Homework 6 [TI8419 - Multilinear Algebra]

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Unfolding, folding, and n-mode product

Problem 1

For a third-order tensor $\mathbf{X} \in \mathbb{C}^{I \times J \times K}$, using the concept of n-mode fibers, implement the function unfold according to the following prototype

$$[\mathcal{X}]_{(n)} = \text{unfold}(\mathcal{X}, n)$$

Hint: Use the file "unfolding_folding.mat" to validate your function.

Results

Simulation setup

- The algorithm was applied to reshape the original data into a N-mode tensor;
- N in range $\{1, 2, 3\}$.

Discussion

- Experiment proposed in the example 2.6 of the book Multi-way Analysis With Applications in the Chemical Sciences (Smilde, 2004).

Tensor X

```
X(:, :, 1)
1  2  3;
4  5  6;
7  8  9;
3  2  1;

X(:, :, 2)
5  6  7;
8  9  4;
5  3  2;
4  5  6;
```

Tensor X (mode-1)

```
X(4, 6)
1  2  3  5  6  7;
4  5  6  8  9  4;
7  8  9  5  3  2;
3  2  1  4  5  6;
```

Tensor X (mode-2)

```
X(3, 8)
1  4  7  3  5  8  5  4;
2  5  8  2  6  9  3  5;
3  6  9  1  7  4  2  6;
```

Tensor X (mode-3)

```
X(2, 12)
1  4  7  3  2  5  8  2  3  6  9  1;
5  8  5  4  6  9  3  5  7  4  2  6;
```

- Validation

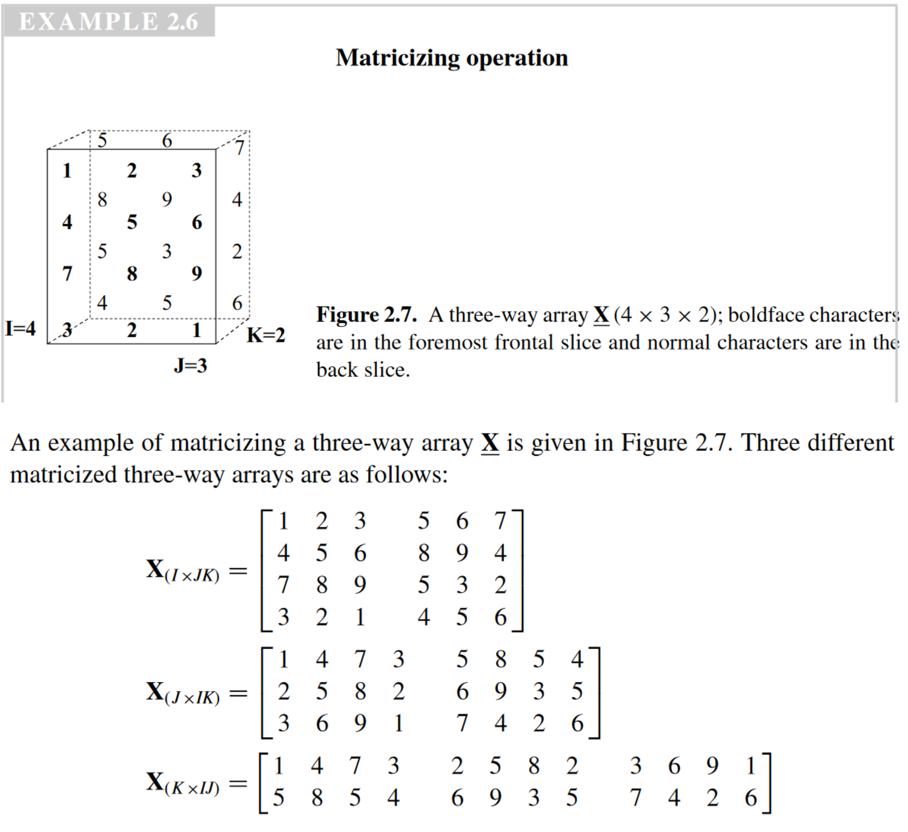
Unfold difference

```
sum(X1 - unfold(X, 1)) = 0.00
sum(X2 - unfold(X, 2)) = 0.00
sum(X3 - unfold(X, 3)) = 0.00
```

We assess the difference between the given data and the algorithm output, and we can see that the residuals sum leads to zero.

Problem script.

Fold experiment output log: Unfold Txt File.


EXAMPLE 2.6

Matricizing operation

Figure 2.7. A three-way array $\underline{\mathbf{X}}$ ($4 \times 3 \times 2$); boldface characters are in the foremost frontal slice and normal characters are in the back slice.

An example of matricizing a three-way array $\underline{\mathbf{X}}$ is given in Figure 2.7. Three different matricized three-way arrays are as follows:

$$\mathbf{X}_{(I \times JK)} = \begin{bmatrix} 1 & 2 & 3 & 5 & 6 & 7 \\ 4 & 5 & 6 & 8 & 9 & 4 \\ 7 & 8 & 9 & 5 & 3 & 2 \\ 3 & 2 & 1 & 4 & 5 & 6 \end{bmatrix}$$

$$\mathbf{X}_{(J \times IK)} = \begin{bmatrix} 1 & 4 & 7 & 3 & 5 & 8 & 5 & 4 \\ 2 & 5 & 8 & 2 & 6 & 9 & 3 & 5 \\ 3 & 6 & 9 & 1 & 7 & 4 & 2 & 6 \end{bmatrix}$$

$$\mathbf{X}_{(K \times IJ)} = \begin{bmatrix} 1 & 4 & 7 & 3 & 2 & 5 & 8 & 2 & 3 & 6 & 9 & 1 \\ 5 & 8 & 5 & 4 & 6 & 9 & 3 & 5 & 7 & 4 & 2 & 6 \end{bmatrix}$$

Problem 2

Implement the function fold that converts the unfolding $[\mathcal{X}]_{(n)}$ obtained with $\text{unfold}(\mathcal{X}, n)$ back to the tensor $\mathcal{X} \in \mathbb{C}^{I \times J \times K}$ (i.e., a 3-d array in Matlab/Octave), according to the following prototype:

$$\mathcal{X} = \text{fold}([\mathcal{X}]_{(n)}, [IJK], n)$$

Hint: Use the file "unfolding_folding.mat" to validate your function.

Results

Simulation setup

- The algorithm was applied to build a tensor from a N-mode tensor;
- N in range $\{1, 2, 3\}$.

Discussion

- Experiment proposed in the example 2.6 of the book Multi-way Analysis With Applications in the Chemical Sciences (Smilde, 2004).

Tensor X (mode-1)

```
X(4, 6)
1  2  3  5  6  7;
4  5  6  8  9  4;
7  8  9  5  3  2;
3  2  1  4  5  6;
```

Tensor X (mode-2)

```
X(3, 8)
1  4  7  3  5  8  5  4;
2  5  8  2  6  9  3  5;
3  6  9  1  7  4  2  6;
```

Tensor X (mode-3)

```
X(2, 12)
1  4  7  3  2  5  8  2  3  6  9  1;
5  8  5  4  6  9  3  5  7  4  2  6;
```

Tensor X from (mode-1)

```
X(:, :, 1)
1  2  3;
4  5  6;
7  8  9;
3  2  1;

X(:, :, 2)
5  6  7;
8  9  4;
5  3  2;
4  5  6;
```

Tensor X from (mode-2)

```
X(:, :, 1)
1  2  3;
4  5  6;
7  8  9;
3  2  1;

X(:, :, 2)
5  6  7;
8  9  4;
5  3  2;
4  5  6;
```

Tensor X from (mode-3)

```
X(:, :, 1)
1  2  3;
4  5  6;
7  8  9;
3  2  1;

X(:, :, 2)
5  6  7;
8  9  4;
5  3  2;
4  5  6;
```

- Validation

Fold difference

```
sum(tenX - fold(X1)) = 0.00
sum(tenX - fold(X2)) = -0.00
sum(tenX - fold(X3)) = -0.00
```

We assess the difference between the given data and the algorithm output, and we can see that the residuals sum leads to zero.

Problem script.

Fold experiment output log: Fold Txt File.

Problem 3

For given matrices $\mathbf{A} \in \mathbb{C}^{P \times I}$, $\mathbf{B} \in \mathbb{C}^{Q \times J}$, $\mathbf{C} \in \mathbb{C}^{R \times K}$ and tensor $\mathcal{X} \in \mathbb{C}^{I \times J \times K}$, calculate the tensor $\mathcal{Y} \in \mathbb{C}^{P \times Q \times R}$ via the following multilinear transformation:

$$\mathcal{Y} = \mathcal{X} \times_1 \mathbf{A} \times_2 \mathbf{B} \times_3 \mathbf{C}$$

Hint: Use the file "multilinear_product.mat" to validate your result.

Results

Simulation setup

- The algorithm was applied to compute the N-mode product between a given tensor and factor matrices.

Discussion

The results are consistent with the proposed scenario, since given data after the algorithm succeds to obtain a very low NMSE (dB) value.

```
NMSE between a given tensor and its version afected by the N-mode product: -666.47 dB
```

Problem script