[STAT-14] The one-sample t test

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An example

The logic of hypothesis testing is not obvious for the beginner. But practice teaches us how efficient is a setting that allows us to use the same argument in many different situations. So, I skip the theoretical discussion and, instead of a formal definition, I start with an example.

Example 1. The `lwages` data set includes data on wages in years 1980 and 1987. The sample size is $n = 545$. The variables are: (a) `nr`, an identifier, (b) `lwage0`, wages in 1980, in thousands of US dollars and (c) `lwage7`, the same for 1987. The wages come in log scale, which helps to correct the skewness. Do these data support that there has been a change in the wages? Note that we don't care about individual changes, but about the average change.

To examine this question, I introduce a new variable X, corresponding to the difference between these two years (1987 minus 1980). I search for evidence that the mean of X is different of zero. My first analysis is based on a confidence interval. The basic information is

$$\bar{x} = 0.473, \qquad s = 0.606, \qquad n = 545.$$

Given the sample size, we shouldn't worry about the skewness, but, anyway, the histograms of Figure 1 support the use of the log scale. With the appropriate t factor ($t_{0.025}(544) = 1.964$), I get the 95% confidence limits 0.422 and 0.524. Since this interval does not contain zero, it can be concluded (95% confidence) that there has been a change. it is said then that the mean difference 0.473 is significant or, more specifically, that it is significantly different from zero.

¶ Source: F Vella & M Verbeek (1998), Whose wages do Unions raise? A dynamic model of unionism and wage rate determination for young men, *Journal of Applied Econometrics* **13**, 163-183. □

The one-sample t test

Let me now tell you the story in a different way. I denote by μ the population mean of the wage increase (in log scale). Then, the conclusion of the above argument can be stated saying that I tested the **null hypothesis** $H_0 : \mu = 0$. Applied to the actual data, the test rejected H_0. So, I concluded that $\mu \neq 0$, with 95% confidence.

An alternative way to perform the test is based on the **test statistic**

$$T = \frac{\bar{X}}{S/\sqrt{n}}\,,$$

which, under the null (i.e. assuming that H_0 is valid) and an implicit normality assumption, follows a $t(n-1)$ distribution. We call this a t **test**. The absolute value of the statistic is compared with the critical value $t_{0.025}$, which corresponds to a 95% interval. If the critical value is exceeded, H_0 is rejected, with 95% confidence. We say then that the t value is significant.

Although the 95% level is a standard, we can change it by replacing $t_{0.025}$ by the t_α critical value corresponding to the $1 - 2\alpha$ confidence level. Mind that the use of other levels than the usual 95%

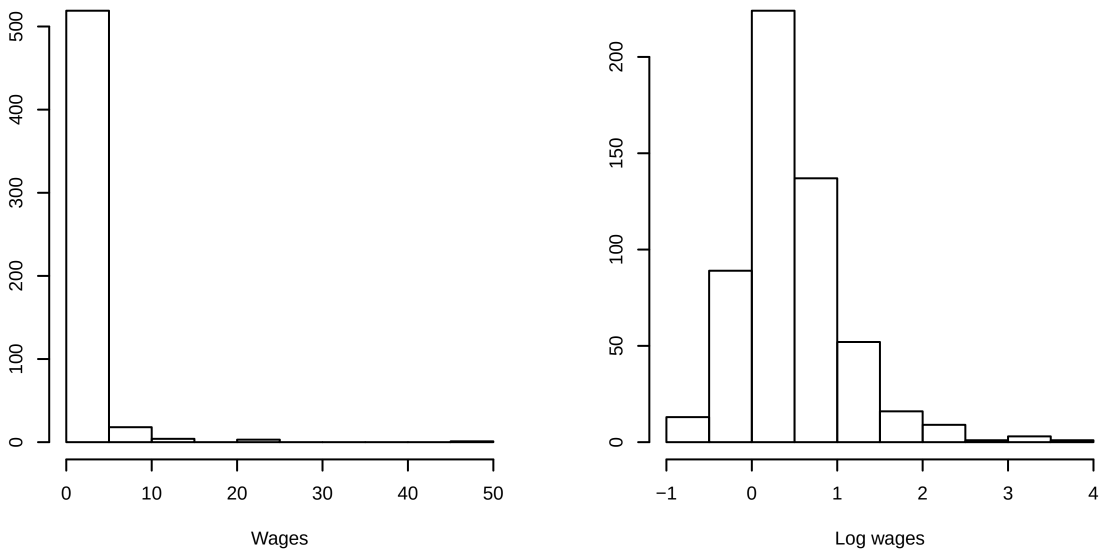

Figure 1. Distribution of wage differences (Example 1)

has to be justified. Here, the value

$$t = \frac{0.473}{0.606/\sqrt{545}} = 18.22$$

exceeds the critical value 1.964. Again, we reject H_0 and conclude that there is a change in mean wages.

The result of this test is usually presented in terms of a p-**value**. This is the 2-tail probability P associated, under the null, to the actual value of the t statistic. It is taken as a measure of the extent to which the actual results are significant (the lower the p-value, the higher the significance). With a 95% confidence level, we consider that there is significance when $P < 0.05$. In the example,

$$P = \mathrm{p}\big[|T| > 18.22\big] = 1.54 \times 10^{-58}.$$

By replacing $\bar{x}$ by $\bar{x} - \mu_0$, this test can be applied to a null $H_0 : \mu = \mu_0$, in which μ_0 is a prespecified value.

Without normality, the same methods are approximately valid for big samples. It is generally agreed that $n \geq 50$ is big enough for that. Of course, for big samples, the difference between the critical value t_α and $z_\alpha = 1.960$ becomes irrelevant.

Homework

A. Reanalyze the data of Example 1 after the wages putting back in the original scale (no logs). How does the interpretation of the mean difference change?