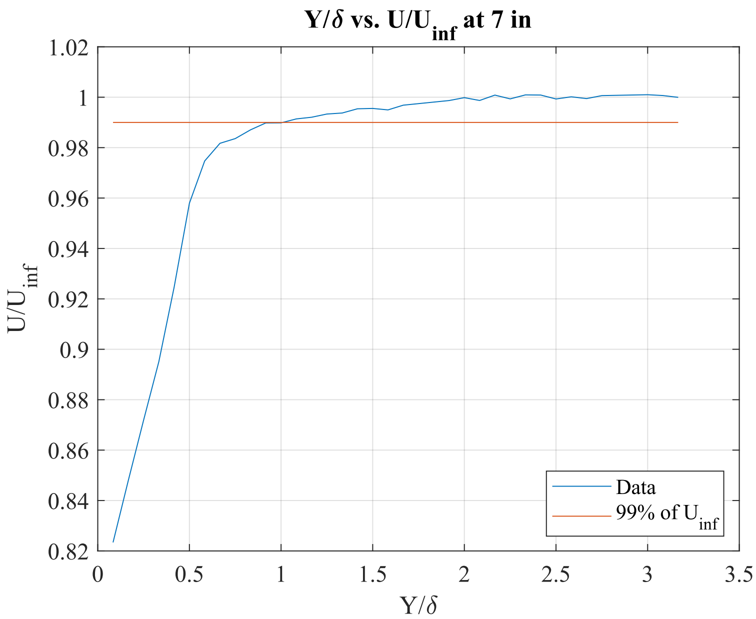

Figure A.7: *Ydelta vs. UUinf at 7 in*

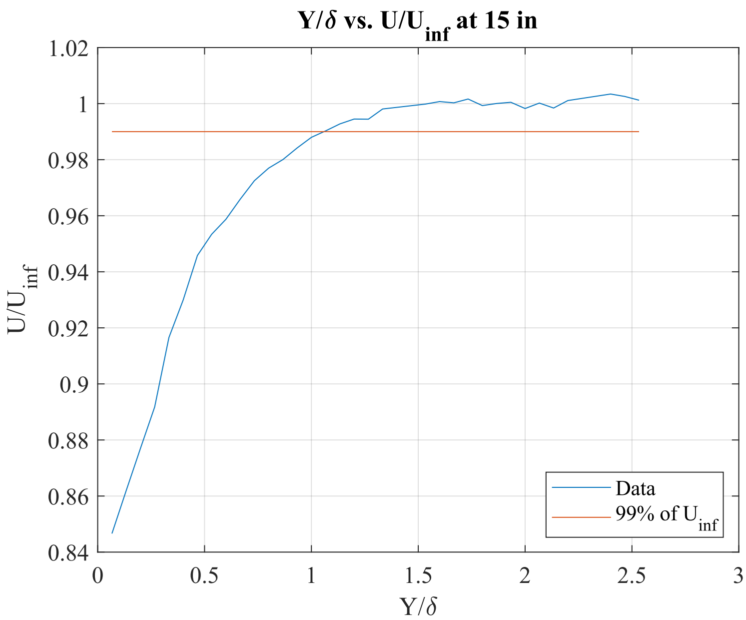

Figure A.8: *Ydelta vs. UUinf at 15 in*

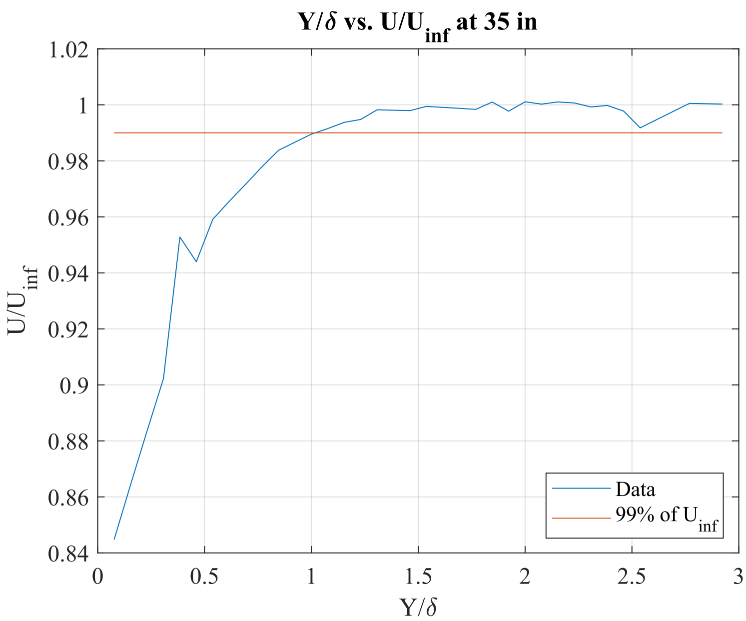

Figure A.9: *Ydelta vs. UUinf at 35 in*

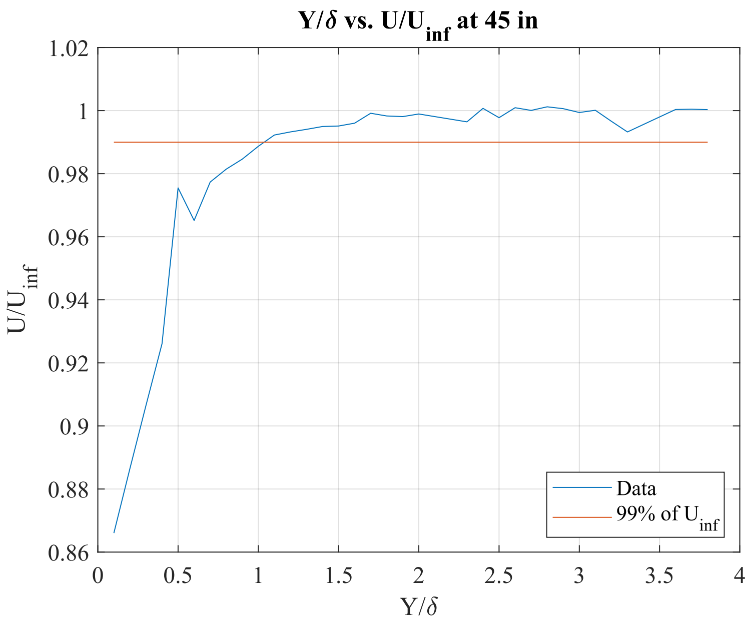

Figure A.10: *Ydelta vs. UUinf at 45 in*

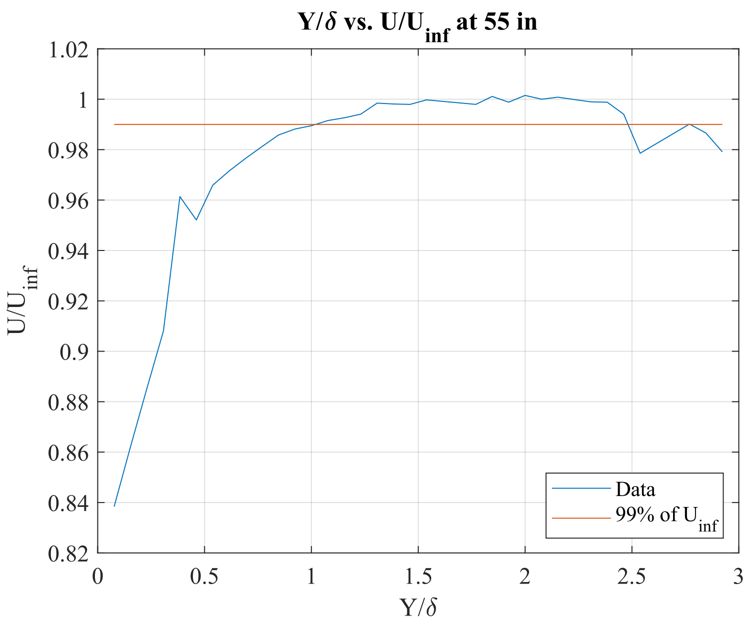

Figure A.11: *Ydelta vs. UUinf at 55 in*

B

Appendix B

B.1 Lab8Analysis.m

```
clear, clc, close all
%% Kyle Ostendorf Lab 8
%% Constants
figure_dir = "../Figures/";

x = [0:10,15:5:65];
n_x = length(x); % number of tests
taps = 1:38;
n_taps = length(taps); % number of taps
bad_taps = [1,2,21,22,34,35];
good_taps = taps;
good_taps(bad_taps) = [];
d_m = .004; % [m]
y = taps * d_m;

mu = 1.8e-5; % [N*s/m^2]
rho = 1.225; % [kg/m^3]
IN0 = readmatrix("0in.csv");
chosen = 2:2:21; %% Data chosen to be graphed
%% Reading Cell array
total_p = zeros(n_x, n_taps);
static_p = zeros(n_x, 1);
for i = 1:length(x)

MAREIX = readmatrix(sprintf("%din.csv",x(i)));
total_p(i,:) = mean([MAREIX(:,2:17),MAREIX(:,35:50),MAREIX(:,68:73)],"omitmissing");
static_p(i,:) = mean(MAREIX(:,74));
end

%% Interpolating Data
interp_data = zeros(n_x, length(bad_taps));
i_total_p = zeros(n_x, n_taps);
for i = 1:n_x
good_data = total_p(i,good_taps);
```

```
interp_data(i,:) = interp1(good_taps,good_data,bad_taps,"linear","extrap");
i_total_p(i,:) = [interp_data(i,1:2),good_data(1:18),interp_data(i,3:4),good_data(19:2
↪ 9),interp_data(i,5:6),good_data(30:32)];
end

%% Velocity
Q = i_total_p - static_p;
U = sqrt(2*Q/rho);
U_inf = zeros(1,n_x);
U_Uinf = zeros(n_x, n_taps);
for i = 1:n_x
    U_inf(i) = mean(U(i,25:1:30));
    for j = 1:n_taps
        U_Uinf(i,j) = U(i,j)/U_inf(i);
    end
end


%% Delta Estimations
delta = zeros(1,n_x);
y_delta = zeros(n_x,n_taps);
for i = 1:n_x
    delta_loc = 1;
    u = 0.99*U_inf(i);
    while (U(i,delta_loc) < u)
        delta_loc = delta_loc +1;
    end
    delta(i) = d_m*(delta_loc-1);
    y_delta(i,:) = y / delta(i);

end

%% Plot y_delta vs. U_Uinf
for i  = chosen
    figure(i)
    plot(y_delta(i,:), U_Uinf(i,:))
    hold on
    plot([y_delta(i,1),y_delta(i,n_taps)],[.99,.99])

    fontname("Times New Roman");
    fontsize(12, "points");
    title_str = "Y/\delta vs. U/U_{inf} at "+ x(i) +" in";
    title(title_str);
    xlabel("Y/\delta");
    ylabel("U/U_{inf}");
    grid on;
    legend({'Data', '99% of U_{inf}'},"Location",'southeast')
    saveas(gcf, figure_dir + "Ydelta vs. UUinf at "+ x(i) +" in" + ".svg");
end

%% Displacement and Momentum thickness
```

```
theta = zeros(1,n_x);
delta_star = zeros(1,n_x);
for i = 1:n_x
    for j = 1:n_taps-1
        f1 = U_Uinf(i,j)*(1- U_Uinf(i,j));
        f2 = U_Uinf(i,j)*(1- U_Uinf(i,j));
        theta(i) = theta(i) + d_m*((f1 + f2)/2);

        f3 = (1 - U_Uinf(i,j));
        f4 = (1 - U_Uinf(i,j));
        delta_star(i) = delta_star(i) + d_m*((f3 + f4)/2);
    end
end

%% Theoretical vs Experimental delta
V_inf = mean(U_inf); % [m/s] <-- this is for 10 Hz
Re_transition = 5*10^5; % []
m_to_in = 39.3700787;
x_transition = Re_transition * mu / (rho * V_inf); % [m]
x_transition = x_transition * m_to_in; % [in]
x_laminar = 0 : 0.1 : x_transition; % [in]
x_turbulent = x_transition : 0.1 : 70; % [in]
Re_laminar = rho * V_inf * (x_laminar* 39.3700787^-1) / mu; % []
Re_turbulent = rho * V_inf * (x_turbulent* 39.3700787^-1) / mu; % []
boundary_layer_laminar = 5.0 * x_laminar ./ sqrt(Re_laminar); % [in]
boundary_layer_turbulent = 0.37 * x_turbulent ./ Re_turbulent.^(1 / 5); % [in]

% Output
figure(23);
yyaxis left
plot(x_laminar, boundary_layer_laminar*(m_to_in^-1));
hold on;
plot(x_turbulent, boundary_layer_turbulent*(m_to_in^-1));
hold on
plot(x,delta);
hold on
ylabel("\delta [m]");
yyaxis right
plot(x, theta);
ylabel("\theta");
hold off
title_str = "Boundary Layer Thickness vs. Distance from LE";
title(title_str);
xlabel("x [in]");
legend({'Laminar','Turbulent','Experimental \delta','Experimental
↪  \theta'},"Location",'southeast')
grid on;
saveas(gcf, figure_dir + title_str + ".svg");

%% Shear Stress Coefficient
cf = 2 * gradient(theta) ./ gradient(x*39.3700787^-1);
```

```
cf_rel = [(0.0583 ./ Re_laminar.^0.2), (0.0583 ./ Re_turbulent.^0.2)];
% tau_w = zeros(n_x,n_taps);
% cf = zeros(n_x,n_taps);
% for i = 1:n_x
%     tau_w(i,:) = mu * gradient(U(i,:),y);
%     cf(i,:) = tau_w(i,:) / (1/2 * rho * U_inf(i)^2);
% end
figure(24)
plot(x,cf)
hold on
plot([x_laminar, x_turbulent], cf_rel)
title_str = "Local Shear Stress Coefficient vs. Distance from LE";
title(title_str);
xlabel("x [in]");
ylabel("c_{f}")
legend({'Experimental','Empirical Relation'},"Location",'best')
grid on;
saveas(gcf, figure_dir + title_str + ".svg");

%% Drag Coefficient
Cd_rel = [(0.074 ./ Re_laminar.^0.2), (0.074 ./ Re_turbulent.^0.2)];
Cd = 2 .* theta ./ (x.*39.3700787^-1);
figure(25)
plot(x,Cd)
hold on
plot([x_laminar, x_turbulent], Cd_rel)
title_str = "Drag Coefficient vs. Distance from LE";
title(title_str);
xlabel("x [in]");
ylabel("C_{d}")
legend({'Experimental','Empirical Relation'},"Location",'best')
grid on;
saveas(gcf, figure_dir + title_str + ".svg");

%% Re Graph
figure(26)
plot([x_laminar, x_turbulent],[Re_laminar,Re_turbulent])
ylabel("Re");
xlabel("x [in]")
title_str = "Reynold's Number vs. Distance from LE";
title(title_str);
grid on;
saveas(gcf, figure_dir + title_str + ".svg");
```