Homework 2 [TI8419 - Multilinear Algebra]

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Problem 1

Generate $\mathbf{X} = \mathbf{A} \diamond \mathbf{B} \in \mathbb{C}^{I \times R}$, for randomly chosen $\mathbf{A} \in \mathbb{C}^{I \times R}$ and $\mathbf{B} \in \mathbb{C}^{I \times R}$. Compute the left pseudo-inverse of $\mathbf{X}$ and obtain a graph that shows the run time vs. number of rows (I) for the following methods.

Method 1:

Matlab/Octave function: $pinv(\mathbf{X}) = pinv(\mathbf{A} \diamond \mathbf{B})$

Method 2:

$\mathbf{X}^{\dagger} = (\mathbf{X}^{\top}\mathbf{X})^{-1}\mathbf{X}^{\top} = [(\mathbf{A} \diamond \mathbf{B})^{\top}(\mathbf{A} \diamond \mathbf{B})]^{-1}(\mathbf{A} \diamond \mathbf{B})^{\top}$

Method 3:

$\mathbf{X}^{\dagger} = [(\mathbf{A}^{\top}\mathbf{A}) \odot (\mathbf{B}^{\top}\mathbf{B})]^{-1}(\mathbf{A} \diamond \mathbf{B})^{\top}$

Note: Consider the range of values $I \in \{2, 4, 8, 16, 32, 64, 128, 256\}$ and plot the curves for $R = 2$ and $R = 4$.

Results

Simulation setup

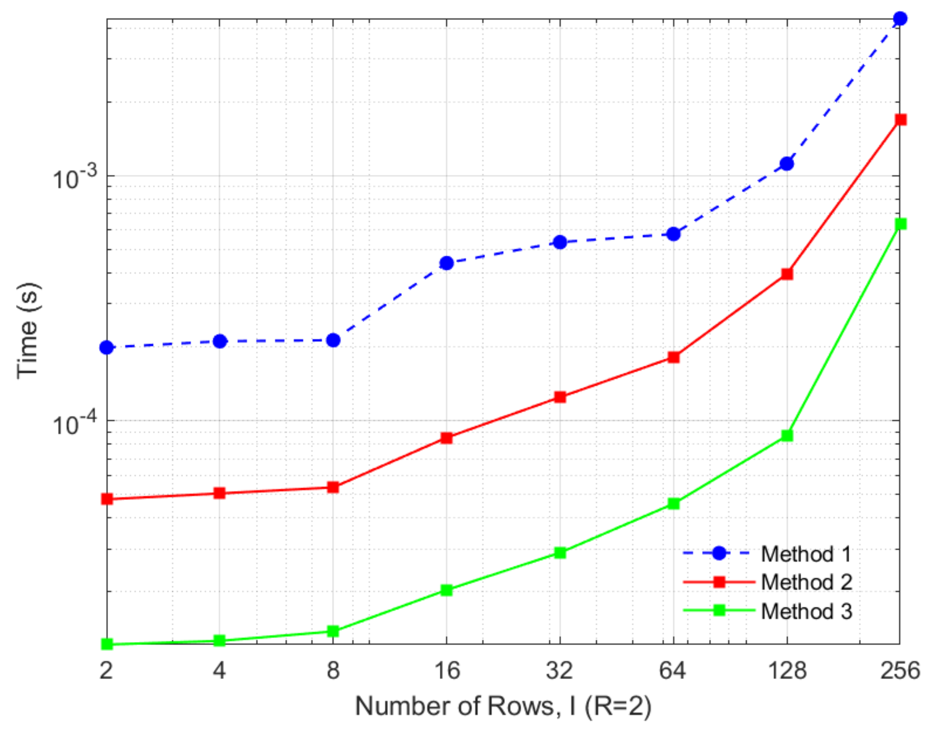
- 500 Monte Carlo Runs;
- Each Monte Carlo iteration uses a new matrix initialization from a Normal distribution $\mathcal{N}(0,\, 1)$;
- Compute the mean for each value, for $N = \{2, 4, 6, 8, 16, 32, 64, 128, 256\}$.

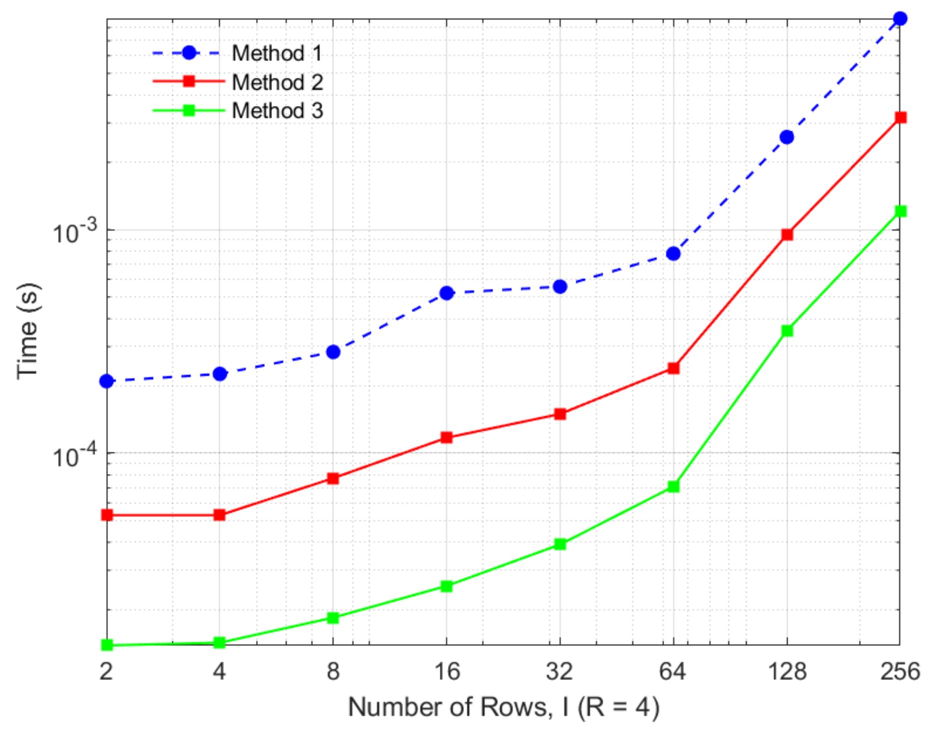
Discussion

We can see that for all values of I, Matlab's method is outperformed by the methods 2 and 3. All methods present a subtle gap between their cost, approximately constant. Method 2 is two times faster then Matlab, while method 3 is ten times faster.

The experiment with $R = 4$ also supports the results presented for $R = 2$, with very similar plots.

Problem 1 script and Figures.





Problem 2

Generate $\overset{N}{\underset{n=1}{\diamond}} \mathbf{A}_{(n)} = \mathbf{A}_{(1)} \diamond \cdots \diamond \mathbf{A}_{(N)}$, where every $\mathbf{A}_{(n)}$ has dimensions 4×2, $n = 1, \ldots, N$. Evaluate the run time associated with the computation of the Khatri-Rao product as a function of the number N of matrices for the above methods.

Note: Consider the range of values $N \in \{2, 4, 6, 8, 10\}$.

The symbols $\odot$ and $\diamond$ denotes the Hadamard and the Khatri-Rao Product, respectively.

Results

Simulation setup

- 500 Monte Carlo Runs;
- Each Monte Carlo iteration uses a new matrix initialization from a Normal distribution $\mathcal{N}(0,\, 1)$;
- Each matrix has 4×2 dimension;
- Compute the mean for each value, for $N = \{2, 4, 6, 8, 10\}$.

Discussion

The results are consistent with the experiment perfomed in HW1, that for randomly generated $\mathbf{A}$ and $\mathbf{B} \in \mathbb{C}^{N \times N}$, an algorithm to compute the Khatri-Rao Product $\mathbf{A} \diamond \mathbf{B}$ was created according with the following prototype function:

$$R = kr(A, B).$$

Problem 2 script