Causal Inference: Problem Set 11

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Based on Chapter 11 problems of Wooldridge.

1. Let $\{x_t\}_{t=1}^{\infty}$ be a covariance stationary process and define $\gamma_h = Cov(x_t, x_{t+h})$ for $h \geq 0$. Show that

$$Corr(x_t, x_{t+h}) = \gamma_h/\gamma_0.$$

2. Let $\{e_t\}_{t=-2}^{\infty}$ be a sequence of i.i.d. random variables with zero mean and variance of one. Define a stochastic process by (btw, this is an MA(2) process)

$$x_t = e_t - \frac{1}{2}e_{t-1} + \frac{1}{2}e_{t-2}, \quad t = 1, 2, \ldots.$$

   (i) Find $\mathbb{E}[x_t]$ and $Var(x_t)$. Do either of these depend on t?

   (ii) Show that $Corr(x_t, x_{t+1}) = -1/2$ and $Corr(x_t, x_{t+2}) = 1/3$.

   (iii) What is $Corr(x_t, x_{t+h})$ for $h > 2$?

   (iv) Is x_t weakly dependent? Is it covariance stationary?

3. Let $\{y_t\}_{t=1}^{\infty}$ be a random walk, i.e. $y_t = y_{t-1} + e_t$ where $y_0 = 0$ and $\{e_t\}_{t=1}^{\infty}$ is i.i.d. with mean zero and variance σ_e^2. Show that

$$Corr(y_t, y_{t+h}) = \sqrt{t/t+h} \quad \text{for} \quad t \geq 1, h > 0.$$

4. (i) Following the efficient market hypothesis example on stock returns in Slides/Wooldridge, let's check the possibility that there may be a quadratic functional form. Use the data in NYSE to estimate

$$return_t = \beta_0 + \beta_1 return_{t-1} + \beta_2 return_{t-1}^2 + u_t.$$

   Report the results in standard form.

   (ii) State and test the null hypothesis that $\mathbb{E}[return_t | return_{t-1}]$ does not depend on $return_{t-1}$. (Hint: There are two restrictions to test here.) What do you conclude?

   (iii) Drop $return_{t-1}^2$ from the model, but add interaction term $return_{t-1}return_{t-2}$. Now test the efficient markets hypothesis.

(iv) What do you conclude about predicting weekly stock returns based on past stock returns.

5. Use the data in PHILLIPS for this exercise.

   (i) Estimate an AR(1) model for the unemployment rate using only the data through the year 2003. Use this equation to predict the unemployment rate for 2004. Compare this with the actual unemployment rate for 2004 which is 5.5.

   (ii) Add a lag of inflation to the AR(1) model from part (i), again being sure to use only the data through 2003. Is the inf_{t-1} variable statistically significant?

   (iii) Use the equation from part (ii) to predict the unemployment rate for 2004. Is the result better or worse than in the model from part (i)?

6. Use the data in APPROVAL to answer the following questions. See also the related question from Problem Set 10.

   (i) Compute the first order autocorrelations for the variables *approve* and *lrgasprice*. Do they seem close enough to unity to worry about unit roots?

   (ii) Consider the model

   $$approve_t = \beta_0 + \beta_1 lcpifood + \beta_2 lrgasprice_t + \beta_3 unemploy_t + \beta_4 Sep11 + \beta_5 iraqinvade + u_t,$$

   where the first two variables are in logarithmic form. Given what you found in part (i), why might you hesitate to estimate this model by OLS?

   (iii) Estimate the equation in part (ii) by differencing all variables (including the dummy variables). How do you interpret your estimate of β_2? Is it statistically significant?

   (iv) Interpret your estimate of β_4 and discuss its statistical significance.

   (v) Add $lsp500$ to the model in part (ii) and estimate the equation by first differencing. Discuss what you find for the stock market variable.