f-20-jupyter-basisskift

April 15, 2021

1 En befolkningsmodel

500000 menesker i storbyen

700000 menesker i landområder

Hvert år flyttes:

3% af landbefolkning til byen

2% af bybefolkning på landet

Lad x koordinat angive bybefolkningen, og y koordinat giver landbefolkningen. Uviklingen fra et år til den næste er givet ved en lineær transformation $L\colon \mathbb{R}^2 \to \mathbb{R}^2$

$L(x, y) = (0.98x + 0.03y, 0.02x + 0.97y)$

[1]:
```
import numpy as np
```

Start med fordeling $b_0 = (x_0, y_0) = (500000, 700000)$

[2]:
```
b0 = np.array([500000, 700000])[:, np.newaxis]
```

[3]:
```
a = np.array([[0.98, 0.03],
              [0.02, 0.97]])
```

[4]:
```
# efter 1 år
b1 = a @ b0
b1
```

[4]:
```
array([[511000.],
       [689000.]])
```

[5]:
```
# De første 99 år
n = 100
b = np.empty((2,n))
b[:, [0]] = b0
for i in range(1,n):
    b[:, [i]] = a @ b[:, [i-1]]
```

[6]:
```
b[:, 10]
```

[6]:
```
array([588277.87336756, 611722.12663244])
```

[7]:
```
b[:, 20]
```

[7]:
```
array([641133.09707012, 558866.90292988])
```

[8]:
```
b[:, 30]
```

[8]:
```
array([672779.47193255, 527220.52806745])
```

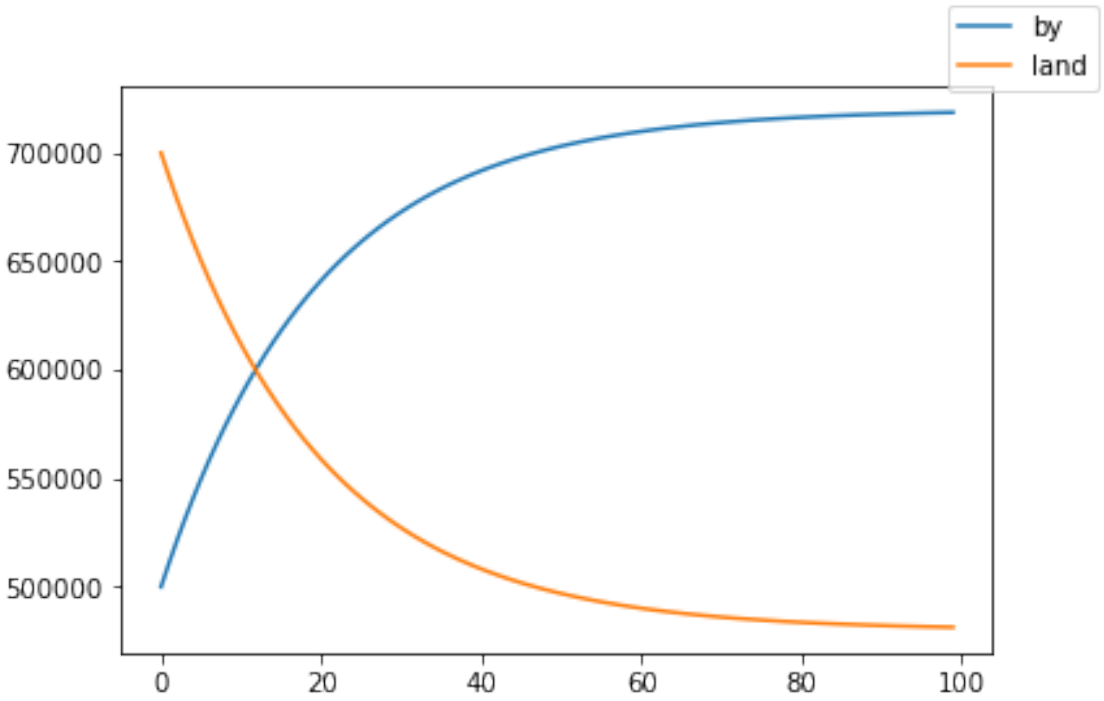
[9]:
```
b[:, 99]
```

[9]:
```
array([718628.93007529, 481371.06992471])
```

[10]:
```
import matplotlib.pyplot as plt
```

[11]:
```
fig, ax = plt.subplots()
ax.plot(b[0,:], label='by')
ax.plot(b[1,:], label='land')
fig.legend()
```

[11]:
```
<matplotlib.legend.Legend at 0x10f08c5b0>
```



Nye basis F: $(3,2),(1,-1)$

Basisskiftsmatricen fra F til standardbasis af $\mathbb{R}^2$ er

[12]:
```
t = np.array([[3.0,  1.0],
              [2.0, -1.0]])
```

[13]:
```
# ny matrix af L ift. F
c = np.linalg.inv(t) @ a @ t
c
```

[13]:
```
array([[ 1.00000000e+00, -2.77555756e-17],
       [ 0.00000000e+00,  9.50000000e-01]])
```

[14]:
```
c = (1/(3*(-1)-1*2.)) * np.array([[-1.0, -1.0], [-2.0, 3.0]]) @ a @ t
c
```

[14]:
```
array([[1.  , 0.  ],
       [0.  , 0.95]])
```

$$C = T^{-1}AT$$

$$A = TCT^{-1}$$

$$A^2 = AA = TCT^{-1}TCT^{-1} = TCCT^{-1} = TC^2T^{-1}$$

$$A^k = TC^kT^{-1}$$

[15]:
```
c @ c
```

[15]:
```
array([[1.    , 0.    ],
       [0.    , 0.9025]])
```

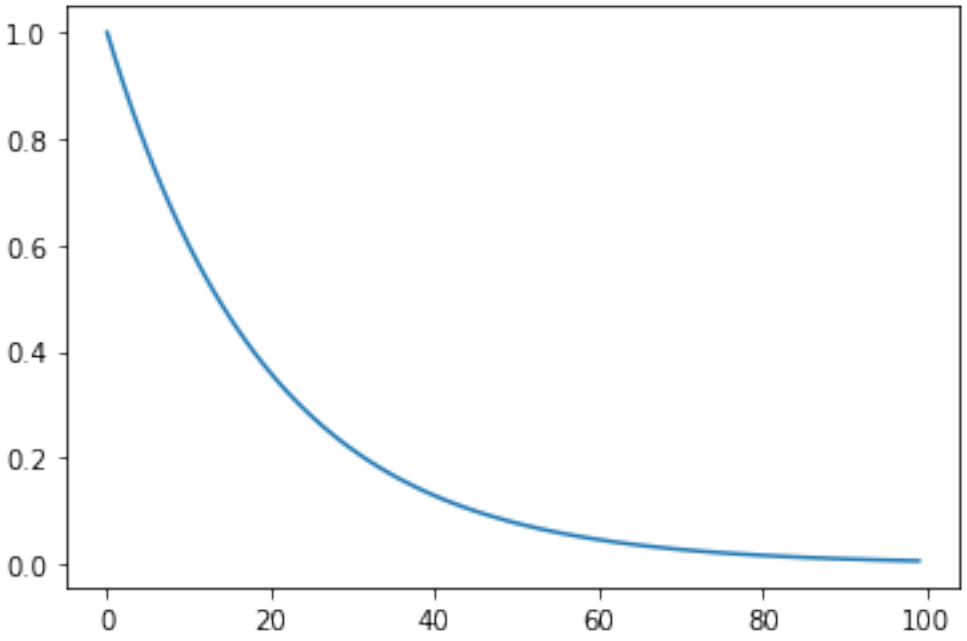
[16]:
```
c @ c @ c
```

[16]:
```
array([[1.      , 0.      ],
       [0.      , 0.857375]])
```

[17]:
```
y = np.empty(100)
for k in range(100):
    y[k] = (0.95)**k

fig, ax = plt.subplots()
ax.plot(y)
```

[17]:
```
[<matplotlib.lines.Line2D at 0x10f19a3d0>]
```



$b_k = A^k b_0 = TC^k T^{-1} b_0 \to T\,\mathrm{diag}(1,0)T^{-1}b_0$

$C^k = \mathrm{diag}(1^k, (0.95)^k) \to \mathrm{diag}(1,0)$

[18]:
```
grænse_b = t @ np.diag([1.0, 0.0]) @ np.linalg.inv(t) @ b0
```

[19]:
```
grænse_b
```

[19]:
```
array([[720000.],
       [480000.]])
```

[ ]:
```

```