Noninvasive Assessment of Atrial Fibrillation Complexity Using Tensor Decomposition Techniques

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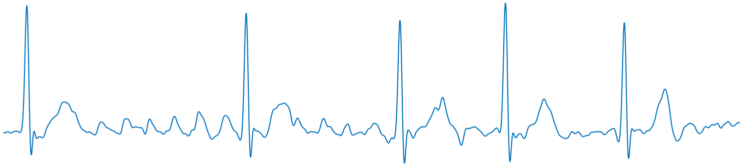
Atrial Fibrillation

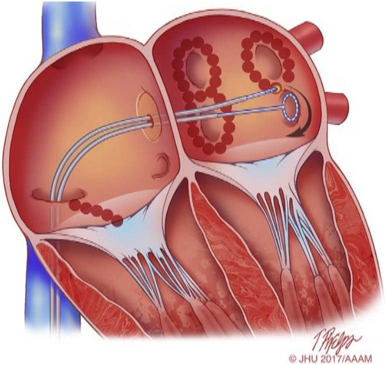
- Atrial Fibrillation (AF) is the most common sustained cardiac arrhythmia encountered in clinical practice.
  - In the EU, the number of adults with AF will double from 2010 to 2060[1].



- The complex electrophysiological mechanisms underlying AF are not completely understood.

[1] Krijthe *et al.*, "Projections on the number of individuals with atrial fibrillation in the European Union, from 2000 to 2060," *Eur Heart J*. 2013.

Step-wise Catheter Ablation (CA)



- Noninvasive techniques to assess AF electrophysiological complexity can help guide step-wise CA in real time.
  - Impact of pulmonary vein isolation (PVI) and other widely used techniques on atrial activity (AA) complexity.

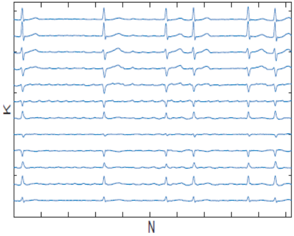
Figure from Tim Helps © 2017 Johns Hopkins University, AAM

Matrix Approach

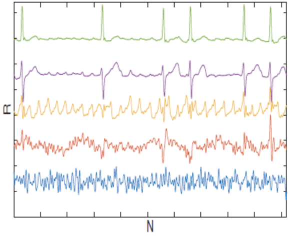
The ECG data matrix can be modeled as:

$$\mathbf{Y} = \mathbf{MS} \in \mathbb{R}^{K \times N} \,, \tag{1}$$

where $\mathbf{M} \in \mathbb{R}^{K \times R}$ is a mixing matrix and $\mathbf{S} \in \mathbb{R}^{R \times N}$ is the source matrix.

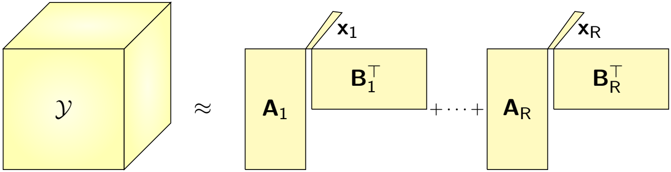


$$= \begin{bmatrix} m_{1,1} & \cdots & m_{1,R} \\ \vdots & \ddots & \vdots \\ m_{K,1} & \cdots & m_{K,R} \end{bmatrix} \times$$



Tensor Approach

- The ECG data can be modeled as a 3rd-order tensor $\mathcal{Y}$ via row-Hankelization.
  - Tensor decompositions factorize data as a sum of simpler tensors.



- Block Term Tensor Decomposition (BTD) based on Hankel structure[2].

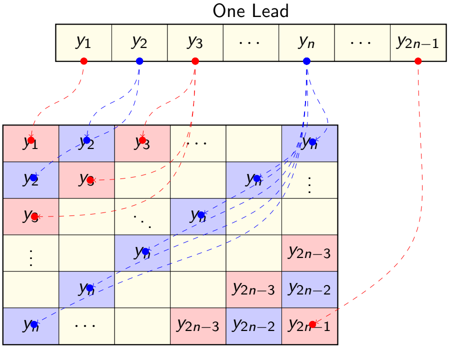
[2] De Lathauwer, "Blind separation of exponential polynomials and the decomposition of a tensor in rank-$(L_r, L_r, 1)$ terms," *SIAM J. Matrix Anal. Appl.*, 2011.

BTD-Hankel Model

Low-rank Hankel structure

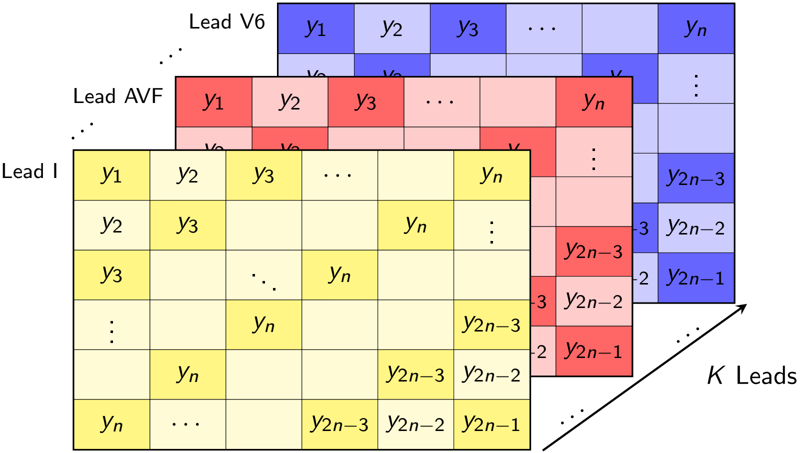
- AA signal during AF can be represented by an all-pole model (2)
- The structured Hankel matrix has a rank equal to the number of poles (L_r)

$$s(n) = \sum_{\ell=1}^{L_r} \alpha_\ell z_\ell^n \qquad (2)$$

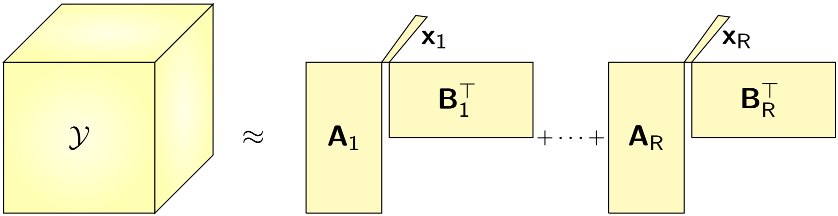


Tensor Approach

- Stack each Hankel matrix in the 3rd-mode of the tensor $\mathcal{Y}$.



BTD Approach



Challenge

- Parameter estimation
  - R, L_r
- Factor estimation
  - $\mathbf{A}$, $\mathbf{B}$, $\mathbf{X}$

Constrained Alternating Group Lasso

Classical BTD Approach

- Fixed structure minimizing $f(\mathbf{A}, \mathbf{B}, \mathbf{X})$ with prior knowledge of (R, L_r)

$$f(\mathbf{A}, \mathbf{B}, \mathbf{X}) \triangleq \left\| \mathcal{Y} - \sum_{r=1}^{R} \left(\mathbf{A}_r \mathbf{B}_r^\top \right) \circ \mathbf{x}_r \right\|_F^2 \tag{3}$$

Constrained Alternating Group Lasso (CAGL) Approach

- Non-fixed structure minimizing $F(\mathbf{A}, \mathbf{B}, \mathbf{X})$ ensuring the Hankel structure
- Penalization term (γ) and $g(\mathbf{A}, \mathbf{B}, \mathbf{X})$ limiting the multilinear ranks and number of blocks
- Allows simultaneous estimation of (R, L_r) and model factors

$$F(\mathbf{A}, \mathbf{B}, \mathbf{X}) \triangleq f(\mathbf{A}, \mathbf{B}, \mathbf{X}) + \gamma\, g(\mathbf{A}, \mathbf{B}, \mathbf{X}) \tag{4}$$

AF Complexity Index

Signal Complexity

The more poles the signal contains, the more complex it can be considered

- The complexity index proposed in this work is based on the number of poles L_r contained in a signal.
- The Hankel matrix rank is equal to number of poles L_r.

Atrial Source Classification

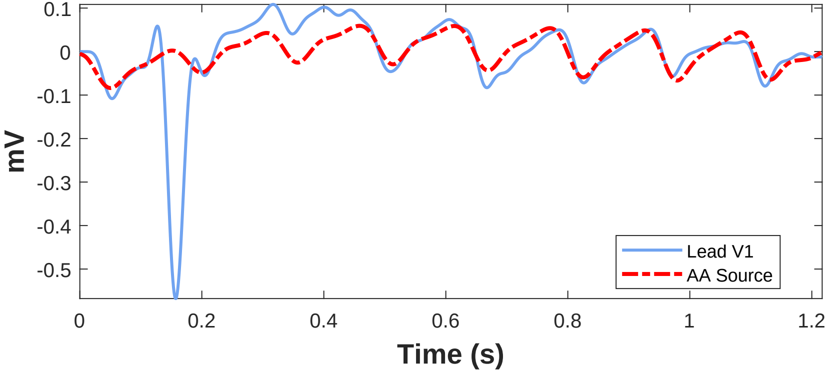
Challenge

After performing CAGL, the automated AA source classification is still a problem

- Spectral concentration (SC), dominant frequency (DF), kurtosis and visual inspection to evaluate AA extraction[3].

[3] De Oliveira and Zarzoso, "Source analysis and selection using block term decomposition in atrial fibrillation", in *Proc. LVA/ICA*, 2018.

AA Source Estimation



- SC = 74.3%
- DF = 6.4 Hz
- Kurtosis = 177.0
- AA Hankel Matrix Rank = 33

Database and Experimental Setup

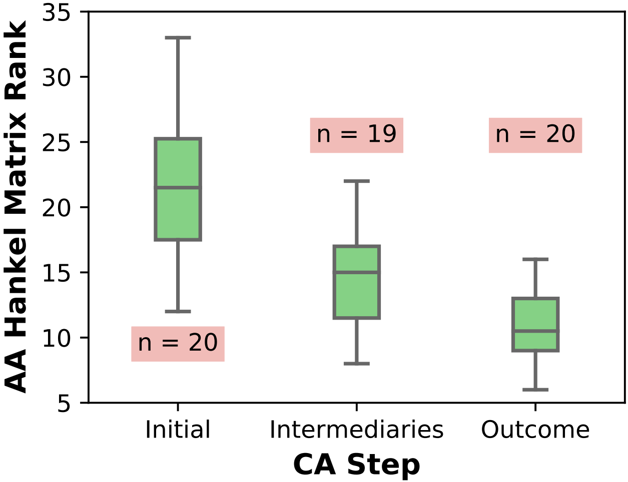
Database

- 20 patients suffering from persistent AF
- 59 ECG segments from 0.72 to 1.42 seconds

Cardiology Department of Princess Grace Hospital Center, Monaco

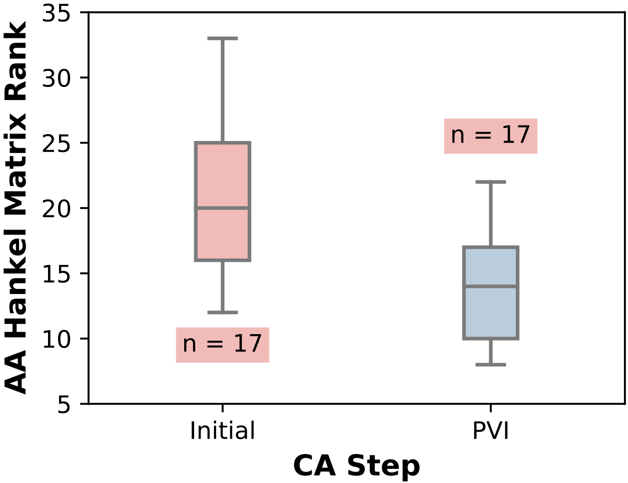
- Hankel-based BTD was implemented using CAGL.

Impact of CA step on AA complexity



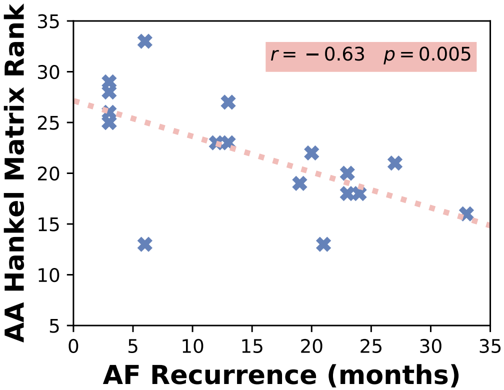
- 20 patients undergoing various CA steps
- 59 ECG segments (1.06 ± 0.2 s)

Impact of PVI on AA complexity



- 17 patients undergoing PVI
- 34 ECG segments

AF Recurrence vs. Complexity Before CA



Relationship

A significant Pearson correlation between AF recurrence and the proposed index

- 18 patients with complete follow-up information

Conclusions

Contributions

- Jointly extract the AA signal and measure AF complexity via tensor decomposition
- Very short ECG recordings (1.06 ± 0.20 s)
- Validation in 20 patients undergoing CA
  - Expected decreasing AF complexity throughout CA steps
  - Significant correlation with AF recurrence after CA

Clinical Impact

- A potential tool to help guide CA in real time

Future Work

- Increase number of patients in the database
- Compare the proposed index with other state-of-the-art indices