Noninvasive Assessment of Atrial Fibrillation Complexity Using Tensor Decomposition Techniques

Lucas Abdalah, Pedro M. R. de Oliveira, Walter Freitas Jr, Vicente Zarzoso

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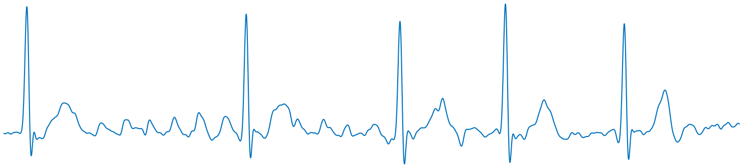
Atrial Fibrillation

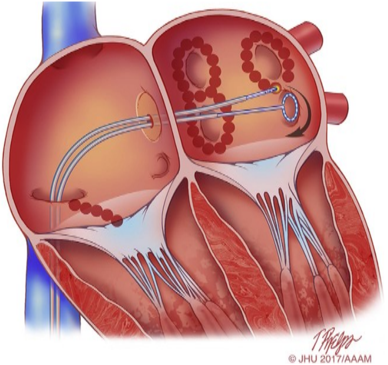
- Atrial Fibrillation (AF) is the most common sustained cardiac arrhythmia encountered in clinical practice.
  - In the EU, the number of adults with AF will double from 2010 to 2060[1].



- The complex electrophysiological mechanisms underlying AF are not completely understood.

[1] Krijthe *et al.*, "Projections on the number of individuals with atrial fibrillation in the European Union, from 2000 to 2060," *Eur Heart J*. 2013.

Step-wise Catheter Ablation (CA)



- Noninvasive techniques to assess AF electrophysiological complexity can help guide step-wise CA in real time.
  - Impact of pulmonary vein isolation (PVI) and other widely used techniques on atrial activity (AA) complexity.

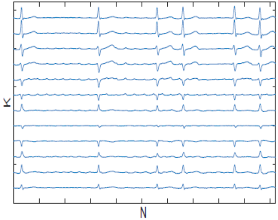
Figure from Tim Helps © 2017 Johns Hopkins University, AAM

BSS Model

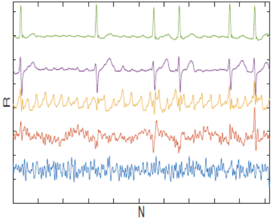
The ECG data matrix can be modeled as:

$$\mathbf{Y} = \mathbf{MS} \in \mathbb{R}^{K \times N}, \tag{1}$$

where $\mathbf{M} \in \mathbb{R}^{K \times R}$ is a mixing matrix and $\mathbf{S} \in \mathbb{R}^{R \times N}$ is the source matrix.

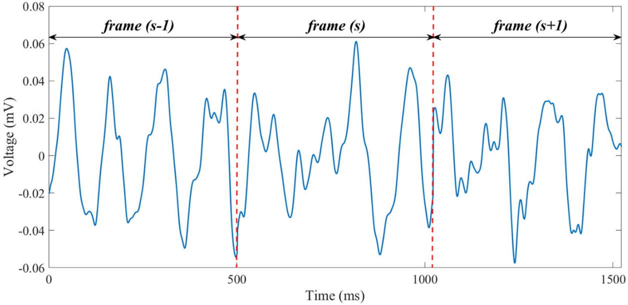


$$= \begin{bmatrix} m_{1,1} & \cdots & m_{1,R} \\ \vdots & \ddots & \vdots \\ m_{K,1} & \cdots & m_{K,R} \end{bmatrix} \times$$



Matrix Approach

- Nondipolar Component Index (NDI)[2]
  - PCA applied to TQ Intervals
  - AA is represented by the 3D subspace spanned by its first 3 PCs



[2] M. Meo et. al, "Noninvasive assessment of atrial fibrillation complexity in relation to ablation characteristics and outcome," *Frontiers in Physiology*, 2018. Figure from M. Meo et al., 2018.

Nondipolar Component Index (NDI)

- Compute PCA on the preprocessed ECG

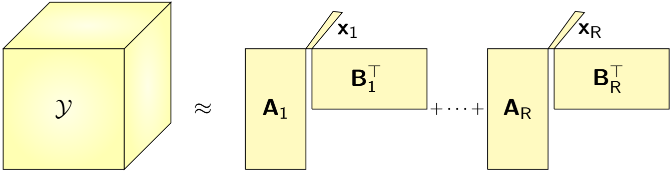
$$\mathbf{Y}_{TQ} = \mathbf{U\Sigma V}^T \tag{2}$$

- The proportion of power not explained by the 3 dominant PCs

$$\text{NDI} = 1 - \frac{\sum_{l=1}^{3} \sigma_l^2}{\sum_{l=1}^{L} \sigma_l^2}. \tag{3}$$

Tensor Approach

- The ECG data can be modeled as a 3rd-order tensor $\mathcal{Y}$ via row-Hankelization.
  - Tensor decompositions factorize data as a sum of simpler tensors.



- Block Term Tensor Decomposition (BTD) based on Hankel structure[3].

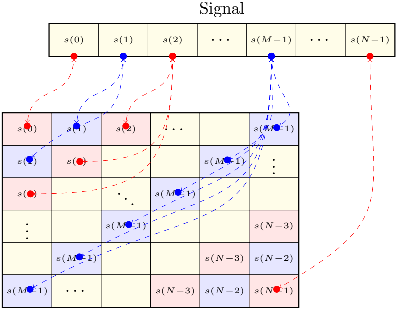
[3] De Lathauwer, "Blind separation of exponential polynomials and the decomposition of a tensor in rank-$(L_r, L_r, 1)$ terms," *SIAM J. Matrix Anal. Appl.*, 2011.

BTD-Hankel Model

Low-rank Hankel Structure

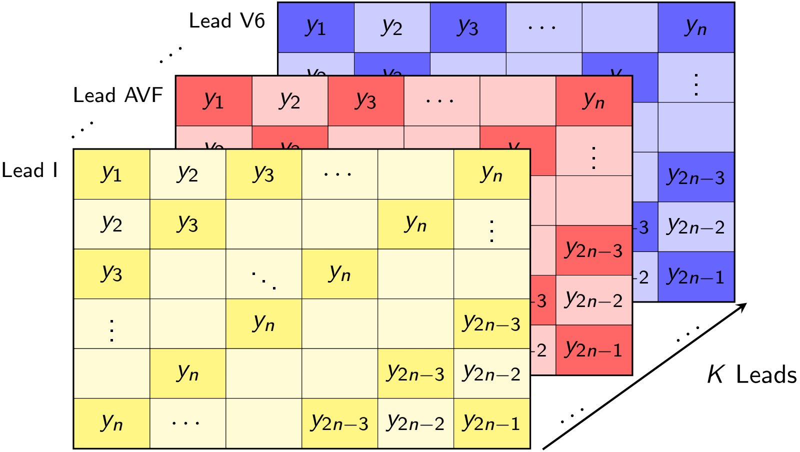
- The signal $s(n)$ is represented by an all-pole model (4)
- A Hankel matrix has a rank equal to the number of poles (L)

$$s(n) = \sum_{l=1}^{L} c_l z_l^n, \quad 0 \leq n \leq N-1 \tag{4}$$

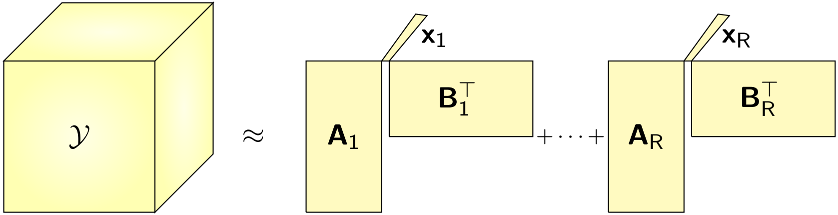


Tensor Approach

- Stack each Hankel matrix in the 3rd-mode of the tensor $\mathcal{Y}$.



BTD Approach



Challenge

- Parameter estimation
  - R, L_r
- Factor estimation
  - $\mathbf{A}$, $\mathbf{B}$, $\mathbf{X}$

Algorithm

Classical BTD Approach

- Fixed structure minimizing $f(\mathbf{A}, \mathbf{B}, \mathbf{X})$ with prior knowledge of (R, L_r)

$$f(\mathbf{A}, \mathbf{B}, \mathbf{X}) \triangleq \left\| \mathcal{Y} - \sum_{r=1}^{R} \left(\mathbf{A}_r \mathbf{B}_r^\top \right) \circ \mathbf{x}_r \right\|_F^2 \quad (5)$$

Constrained Alternating Group Lasso (CAGL) Approach

- Non-fixed structure minimizing $F(\mathbf{A}, \mathbf{B}, \mathbf{X})$ ensuring the Hankel structure
- Penalization term (γ) and $g(\mathbf{A}, \mathbf{B}, \mathbf{X})$ limiting the multilinear ranks and number of blocks
- Allows simultaneous estimation of (R, L_r) and model factors

$$F(\mathbf{A}, \mathbf{B}, \mathbf{X}) \triangleq f(\mathbf{A}, \mathbf{B}, \mathbf{X}) + \gamma\, g(\mathbf{A}, \mathbf{B}, \mathbf{X}) \quad (6)$$

Constrained Alternating Group Lasso

$$g(\mathbf{A},\mathbf{B},\mathbf{C}) \triangleq \|\mathbf{A}\|_{2,1} + \|\mathbf{B}\|_{2,1} + \|\mathbf{C}\|_{2,1} \tag{7}$$

- Structured low-rank approximation (SRLA)
- Geometric properties of the mixed $\ell_{2,1}$-norm allows one to select the relevant low-rank blocks.
- The problem is nonconvex (and nonsmooth), but convex by blocks, so a block coordinate descent (BCD) approach is employed[4].
- Cadzow's Algorithm ensures Hankel Structure:

$$\hat{\mathbf{H}}_r \approx \hat{\mathbf{A}}_r \hat{\mathbf{B}}_r^\intercal \tag{8}$$

[4] Goulart et al., "Alternating group lasso for block-term tensor decomposition with application to ECG source separation", in *IEEE Transactions on Signal Processing*, vol. 68, pp. 2682-2696, 2020.

AF Complexity Index

Signal Complexity

The more poles the signal contains, the more complex it can be considered

- The complexity index proposed in this work is based on the number of poles L_r contained in a signal.
- The Hankel matrix $(\hat{\mathbf{H}}_r)$ rank is equal to number of poles L_r.

AA Hankel Matrix Rank

Compute $rank(\hat{\mathbf{H}}_r)$ for the AA estimated block

Atrial Source Classification

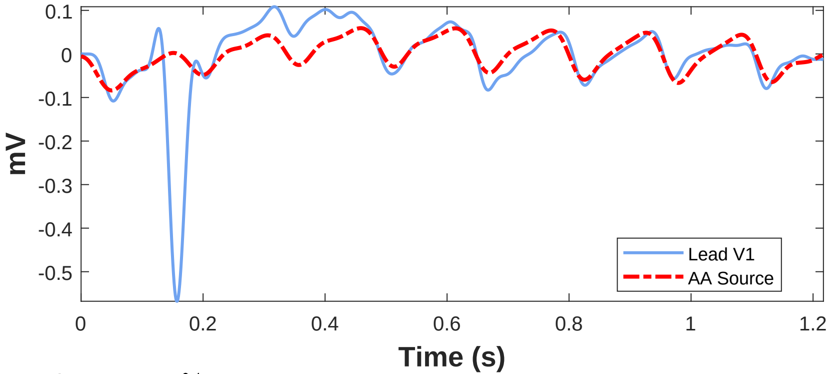
Challenge

After performing CAGL, the automated AA source classification is still a problem

- Spectral concentration (SC), dominant frequency (DF), kurtosis and visual inspection to evaluate AA extraction[5].

[5] De Oliveira and Zarzoso, "Source analysis and selection using block term decomposition in atrial fibrillation", in *Proc. LVA/ICA*, 2018.

AA Source Estimation



- SC = 74.3%
- DF = 6.4 Hz
- Kurtosis = 177.0
- AA Hankel Matrix Rank = 33

Database and Experimental Setup

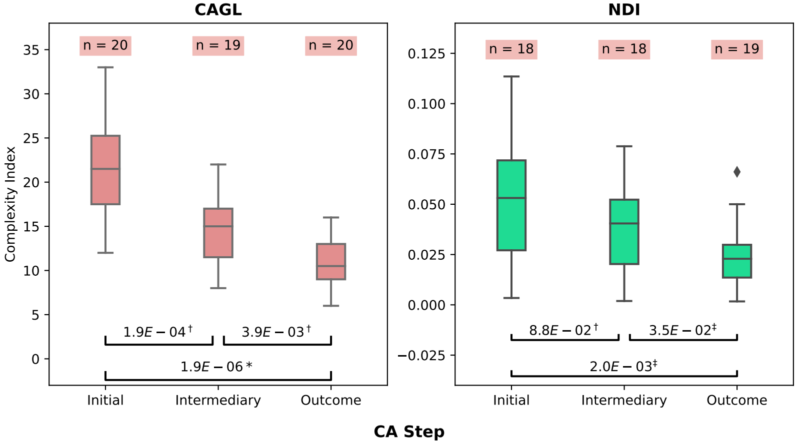
Database

- 20 patients suffering from persistent AF
- 59 ECG segments from 0.72 to 1.42 seconds

Cardiology Department of Princess Grace Hospital Center, Monaco

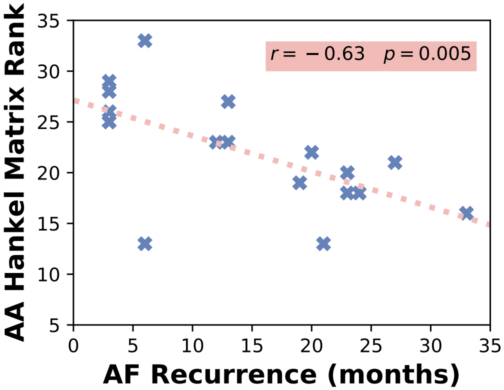
- Hankel-based BTD was implemented using CAGL.

Impact of CA step on AA complexity



- 20 patients undergoing various CA steps
- 59 ECG segments (1.06 ± 0.2 s)

AF Recurrence vs. Complexity Before CA

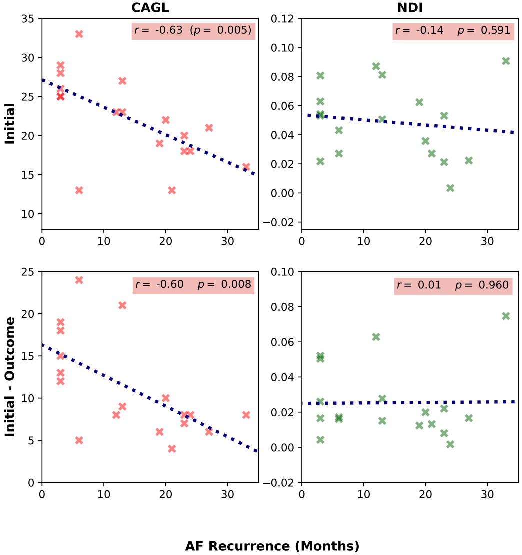


Relationship

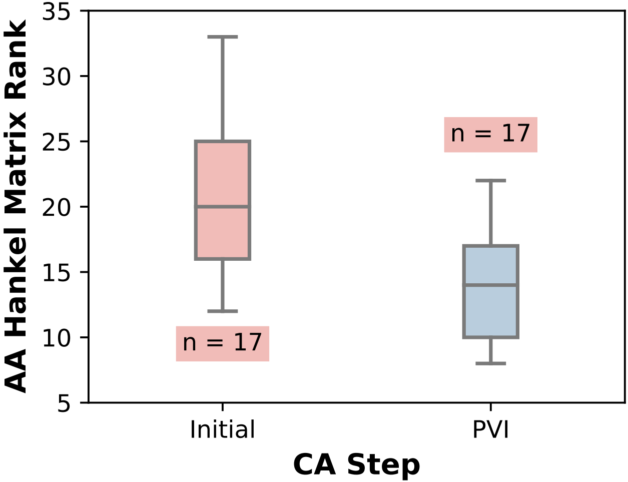
A significant Pearson correlation between AF recurrence and the proposed index

- 18 patients with complete follow-up information

AF Recurrence vs. Complexity Before CA



Impact of PVI on AA complexity



- 17 patients undergoing PVI
- 34 ECG segments

Conclusions

Contributions

- Jointly extract the AA signal and measure AF complexity via tensor decomposition
- Very short ECG recordings (1.06 ± 0.20 s)
- Validation in 20 patients undergoing CA
  - Expected decreasing AF complexity throughout CA steps
  - Significant correlation with AF recurrence after CA

Clinical Impact

- A potential tool to help guide CA in real time

Future Work

- Increase number of patients in the database
- Compare the proposed index with other state-of-the-art indices

References

[1] Krijthe *et al.*, "Projections on the number of individuals with atrial fibrillation in the European Union, from 2000 to 2060," *Eur Heart J.* 2013.
[2] M. Meo *et al.*, "Noninvasive assessment of atrial fibrillation complexity in relation to ablation characteristics and outcome," *Frontiers in Physiology*, 2018.
[3] De Lathauwer, "Blind separation of exponential polynomials and the decomposition of a tensor in rank-(L_r,L_r,1) terms," *SIAM J. Matrix Anal. Appl.*, 2011.
[4] Goulart *et al.*, "Alternating group lasso for block-term tensor decomposition with application to ECG source separation", in *IEEE Transactions on Signal Processing*, vol. 68, pp. 2682-2696, 2020.
[5] De Oliveira and Zarzoso, "Source analysis and selection using block term decomposition in atrial fibrillation", in *Proc. LVA/ICA*, 2018.

Previous Work

CinC 2020

L. S. Abdalah, P. M. R. de Oliveira, W. Freitas Jr, and V. Zarzoso, "Tensor-based noninvasive atrial fibrillation complexity index for catheter ablation," in Proc. Computing in Cardiology, vol. 47, Rimini, Italy, Sep. 2020.

SBRT 2021

L. Abdalah, W. Freitas Jr, P. M. R. de Oliveira, and V. Zarzoso, "Low-Rank Hankel Signal Model: Numerical Results," in Proc. Simpósio Brasileiro de Telecomunicações e Processamento de Sinais, vol. 39, Fortaleza, Brazil, Sep. 2021.

Thank You!