TIP8419 - Tensor Algebra
Homework 3

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Least-Squares Khatri-Rao Factorization (LSKRF)

Generate $\mathbf{X} = \mathbf{A} \diamond \mathbf{B} \in \mathbb{C}^{20\times 4}$, for randomly chosen $\mathbf{A} \in \mathbb{C}^{5\times 4}$ and $\mathbf{B} \in \mathbb{C}^{4\times 4}$. Then, implement the Least-Squares Khatri-Rao Factorization (LSKRF) algorithm that estimate $\mathbf{A}$ and $\mathbf{B}$ by solving the following problem

$$(\hat{\mathbf{A}}, \hat{\mathbf{B}}) = \min_{\mathbf{A},\mathbf{B}} \|\mathbf{X} - \mathbf{A} \diamond \mathbf{B}\|_F^2.$$

Compare the estimated matrices $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ with the original ones. What can you conclude? Explain the results.

Hint: Use the file "krf_matrix.mat" to validate your result.

Assuming 1000 Monte Carlo experiments, generate $\mathbf{X}_0 = \mathbf{A} \diamond \mathbf{B} \in \mathbb{C}^{IJ\times R}$, for randomly chosen $\mathbf{A} \in \mathbb{C}^{I\times R}$ and $\mathbf{B} \in \mathbb{C}^{J\times R}$, with $R = 4$, whose elements are drawn from a normal distribution. Let $\mathbf{X} = \mathbf{X}_0 + \alpha\mathbf{V}$ be a noisy version of $\mathbf{X}_0$, where $\mathbf{V}$ is the additive noise term, whose elements are drawn from a normal distribution. The parameter α controls the power (variance) of the noise term, and is defined as a function of the signal to noise ratio (SNR), in dB, as follows

$$\mathrm{SNR}_{\mathrm{dB}} = 10\log_{10}\left(\frac{||\mathbf{X}_0||_F^2}{||\alpha\mathbf{V}||_F^2}\right). \tag{1}$$

Assuming the SNR range [0, 5, 10, 15, 20, 25, 30] dB, find the estimates $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ obtained with the LSKRF algorithm for the configurations $(I, J) = (10, 10)$ and $(I, J) = (30, 10)$.

Let us define the normalized mean square error (NMSE) measure as follows

$$\mathrm{NMSE}(\mathbf{X}_0) = \frac{1}{1000}\sum_{i=1}^{1000} \frac{\|\hat{\mathbf{X}}_0(i) - \mathbf{X}_0(i)\|_F^2}{\|\mathbf{X}_0(i)\|_F^2}, \tag{2}$$

where $\mathbf{X}_0(i)$ e $\hat{\mathbf{X}}_0(i)$ represent the original data matrix and the reconstructed one at the ith experiment, respectively. For each SNR value and configuration, plot the NMSE vs. SNR curve. Discuss the obtained results.

Note: For a given SNR (dB), the parameter α to be used in your experiment is determined from equation (1).