EXTENDED ESSAY 2021

Subject field:
Physics

Title:
Investigating the physics of rolling cylinders

Research question:

How does the internal diameter of a hollow cylinder affect the time it takes to roll down an incline?

Candidate Number:
hzp230

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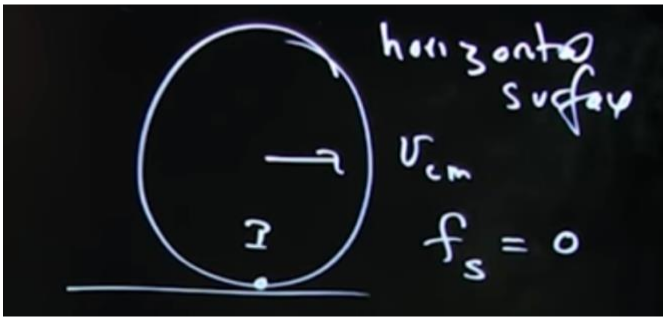
Table of Contents

Introduction

Mechanics is a crucial part of physics which gives an explanation to how physical object move. When I was younger, I was shocked by the fact that two objects of different mass that would be dropped at the same time would touch the floor at the same time. As I continued studying physics, I learnt that this was because of the law of conservation of every and more especially the gravitational potential energy being transferred to kinetic energy. Furthermore, in my IB physics class we looked at objects like a cube sliding down an incline and thought about this idea of conservation of energy. However, those two objects with different mass did not arrive at the same time due to the friction that affected the different mass in a different way. I questioned myself on how this would happen on a frictionless surface, but the problem was that it wouldn’t be possible to create this. After thought the idea occurred that by looking at rolling object it would be possible to obtain this idealisation of a frictionless surface as the cylinder will roll without static friction.



$I = the\ contact\ point$

$V_{cm} = velocity$

$f_s = force\ of\ static\ friction$

Figure 1 Image displaying the force of friction acting on a rolling object ("36.1 Friction" 1.01 minutes)

Figure one is part of a video which shows that on a horizontal surface a cylinder going at a certain velocity will have no static friction. Nevertheless, as the cylinder is going down an incline this drastically changes as the there starts to be static friction. This led the research to look at the rolling motion of cylinder and question the variables of the time it takes for a cylinder to roll down a ramp. One factor which stood out was the internal diameter of a cylinder which.

As a result, the research chooses to investigate the research question **"How does the internal diameter of a hollow cylinder affects the time it takes to roll down an incline?"** throughout this essay the paper will approach this question from two perspective; first by calculating the values theoretically using theory and by deriving different equation. Secondly the paper will compare those result to an experiment and establish if the result match theoretical data.

Background information

Important definition

Static friction – This occurs when the objects are at rest. This force is experienced when a force is applied but is not sufficient to make the object move. The force of static friction follows newton first law as it changes depending on the force applied.

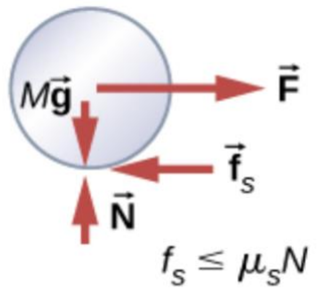
Dynamic friction – The force or friction as 2 objects are moving relative to each other.

Translational kinetic energy – Energy making a cylinder go in a straight line

Rotational kinetic energy – the energy making an object rotate in a circular motion.

Rolling motion of a cylinder

A cylinder rolls due to its static friction. As it is released from the top of the incline torque motion is created from the force of static friction.



μ_s = coefficient of static friction

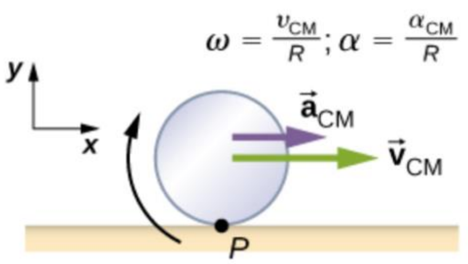
mg = Vector force of gravity

N = Vector of normal to the incline

$f_s = force\ of\ static\ friction$

Figure 2 Force acting on cylinder preventing it from slipping ("University Physics")

In figure 2 it is possible to observe the vectors of force that are involved not to make the cylinder slip. The point of contact with the incline does not slip and therefore a torque motion is created.



P = point of contact between cylinder and surface

$\omega = angular\ velocity$

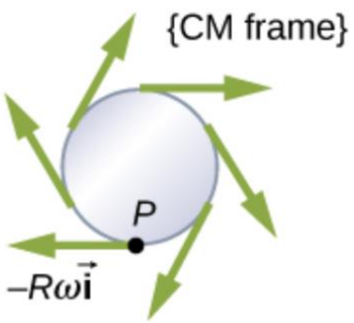
v = vector of velocity

α *= angular acceleration*

a = acceleration vector

Figure 3 Linear velocity and acceleration vectors acting on cylinder ("University Physics")

Figure 3 shows that relative to the surface P does not move and both vectors *a* and *v* show the linear movement of the cylinder. Both vector force is caused by the transfer of gravitational potential energy to kinetic energy in its two forms: translational and rotational.



-R = radius of cylinder

P = point of contact between cylinder and surface

$\omega = angular\ velocity$

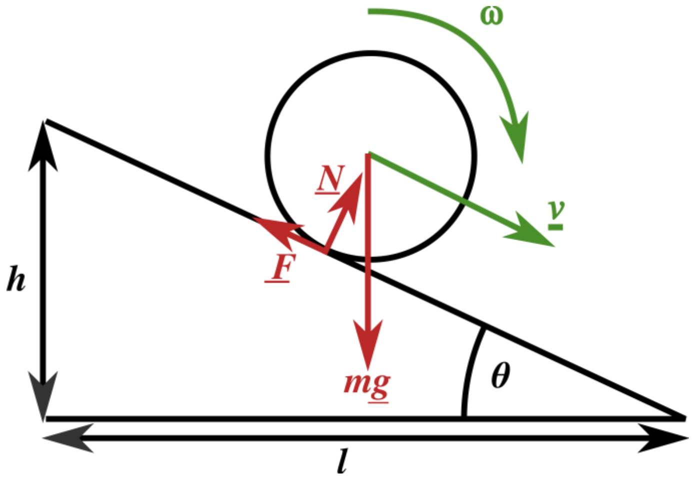
i = Horizontal component

Figure 4 velocity at point of contact ("University Physics")

Figure 4 shows the velocity of P relative to the centre of mass of the cylinder and as show the vector velocity $-R\omega i$. Because friction causes the cylinder to roll and not but does not take energy it is considerable to say that cylinder is an appropriate object to look at the conservation of energy.

Law of conservation of energy

To understand how different internal diameter impact the time a cylinder takes to roll down an incline it is important to look at the conservation of energy of a rolling cylinder. When an object is release from a certain hight it gains gravitational potential energy and this energy is converted to kinetic energy. As a cylinder is released from the top of an incline its transvers it's potential energy into kinetic energy however as a cylinder is released the kinetic energy is divided. The energy is split between transversal kinetic energy and rotational kinetic energy.



$h = height$

$l = length$

$mg = force\ of\ gravity$

$N = Normal\ to\ the\ incline$

$\omega = angular\ velocity$

$v = vector\ velocity$

$\theta = angle\ of\ the\ slope$

$F = force\ of\ static\ friction$

Figure 5 Free body diagram of cylinder on incline ("Rolling Sphere")

As it can be seen from figure 5 The green arrows (v, ω) show the two ways kinetic energy has been transferred from the gravitational potential energy. *v* demonstrates the kinetic energy transferred into velocity to and ω demonstrates the rotational kinetic energy being transferred into angular velocity. In addition, it can be deduced that the mass will not be a factor to this investigation as the conservation of energy can be shown from the equation below:

$$mgh = \frac{1}{2}mv^2 -> gh = \frac{1}{2}v^2$$

Equation 1

From equation 1 it can be seen that the mass cancels out therefore proving that mass is not a factor in this experiment. This will be proven for cylinders later in this paper (equation 11)

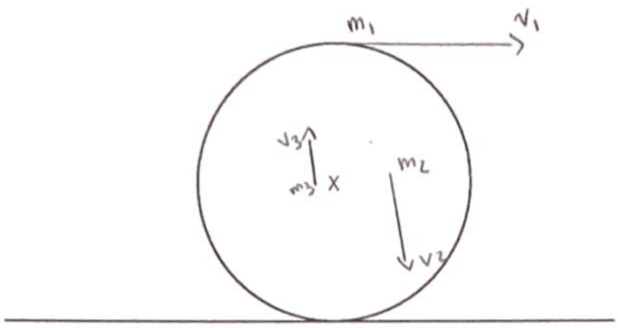
Application

There is a very wide range of application to the physics of cylinders. The principal use for this field is used to understand the motion of a wheel. Bikes, cars and motorcycles all use this field to optimise their performance. By learning the physics of cylinders, it is possible to understand the motion and optimize how wheels convert potential energy into kinetic energy.

Theory

Equation for the conservation of energy for a rolling cylinder

Cylinders ideally transfer gravitational energy to kinetic energy down an incline with very little losses in energy. This is caused by the cylinder rolling because of friction without slipping.



m = *point of mass*

v = *velocity*

Figure 6 Rotational kinetic energy on a cylinder image created by candidate

Figure 6 show the velocity at different point on a cylinder. In order to calculate the energy being transferred it is required to look at the value of the kinetic energy stated by equation 2.

$$Kinetic\ enrgy = \frac{1}{2}mv^2$$

Equation 2 Kinetic energy

However as shown in figure 6 the velocity is different at each point and therefore to find the total rotational kinetic energy it is required to evaluate the sum of the kinetic energy at every point on the cylinder which can be depicted in equation 3

$$K_{rot} = \sum \frac{1}{2} m v^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 + \cdots$$

Equation 3

This equation shows that the rotational kinetic energy is equal to the sum of kinetic energy at every single point on the cylinder. Nevertheless, as outlined the problem is that the velocity is different at every point because it depends on how far it is from the centre of mass. In order to overcome this difficulty this equation needs to be derived.

First using $v = r\omega$:

$$\sum \frac{1}{2} m (r\omega)^2$$

Equation 4

As the radius is also not the same on different part of the cylinder the following equation $I = \sum mr^2$can be used to get:

$$\frac{1}{2} (\sum mr^2) \omega^2$$

Equation 5

After using this last equation, the following equation is found demonstrating the final equation for the rotational kinetic energy

$$K_{rot} = \frac{1}{2} I \omega^2$$

Equation 6

Now in order to find the total kinetic energy for a rolling object the two equation for the transversal kinetic energy and the rotational kinetic energy are added to each other. As the experiment looks at the conservation of energy to calculate the time for the cylinder to roll down the incline, the equation can be displayed as:

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

Equation 7

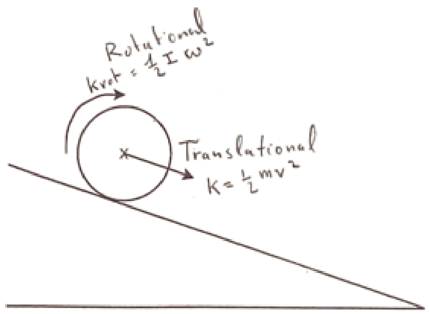


Figure 7 Kinetic energy on a rolling cylinder

The two forms of kinetic energy from Equation 7 are shown on Figure 7. As it is depicted the translational kinetic energy is making the cylinder go forward and the rotational kinetic energy makes the cylinder turn in a circular motion.

Substituting moment of inertia for different radius

As it can be perceived the rotational kinetic energy also depends on the amount of point which add to the total kinetic energy, it is for this reason that a hollow cylinder will obtain less rotational energy. The research has therefore chosen to focus on this variable and look at the change of time it takes a cylinder to roll down a ramp depending on the internal diameter. Assuming the cylinder does not slip the research will be able to change equation 7 in order to make it dependent on the internal diameter. In order to fit the question, the equation will also need to be rearranged to find the time for the cylinder to roll down the ramp.

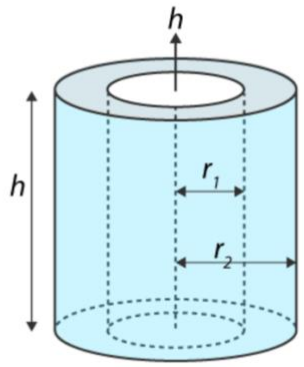


Figure 8 picture of hollow cylinder (Byju's the Learning)

The moment of inertia is calculated by doing the sum of all the point on the cylinder at different radius and obtained by differentiating the equation of very thin cylindrical shells. The equation obtained from this process and for the moment of inertia of a hollow cylinder is shown as:

$$I = \frac{1}{2}m({r_2}^2 + {r_1}^2)$$

Equation 8

*$r_1$and r_2 are labelled in figure 3

Using this knowledge, it was then possible to generate an equation linking gravitational potential energy and the rotational kinetic energy dependent on the internal diameter (r_1).

Substituting the equation for *I* and ω in the equation of conservation of energy

Starting with the equation for the conservation of the total kinetic energy from the gravitational potential energy:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Equation 7

Where m is the mass, g is the gravitational potential energy, h is the height, v is the velocity, *I* the moment of inertia and ω the angular velocity.

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(\frac{1}{2}m(r_2^2 + r_1^2))\omega^2$$

Equation 9

By substituting $\omega = \frac{v}{r_2}$ in order to get rid of the ω

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(\frac{1}{2}m(r_2^2 + r_1^2)(\frac{v}{r_2})^2)$$

Equation 10

The masses then cancel and simplify to give

$$gh = \frac{1}{2}v^2 + (\frac{1}{4}(r_2^2 + r_1^2)\frac{v^2}{r_2^2})$$

Equation 11

Finding equation in term of velocity

The equation then simplifies to be rearranged in term of *v*

$$gh = \frac{1}{2}v^2 + (\frac{v^2}{4r_2^2}(r_2^2 + r_1^2))$$

Equation 12

$$\frac{gh}{\frac{1}{2} + \frac{1}{4r_2^2}(r_2^2 + r_1^2)} = v^2$$

Equation 13

$$v = \pm\sqrt{\frac{gh}{\frac{1}{2}\left(1 + \frac{1}{2r_2^2}(r_2^2 + r_1^2)\right)}}$$

Equation 14

Creating a function of time

This equation is a function of the final velocity; however, the aim of this experiment is to determine the amount of time it takes for different cylinders with different hollowness therefore the equation has to be rearranged in term of t and not v. This can be done by dividing the final velocity by 2 in order to get the average velocity and using the formula of distance / time we can find the following equation.

$$t = \frac{d}{\left(\frac{\pm\sqrt{\frac{gh}{\frac{1}{2}(1+\frac{1}{2r_2{}^2}(r_2{}^2+r_1{}^2))}}}{2}\right)}$$

Equation 15

For the experiment conducted only the only variable will be the internal diameter and the rest will be controlled. Therefore, the following can be substituted distance will be 2.22 metres, $g = 9.81ms^{-2}$, h = 0.068m, r_2= 0.0626m a new equation can be written with the equation having only one unknown r_1.

$$t = \frac{2.22}{\left(\frac{\pm\sqrt{\frac{9.81\times 0.068}{\frac{1}{2}(1+\frac{1}{2(0.0626)^2}(0.0626^2+r_1{}^2))}}}{2}\right)}$$

Equation 16 final equation when applied to experiment

Equation 15 is the final equation which fits specifically the experiment which will be conducted in order to generate a graph for the variable of the internal diameter against time.

Theoretical data

Using the equation above theoretical data can be generated. The data being generated does not take into consideration the air resistance nor the cylinder slipping.

<u>Sample calculation:</u>

Internal radius in m = 0

$$v = \pm\sqrt{\frac{9.81 \times 0.068}{\frac{1}{2}(1 + \frac{1}{2(0.0626)^2}(0.0626^2 + (\mathbf{0})_1^2))}}$$

$$v = 0.9431$$

Average vel:

$$v = \frac{0.9431}{2}$$

Finding total time for cylinder to roll down ramp

$$t = \frac{2.22}{0.4116}$$

$$t = 4.7078$$

<u>Sample calculation for absolute uncertainty of time:</u>

For cylinder with internal radius of 0.01005 m
Take biggest value

$$t = \frac{2.22}{\left(\frac{\pm\sqrt{\frac{9.81 \times 0.068}{\frac{1}{2}(1 + \frac{1}{2(0.0626)^2}(0.0626^2 + (0.01010)^2))}}}{2}\right)}$$

$$t = 4.72825$$

Take minimum value

$$t = \frac{2.22}{\left(\frac{\pm \sqrt{\frac{9.81 \times 0.068}{\frac{1}{2}(1 + \frac{1}{2(0.0626)^2}(0.0626^2 + (0.01000)^2))}}}{2} \right)}$$

$$t = 0.46956$$

Subtract both values to find the absolute uncertainty for time:

$$4.72825 - \ 0.46956 = 0.00040$$

Table 1 Theoretical data calculating time for cylinder with different internal radius

Internal radius (mm) ±0.05mm	Internal radius (m) ± 0.00005	final vel/ ms^{-1}	average vel/ ms^{-1}	Final time Seconds	Absolute uncertainty of time
0.00	0.00000	0.94310	0.47155	4.70787	0.00000
10.05	0.01005	0.93901	0.46954	4.72806	0.00040
19.70	0.01970	0.92791	0.46395	4.78495	0.00082
30.10	0.03010	0.90873	0.45437	4.88591	0.00116
39.95	0.03995	0.88613	0.44307	5.01055	0.00153
50.10	0.05010	0.85612	0.42806	5.18615	0.00182

The values in table 1 was rounded to 5 decimal places due to the work being done in meters and the measurement were limited to an uncertainty of $\pm 0.00005\ m$. In addition, the uncertainty will be negliged when creating the graph as they can be rounded to 0 and error bars will not be visible on graph.

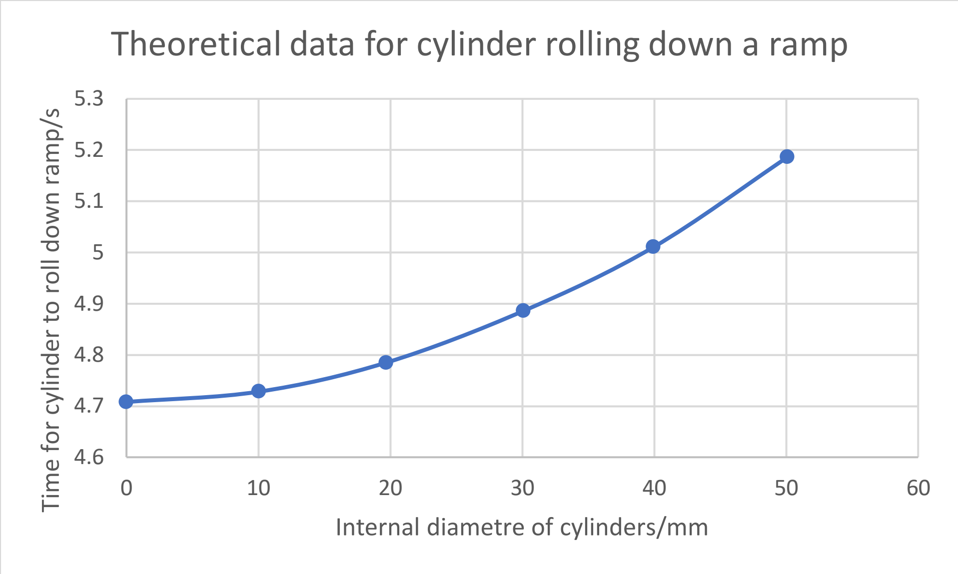


Figure 9 Graph of theoretical data made by candidate

Table 1 are graphed on figure 9. The data was obtained using equation 16. Table 1 show the different process conducted as the measurement where not done in SI unit. Through this table the work is divided in order to avoid mistake using excel. The first step is done to calculate the final velocity which is then divided by 2 in order to get the average velocity and finally the distance ($2.22m \pm 0,01$) is divided by the average velocity to get the final time for the cylinder to roll down the ramp.

Investigation/ Experiment

The purpose of this experiment will be to determine and verify the relationship of the time it takes to roll down an incline. This experiment is fundamental, however there are many factors in this experiment which will have to be controlled to make the result accurate and reliable

Experiment variables

Table 2 Independent and Dependent Variable

Variable	**Unit**	**Absolute uncertainty**	**How to measure**
Independent: Internal diameter of cylinder	Millimetres	± 0.05	Vernier scale
Depended: Time for the cylinder to roll down the incline	Seconds	± 0.01	Stopwatch

Table 3 Controlled variable

Controlled variable	**How might it affect dependent variable**	**Attempt to control**
Height of the incline	Every cylinder should start with the same amount of gravitational potential energy	Keep the same height throughout the experiment
Length of the incline	A longer ramp will result in the cylinder taking more time to roll down the ramp and thus affect the aim of the experiment which is to focus on only the internal diameter.	Start the cylinder at the same place every time by placing a mark on the ramp of where the centre of mass of the cylinder start
Direction of the cylinder	As the inclined used in the experiment has border for the cylinder not to fall, as the cylinder is released it systematically will collide with the border of the ramp and will slow down the cylinder	By trying to align the cylinder before releasing and being very careful of it being allied with the ramp before release. It will systematically collide but for some trial more than other and a trial will be ignored if cylinder was slowed to a great extent

Experimental set up

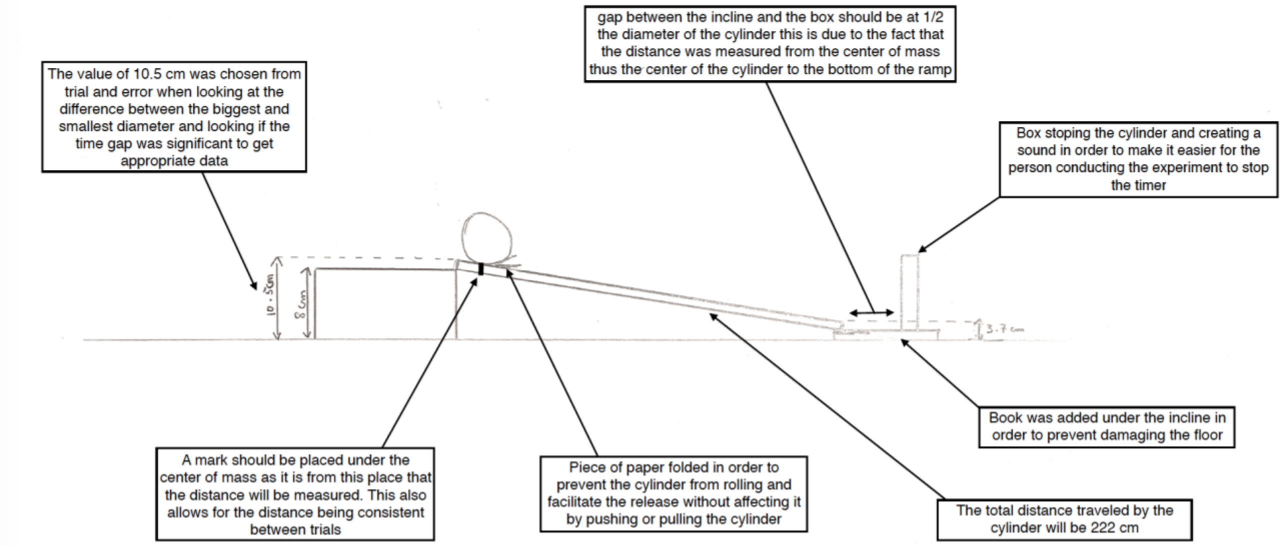


Figure 10 Sketch of experiment set up made by candidate

Figure 11 Picture of cylinder used in experiment made by candidate

Figure 12 Image of piece of paper blocking cylinders made by candidate

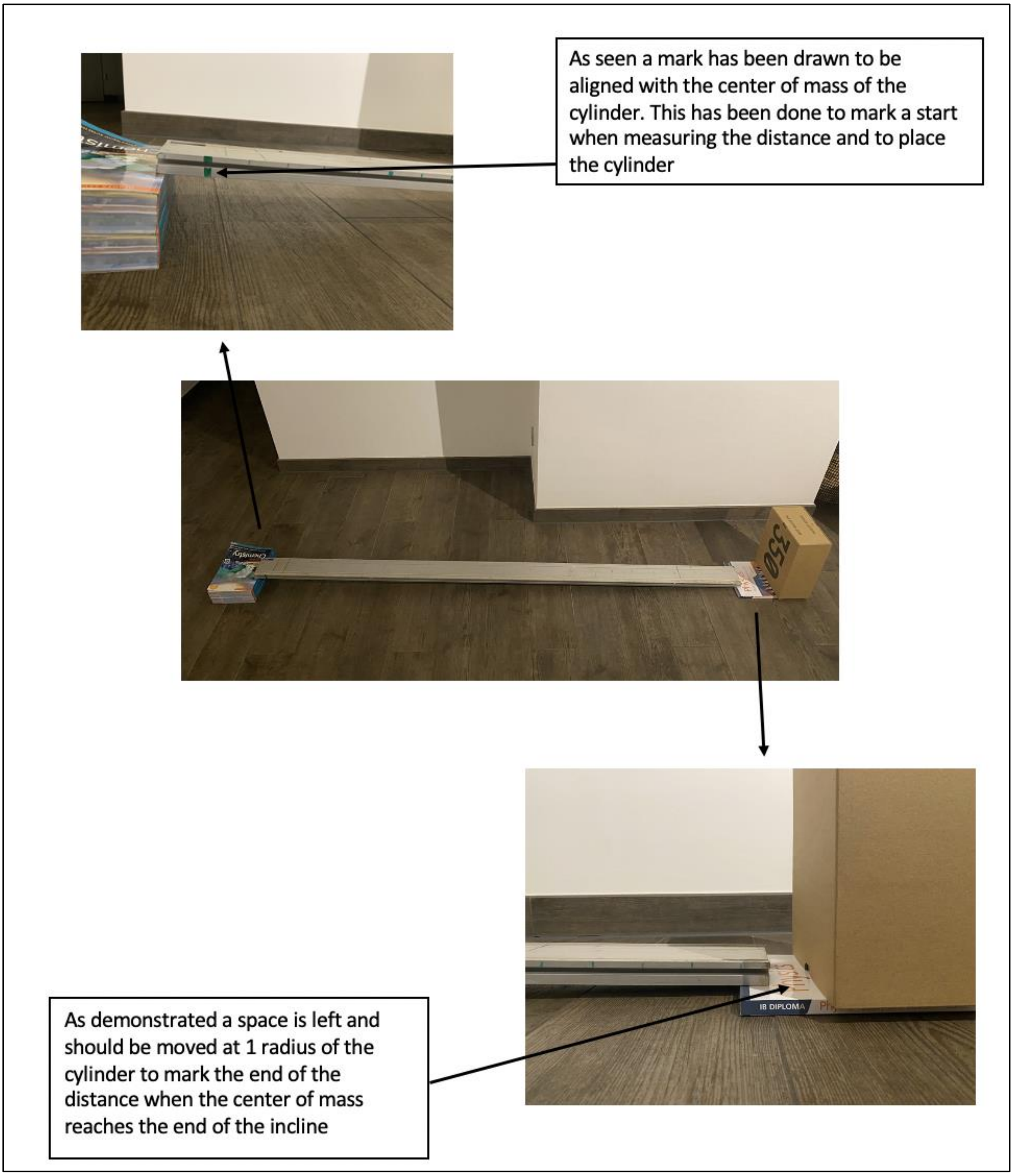


Figure 13 Annotated picture of experiment made by candidate

Rationale for chosen experiment

The final experimental set up was sketched and created from a previous set up. The first experiment did not have the piece of paper in figure 12 or the box at the end of the incline in figure 13. After collecting the results of the first experiment, it could be seen that the result where not accurate and did not match the theoretical data.

Table 4 Raw data of initial experiment (data extracted full table with every trial shown in appendix 1)

Internal diameter (mm) ± 0.05	Average time (s) of 10 trials for time taken for cylinder to roll down incline ± 0.01	Uncertainty of the time (s) taken for cylinder to roll down incline
0.00	4.30	0.07
10.05	4.60	0.15
19.70	4.77	0.25
30.10	4.95	0.13
39.95	5.13	0.12
50.10	5.43	0.36

As this experiment has shown to have a large number of uncertainties the initial experiment was modified. The final experiment was created from modification to the first experiment. First, as the cylinder where released on the incline without the use of the piece of paper in figure 12 the cylinder was pushed or pull as released by hand. This led the cylinder to start with a positive or negative velocity as a result of being pushed or pulled. Secondly, a box as added to the end of the incline. As the cylinder reaches the end of the ramp it is complicated to see with precision when to stop the timer. Therefore, a sound was thought to be a lot more distinct and result in a more precise recording of time. In addition, the incline had border which slowed down the cylinder as shown below on figure 14. This is very had to prevent and will be maintained in the final experiment although in order to lower the error some trial will be disregarder depending on the extent it has been slowed down.

As shown figure 14 the incline has small border which slowed down the cylinder.

Figure 14 Image of the incline taken by candidate

Collected data

Sample calculation for uncertainty:
Data extrapolated from raw data in appendix.

$$\frac{(Largest\ speed - Smallest\ speed)}{2} = Absolute\ uncertainty\ for\ time$$

For 10.05mm radius cylinder:

$$\frac{4.94 - 4.72}{2} = 0.11\ seconds$$

Table 5 Selected experimental result raw data in appendix 2

Internal radius (mm) ±0.05mm	Average time (*s*) for cylinder to roll down incline	Absolute Uncertainty for time (*s*)	Theoretical result for the time (*s*) of a cylinder to roll down incline	Absolute Uncertainty for time (*s*)
0.00	4.83	0.17	4.7079	0.00000
10.05	4.86	0.11	4.7281	0.00040
19.70	4.90	0.13	4.7849	0.00082
30.10	4.98	0.18	4.8859	0.00116
39.95	5.15	0.20	5.0105	0.00153
50.10	5.33	0.20	5.1862	0.00182

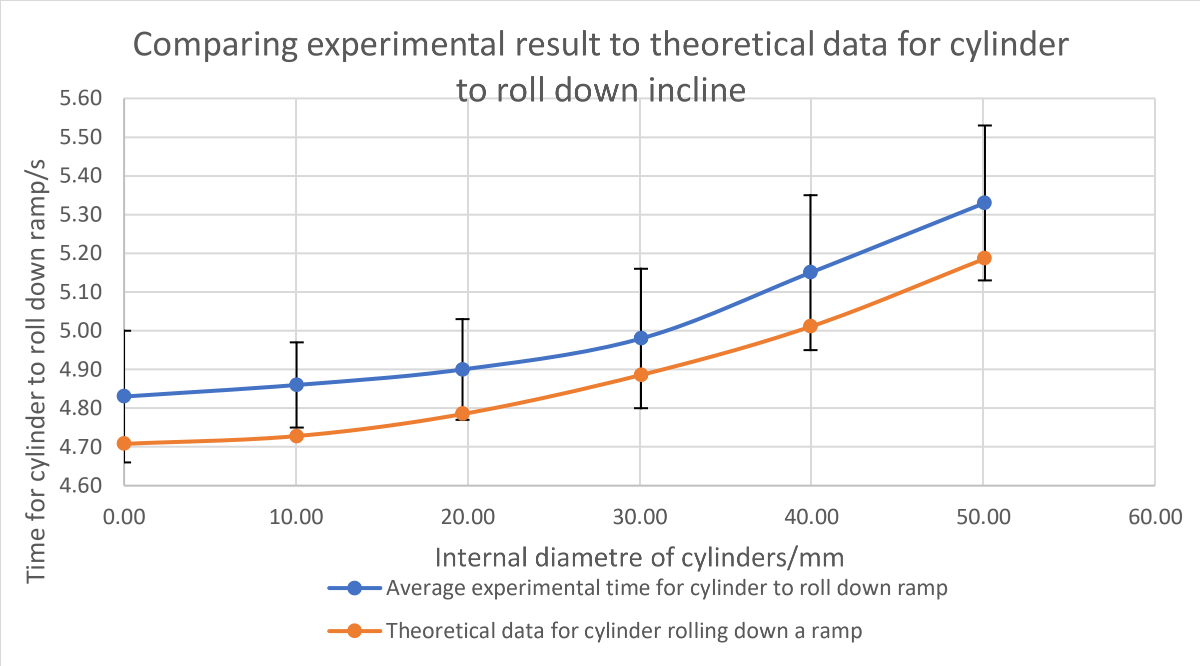


Figure 15 Graph made by candidate comparing values obtained experimentally and theoretically calculated data

The lines show in figure 15 compare the theoretical data and the data collected experimentally. Both curves have very similar shape showing the experiment's success. The lines are in an exponential form and underline the relationship between the internal diameter and the time a cylinder takes to roll down the ramp. In addition, the uncertainty error bars show the theoretical data matching the experimental data.

Conclusion and evaluation

Limitation to the procedure

As demonstrated in the graph it can be see that the line of the experimental data is shifted up compared to the line of the theoretical data. This shows a systematic error in the experiment. This error can be explained through the fact that resistance was applied in the cylinder as a form of friction. This friction was caused by two different reason. First as shown in figure 14 the cylinder systematically collided with the wall of the incline. An attempt to reduce this error was to disregard results depending on the extent the cylinder has been slowed down. Secondly a potential error which would've led to a shift to the right could be the air resistance which was the neglected in the theoretical calculations. In addition, the experiment was limited by human error as the timer was being stopped after the sound was heard and saw the cylinder colliding with the box.

The aim of this experiment was to find the relationship for the time taken for cylinder to roll down an incline dependent on different size internal radius. The results obtained were shown to be precise but not lacked accuracy. They are precise to see the change depending on the internal radius as the curve is similar to the theory in gradient, but it just shifted upwards. This shift caused by either air resistance or the fact that the cylinder scraped the ramp increased the time. However, as the shift was similar to a vertical translation and the slope was the part this essay is focusing on, this shift could be ignored resulting in the result being reliable to look at the change of time a cylinder takes to go down an incline depending on its internal diameter.

Conclusion

To answer the research question of "**How does the internal diameter of a hollow cylinder affect the time it takes to go down an incline?"** the following formula was derived for my experiment.

$$t = \frac{d}{\left(\frac{\pm\sqrt{\frac{gh}{\frac{1}{2}(1+\frac{1}{2r_2{}^2}(r_2{}^2+r_1{}^2))}}}{2}\right)}$$

Equation 15

This is the general formula with all the unknown and the formula below is shown specific for my experiment with the value for distance and the diameter of the cylinders which remains constant.

$$t = \frac{2.22}{\left(\frac{\pm\sqrt{\frac{9.81\times 0.068}{\frac{1}{2}(1+\frac{1}{2(0.0626)^2}(0.0626^2+r_1{}^2))}}}{2}\right)}$$

Equation 16

Throughout this experiment the purpose was to figure out what was the relationship between the hollowness of a cylinder and the time it takes to go down an incline. As this experiment has shown the mass does not have an effect on the time the cylinder will take but only the moment of inertia therefore the geometry for the shape. In view of a future experiment the apparatus might need to be improved as the cylinder collided with the wall of, the ramps and maybe consider the air resistance in order to subtract this from the values. The experiment has proven to support theoretical formula as all the theoretical result except 1 fall into the error bars of the experimental data. In addition, the error was systematic and shows a regular shift. The gradient showing the relationship is seen to be close to the theoretical data supporting the derived function for time shown in equation 15 and 16.

<u>What's next:</u>

In the outlook of a future experiment some suggestion could guide this experiment to a better result. A main issue was the friction of the cylinder which collided with the border of the slope and by finding a way to reduce this friction would make a considerable difference. Furthermore, it has been shown that measuring the time by hand with a stopwatch lead to a large amount of uncertainty. This could be replaced by using a phone to film the cylinder rolling and precisely seeing when the cylinder reached the end of the incline.

Work cited

Works Cited

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Appendices

Appendix 1 Table for first experiment

Internal radius (mm) ±0.05mm	1	2	3	4	5	6	7	8	9	10	average
0.00	4.27	4.27	4.29	4.33	4.23	4.28	4.35	4.36	4.31	4.30	4.30
10.05	4.67	4.77	4.58	4.75	4.58	4.56	4.54	4.45	4.47	4.58	4.60
19.70	4.51	5.00	4.73	4.70	4.91	4.82	4.71	4.70	4.85	4.78	4.77
30.10	5.07	4.93	4.91	4.96	4.94	5.04	4.83	5.02	4.94	4.81	4.95
39.95	5.05	4.99	5.23	5.19	5.01	5.25	5.14	5.18	5.13	5.10	5.13
50.10	5.98	5.34	5.44	5.37	5.42	5.30	5.38	5.36	5.41	5.29	5.43

Appendix 2 Raw data for experiment

Internal radius (mm) ±0.05mm	1	2	3	4	5	6	7	8	9	10	average
0.00	4.97	4.87	4.90	4.68	4.86	4.63	4.86	4.79	4.92	4.84	4.83
10.05	4.93	4.89	4.92	4.89	4.72	4.87	4.80	4.83	4.94	4.84	4.86
19.70	4.96	4.93	5.02	4.82	4.94	4.76	4.91	4.84	4.92	4.92	4.90
30.10	5.21	4.99	4.96	4.86	4.97	4.89	5.04	4.91	5.06	4.93	4.98
39.95	4.92	5.12	5.22	5.26	4.95	5.23	5.17	5.31	5.11	5.21	5.15
50.10	5.32	5.27	5.41	5.34	5.40	5.36	5.24	5.49	5.10	5.33	5.33