f-26-jupyter-tri-qr

May 6, 2021

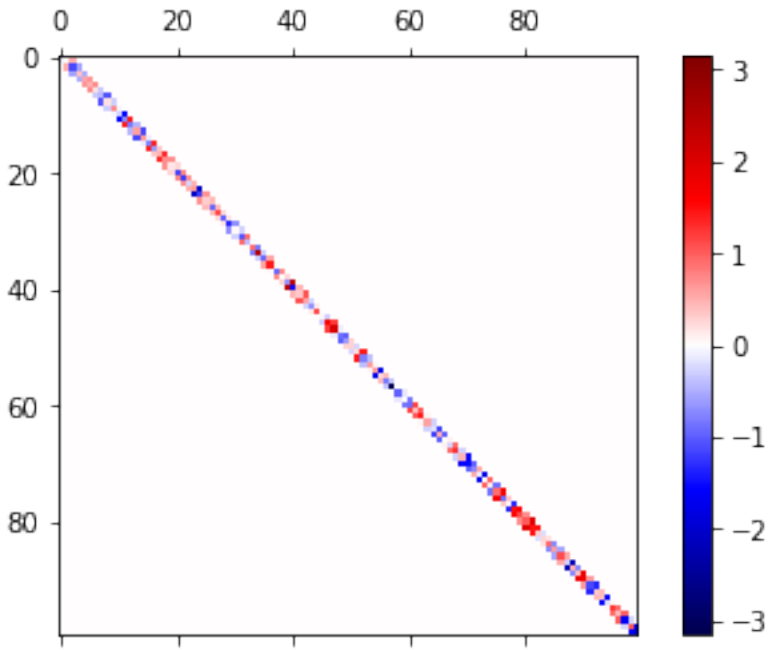
[1]:
```
import matplotlib.pyplot as plt
import numpy as np
```

[20]:
```
def heat_map(a):
    fig, ax = plt.subplots()
    ax.set_aspect('equal')
    mx = a.max()
    mi = a.min()
    r = np.max([mx, -mi])
    if r < 20 * np.finfo(float).eps:
        r = 1
    im = ax.matshow(a, cmap='seismic', clim = (-r, r))
    fig.colorbar(im)
```

[21]:
```
rng = np.random.default_rng()
```

[22]:
```
n = 100
d = rng.standard_normal(n)
u = rng.standard_normal(n-1)
a = np.diag(d) + np.diag(u, 1) + np.diag(u, -1)
```

[23]:
```
heat_map(a)
```



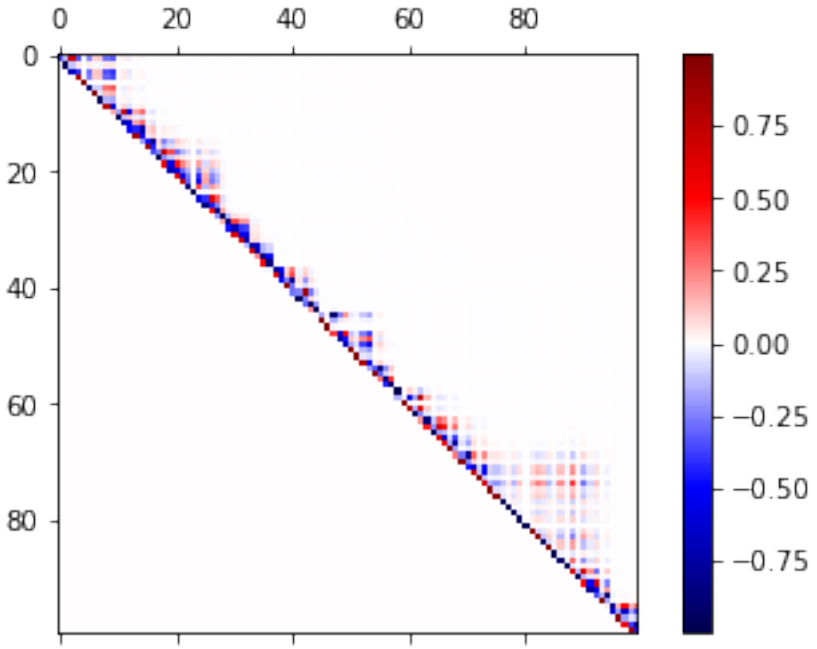
[24]:
```
def house(x):
    norm_x = np.linalg.norm(x)
    if norm_x == 0:
        v = np.zeros_like(x)
        v[0] = 1
        s = 0
    else:
        u = x / norm_x
        eps = -1 if u[0] >= 0 else +1
        s = 1 + np.abs(u[0])
        v = - eps * u
        v[0] += 1
        v /= s
    return v, s

def householder_qr_data(a):
    data = np.copy(a)
    k = a.shape[1]
    s = np.empty(k)
    for j in range(k):
        v, s[j] = house(data[j:, [j]])
        data[j:, j:] -= s[j] * v @ (v.T @ data[j:, j:])
        data[j+1:, [j]] = v[1:]
    return data, s
```

```
def householder_qr(a):
    data, s = householder_qr_data(a)
    n, k = a.shape
    r = np.triu(data[:k, :k])
    q = np.eye(n, k)
    for j in reversed(range(k)):
        x = data[j+1:, [j]]
        v = np.vstack([[1], x])
        q[j:, j:] -= s[j] * v @ (v.T @ q[j:, j:])
    return q, r
```

[25]:
```
q, r = householder_qr(a)
```

[26]:
```
heat_map(q)
```



[27]:
```
np.diag(q, -1)
```

[27]:
```
array([-0.36022396, -0.98886188, -0.663464  ,  0.83475541,  0.99975293,
       -0.88818622,  0.97866166, -0.97524032,  0.76590158,  0.16505643,
       -0.98260121, -0.71628775, -0.5018436 ,  0.86678428, -0.19839219,
        0.88141037, -0.35495346, -0.96393336,  0.56875911, -0.71428554,
        0.61482539,  0.73797247, -0.89045042, -0.99999149, -0.58780744,
       -0.86206892,  0.4870187 , -0.2064181 ,  0.16137599, -0.47431932,
        0.6186366 ,  0.68912392, -0.13119791,  0.68727316,  0.24007252,
        0.49954097, -0.01665477,  0.60330319, -0.49039053,  0.96034573,
```

```
-0.27973754, -0.96684014, -0.29693698,  0.21137974,  0.01965537,
 0.98447901,  0.99972682,  0.39781514, -0.76716249, -0.69084709,
 0.96867824,  0.99561172,  0.83756207, -0.1682325 ,  0.94104029,
 0.27720076, -0.4363691 ,  0.00414989, -0.14464588,  0.99951955,
 0.80381031, -0.97043065,  0.20927662,  0.4519901 , -0.97122344,
 0.65161918,  0.54830769,  0.98152166,  0.26294124,  0.96320332,
-0.46588751,  0.72047167,  0.98914733,  0.63437708,  0.97906459,
 0.95845444, -0.98631013, -0.99766899, -0.97992606, -0.98549102,
-0.99984052,  0.97915446,  0.92558287, -0.95734743,  0.9647186 ,
-0.99712613, -0.91762629, -0.99787021,  0.62630122, -0.98836586,
 0.48618846, -0.82072836, -0.34903183,  0.98604536,  0.03726919,
-0.59883682,  0.97522959,  0.53721077,  0.88976673])
```

[28]:
```
np.diag(q,-2)
```

[28]:
```
array([0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.])
```

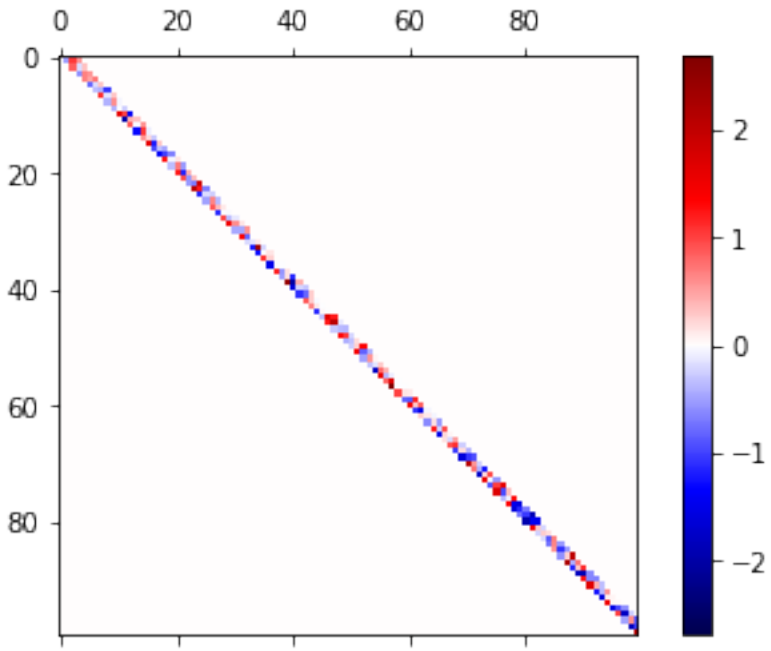
[29]:
```
np.diag(q,10)
```

[29]:
```
array([ 3.92562139e-03,  3.75309166e-02,  5.37639643e-03,  1.17865329e-02,
        1.77020576e-02, -6.83416385e-05,  2.77475330e-03, -1.28120219e-04,
       -4.21202958e-04, -7.73777988e-04,  5.78840969e-03, -5.83595912e-04,
        1.52340316e-03,  3.03768794e-05, -3.96180562e-03, -1.44523769e-02,
        1.17399927e-02, -3.57166285e-02, -2.19611450e-03, -1.71780607e-03,
       -8.66223726e-04, -9.05895357e-04, -9.90247628e-04, -7.20940847e-05,
        6.00333935e-07, -4.29031534e-05,  1.79752367e-05, -8.44977555e-07,
        3.02343365e-06, -2.96463617e-06,  1.84378151e-05, -1.97923357e-06,
        9.57635591e-06,  3.03501277e-05,  6.99651915e-06,  1.34376104e-07,
        3.14728305e-08, -8.55927488e-05, -3.14742232e-05, -6.06465393e-05,
        4.79373981e-06, -2.15413387e-05,  3.44486724e-05, -6.55528775e-04,
        1.83267386e-04,  2.54916290e-02,  1.17989785e-03, -7.62267640e-05,
       -3.08856917e-05, -1.27535494e-07,  3.99171389e-06,  4.61607615e-07,
       -6.86903939e-07, -9.14071221e-07,  1.18307226e-06, -1.33458513e-06,
       -9.82329292e-06,  2.64484594e-06,  3.50542893e-03, -1.75650922e-03,
       -1.74556878e-04,  1.52187618e-03, -9.71325594e-05, -9.78520087e-03,
       -3.29888477e-03,  1.24428241e-03,  3.36973249e-03, -2.76543383e-03,
        1.87904716e-03, -3.00604577e-02,  9.01515040e-04,  7.24432589e-02,
        1.43804808e-01,  2.17611530e-02, -1.57438699e-01, -1.17541760e-02,
        8.98998341e-02,  7.93581495e-03,  3.92553407e-02, -1.43004452e-02,
       -7.01535718e-02,  2.34672164e-03, -3.66959399e-02,  4.58143778e-03,
       -2.16203341e-02, -6.09947790e-04, -2.91123556e-05,  6.18929303e-04,
       -2.95930672e-05, -1.10085798e-03])
```

[30]:
```
np.diag(q, 30)
```

[30]:
```
array([-1.94679032e-07,  2.87661413e-07, -4.37560110e-08, -3.18718290e-08,
       -2.58072489e-08, -2.23249431e-10,  2.99575333e-09, -1.81846772e-11,
        1.32305427e-11, -7.87932951e-12, -2.42157832e-10, -3.45328334e-12,
       -6.54819928e-11, -4.91582539e-11, -7.07001527e-12,  2.41657097e-13,
        1.73235593e-14, -3.79072151e-12,  3.11379204e-13, -1.46300095e-12,
       -4.13519373e-13, -2.76682970e-13, -1.86472578e-12,  2.13410720e-12,
        1.11708187e-15, -9.95948188e-13,  1.87871597e-13, -3.22415083e-13,
        7.18542273e-15, -2.03481463e-16,  7.34136656e-15,  3.45422179e-15,
       -1.81748679e-14,  3.61802273e-14,  4.65508586e-15, -8.01555274e-14,
       -1.02867443e-13,  8.94377396e-13,  5.85186376e-12, -9.55183385e-13,
       -1.00901909e-12, -4.53517693e-12,  1.90461276e-13, -1.24813065e-11,
       -1.00957745e-11, -7.20875082e-10, -7.12190235e-11,  3.87225130e-12,
       -1.11367330e-09, -8.46740425e-10, -1.43249632e-10,  5.77579660e-10,
        3.98950606e-10,  1.96465301e-09,  1.83807374e-08,  1.86079349e-09,
       -9.97325886e-08, -3.22342161e-08,  1.02950682e-04, -8.60543240e-05,
        1.53144996e-05,  1.16235358e-04,  6.54265081e-05, -7.85298248e-05,
       -9.38097099e-04,  7.70290192e-06, -6.22696686e-06, -4.65596709e-05,
       -3.15527911e-06, -1.17952320e-04])
```

[31]:
```
heat_map(r)
```



[32]:
```
np.diag(r,-1)
```

[32]:
```
array([0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.])
```

[33]:
```
np.diag(r,2)
```

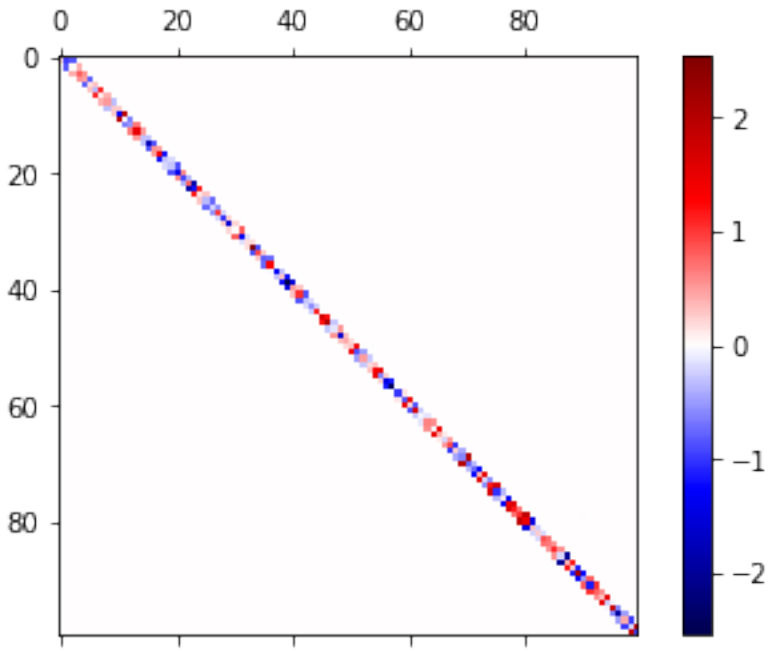
[33]:
```
array([-1.85807461e-01,  5.64554949e-01,  3.36373678e-01,  5.48645318e-01,
        7.20097588e-01,  3.05927042e-01, -1.06002132e+00,  3.00842065e-01,
       -5.27577235e-02, -2.59146321e-01, -1.39990685e+00,  3.19940962e-01,
        5.76004954e-01, -9.53747614e-02, -2.78616459e-01,  3.05096522e-01,
       -4.79507363e-01, -6.84895674e-01,  1.41915887e-01, -5.41244414e-01,
        4.34321459e-01,  3.49597111e-01,  1.85829888e+00, -6.86427586e-01,
       -2.53742456e-01, -3.77570126e-01,  1.14968898e-01, -3.32349326e-02,
       -1.13369009e-01,  1.33517988e-01,  5.76113458e-01,  1.84703487e-02,
        1.10100237e-01, -2.15086322e-01,  1.33127947e-01,  1.24843774e-02,
       -1.20400845e-02,  1.52385263e-01, -1.15205709e+00,  4.31230990e-01,
       -2.99770259e-01,  2.55800014e-01, -3.99056899e-02, -4.74880600e-03,
       -6.24354798e-03,  1.21249038e+00, -1.14576127e-01, -3.98920586e-01,
       -2.26329563e-01,  1.81440037e-01,  1.36422288e+00, -4.16464048e-01,
        4.52964950e-02,  2.84837606e-01,  4.05522860e-01, -1.00180044e-01,
       -4.87381065e-03, -5.98661555e-04,  1.21531334e-01,  1.15984864e+00,
        8.44867947e-01, -5.46099739e-02, -5.92610689e-02, -5.42266684e-01,
        9.66327665e-01,  5.81443445e-02,  5.04879780e-01, -2.67017209e-01,
       -4.06048398e-01, -9.34348035e-01, -2.48149524e-01, -1.25497285e+00,
        9.67311070e-01, -5.61634074e-01,  1.59521038e+00,  3.47880910e-01,
        1.32399511e+00, -1.58087720e+00, -6.94026977e-01, -1.90813394e+00,
       -1.45951637e+00, -2.14924466e-01,  1.74547948e-01,  6.93760542e-01,
       -5.35832047e-01, -5.28178206e-01,  2.03854194e+00,  7.31922078e-01,
        1.12111010e+00, -6.14872278e-01, -6.59846139e-01, -3.56365666e-01,
        5.55369464e-01,  5.51942700e-02,  3.45561173e-02, -6.01281983e-01,
       -2.58623582e-01, -8.83692624e-01])
```

[34]:
```
np.diag(r, 3)
```

[34]:
```
array([0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.])
```

[35]:
```
b = r @ q
```

[36]:
```
heat_map(b)
```



[37]:
```
np.diag(b,-2)
```

[37]:
```
array([0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.])
```

[38]:
```
np.diag(b,2)
```

[38]:
```
array([-1.38777878e-17,  0.00000000e+00, -8.67361738e-19,  8.76035355e-17,
        1.38777878e-17,  2.25514052e-17, -8.32667268e-17,  1.38777878e-16,
       -1.73472348e-18, -2.77555756e-17,  1.11022302e-16,  2.77555756e-17,
       -5.55111512e-17, -2.77555756e-17, -4.16333634e-17,  5.55111512e-17,
       -2.77555756e-17,  0.00000000e+00, -1.38777878e-17,  5.55111512e-17,
        8.32667268e-17,  0.00000000e+00,  5.20417043e-17,  1.11022302e-16,
        6.93889390e-17, -5.55111512e-17,  0.00000000e+00, -3.46944695e-18,
        0.00000000e+00, -2.77555756e-17, -2.22044605e-16, -1.38777878e-17,
        1.38777878e-17, -8.32667268e-17,  1.38777878e-17, -1.73472348e-18,
       -3.46944695e-18,  0.00000000e+00,  1.66533454e-16, -2.77555756e-17,
       -1.38777878e-17,  0.00000000e+00, -6.93889390e-18,  2.16840434e-19,
       -2.37169225e-20, -5.89805982e-17, -1.38777878e-17,  5.55111512e-17,
       -4.85722573e-17,  0.00000000e+00, -2.77555756e-16,  2.77555756e-17,
        0.00000000e+00, -2.77555756e-17,  5.55111512e-17,  1.38777878e-17,
```

```
 1.73472348e-18, -2.71050543e-20, -2.34187669e-17,  0.00000000e+00,
-1.38777878e-16,  7.63278329e-17,  0.00000000e+00,  2.63677968e-16,
-1.11022302e-16,  0.00000000e+00, -1.38777878e-17,  0.00000000e+00,
 5.55111512e-17,  2.22044605e-16, -2.08166817e-17, -1.11022302e-16,
 5.55111512e-17, -1.38777878e-17, -2.77555756e-17,  1.56125113e-17,
 7.63278329e-17,  6.93889390e-18, -4.33680869e-19,  1.12757026e-16,
-2.77555756e-17,  6.24500451e-17,  1.90819582e-17,  8.67361738e-18,
 3.46944695e-17,  1.04083409e-17,  1.80411242e-16, -1.38777878e-17,
-2.77555756e-17,  1.11022302e-16,  5.55111512e-17,  6.93889390e-18,
 8.32667268e-17,  0.00000000e+00, -8.67361738e-19, -8.32667268e-17,
 5.55111512e-17, -2.77555756e-16])
```

[39]:
```
np.max(np.abs(np.diag(b, 2)))
```

[39]:
```
2.7755575615628914e-16
```

[40]:
```
def qr_metode_ide(a):
    b = np.copy(a)
    for i in range(20):
        q, r = householder_qr(b)
        b = r @ q
    return b
```

[41]:
```
a = np.diag(np.full(10, 2.0)) + np.diag(np.full(9, -1.0), 1) + np.diag(np.
↪full(9, -1.0), -1)
```

[42]:
```
a
```

[42]:
```
array([[ 2., -1.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [-1.,  2., -1.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0., -1.,  2., -1.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0., -1.,  2., -1.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0., -1.,  2., -1.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0., -1.,  2., -1.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0., -1.,  2., -1.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0., -1.,  2., -1.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0., -1.,  2., -1.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0., -1.,  2.]])
```

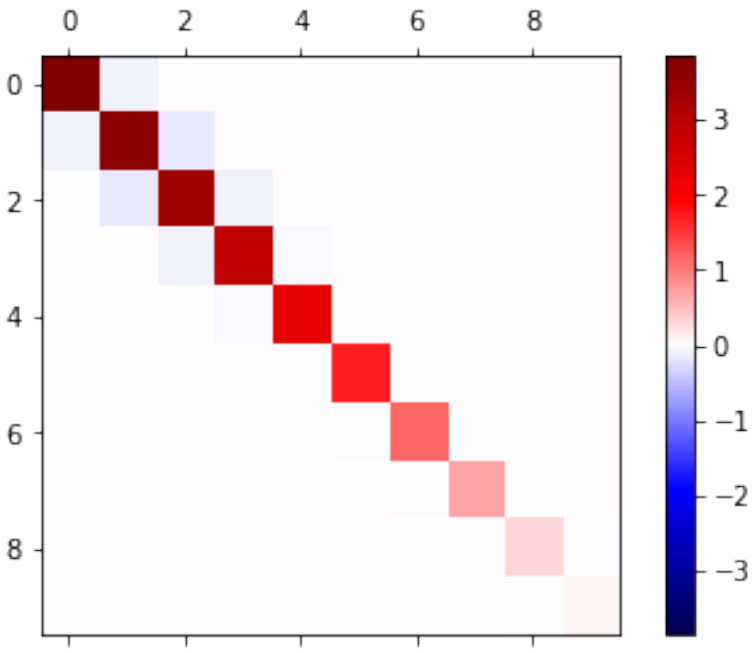
[43]:
```
b = qr_metode_ide(a)
```

[44]:
```
b
```

[44]:
```
array([[ 3.86011489e+00, -1.09161607e-01,  8.44807597e-17,
         2.86033281e-16,  2.73067337e-16,  2.69092696e-17,
        -1.03245780e-16, -1.60032334e-17, -3.32471999e-17,
        -1.16754420e-17],
       [-1.09161607e-01,  3.67326756e+00, -1.54863157e-01,
```

```
  2.87445786e-16,  7.81101374e-17, -4.78791728e-17,
  1.76526103e-16, -1.49801159e-16, -6.22751373e-17,
 -5.24423619e-17],
[ 0.00000000e+00, -1.54863157e-01,  3.35419157e+00,
 -1.08303279e-01, -6.46986586e-17,  6.12156592e-16,
  9.54403057e-17,  3.77003045e-17,  6.82191767e-17,
  8.26028603e-17],
[ 0.00000000e+00,  0.00000000e+00, -1.08303279e-01,
  2.85100306e+00, -4.38814554e-02, -1.08840848e-16,
  5.50469935e-16,  7.15021816e-16,  2.54401666e-16,
  1.21897649e-16],
[ 0.00000000e+00,  0.00000000e+00,  0.00000000e+00,
 -4.38814554e-02,  2.28787915e+00, -1.11540140e-02,
 -3.84638736e-16,  8.69746479e-17,  4.91478204e-16,
  1.87690091e-16],
[ 0.00000000e+00,  0.00000000e+00,  0.00000000e+00,
  0.00000000e+00, -1.11540140e-02,  1.71558433e+00,
 -1.46856659e-03, -8.13334317e-16, -6.36725476e-16,
 -1.68162485e-16],
[ 0.00000000e+00,  0.00000000e+00,  0.00000000e+00,
  0.00000000e+00,  0.00000000e+00, -1.46856659e-03,
  1.16917391e+00, -6.22818934e-05, -5.61325307e-17,
 -4.73942813e-17],
[ 0.00000000e+00,  0.00000000e+00,  0.00000000e+00,
  0.00000000e+00,  0.00000000e+00,  0.00000000e+00,
 -6.22818934e-05,  6.90278540e-01, -2.41065834e-07,
 -2.55605470e-17],
[ 0.00000000e+00,  0.00000000e+00,  0.00000000e+00,
  0.00000000e+00,  0.00000000e+00,  0.00000000e+00,
  0.00000000e+00, -2.41065834e-07,  3.17492934e-01,
 -6.21302329e-13],
[ 0.00000000e+00,  0.00000000e+00,  0.00000000e+00,
  0.00000000e+00,  0.00000000e+00,  0.00000000e+00,
  0.00000000e+00,  0.00000000e+00, -6.21426878e-13,
  8.10140528e-02]])
```

[45]:
```
heat_map(b)
```



[46]:
```
np.diag(b)
```

[46]:
```
array([3.86011489, 3.67326756, 3.35419157, 2.85100306, 2.28787915,
       1.71558433, 1.16917391, 0.69027854, 0.31749293, 0.08101405])
```

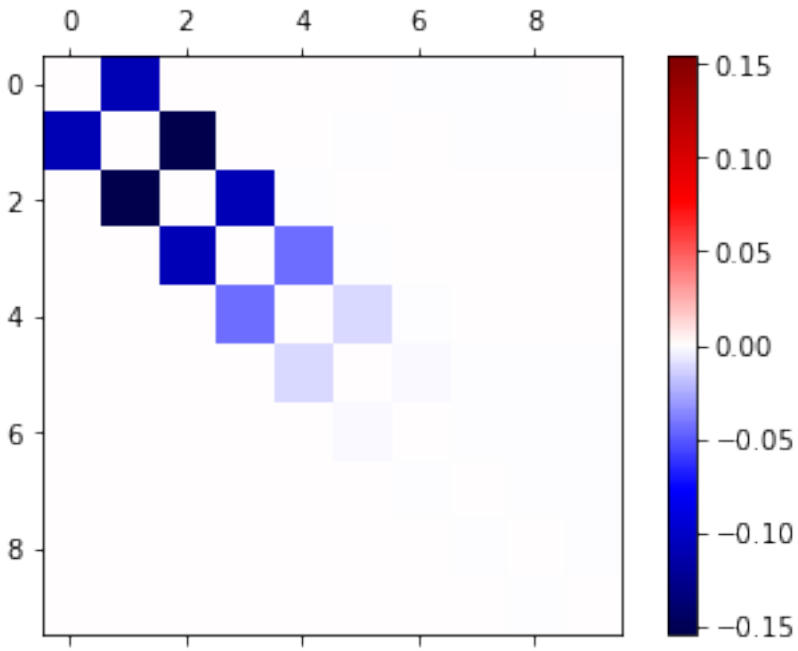
[47]:
```
np.diag(b, 1)
```

[47]:
```
array([-1.09161607e-01, -1.54863157e-01, -1.08303279e-01, -4.38814554e-02,
       -1.11540140e-02, -1.46856659e-03, -6.22818934e-05, -2.41065834e-07,
       -6.21302329e-13])
```

[48]:
```
np.diag(b, 2)
```

[48]:
```
array([ 8.44807597e-17,  2.87445786e-16, -6.46986586e-17, -1.08840848e-16,
       -3.84638736e-16, -8.13334317e-16, -5.61325307e-17, -2.55605470e-17])
```

[49]:
```
heat_map(b - np.diag(np.diag(b)))
```


0
2
4
6
8
0
2
4
6
8
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15

[ ]: