

# TIP8419 - Tensor Algebra

## Homework 4

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### Least Squares Kronecker Product Factorization (LSKronF)

**Problem 1** Generate  $\mathbf{X} = \mathbf{A} \otimes \mathbf{B} \in \mathbb{C}^{24 \times 6}$ , for randomly chosen  $\mathbf{A} \in \mathbb{C}^{4 \times 2}$  e  $\mathbf{B} \in \mathbb{C}^{6 \times 3}$ . Then, implement the Least Square Kronecker Product Factorization (LSKronF) algorithm that estimate  $\mathbf{A}$  and  $\mathbf{B}$  by solving the following problem

$$(\hat{\mathbf{A}}, \hat{\mathbf{B}}) = \min_{\mathbf{A}, \mathbf{B}} \|\mathbf{X} - \mathbf{A} \otimes \mathbf{B}\|_F^2.$$

Compare the estimated matrices  $\hat{\mathbf{A}}$  and  $\hat{\mathbf{B}}$  with the original ones. What can you conclude? Explain the results.

Hint: Use the file “kronf\_matrix.mat” to validate your result.

**Problem 2** Assuming 1000 Monte Carlo experiments, generate  $\mathbf{X}_0 = \mathbf{A} \otimes \mathbf{B} \in \mathbb{C}^{IJ \times PQ}$ , for randomly chosen  $\mathbf{A} \in \mathbb{C}^{I \times P}$  and  $\mathbf{B} \in \mathbb{C}^{J \times Q}$ , whose elements are drawn from a normal distribution. Let  $\mathbf{X} = \mathbf{X}_0 + \alpha \mathbf{V}$  be a noisy version of  $\mathbf{X}_0$ , where  $\mathbf{V}$  is the additive noise term, whose elements are drawn from a normal distribution. The parameter  $\alpha$  controls the power (variance) of the noise term, and is defined as a function of the signal to noise ratio (SNR), in dB, as follows

$$\text{SNR}_{\text{dB}} = 10 \log_{10} \left( \frac{\|\mathbf{X}_0\|_F^2}{\|\alpha \mathbf{V}\|_F^2} \right). \quad (1)$$

Assuming the SNR range [0, 5, 10, 15, 20, 25, 30] dB, find the estimates  $\hat{\mathbf{A}}$  and  $\hat{\mathbf{B}}$  obtained with the LSKronF algorithm for the configurations: I.  $(I, J) = (2, 4)$ ,  $(P, Q) = (3, 5)$  and II.  $(I, J) = (4, 8)$ ,  $(P, Q) = (3, 5)$ .

Let us define the normalized mean square error (NMSE) measure as follows

$$\text{NMSE}(\mathbf{X}_0) = \frac{1}{1000} \sum_{i=1}^{1000} \frac{\|\hat{\mathbf{X}}_0(i) - \mathbf{X}_0(i)\|_F^2}{\|\mathbf{X}_0(i)\|_F^2},$$

where  $\mathbf{X}_0(i)$  e  $\hat{\mathbf{X}}_0(i)$  represent the original data matrix and the reconstructed one at the  $i$ th experiment, respectively. For each SNR value and configuration, plot the NMSE vs. SNR curve. Discuss the obtained results.