Homework 5 [TI8419 - Multilinear Algebra]

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Kronecker Product Singular Value Decomposition (KPSVD)

Problem 1

Generate a block matrix according to the following structure

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_{1,1} & \dots & \mathbf{X}_{1,N} \\ \vdots & \ddots & \vdots \\ \mathbf{X}_{M,1} & \dots & \mathbf{X}_{M,N} \end{pmatrix}, \mathbf{X}_{i,j} \in \mathbb{C}^{P \times Q}, 1 \leq i \leq M, 1 \leq j \leq N,$$

Implement the KPSVD for the matrix $\mathbf{X}$ by computing σ_k, $\mathbf{U}_k$, and $\mathbf{V}_k$ such that

$$\mathbf{X} = \sum_{k=1}^{r_{KP}} \sigma_k \mathbf{U}_k \otimes \mathbf{V}_k$$

Results

Simulation setup

- The algorithm that uses the SVD was applied to estimate the original data;
- $M = N = P = Q = 3$;
- Randomly generate $\mathbf{X}_{i,j} = \text{rand}(P, Q), 1 \leq i \leq M, 1 \leq j \leq N$
- Initializated from a Normal distribution $\mathcal{N}(0,\, 1)$.

Discussion

We use the experiment with the real rank to validate the algorithm, by observing the NMSE between the given data and obtained as output to KPSVD.

NMSE with KPSVD

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Original Matrix vs KPSVD estimation (full rank): = -596.10 dB
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The output present a very low NMSE value what, what may be used as evidence to confirm the proper algorithm estimation.

Problem 1 script.

Problem 2

In the above problem, set $M = N = P = Q = 3$ and randomly generate $\mathbf{X}_{i,j} = \text{rand}(P, Q), 1 \leq i \leq M, 1 \leq j \leq N$. Then compute the KPSVD and the Kronecker-rank r_{KP} of $\mathbf{X}$ by using your KPSVD prototype function. Consider $r \leq r_{KP}$. Compute the nearest rank-r for the matrix $\mathbf{X}$.

Results

Simulation setup

- 1000 Monte Carlo Runs;
- The algorithm that uses the SVD was applied to estimate the original data;
- $M = N = P = Q = 3$;
- Randomly generate $\mathbf{X}_{i,j} = \text{rand}(P, Q), 1 \leq i \leq M, 1 \leq j \leq N$
- Each Monte Carlo iteration uses a new matrix initialization from a Normal distribution $\mathcal{N}(0,\, 1)$;
- Compute the KPSVD to assess rank deficiency, for R in range $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, where the matrix presents its full rank for $R = 9$.

Discussion

The results are consistent with the proposed scenario, since that for randomly generated $\mathbf{X}$, the algorithm succeds to obtain the lower NMSE (dB) with the known full-rank. Futhermore, we can see that as the rank decreases, the NMSE increases abruptly.

- Original Matrix vs KPSVD estimation

rank	NMSE (dB)
1	-4.72
2	-9.37
3	-14.33
4	-19.99
5	-26.64
6	-35.08
7	-46.19
8	-66.89
9	-597.43

As we can see results, the NMSE reduces as the rank increases, however it reaches the lowest point when the true rank is applied.

Problem 2 script and Figures.