Controlling a beam of electron using magnetic and electric field

What is the relationship between the current in a Helmholtz coil and the voltage across plates for electron beam to travel undeflected?

Introduction:

Medicine and various other areas study the effect of magnetic and electric fields on hadrons. Research institute such as the CERN in Switzerland use the deflection of hadron for cancer treatment. Research on cancer treatment were published on the CERN website. This technology requires large knowledge on the effect of fields of hadrons as it is essential to have a precise control of the path of hadron.

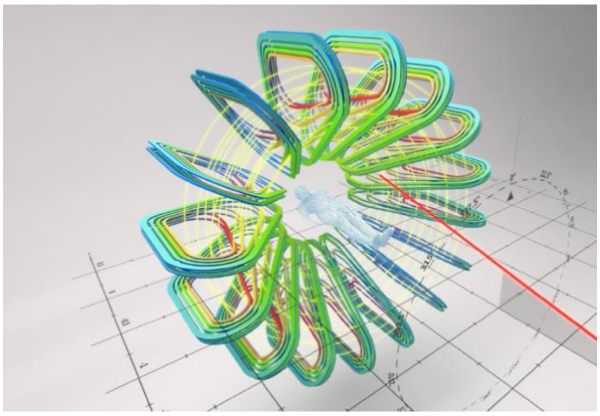


Figure 1(CERN)

Figure 1 shows how hadrons are shot (shown on figure 1 by red beam travelling towards field). They are then precisely deflected by the magnetic field in order to reach specific parts of the body and treat cancer. This essay will discuss the effects of magnetic and electric field on a beam of electron. This relates to cancer treatments as when hadrons are used to treat a body, they need to be precise and therefore the knowledge on the effect of various fields is crucial. This experiment chooses to focus on electron as they are easy to manipulate using an electron gun. Using an electron gun, it is possible to shoot electron in an electron beam tube and connected to plates and Helmholtz coils it is possible to bend the electron beam. The reason for using an electric field (using deflection plates) and a magnetic field (Helmholtz coil) can only bend the beam in one dimension.

Historical Background

Helmholtz coils were created by the germen physicist Hermann von Helmholtz *("Helmholtz Coil*"). The Helmholtz coil are basically two electromagnets on the same axis and the purpose of its creation was to isolate a field and cancel other external magnetic field like the Earth's.

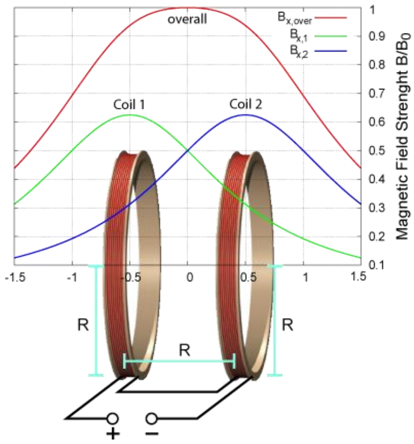


Figure 2 ("Magnetic Field")

The magnetic field in a Helmholtz coil can be represented as shown on figure 2. The combination of the two coils creates a strong uniform magnetic field in between the two coils as shown on figure 3.

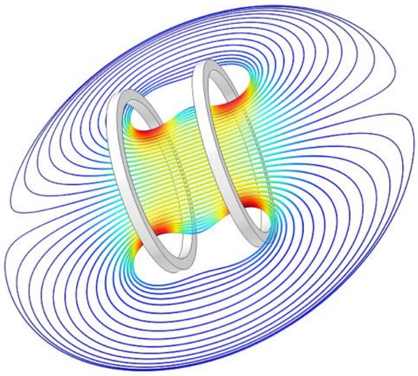


Figure 3 ("Beautiful Streamlines")

Research question

The main aim of this experiment will be to find the ratio at which the electron beam travels undeflected. This will demonstrate the understanding of beam by showing how we can control both fields on order to make them cancel out. This research paper will give an understanding of those fields by showing the ratio for a beam to travel undeflected. This papers research question is **"What is the relationship between the current in a Helmholtz coil and the voltage across plates for electron beam to travel undeflected?"**

Theory

The theory section of this essay will demonstrate the equation for the electric field and then for the magnetic field. Know two equation being known they will be equal to each other in order to find the different ration at which the forces are supposed to cancel each other and result in a undeflected electron beam,

Electric field

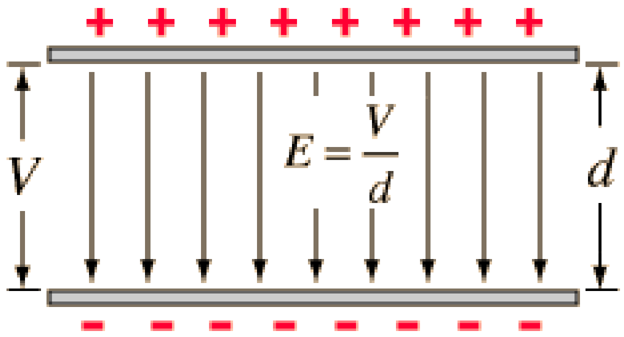


Figure 4 ("Electric Field")

During the experiment two plates will be disposed as show on figure 4. Between the plate an electric field affected the beam of electron in a parabolic shape. The strength of the field is be calculated using the equation:

$$E = \frac{V}{d}$$

Equation 1

Where E is the Electric field strength, V the voltage and d the distance between the two plates. When the beam of electron enters the electric field the negatively charged electron are attracted towards the positive plate. As the electron are short from the electron gun it will result in the election traveling in a parabolic shape as show below in figure 5.

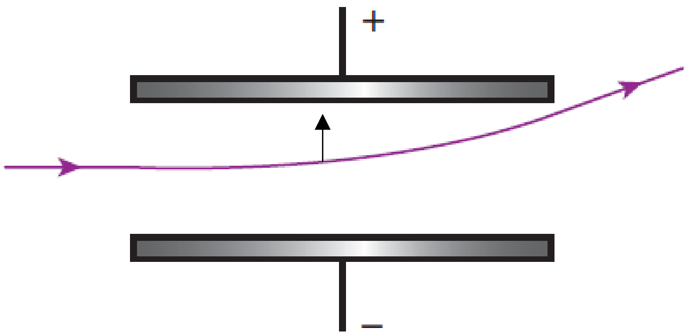


Figure 5 ("Electric Fields")

Figure 5 was modified, and an arrow was added to show in which direction the force of the field Is acting. The beam of electron has an initial velocity which gives its parabolic shape. This is also shown by the beam bending towards the positive end of the deflection plates. The force can therefore be calculated by multiplying the electric field strength by the charge of an electron (e). Giving the equation to find (…):

$$F_E = \frac{V}{d} \times e$$

Equation 2

Magnetic field

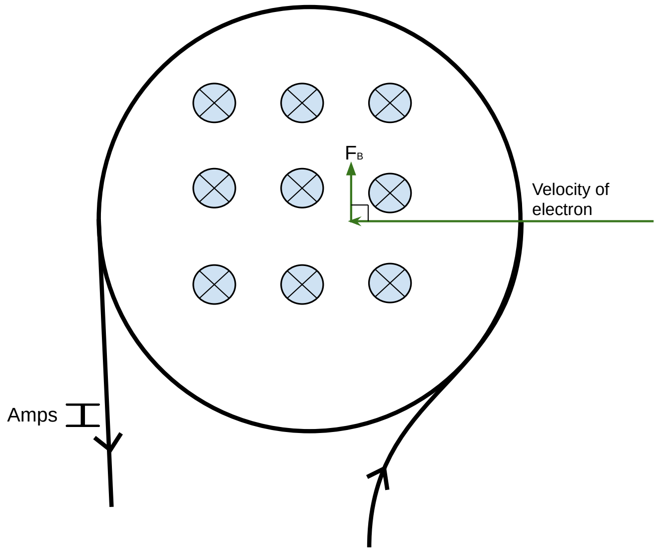


Figure 6 Image created by candidate

Figure 6 shows the magnetic field inside the Helmholtz coils. The diagram represents the coil being viewed from the side. The circles with cross represent the magnetic field going inside the page. Using Flemings Left hand rule we can point our first finger into the page and as we know the current is going from left to right it is possible to denote the motion of the electron beam to be up shown on figure 6 as F_B. The force of the magnetic field is always at right angle to the direction of the electron and therefore we can conclude that the electron beam will be affected in a circular motion instead of the parabolic of the electric field.

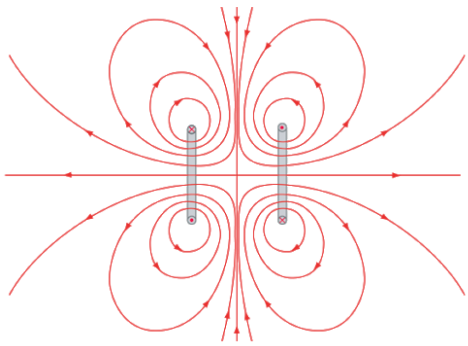


Figure 7 ("Quadrupole Magnetic")

Figure 7 show the magnetic field line from a different angle than figure 6 but as it has been mentioned the force will always act at right angle to the direction of the electron beam. The equation for the magnetic field strength is known as:

$$F_B = Bqv$$

Equation 3

Where B is the magnetic field (in tesla), q the charge of an electron and v the velocity of the electrons in the electron beam. To find the ratio for an undeflected path of the electron beam the two field must be equal to each other in order to cancel out. Therefore equation 2 and equation 3 ($F_B = F_E$). However, from equation 3 B and v are not known and will therefore need to be calculated.

Finding B

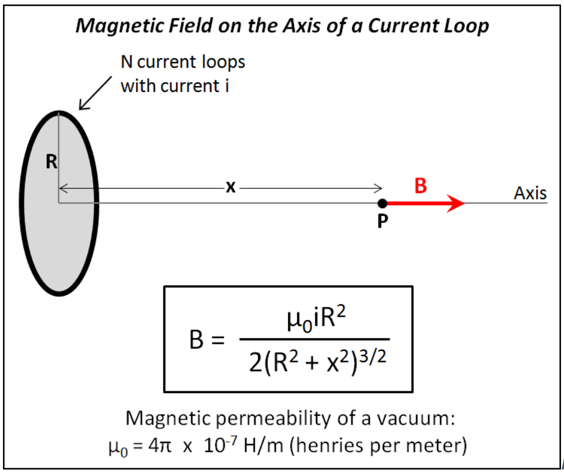


Figure 8 ("A Quantitative")

Using the equation in figure 8:

$$B = \frac{\mu_o I R^2}{2(R^2 + x^2)^{3/2}}$$

Equation 4

Using equation 4 it is possible to find the value of B in terms of R (radius of coil) and I as we know x = 0 and μ is a constant.

$$B = \frac{\mu_o I R^2}{2R^3} = \frac{\mu_o I}{2R}$$

Equation 5

Finding v

Because of the laws of conservation of energy, it is possible to say that in the electron gun:

$$\frac{1}{2}mv^2 = V_g e$$

Where V_g is the volts in the electron gun, e is the elementary charge m is the mass and v is the velocity.

$$v = \sqrt{\frac{2V_g \times e}{m}}$$

Because we know $e = 1.6 \times 10^{-19}$, $V_g = 5000$ and $m = 9.11 \times 10^{-31}$

$$v = \sqrt{\frac{2 \times 5000 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}}} = 4.2 \times 10^7 \ ms^{-1}$$

Combining equations

By substituting back into F_b (equation 2) and putting this equal to F_E (equation 3). Putting both equation equal to each other we are cancelling the forces allowing the beam of electron to travel undeflected.

$$F_b = F_E$$

$$Bqv = \frac{V}{d} \times e$$

Equation 6

By substituting the values found for B and v and the charge is simply the same as the elementary chare *e* we can say:

$$\frac{\mu_o I}{2R} \times e \times 4.2 \times 10^7 = \frac{V}{d} \times e$$

$$I = \frac{2R}{4\pi \times 10^{-7} \times 4.2 \times 10^7 \times d} \times V$$

Equation 7

As R and d are constant throughout the experience, they were measured to be $R = 12cm \pm 0.05$, d=$6.0cm \pm 0.5$ in addition both units will cancel out and therefore don't need to be converted in metres (SI unit). It is then possible to find the theoretical relationship to be:

$$I = \frac{2 \times 12}{4\pi \times 10^{-7} \times 4.2 \times 10^7 \times 6} \times V$$

$$I = \frac{24}{316.67} \times V$$

Equation 8

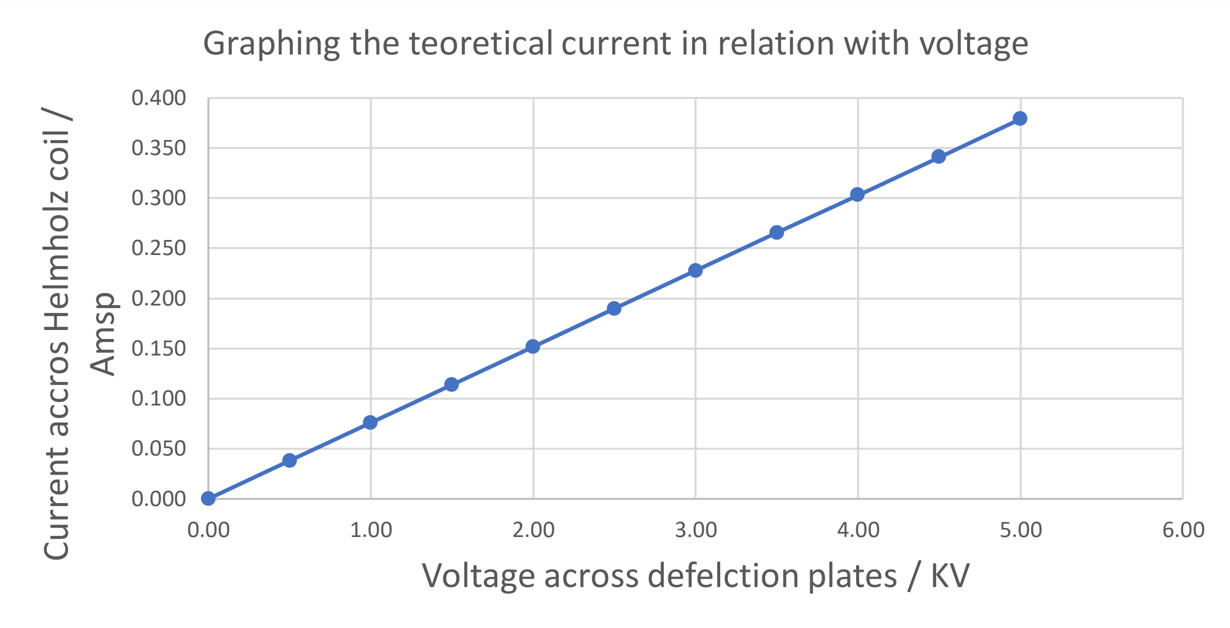
Hypothesis

Table 1 show the value obtained using equation 8

Voltage in KV	Current / Amps
0.00	0.000
0.50	0.038
1.00	0.076
1.50	0.114
2.00	0.152
3.00	0.227
3.50	0.265
4.00	0.303
4.50	0.341
5.00	0.379

No uncertainty was shown due as this data comes from the theory section.

Graph 1. *Theoretical data calculated from equation 8.*



From this graph the relationship is shown to be directly proportional passing though the origin.

Experiment

Variables

Independent:

- Voltage across the deflection plates (measured in Kilovolts KV)
- Range of value used is 0 to 5 KV (going up by 0.50 KV)

Dependent:

- Current across Helmholtz coils (measured in Amps)

Controlled:

1) Time between trials
   a) Reason: As the hight voltage power supply used a capacitor when putting the voltage back to 0 a certain amount of time was required do discharge the capacitor and for the voltage not to affect the beam of electron.
   b) Attempt to control: Some time was required in between trials for the electric field to go back to 0.
2) Voltage electron gun
   a) The voltage of the electron gun affects the speed at which the electrons are shot which was considered as constant to find the relationship in the theory.
   b) Attempt to control: The voltage was kept at 5 KV
3) Radius Helmholtz coil
   a) Radius is also considered a constant in equation ... and therefore, needs to remain constant in order not to affect the predicted relationship

Apparatus

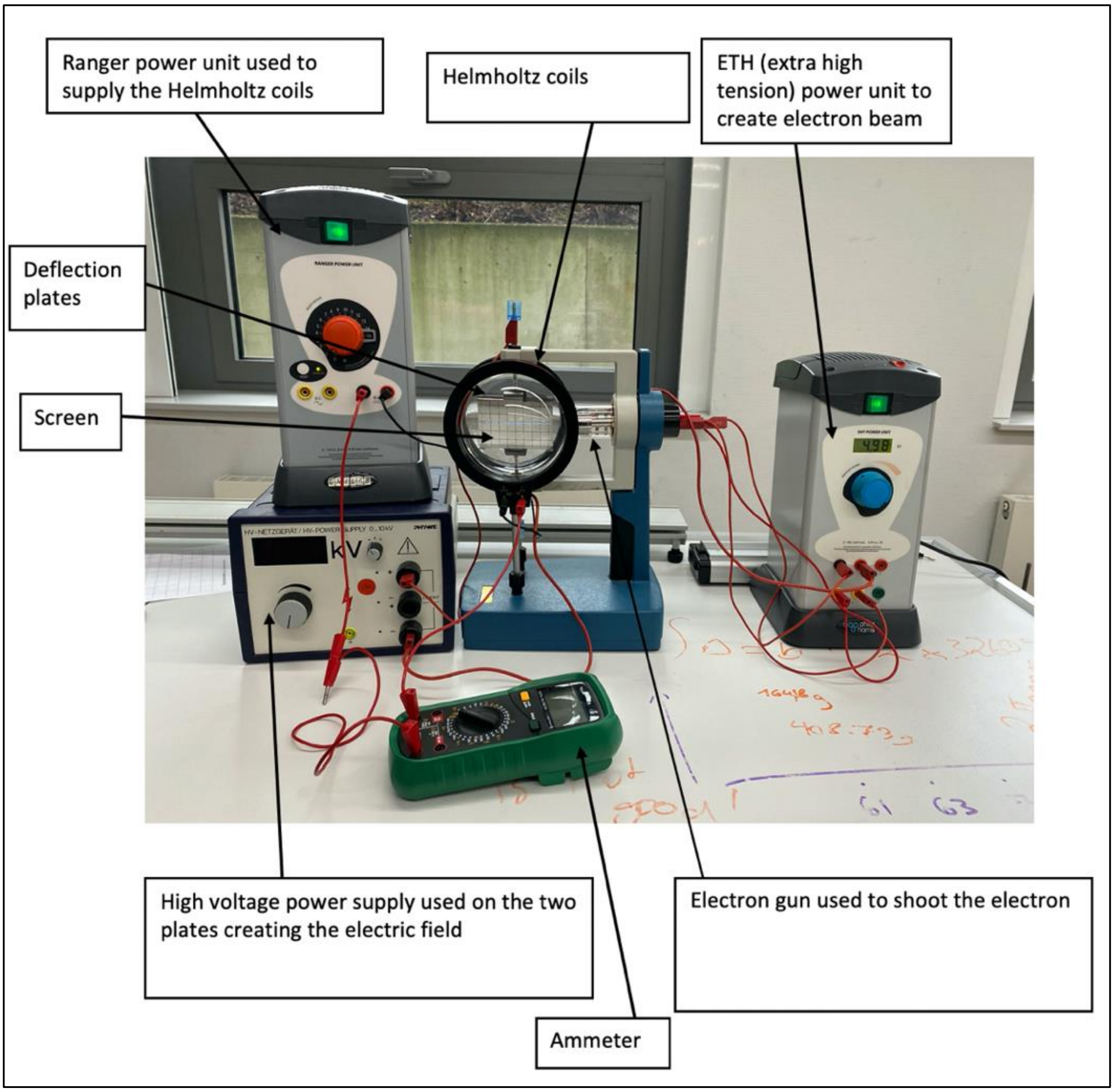


Figure 9 created by candidate

Equipment

- Electron gun with deflection plate
- Helmholtz coils
- Ammeter
- Ranger power unit
- Hight voltage power supply
- ETH extra high-tension power unit
- 11 cables

Methodology:

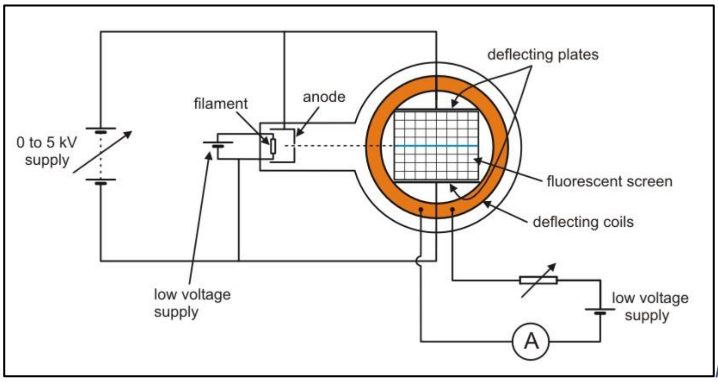
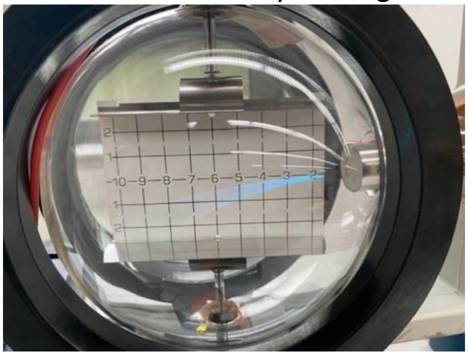
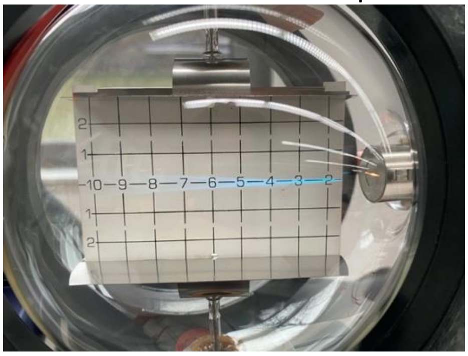
1. Set up the experiment as shown on figure 3. The circuit is also shown below in figure 10
2. 

Figure 10 ("Hoult")
3. Turn on the undeflected electron beam by turning on the ETH power unit to 5 KV.
4. The electron beam should be going thought the tube undeflected
5. Create a deflection by turning the Hight voltage power supply to 0.5 KV
6. 
Figure 11 Picture taken by candidate
7. Adjust the current of the Helmholtz coils coil until the electron beam is traveling undeflected as shown in figure 12. Because of the circular motion effect of the magnetic field and the parabolic motion of the electric field the line will not result in being exactly straight and therefore a range of value will be collected of what is considered an undeflected path.
8. 
Figure 12 picture taken by candidate
9. Repeat steps 6 to 8 increasing the voltage by 0.5 KV until a maximum value of 5 KV is reached
10. Repeat steps 3 to 9 three time to obtain 3 trials in order to generate an average

Risk assessment:

The experiment contained various very high voltage power unit and therefore required shrouded cables which were in good condition.

Raw Data

Table 1. *Data collected during experiment showing the 3 trials and the average of the ratio of voltage and amps resulting in an undeflected path.*

Voltage in KV ± 0.01	Current across Helmholtz coil Trial 1 / Amps ± 0.01	Current across Helmholtz coil Trial 2 / Amps ± 0.01	Current across Helmholtz coil Trial 3 / Amps ± 0.01	Average current across Helmholtz coil ± 0.01	Uncertainty
0.50	0.07	0.08	0.07	0.07	0.01
1.00	0.16	0.15	0.14	0.15	0.02
1.50	0.20	0.21	0.20	0.20	0.01
2.00	0.27	0.28	0.26	0.27	0.02
2.50	0.30	0.32	0.32	0.31	0.03
3.00	0.37	0.37	0.35	0.36	0.02
3.50	0.41	0.41	0.38	0.40	0.03
4.00	0.50	0.47	0.47	0.48	0.02
4.50	0.57	0.56	0.54	0.55	0.02
5.00	0.65	0.61	0.60	0.62	0.04

<u>Sample calculation for uncertainty</u>

This calculating was looking at the maximum and minimum values of all trial. This example is shown for 0.50 KV and values can be found in appendix:

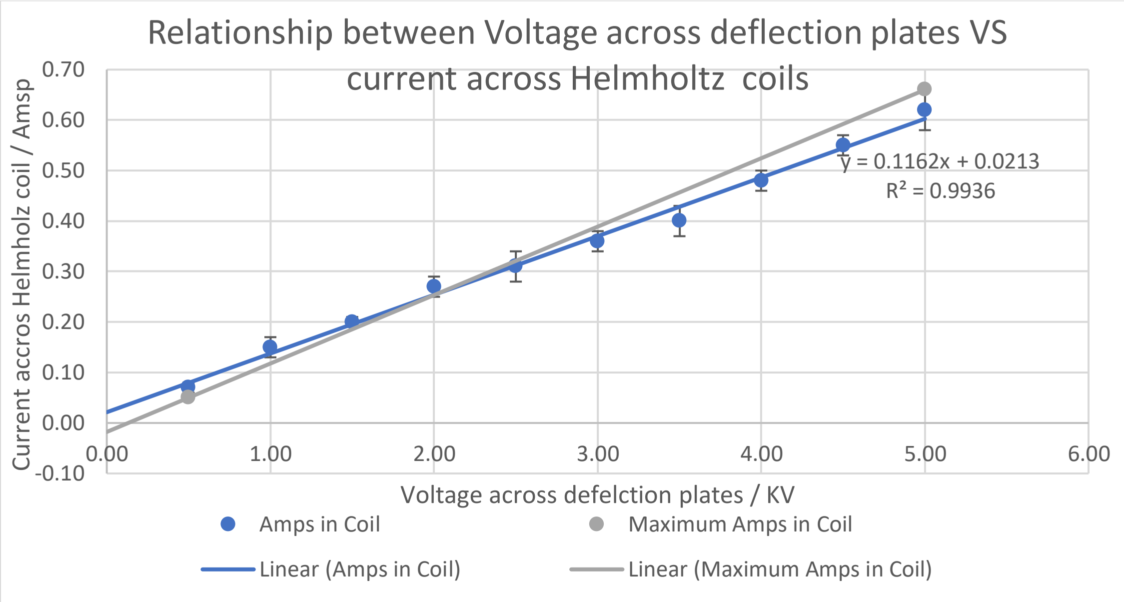
$$= \frac{\max value - \min value}{2}$$

$$\frac{0.8 - 0.6}{2} = 0.1$$

By using the data collected it is possible to see the ratio which result in a beam of electron being undeflected. By graphing these results, it will be possible to find a relationship of the ratios at which a beam of electron is traveling undeflected. The equation of the trendline will give the relationship. As Demonstrated in the hypothesis the result must be directly proportional. As show the uncertainties were not very large and therefore only 3 trials were conducted. Moreover, as a range of answer were taken this lowered uncertainty and lead to more precise results.

Data processing

Graph 2. *Graph of current vs Voltage*



Graph 2 shows that the current and the voltage have a linear fit. However, when the line if best fit was drawn it did not pass through the origin. In order to see if the relationship indeed supported the hypotheses stating a directly proportional relationship a line was potted going from the maximum uncertainty at voltage 5 KV to the minimum uncertainty at the point 0.5 KV. This line showed that by considering the impact of the uncertainty as shown when the line of the maximum amps went below the origin. This proves somewhere in the range of the uncertainty the relationship is directly proportional. This uncertainty could be caused by the error in the measuring tool or by the range of answer chosen for which a beam is considered undeflected.

Conclusion and evaluation

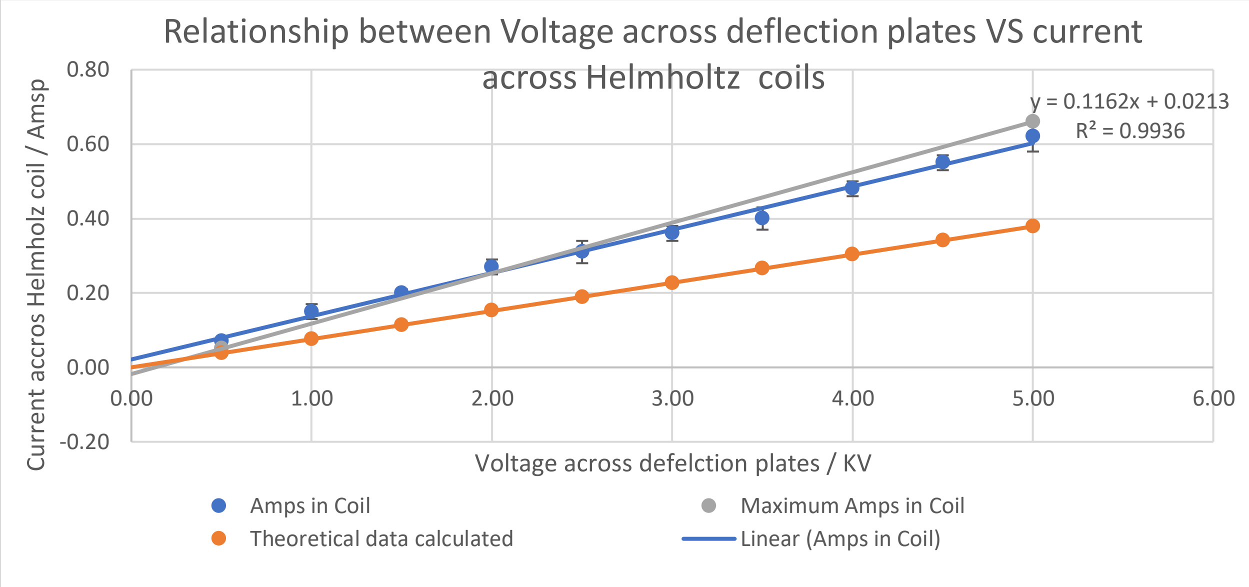
Conclusion:

After collecting the data, it has been clear that there was a relationship between the voltage and the current. As demonstrated in graph 1 the relationship was directly proportional, and this was later proven using an experiment and the data was graphically represented in graph 2. The graph did not directly show the relationship but by looking at the impact if the uncertainty it was possible to prove this relationship and therefore prove the hypothesis.

The relationship can be described by the line $y = 0.1162x + 0.0213$. The results showed precision and accuracy where the uncertainty was kept small as there was small difference between each trial. Coming back to the research question **"What is the relationship between the current in a Helmholtz coil and the voltage across plates for electron beam to travel undeflected?"**. Giving a final answer, a directly proportional relationship was found, however, this doesn't not answer the research question to the extent that the beam of electron was still affected by field and therefore did not travel perfectly undeflected. The fact that the parabolic shape of the electric field did not perfectly cancel out with the effect in circular motion of the magnetic field. This resulted in a beam of electron which not possible to maintain undeflected. In order to find a way to follow on with this investigation the research choose to consider a range of possible solution and average them to give an answer to the research question and find this directly proportional relationship.

Evaluation

Graph 2. *Graph of current vs Voltage*



When comparing the values obtained in the experiment and plot them on the same graph as the theory it is noticeable that there has been a systematic error which could explain why the gradient of the two lines are so different. One reason would be to mention that what was considered an undeflected path in the experiment was not most appropriate decision. This demonstrates a systematic human error which was made in deciding what was considered an undeflected path. An attempt to control this was to take a range of solution however this did conclude to the same gradient as found in theory.

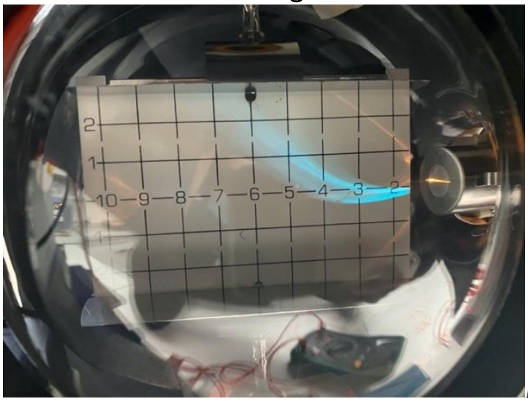


Figure 13 image taken by candidate

On figure 13 it is possible to see an attempt to straighten the path of the electron beam at 5 KV. Because the field do not act in the same way (parabolic and circular motion) it is not possible to get a perfectly straight line and therefore when looking at finding the ratio the experiment ended up looking at straightening only the four first centimetres and the rest was ignored.

<u>What's next:</u>

A further research could be conducted in order to find a more precise relationship. This experiment could use larger coils and using a higher current. By doing this it would be possible to obtain some larger undeflected electron path and therefore have a more accurate relationship. Therefore, obtaining more than the 3-4 cm of undeflected path as shown on figure 13.

Works Cited

"Beautiful Streamlines for Magnetic Field Lines of Helmholtz Coils." *Comsol*, edited by Mohammadamin Tajik, 29 Jan. 2017, www.comsol.com/forum/thread/134841/beautiful-streamlines-for-magnetic-field-lines-of-helmholtz-coils. Accessed 11 Feb. 2021.

CERN. 27 Nov. 2018, home.cern/news/news/knowledge-sharing/using-cern-magnet-technology-innovative-cancer-treatment. Accessed 9 Feb. 2021.

"Electric Field and Distance in Parallel-Plate Capacitor." *Physics Forum*, www.physicsforums.com/threads/electric-field-and-distance-in-parallel-plate-capacitor.760693/. Accessed 11 Feb. 2021.

"Electric Fields." *Revision Science*, revisionscience.com/a2-level-level-revision/physics-level-revision/fields/electric-fields. Accessed 24 Feb. 2021.

"Helmholtz Coil." *Wikipedia*, en.wikipedia.org/wiki/Helmholtz_coil. Accessed 24 Feb. 2021.

Hoult, David. "Atomic and Nuclear Physics." *The Open Door Web Site*, www.saburchill.com/physics/practicals/031.html.

"Magnetic Field of Two Helmholtz Coils." *Virtuelle Experimente*, virtuelle-experimente.de/en/b-feld/b-feld/helmholtzspulenpaar.php. Accessed 11 Feb. 2021.

"Quadrupole Magnetic Field." *Hyperphysics*, edited by Carl Rod Nave, hyperphysics.phy-astr.gsu.edu/hbase/magnetic/magquad.html. Accessed 11 Feb. 2021.

"A Quantitative Study of Helmholtz Coils." *Pocket Lab*, 15 June 2017, www.thepocketlab.com/educators/lesson/quantitative-study-helmholtz-coils. Accessed 6 Mar. 2021.

Appendix:

Trial 1			
kv	min	max	Average
0.5	0.06	0.07	0.065
1	0.15	0.17	0.16
1.5	0.19	0.21	0.2
2	0.25	0.28	0.265
2.5	0.28	0.31	0.295
3	0.36	0.37	0.365
3.5	0.39	0.43	0.41
4	0.49	0.5	0.495
4.5	0.56	0.57	0.565
5	0.63	0.66	0.645

Trial 2			
Kv across	min	max	
0.5	0.07	0.08	0.075
1	0.13	0.16	0.145
1.5	0.2	0.21	0.205
2	0.26	0.28	0.28
2.5	0.31	0.33	0.32
3	0.36	0.38	0.37
3.5	0.4	0.42	0.41
4	0.46	0.48	0.47
4.5	0.55	0.56	0.555
5	0.6	0.62	0.61

Trial 3			
kv	min	max	
0.5	0.06	0.08	0.07
1	0.13	0.14	0.14
1.5	0.19	0.21	0.2
2	0.25	0.27	0.26
2.5	0.31	0.32	0.315
3	0.34	0.35	0.345
3.5	0.37	0.39	0.38
4	0.46	0.48	0.47
4.5	0.53	0.55	0.54
5	0.59	0.61	0.6