[STAT-24] Moderation effects

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Product terms

In general, when the effect of X_1 on Y depends on the value of X_2, we say that there is an interaction effect of X_1 and X_2 on Y. An interaction effect is included in a regression equation through a product term, as in

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_{12} X_1 X_2 + \epsilon.$$

We identify the interaction effect with the coefficient β_{12}. In mathematical terms, the interpretation of the coefficient is easy, taking X_2 as a moderator of the effect of X_1: holding constant X_2, the effect of X_1 on Y is $\beta_1 + \beta_{12} X_2$. Nevertheless, in practice, one has to be careful with product terms. Let me point out some practical issues:

- There is no way to separately interpret the three terms in which X_1 and X_2 are involved. If your research question involves an interaction effect of X_1 and X_2, you should better leave the three terms in the equation, either significant or not.
- The best way to understand an interaction effect is to assign to one of the two factors the role of a moderator of the effect of the other, as suggested above.
- Product terms can be affected by a multicollinearity problem, showing less significance than expected. This is usually fixed by centering the moderator (subtracting the mean).

Interpretation of a moderation effect

The simplest case is that of a binary moderator. Suppose an equation

$$Y = \alpha + \beta X + \gamma Z + \delta X Z + \epsilon,$$

in which Z is a dummy variable which codes two groups (e.g. male/female). To interpret the coefficient of the product term, we look at the two groups separately. In the group $Z = 0$, the model becomes

$$Y = \alpha + \beta X + \epsilon,$$

while in the group $Z = 1$ we have

$$Y = \big(\alpha + \gamma\big) + \big(\beta + \delta\big) X + \epsilon.$$

So, the effects of X in the two groups are β and $\beta + \delta$, respectively. Therefore, δ accounts for the change in the effect of X across groups.

In the case of a continuous moderator, the simplest way to address the moderation issue is to reduce the discussion to a three level scenario. For instance, you can standardize Z and consider the cases $Z = -1$, $Z = 0$ and $Z = 1$. Alternatively, you can pick the Z values so that they allow for an interesting discussion.

Standardized coefficients

Linear transformations can be applied to the variables involved in a regression equation. This does not affect the R-squared and t statistics, but could make the interpretation of the coefficients more appealing. First, it is easy to see that, if we replace the dependent variable Y by $Y^* = (Y - a)/b$, we have to divide all the coefficients on the right side of the equation by b and subtract a/b from the constant term. Second, if we replace an independent variable X by $X^* = (X - a)/b$, this only affects the coefficient of X and the constant term.

Centering all the variables, that is, subtracting the means, is sometimes applied, to get an equation without constant term. Standardizing all the variables ($Z = (X - \bar{x})/s$), we go a bit further. The coefficients of the resulting equation are called **standardized coefficients**, or beta coefficients.

In the regression line, the standardized slope coefficient is the same as the correlation. This is no longer true in multiple regression, although it may seem so. Indeed, the absolute value of a standardized coefficient can exceed 1. But most people look at standardized coefficients as if they were correlations. The correct interpretation of a standardized coefficient β would be: increasing X one standard deviation while holding the other dependent variables constant, the expected value of Y is increased by β standard deviations.

Standardization is usually recommended when each variable in the equation is a perception measure derived from a **Likert scale**, that is, from a set of questions with a small number of response options (typically 1–5 or 1–7). Standardized coefficients can be compared and, with a bit of practice, analysts get used to interpret them at first sight. For instance, a trained analyst may tell you that 0.1 is a weak effect, but 0.75 is a strong one, irrespective of the specific perceptions involved in the model.

But, in many cases, you do not standardize all the variables. Typical examples are:

- Some typical control variables, such as age, are rarely standardized.
- Dummies are never standardized.
- If you include a product term in the equation to account for an interaction effect of two perception measures, you will probably standardize the factors but not the product.

Example 1. The `supermarket` data set was obtained in a large chain of retail supermarkets offering two alternative check-out systems, a self-service option and a traditional, employee-assisted, checkout process. The managers were interested in the customer's perception of service quality and in the extent to which the difference between the two check-out systems made a real difference in the customer's perception. They also wished to examine another point about which there was no consensus: whether the connection between the quality perception by the customers and their fidelity, that quality specialists take for granted, is real.

The sample ($n = 210$) consisted of two groups: one group of customers using the self-service option and another group using the employee-assisted system. The survey was based on short interviews conducted at the store. The questionnaire contained two items related to the overall customer's satisfaction and intention to reuse, and three items for each of the three potential drivers of these outcomes, process performance (speed and effectiveness of the scanning and payment operations), process convenience (service layout and waiting time), employee performance (interaction with employees).

The variables are:

- `type`, type of service, type = 1 for self-service.
- `perfo`, process performance, 1-7 scale (average).
- `conv`, process convenience, 1-7 scale (average).
- `employee`, employee's performance, 1-7 scale (average).
- `sat`, overall satisfaction, 1-7 scale.

- `reuse`, intention to reuse, 1-7 scale.

I fit a linear regression equation to these data, with the intention to reuse as the dependent variable. Except for the dummy, which I leave for the end, the coefficients of the other four regressors can be compared, because the scale is common. The results give a certain support for the role of customer satisfaction as a mediator between the process attributes and the intention to reuse, but we see that, given the satisfaction level, the process performance still contributes. The other two attributes do not contribute much, in comparative terms. The coefficient of the dummy can be interpreted as a mean difference between the two groups (given the other regressors), and is not relevant in a 1–7 scale.

TABLE 1. Linear regression results (Example 1)

Coefficient	Estimate	Std. error	t value	p-value
Intercept	4.34	0.33	13.2	0.000
type	–0.074	0.078	-0.95	0.341
perfo	0.220	0.060	3.66	0.000
conv	–0.013	0.037	–0.36	0.721
employee	0.002	0.054	0.04	0.964
sat	0.169	0.058	2.93	0.004

Since the units of the metric variables used in the analysis are artificial, most analysts would standardize them. I denote with a prefix `z` the standardized version. The regression results obtained after standardization are shown in Table 2. Note that the t statistics and the p-values of the coefficients do not change. The interpretation, for instance of the coefficient of `perfo`, would be: increasing `perfo` one standard deviation while holding the other independent variables constant, `reuse` increases by 0.292 standard deviations (on average). The interpretation of the coefficient of `type` is a bit different: switching from employee-assisted to self-service checkout while holding the other regressors constant, `reuse` decreases by 0.074 standard deviations.

TABLE 2. Linear regression results (standardized variables)

Coefficient	Estimate	Std. error	t value	p-value
Intercept	0.066	0.093	0.71	0.479
type	–0.132	0.138	–0.95	0.341
z.perfo	0.292	0.080	3.66	0.000
z.conv	–0.028	0.078	–0.36	0.721
z.employee	0.004	0.081	0.04	0.964
z.sat	0.246	0.084	2.93	0.004

But, do these four drivers of the intention to reuse have the same effect in both groups? To address this question, I introduce a product term for each one, using the prefix `t` for the product of `type` with any of the other variables. In Table 3, the first line associated to the performance (the coefficient of `perfo`) is related to the effect of this variable in the group type $= 0$, that is, with the employee-assisted checkout, while the second term (the coefficient of `tperfo`) is related to the difference between the effects in the two groups. This shows that the performance matters only in the self-service group. For `conv`, we find effects similar in absolute value, but with opposite sign, so we do not have evidence of this effect. For `employee`, nothing is gained with the addition of the product term. For `sat`, I do not find difference between the two groups. Therefore, three of the product terms may be dropped.

TABLE 3. Linear regression results (with moderation effects)

Coefficient	Estimate	Std. error	t value	p-value
Intercept	–0.029	0.098	–0.30	0.766
type	–0.112	0.137	–0.81	0.417
z.perfo	0.074	.111	0.66	0.508
tperfo	0.414	0.161	2.57	0.011
z.conv	0.159	0.129	1.23	0.219
tconv	–0.301	0.161	–1.87	0.063
z.employee	–0.058	0.115	–0.51	0.613
temployee	0.061	0.165	0.37	0.712
z.sat	0.253	0.124	2.03	0.044
tsat	0.021	0.168	0.13	0.899

So, in my final equation, I get a strong effect of the perfomance in the self-service group. The effect is weaker in the employee-assisted group.

TABLE 4. Final results

Coefficient	Estimate	Std. error	t value	p-value
Intercept	.031	0.093	0.33	0.739
type	–0.138	.137	–1.01	0.314
z.perfo	0.133	0.101	1.31	0.190
tperfo	0.321	0.128	2.50	0.013
z.conv	–0.036	0.077	–0.46	0.647
z.employee	–0.019	0.081	–0.23	0.817
z.sat	0.260	0.083	3.14	0.002

¶ Source: MP Castro-Amorim, PhD Dissertation.

Homework

A. The `wage1` data set used in the example of lecture STAT-21 contains also two dummies, for being female and being married, respectively. Test the effect of education on wages taking into account that this effect may be moderated by gender and/or marital status.