f-15-jupyter-GS-forbedret

March 23, 2021

[1]:
```
import matplotlib.pyplot as plt
import numpy as np
```

[2]:
```
def klassisk_gram_schmidt(a):
    n, k = a.shape
    q = np.empty((n, k))
    r = np.zeros((k, k))
    for j in range(k):
        r[:j, [j]] = q[:, :j].T @ a[:, [j]]
        w = a[:, [j]] - q[:, :j] @ r[:j, [j]]
        r[j, j] = np.linalg.norm(w)
        q[:, [j]] = w / r[j, j]
    return q, r
```

[3]:
```
def forbedret_gram_schmidt(a):
    _, k = a.shape
    q = np.copy(a)
    r = np.zeros((k, k))
    for i in range(k):
        r[i, i] = np.linalg.norm(q[:, i])
        q[:, i] /= r[i,i]
        r[[i], i+1:] = q[:, [i]].T @ q[:, i+1:]
        q[:, i+1:] -= q[:, [i]] @ r[[i], i+1:]
    return q, r
```

[4]:
```
s = 1e-8
a = np.array([[1.0, 1.0, 1.0],
              [  s, 0.0, 0.0],
              [0.0,   s, 0.0],
              [0.0, 0.0,   s]])
a
```

[4]:
```
array([[1.e+00, 1.e+00, 1.e+00],
       [1.e-08, 0.e+00, 0.e+00],
       [0.e+00, 1.e-08, 0.e+00],
       [0.e+00, 0.e+00, 1.e-08]])
```

[5]:
```
q, r = klassisk_gram_schmidt(a)
print(a - q @ r)
print()
print(q.T @ q)
```

```
[[0. 0. 0.]
 [0. 0. 0.]
 [0. 0. 0.]
 [0. 0. 0.]]

[[ 1.00000000e+00 -7.07106781e-09 -7.07106781e-09]
 [-7.07106781e-09  1.00000000e+00  5.00000000e-01]
 [-7.07106781e-09  5.00000000e-01  1.00000000e+00]]
```

[6]:
```
q, r = forbedret_gram_schmidt(a)
print(a - q @ r)
print()
print(q.T @ q)
```

```
[[0. 0. 0.]
 [0. 0. 0.]
 [0. 0. 0.]
 [0. 0. 0.]]

[[ 1.00000000e+00 -7.07106781e-09 -4.08248290e-09]
 [-7.07106781e-09  1.00000000e+00  1.11022302e-16]
 [-4.08248290e-09  1.11022302e-16  1.00000000e+00]]
```

[7]:
```
n = 100
rng = np.random.default_rng()
u, _, vt = np.linalg.svd(rng.random((n, n)))
i = np.arange(n)
s = np.array(2.0 ** (-i))
a = u @ np.diag(s) @ vt
```

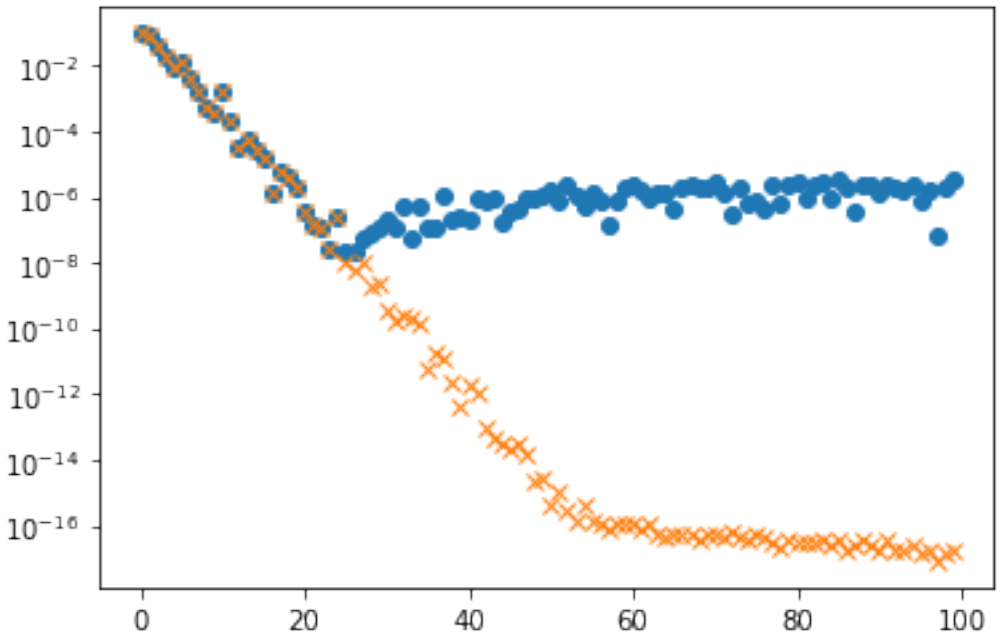
[8]:
```
s[:10]
```

[8]:
```
array([1.        , 0.5       , 0.25      , 0.125     , 0.0625    ,
       0.03125   , 0.015625  , 0.0078125 , 0.00390625, 0.00195312])
```

[9]:
```
qk, rk = klassisk_gram_schmidt(a)
qf, rf = forbedret_gram_schmidt(a)
```

[10]:
```
fig, ax = plt.subplots()
ax.set_yscale('log')
ax.plot(i, rk[i, i], 'o')
ax.plot(i, rf[i, i], 'x')
```

[10]: [<matplotlib.lines.Line2D at 0x10e50be80>]



[11]:
```
2.**-55
```

[11]: 2.7755575615628914e-17

[13]:
```
2.**-25
```

[13]: 2.9802322387695312e-08

[12]:
```
np.finfo(float).eps
```

[12]: 2.220446049250313e-16

[14]:
```
np.sqrt(np.finfo(float).eps)
```

[14]: 1.4901161193847656e-08

[ ]: