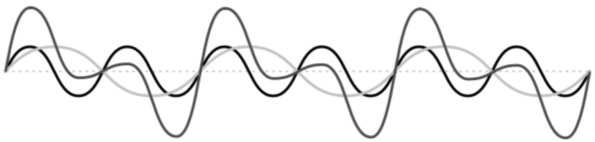
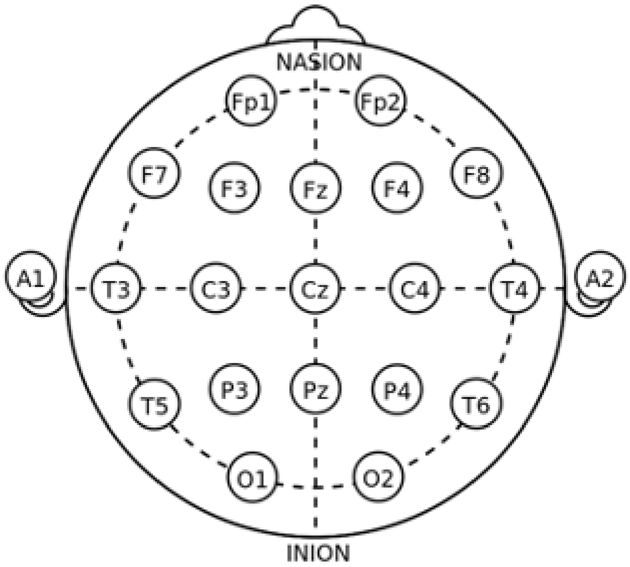
Electroencephalography Frequency Analysis

Taylor Baum

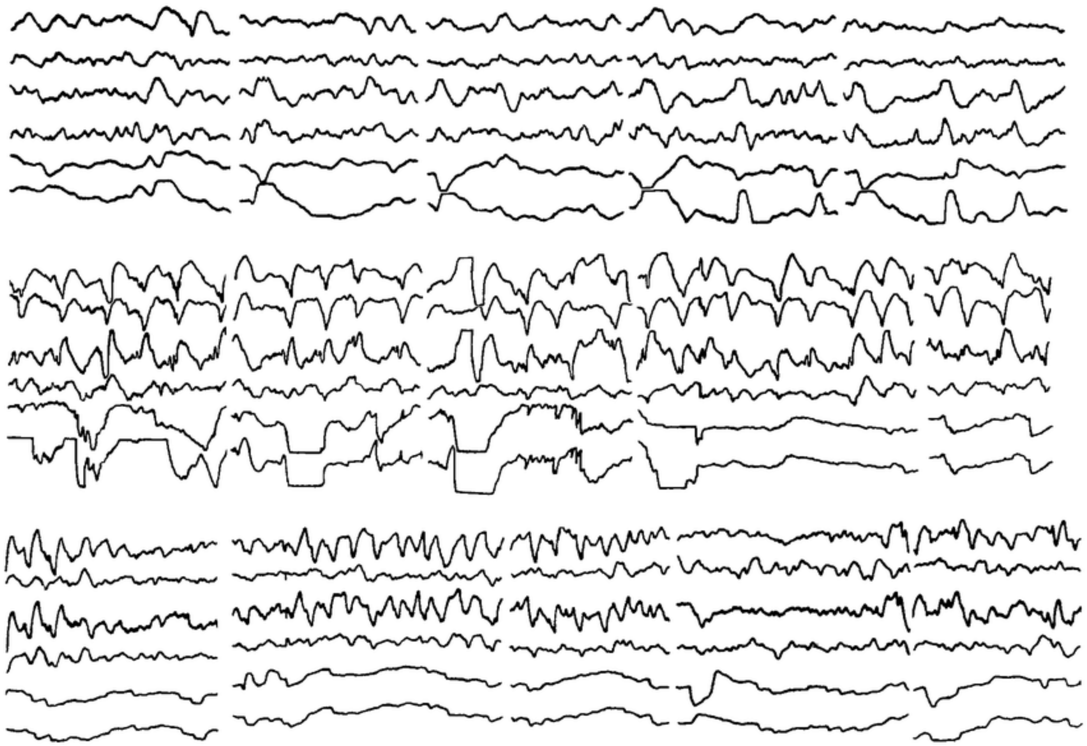


Electroencephalography

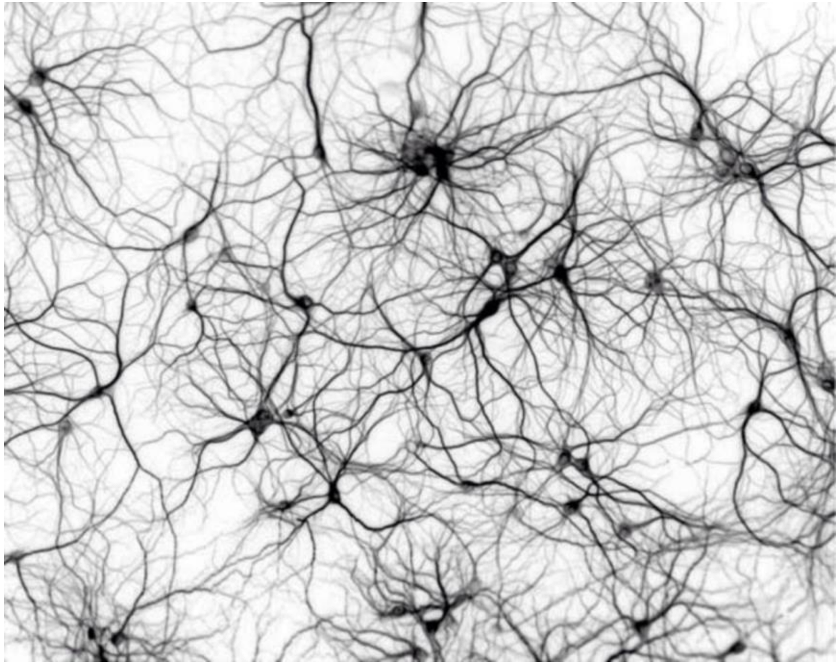
Electroencephalography detects **electrical activity** in the brain.



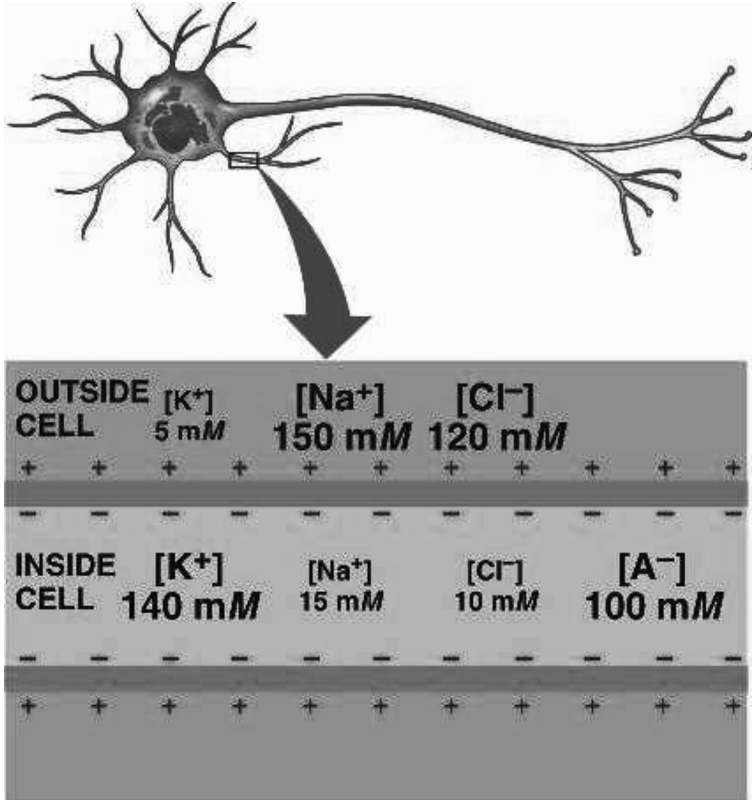
Let’s try and understand what exactly an electroencephalogram is detecting!



In the brain, we have neurons that are oriented in all different directions, and they **transfer information** between one another through **electrical signals**.



The electrical potentials contained in neurons are generated by **differences in concentrations of charge** across cell membranes known as ion gradients.



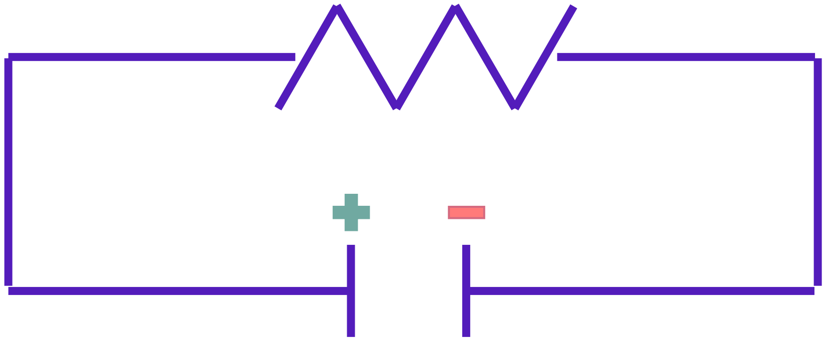
First, let's think about **voltage** and **current**, both which result from the ion gradients.

$$V = IR$$

Voltage = Current * Resistance

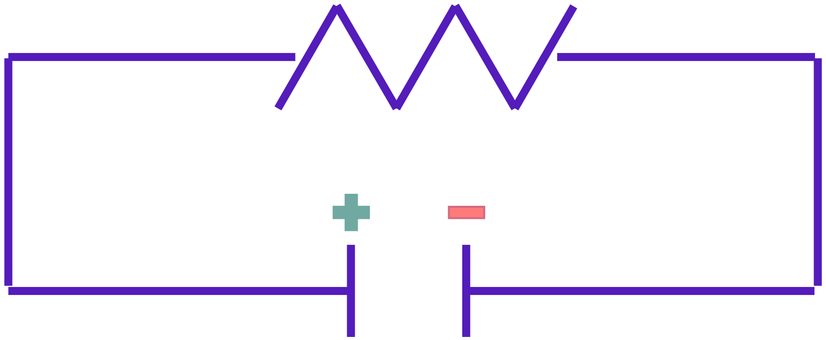
Voltage (V in Ohms) is the difference in electrical potential between any two points in an electrical circuit.

$$V = IR$$

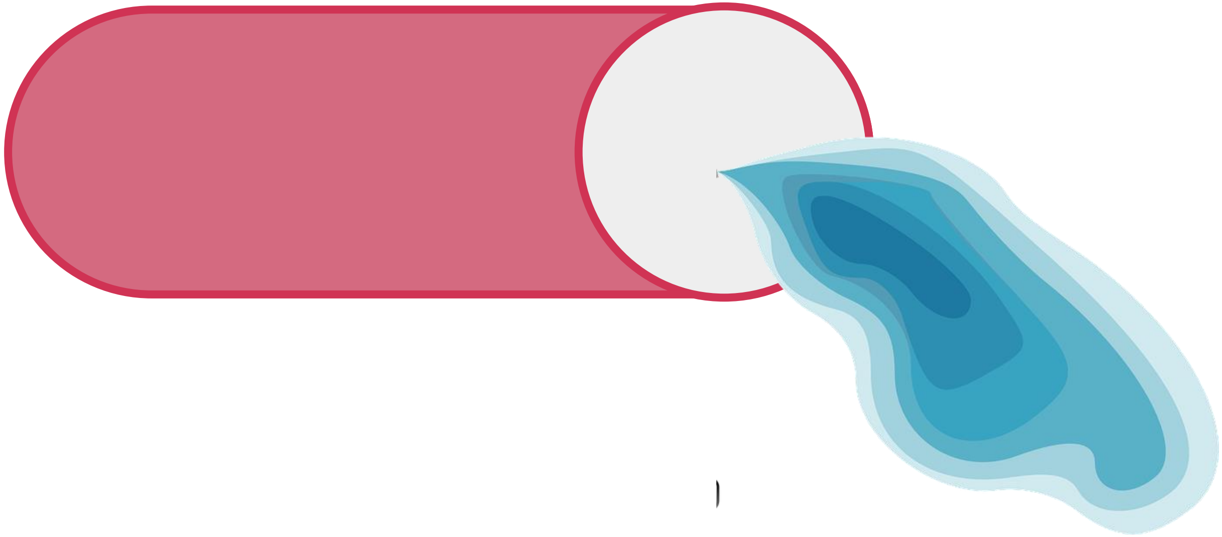


Current (I in amps) is the speed or rate at which the electrons flow through a resistor.

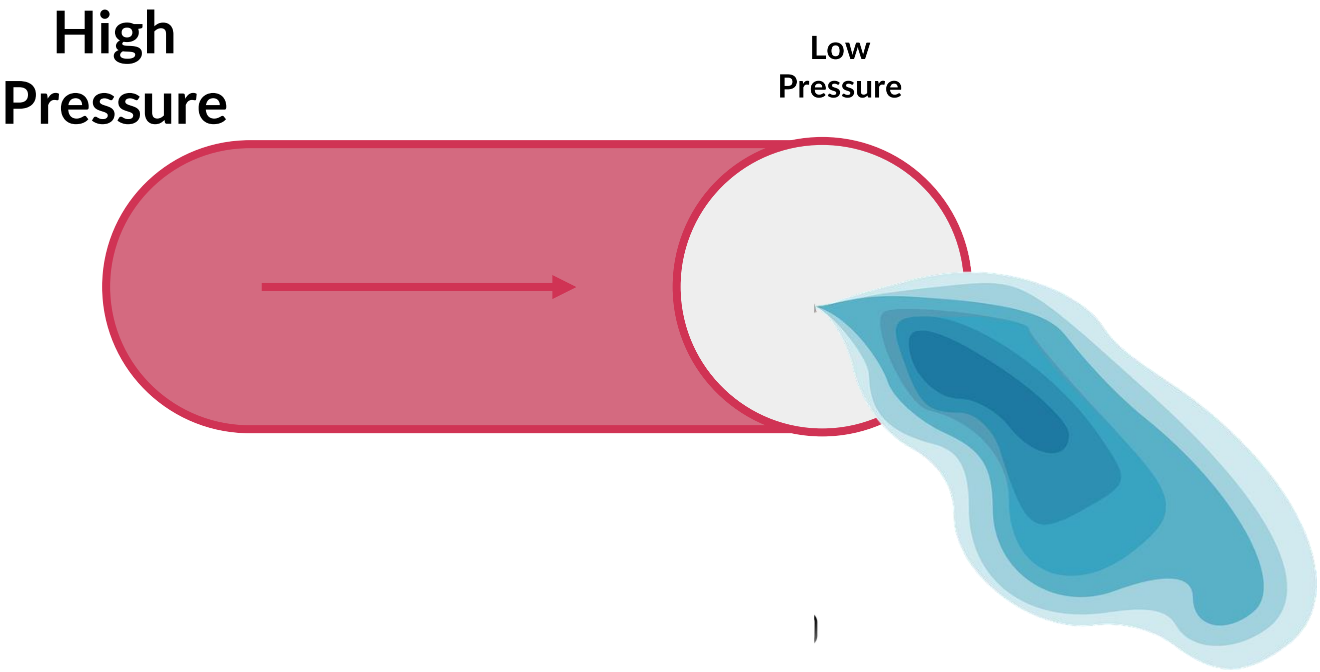
$$V = IR$$



To reemphasize the definition of voltage and current, we consider a **water** analogy.



For **water to move** through a hose **(current to flow)**, there needs to be a **difference in pressure (voltage)** between two locations in the pipe.

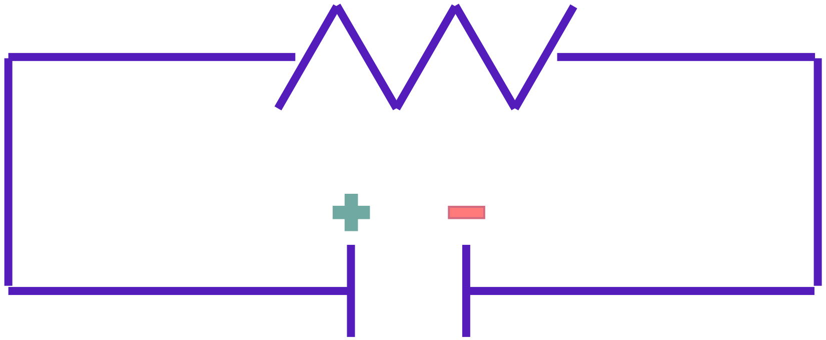


If there is **no difference in pressure (voltage)**, then there is **no flow (current)** through the pipe.

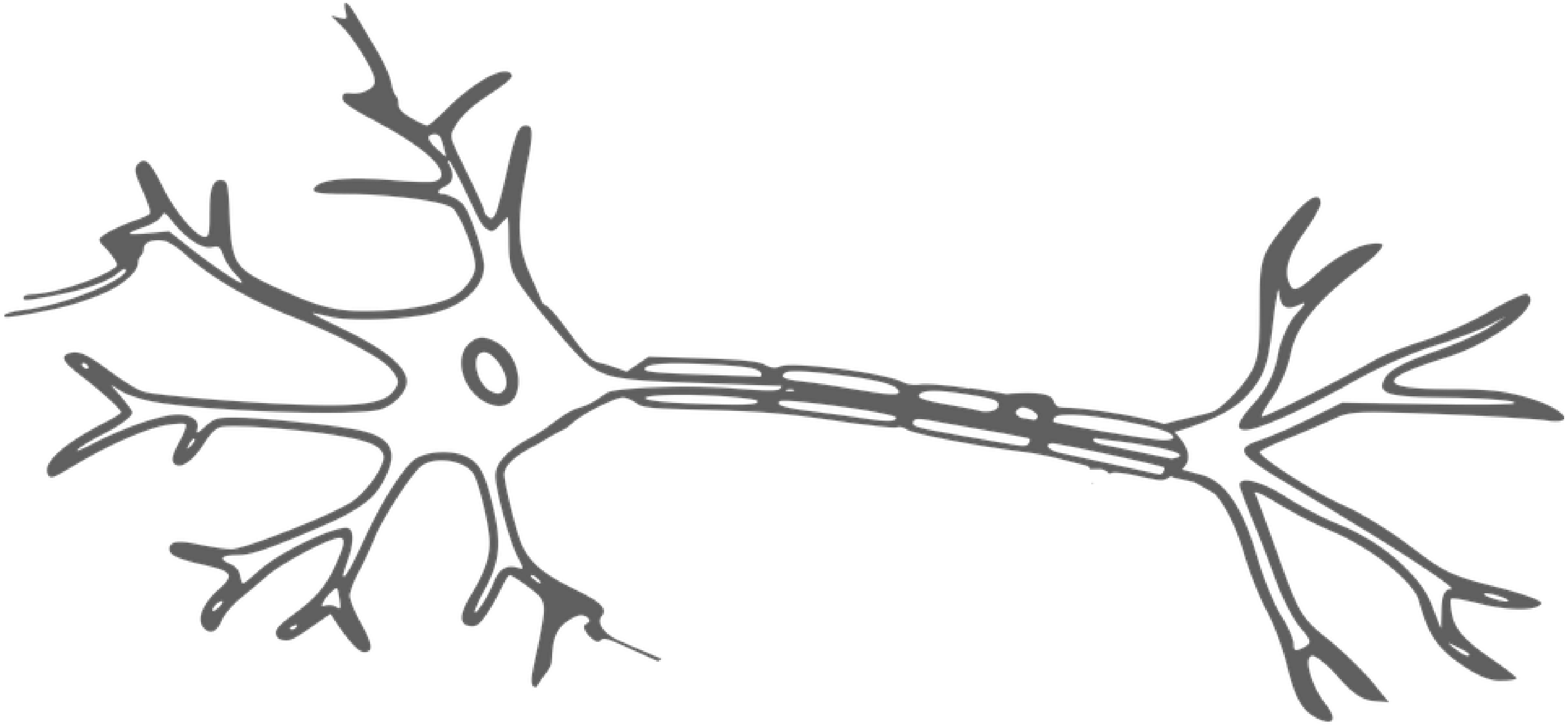
The existence of the pipe is the **resistance** to flow.

So, we can account for all three components of **ohm's law** here.

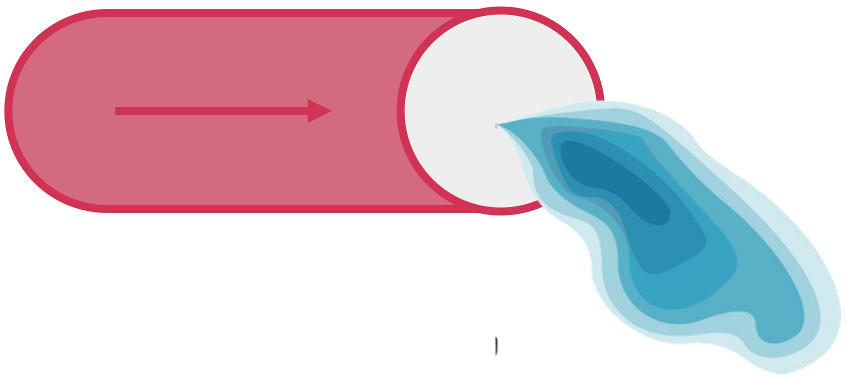
$$V = IR$$

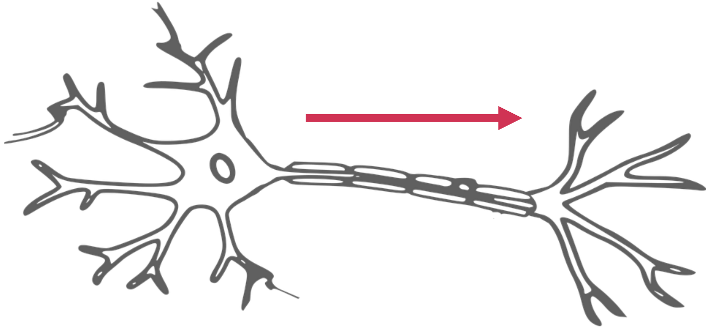


This concept is equivalent for **electrical signals in neurons**.

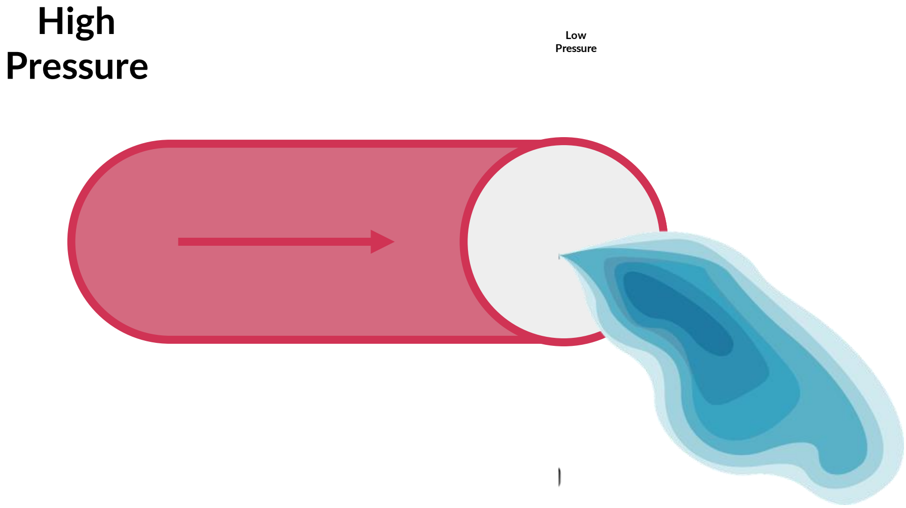


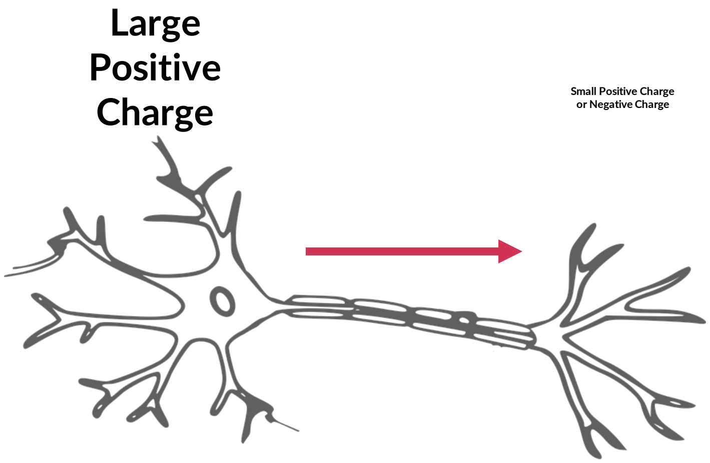
There needs to be some difference in charge concentration between two points in the neuron for electrical charge to flow (**current**).





Think of this flow of charge as a **transfer of information**. With no flow, there is no information being transferred.

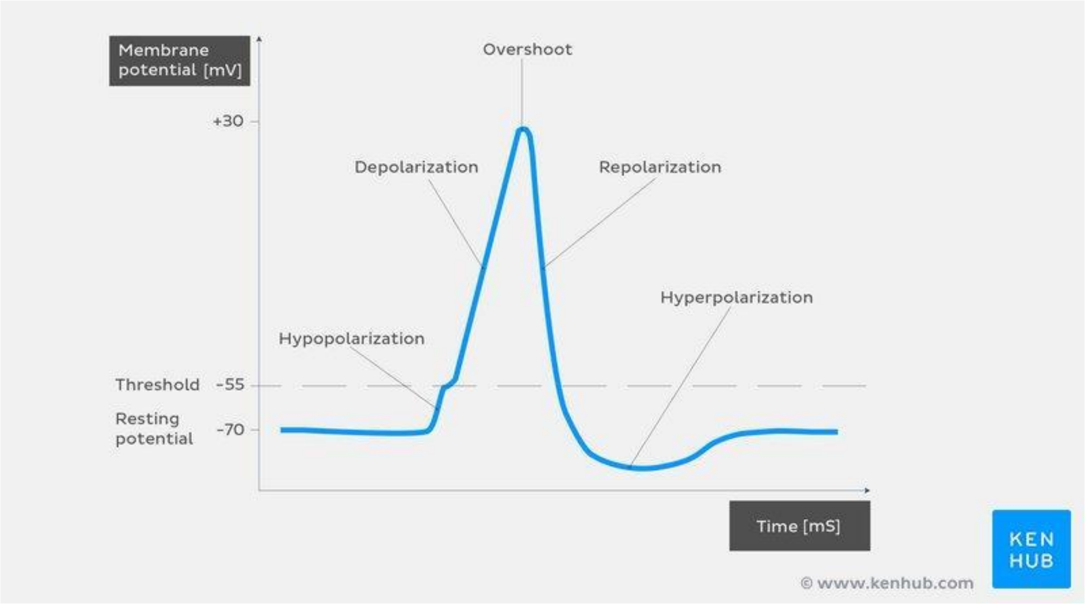




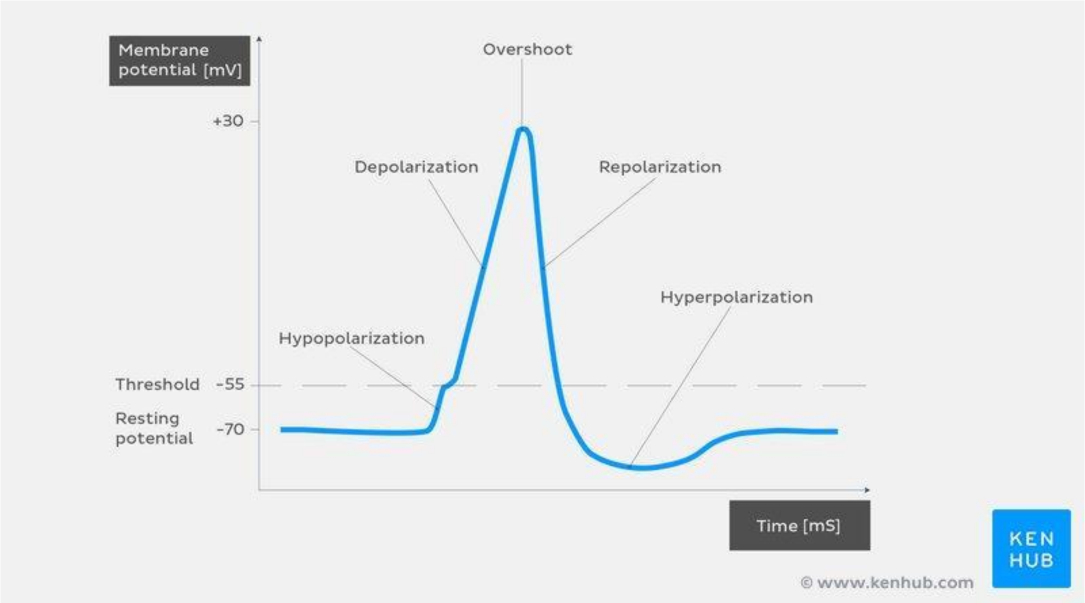
Any questions about:

- electrical signals
- voltage
- current
- neuronal communication

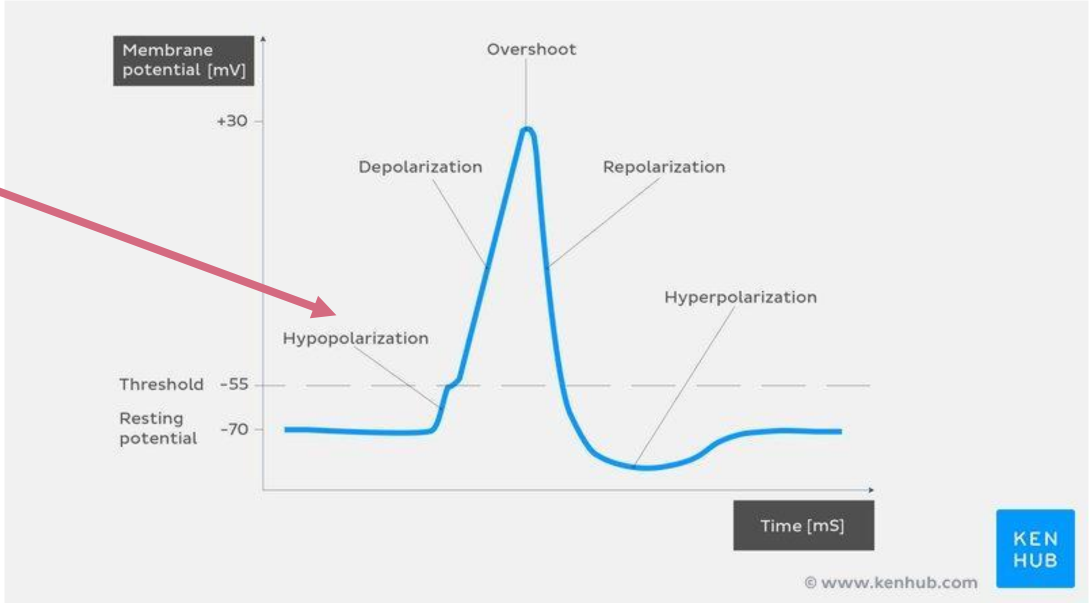
An **action potential** is when the membrane voltage of a neuron gets positive enough that a wave of positive ions moves through the neuron.



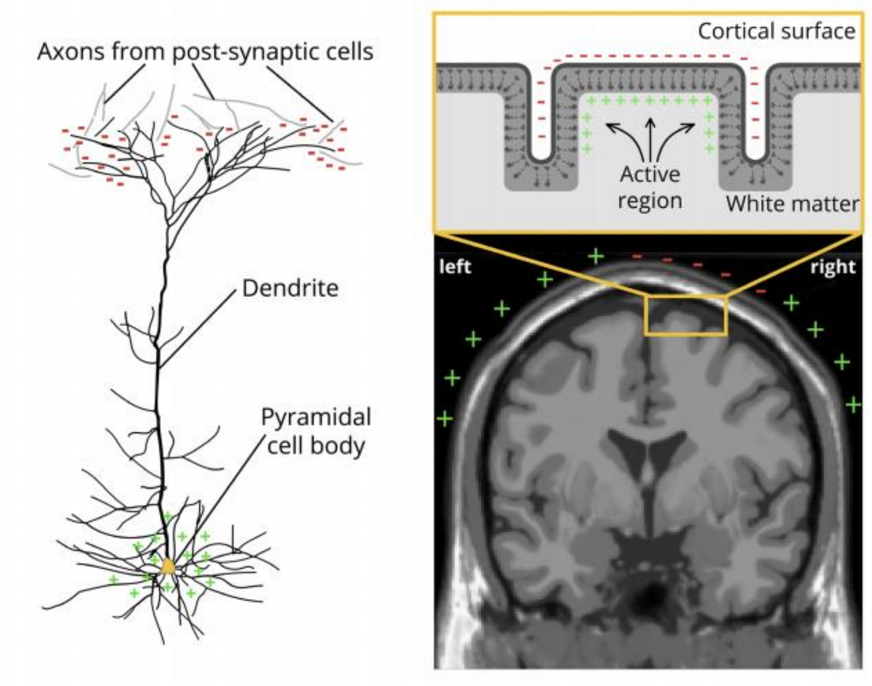
In order for flow or an **action potential** to happen, a big enough membrane potential or difference in charge is needed.



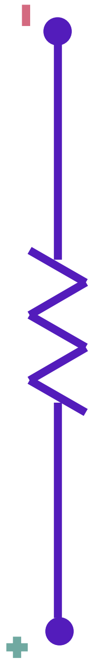
EEGs don’t detect the action potential, but instead **detect charge build-up** in the extracellular space resulting, in the excitatory case, from the **hypopolarization** of the intracellular space.

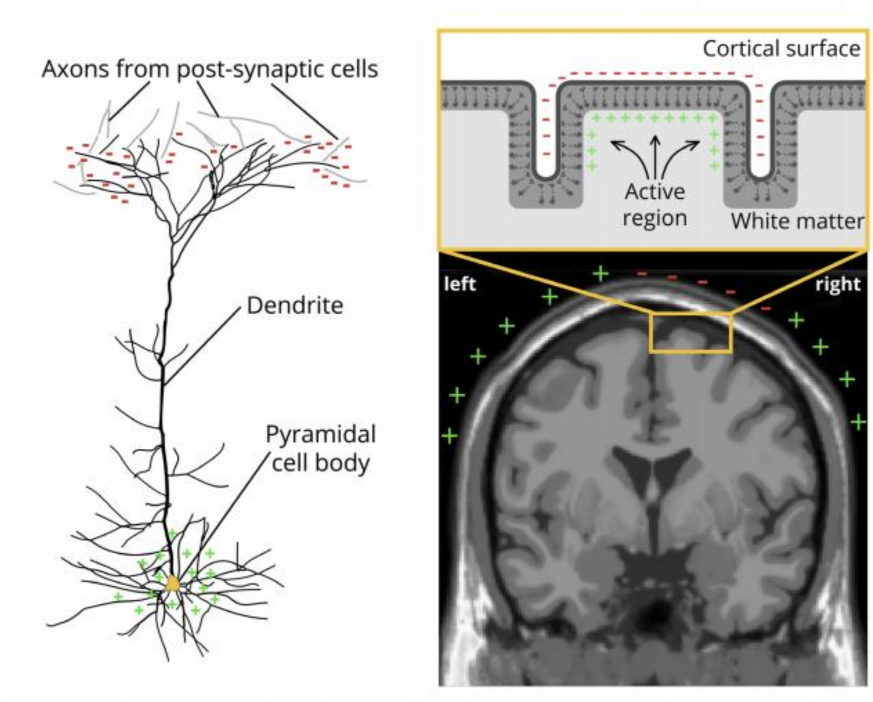


More generally, **EEGs detect the build up of charge** leading either to a neuron firing (Excitatory Postsynaptic Potential EPSP) or a neuron becoming less likely to fire (Inhibitory Postsynaptic Potential IPSP).

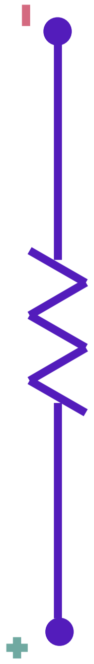


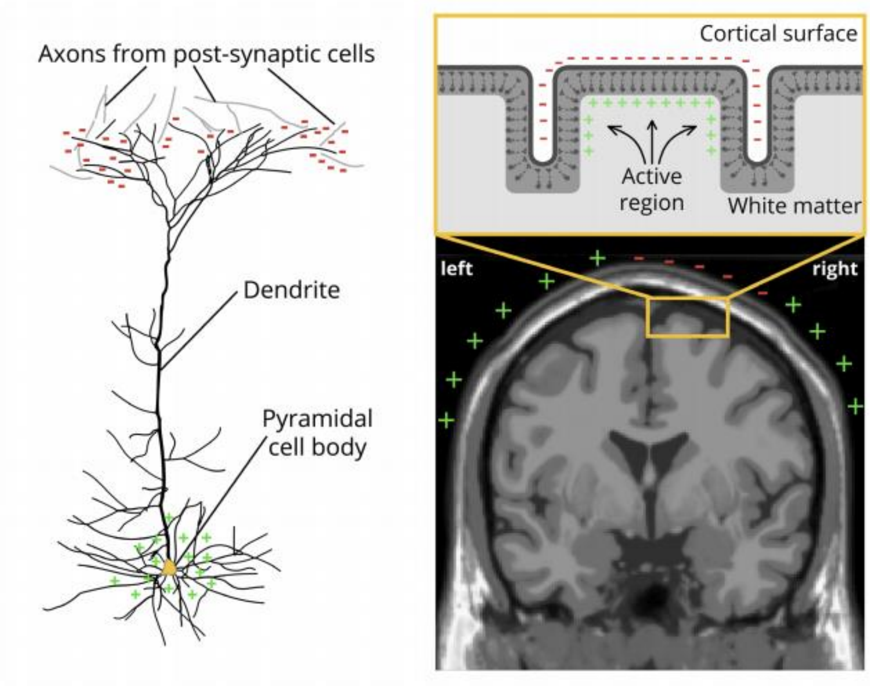
Let’s consider the **Excitatory Postsynaptic Potential** in a cell that is oriented on the cortex according to the figure below.



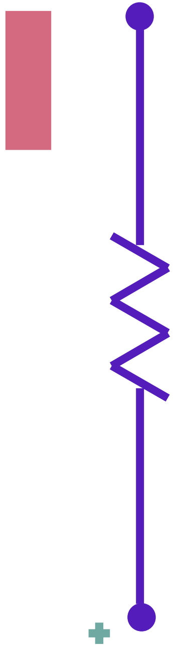


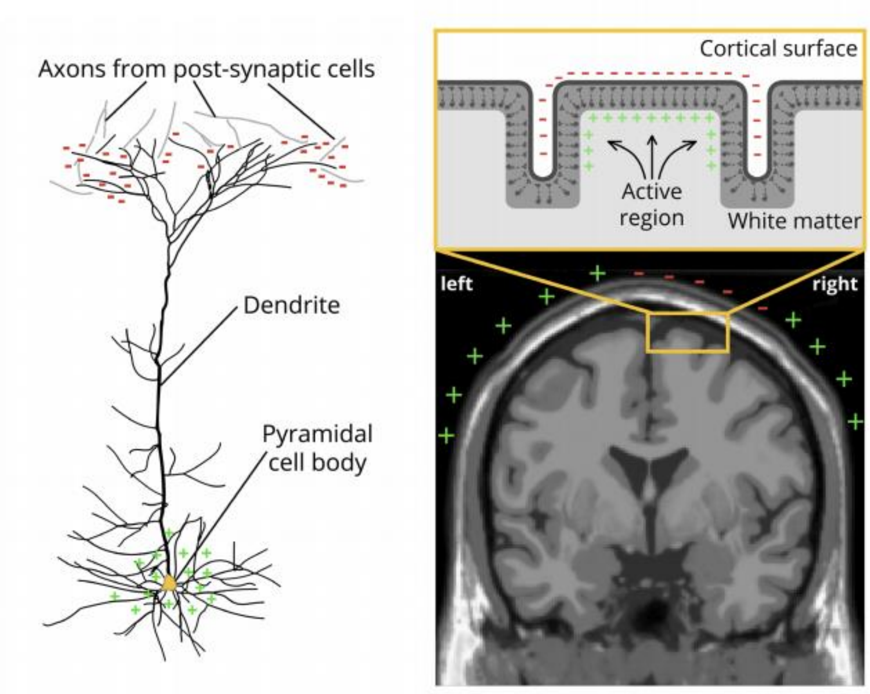
For EPSPs, the post-synaptic potential of a neuron must cross a threshold of **-55 mV** before the neuron will fire.



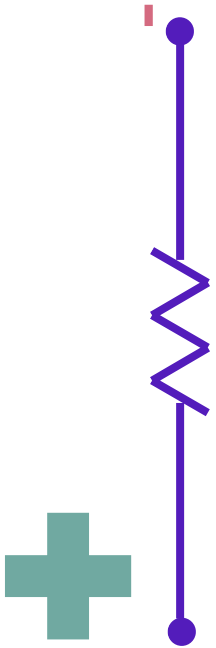


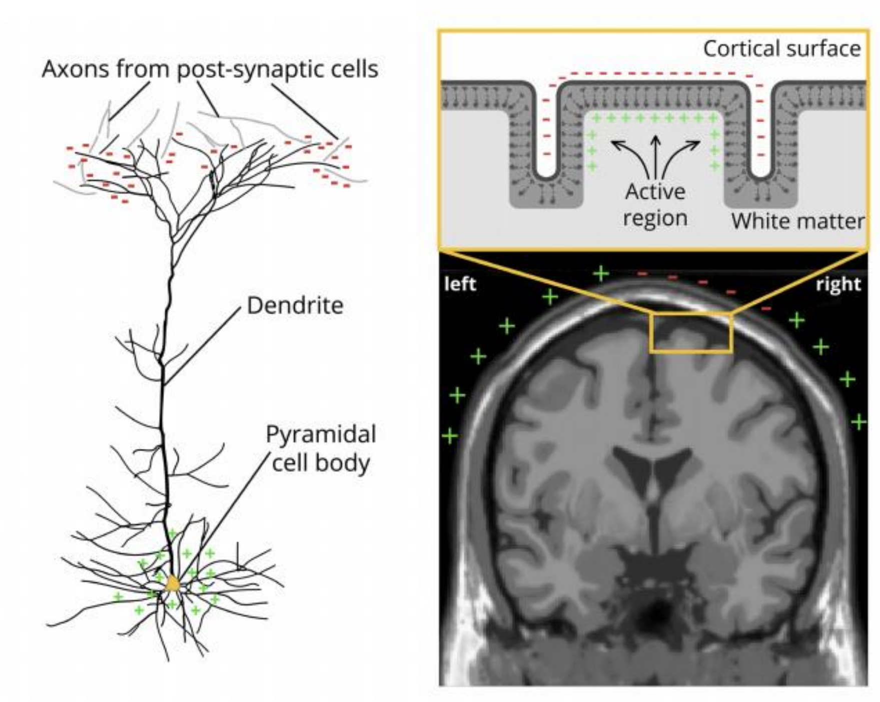
Before a neuron reaches that -55 mV threshold, there are **positive ions rushing into the neuron increasing the membrane potential** and creating a negative charge outside of the cell.





In a portion of a neuron that is away from the negative extracellular region, those positive ions are flowing **out** of the cell, **creating a positive charge externally**.

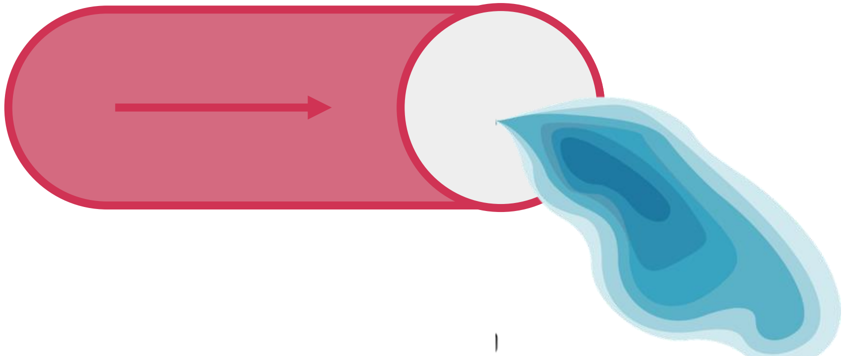




This difference in charge should remind us of what we mentioned earlier.

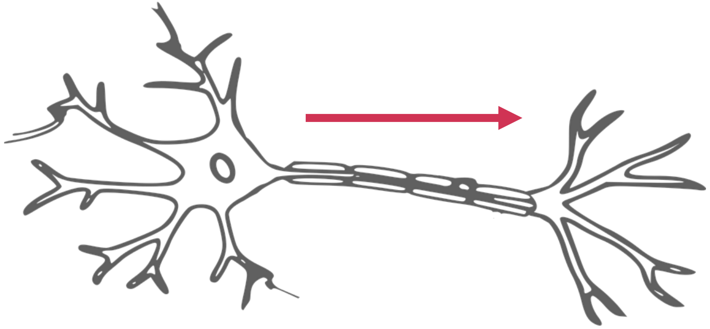
High Pressure

Low Pressure



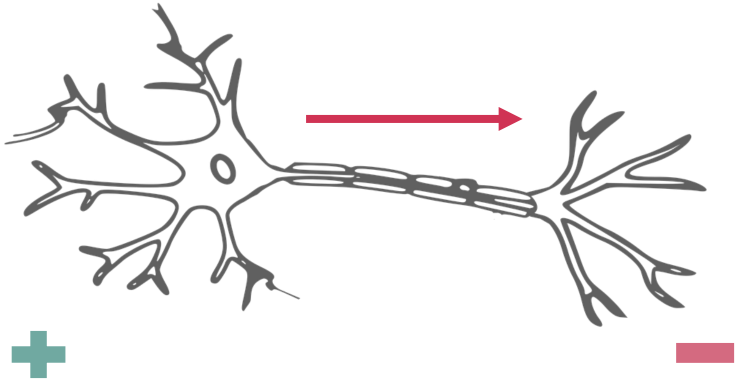
Large Positive Charge

Small Positive Charge or Negative Charge

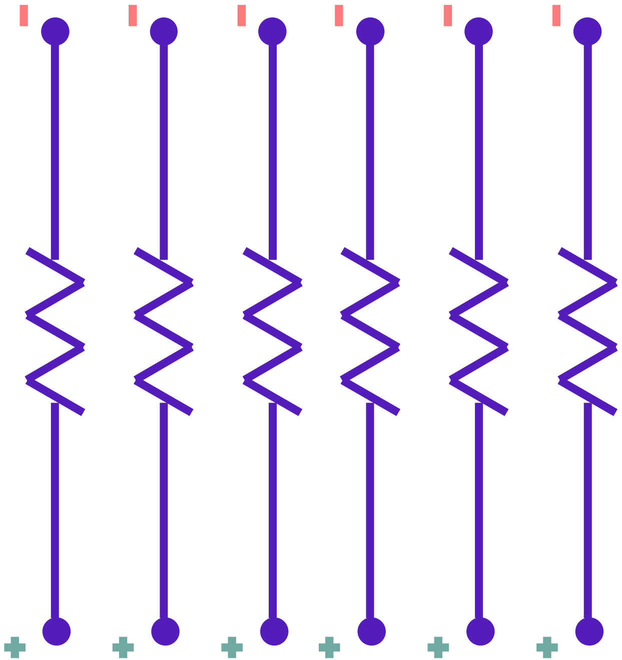


While this difference in charge eventually leads to an action potential, it is the **existence of the difference in charge that the EEG picks up**.

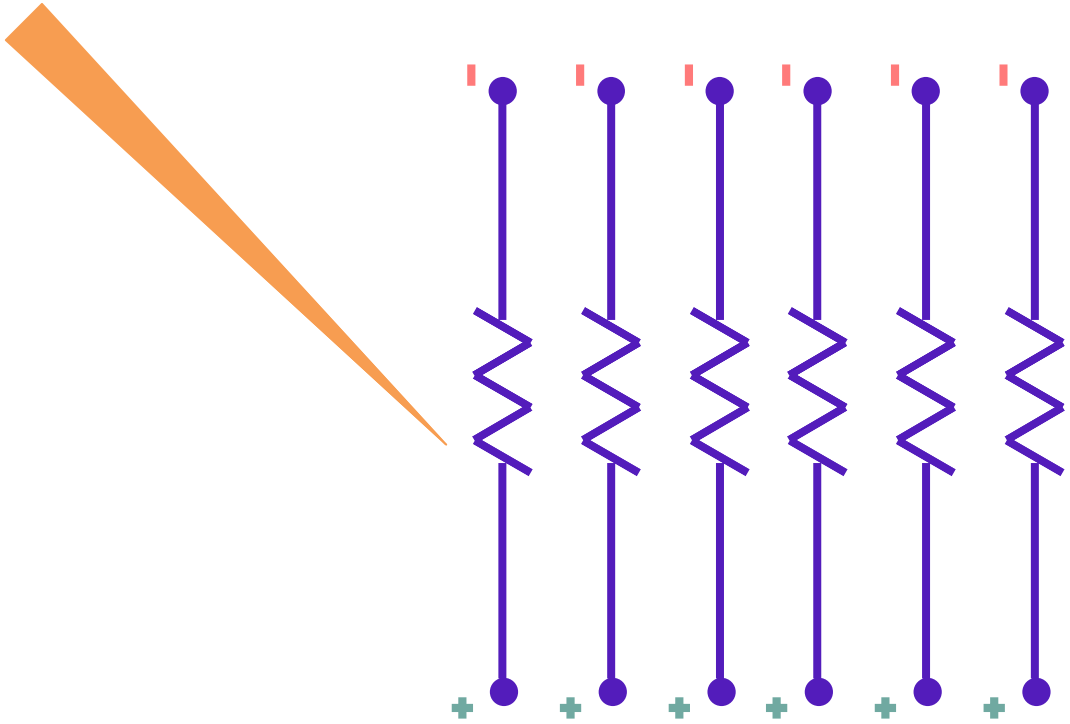
The **single electrical dipole** created in **one neuron** isn't strong enough for an EEG to detect it.



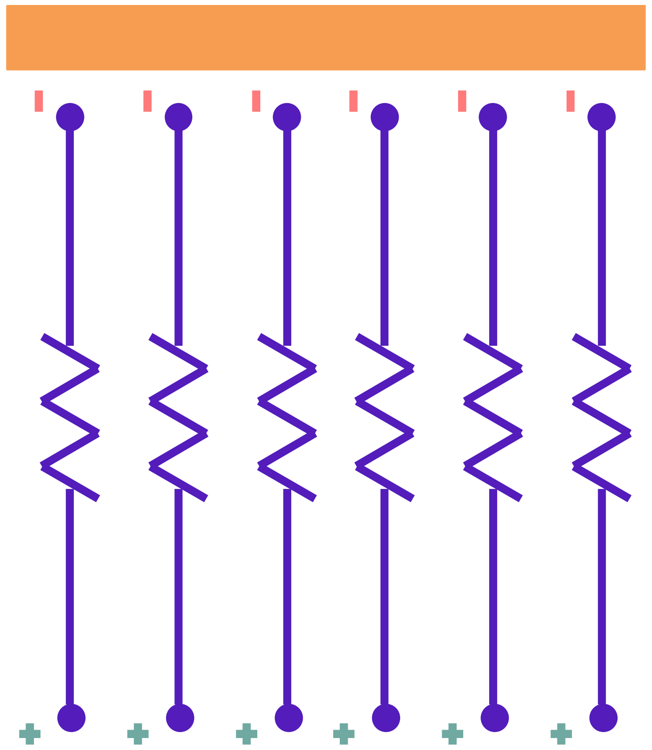
But, there are special neurons near the surface of the brain called **pyramidal** neurons that are all **oriented in the same direction**.



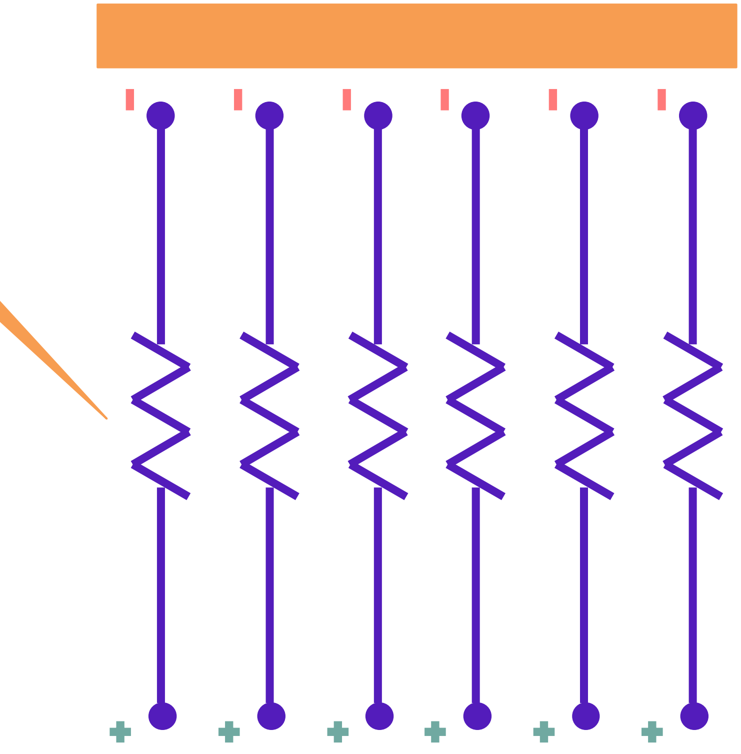
So instead of single neuron recordings, we look at the summed activity of many pyramidal neurons.

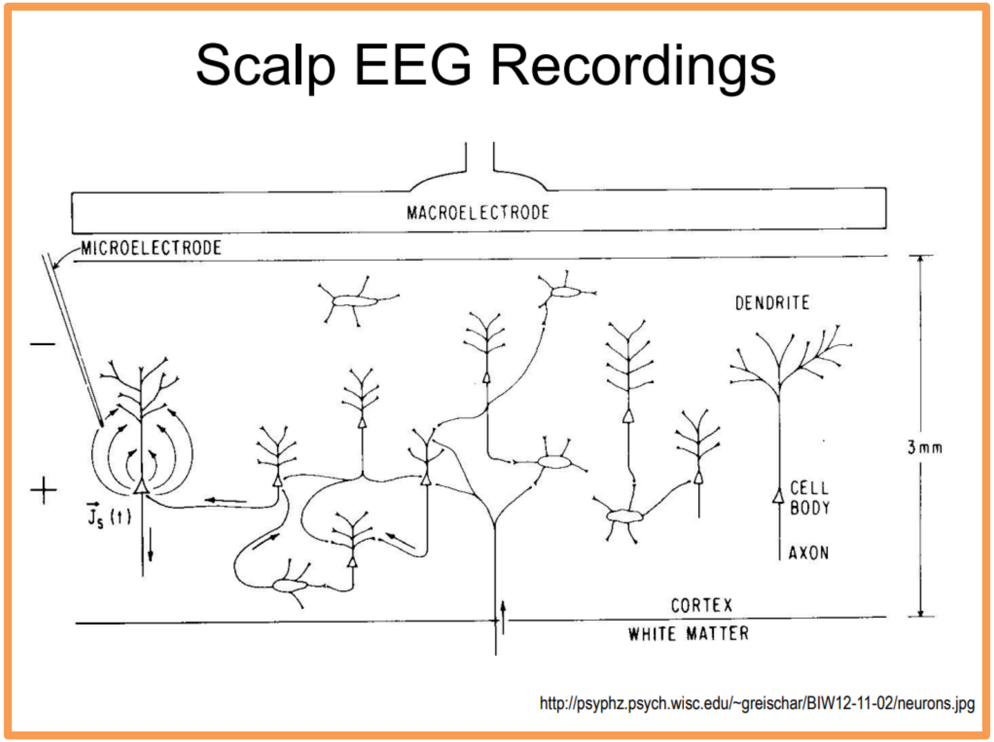


So instead of single neuron recordings, we look at the **summed activity** of many pyramidal neurons.

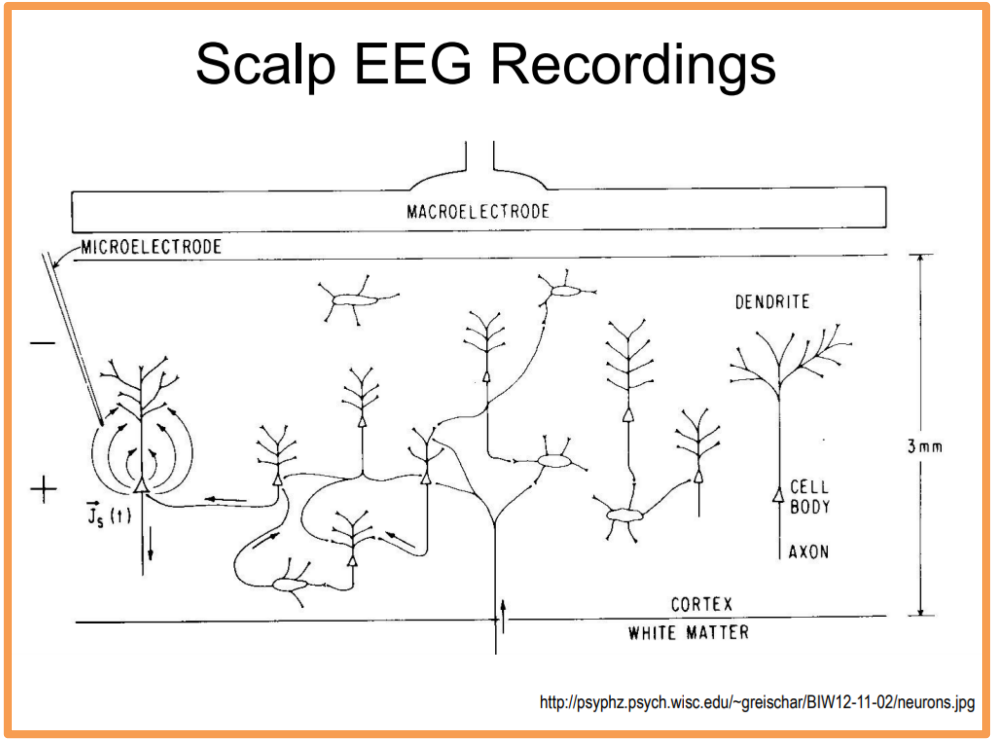


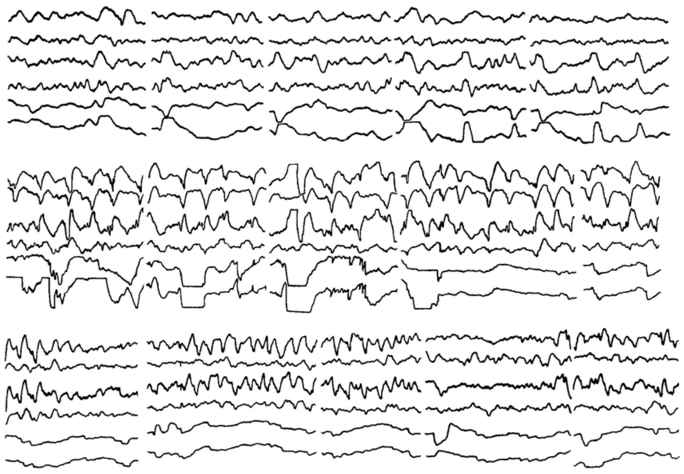
So instead of single neuron recordings, we look at the **summed activity** of many pyramidal neurons.



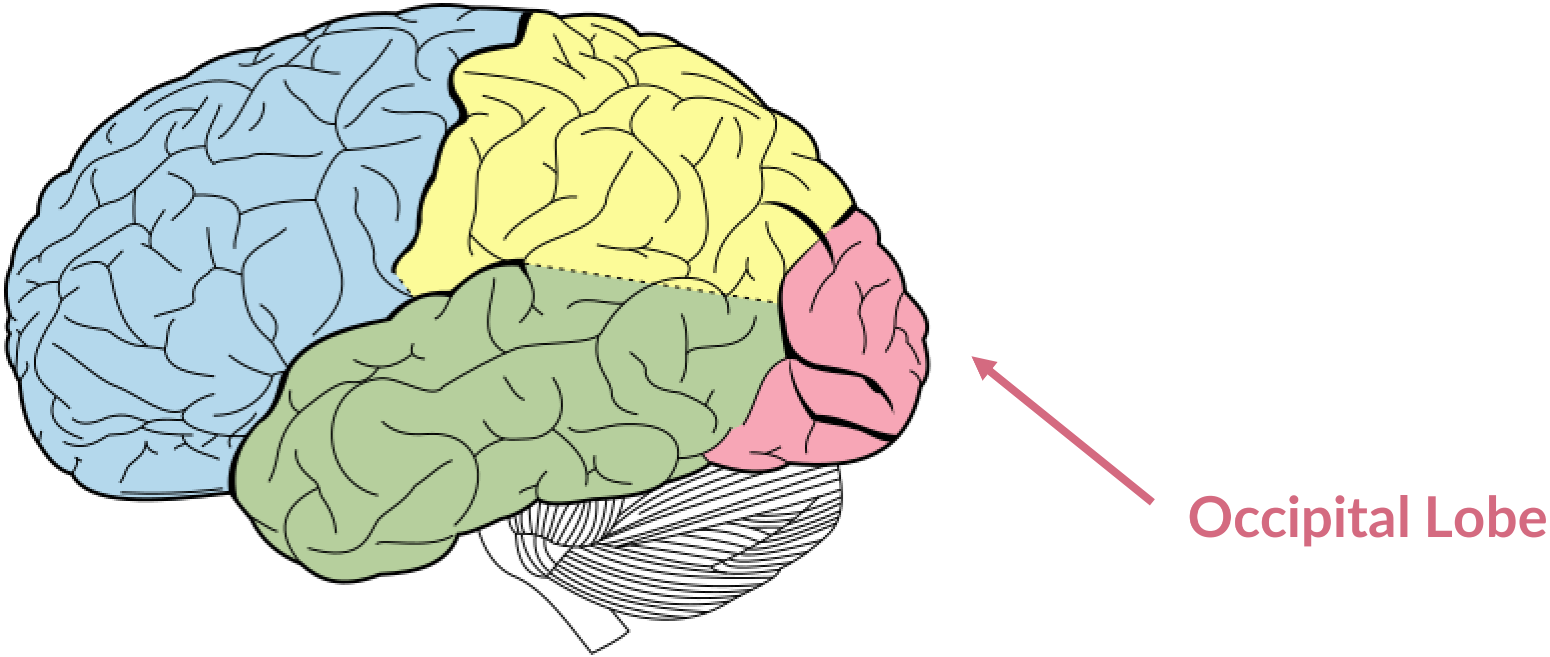


It is also important to keep in mind that EEG signals result from **synchronous activity** within the pyramidal neurons.





For your BCI project, you are interested in EEG signals specifically coming from the occipital lobe, where visual signals are processed.

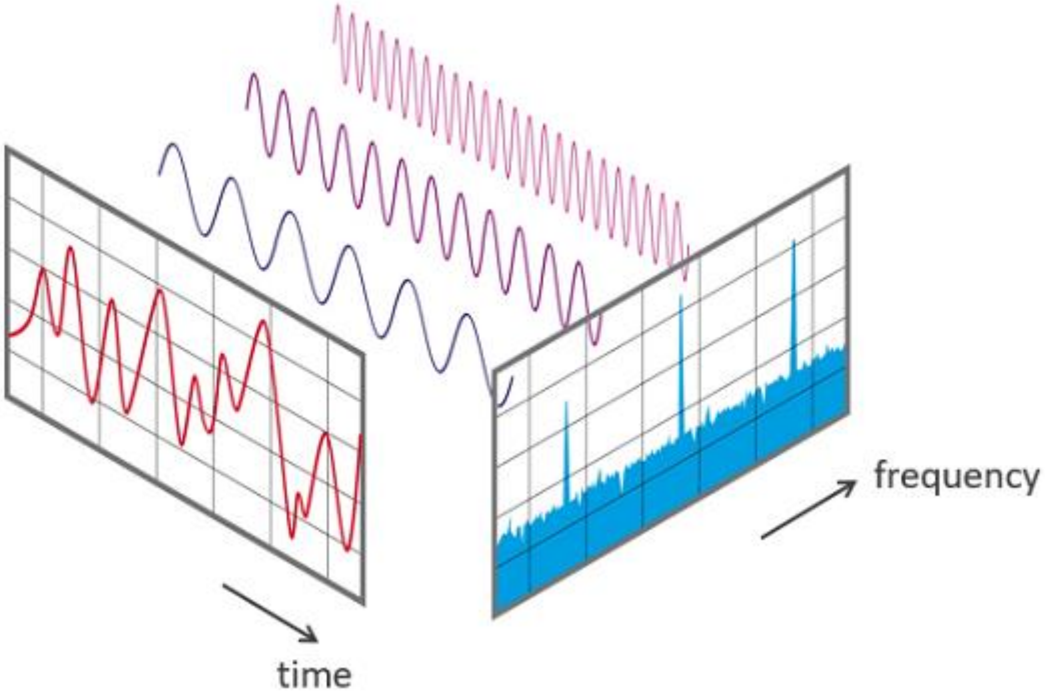


Now we have an idea about where the EEG signal comes from, **physiologically**.

Now let's consider the **frequency domain** features being used in the SSVEP literature we have been reviewing.

Frequency Analysis

Frequency analysis is a field with which we convert a signal from the time-domain into the frequency domain.



The most often understood method for doing this is known as the **Fourier Transform**.

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}dt$$

Here we have a **continuous time signal** being transformed into a **continuous frequency representation.**

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}dt$$

For implementation of the fourier transform, we must consider a different version of this function known as the **Discrete Time Fourier Transform (DTFT)**.

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Here we have a **discrete time signal** being transformed into a **continuous frequency representation.**

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

The DTFT can only be implemented in real life by considering samples of the output of the DTFT, which is known as the **Discrete Fourier Transform (DFT)**.

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

Now, we have a **discrete time signal** being transformed into a **discrete frequency representation.**

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

In order to do **frequency-analysis of a real signal**, we have to window the signal, and then sample the spectrum with the Discrete Fourier Transform.

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

The **Fast Fourier Transform (FFT)** is a method of implementation of the Discrete Fourier Transform.

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi}{N} kn}$$

Any questions about:

- Fourier Transform
- DFT
- DTFT
- FFT

Spectral Estimation

In each of these transformations, we are assuming that we have a **deterministic signal**, meaning that there is **no randomness** in the realizations of the signal.

In our problem, our signals are **inherently random**, and as such, we must take that into account when doing frequency analysis.

We won't go exactly into the details, but a method to estimate the **power spectral density** of random signals is with the **periodogram**.

$$
\begin{aligned}
\hat{\Phi}(\omega) &= \frac{1}{L}\left|X(e^{j\omega})\right|^2 \\
&= \frac{1}{L}\left|\sum_{n=0}^{L-1} x[n]e^{-j\omega n}\right|^2
\end{aligned}
$$

The **periodogram** is the natural way to estimate the power spectral density, but we can do better.

$$\hat{\Phi}(\omega) = \frac{1}{L}\left|X(e^{j\omega})\right|^2$$
$$= \frac{1}{L}\left|\sum_{n=0}^{L-1} x[n]e^{-j\omega n}\right|^2$$

This illustrates an autoregressive process of order 6 used to illustrate the characteristics of the spectral estimation techniques I will show..

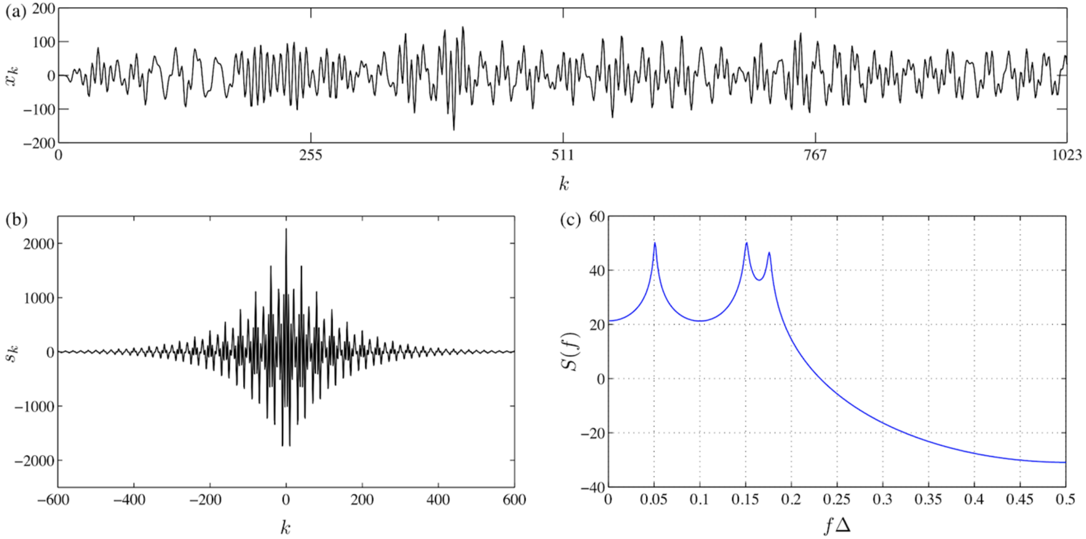


Fig. 1. The AR(6) process given by (7). (a) Sample of length 1024, (b) the autorcovariance sequence, and (c) the PSD.

The periodogram is a **biased estimate** of the PSD because its expected value is the frequency convolution of the true PSD with the DTFT of the autocorrelation of the data window.

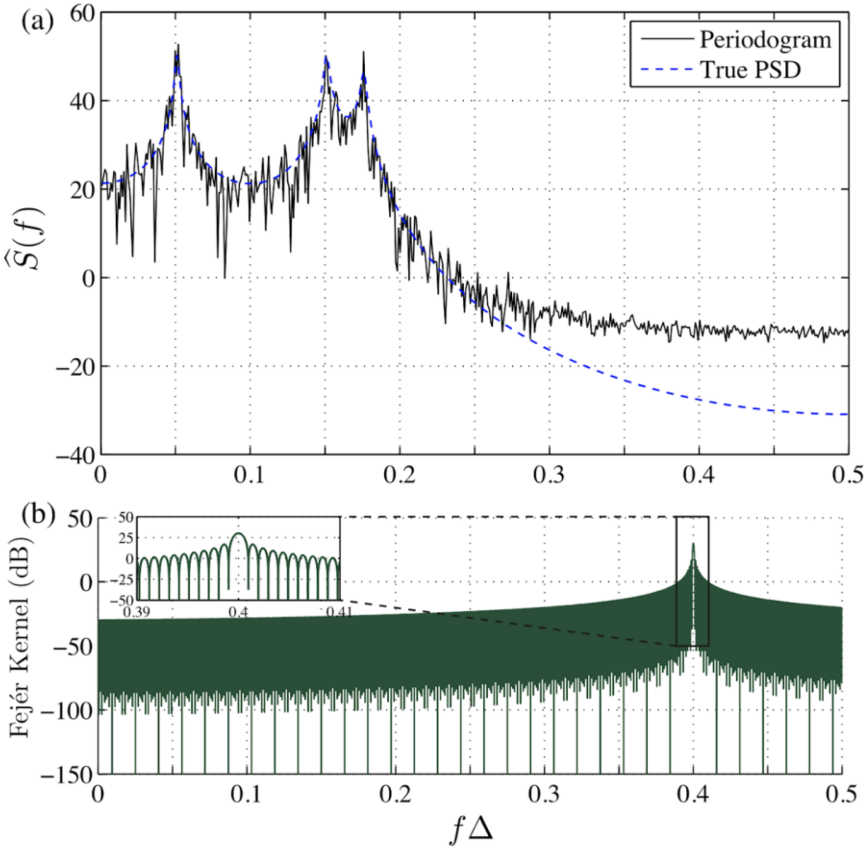


Fig. 2. Periodogram estimate of the PSD of the AR(6) process. (a) Periodogram estimate. (b) Fejér kernel centered at $f\Delta = 0.4$. The zoomed-in view of the main lobe is shown on the top left.

It is, however, asymptotically unbiased.

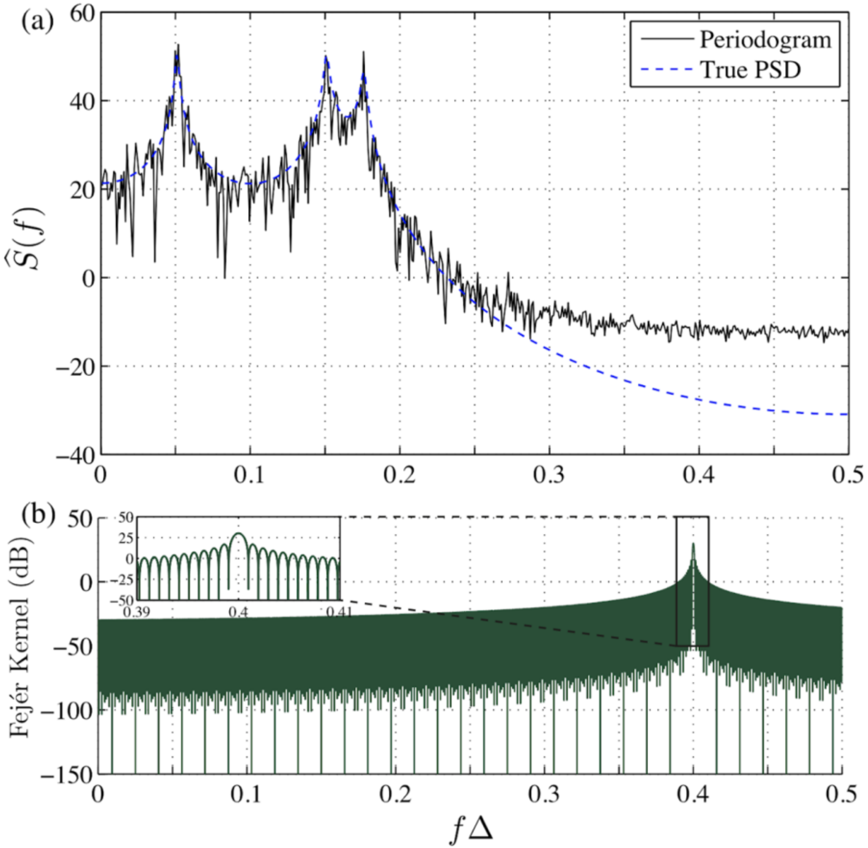


Fig. 2. Periodogram estimate of the PSD of the AR(6) process. (a) Periodogram estimate. (b) Fejér kernel centered at $f\Delta = 0.4$. The zoomed-in view of the main lobe is shown on the top left.

The variance of the periodogram is asymptotically proportional to the square of the true underlying PSD.

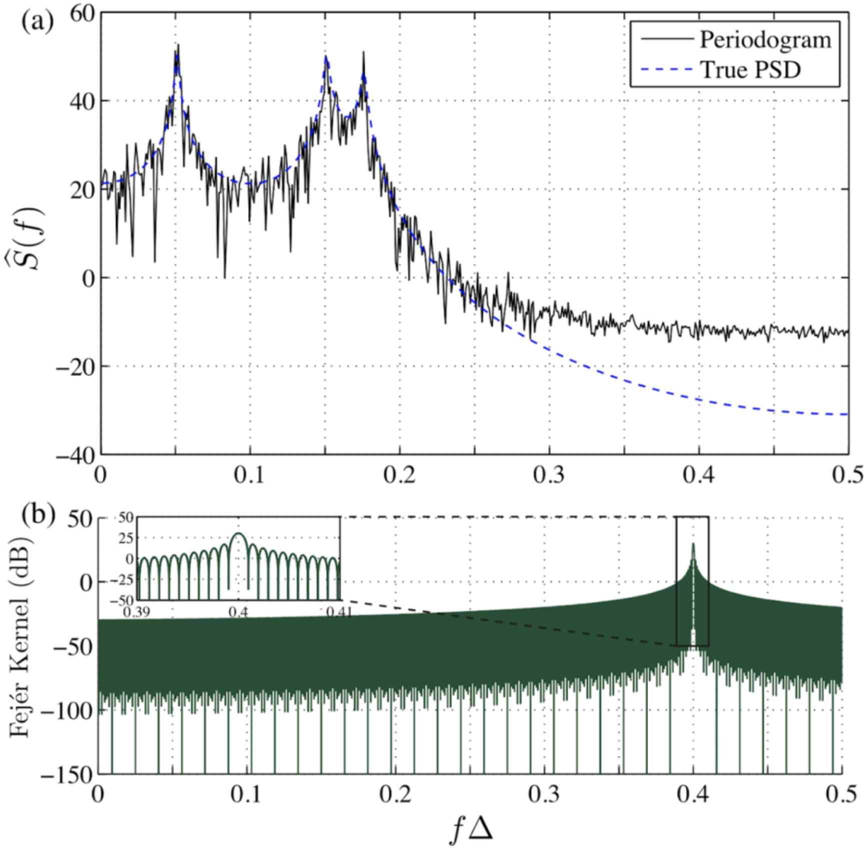
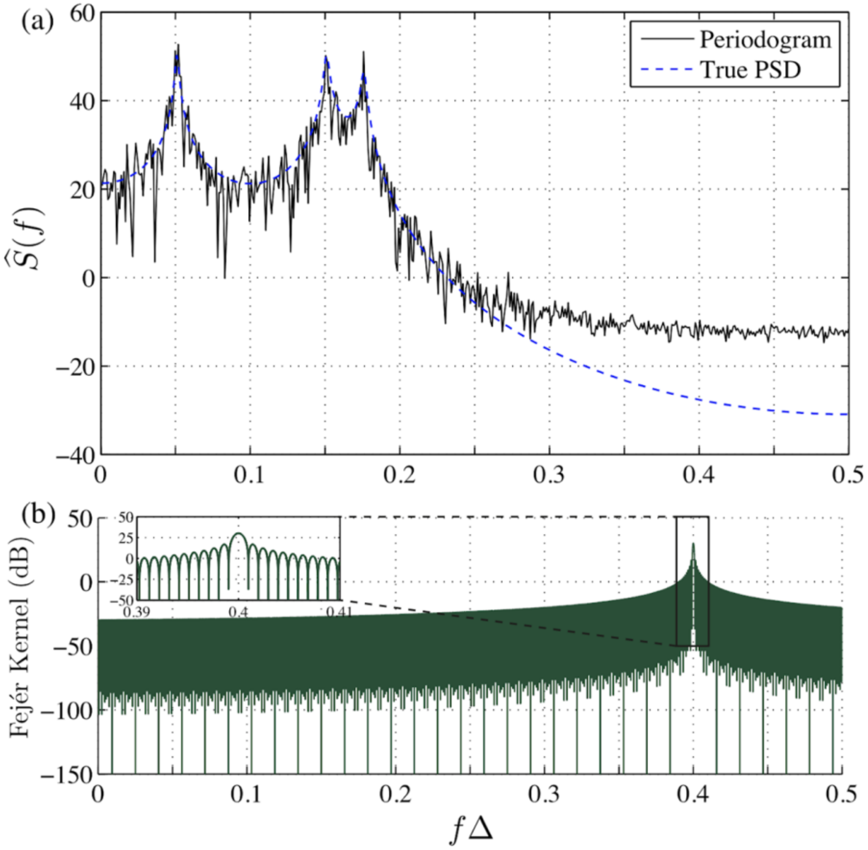


Fig. 2. Periodogram estimate of the PSD of the AR(6) process. (a) Periodogram estimate. (b) Fejér kernel centered at $f\Delta = 0.4$. The zoomed-in view of the main lobe is shown on the top left.

Consequently, the periodogram is asymptotically unbiased but not a consistent estimate.



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Consequently, the periodogram **hits the peaks pretty well**, but is **very spiky throughout the estimate**.

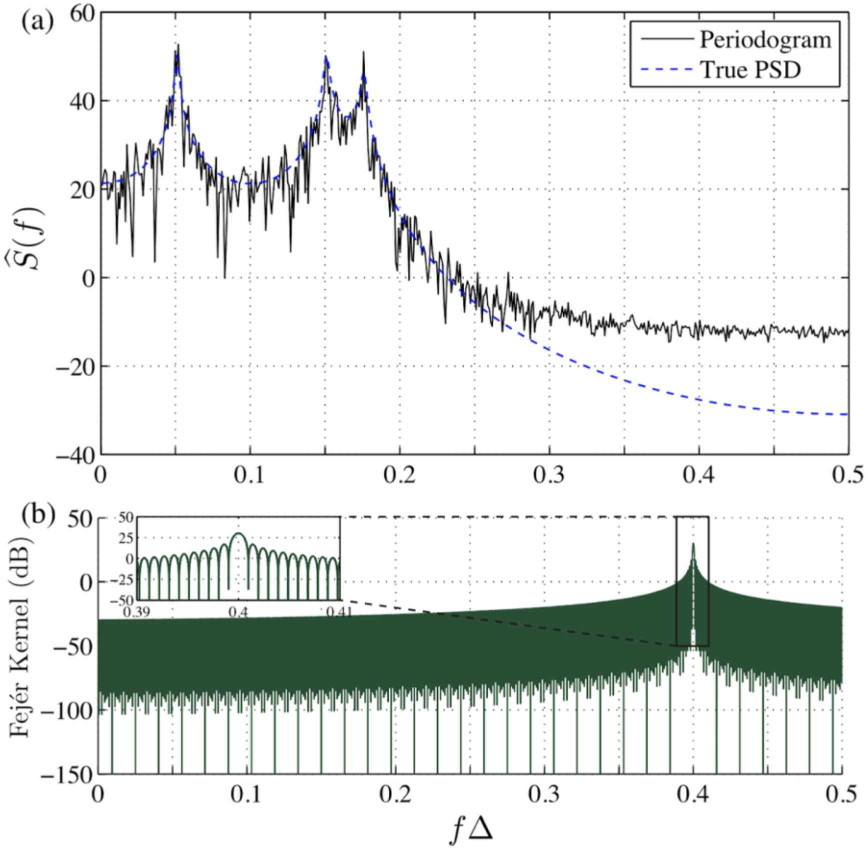

Fig. 2. Periodogram estimate of the PSD of the AR(6) process. (a) Periodogram estimate. (b) Fejér kernel centered at $f\Delta = 0.4$. The zoomed-in view of the main lobe is shown on the top left.

There is a technique known as **multitaper spectral analysis** which is able to tighten up (reduce the variance) the estimate of the power spectrum.

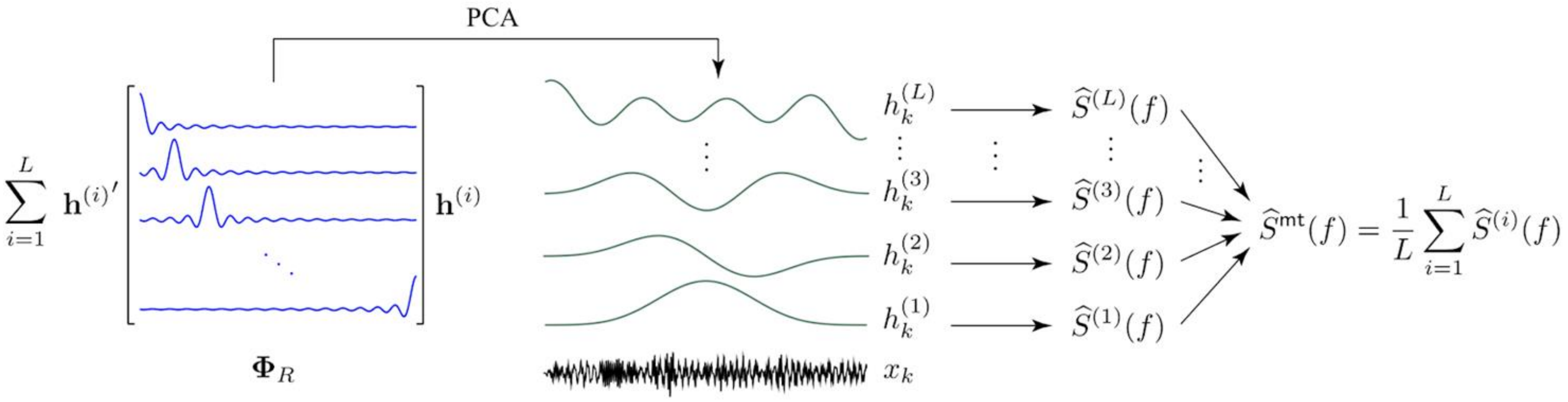

Fig. 4. Schematic depiction of multitaper spectral estimation.

Given L “good” and “uncorrelated” windows, the multitaper spectral estimate formed by averaging the corresponding L tapered estimates would have a variance reduced by a factor L.

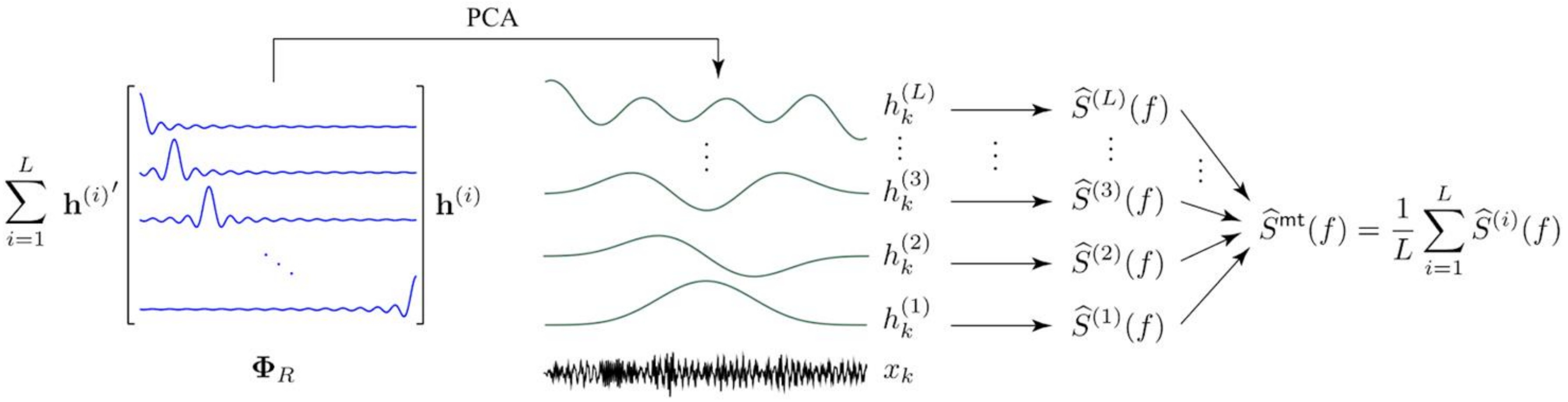

Fig. 4. Schematic depiction of multitaper spectral estimation.

We see a reduction in variance in the resultant spectral estimate.

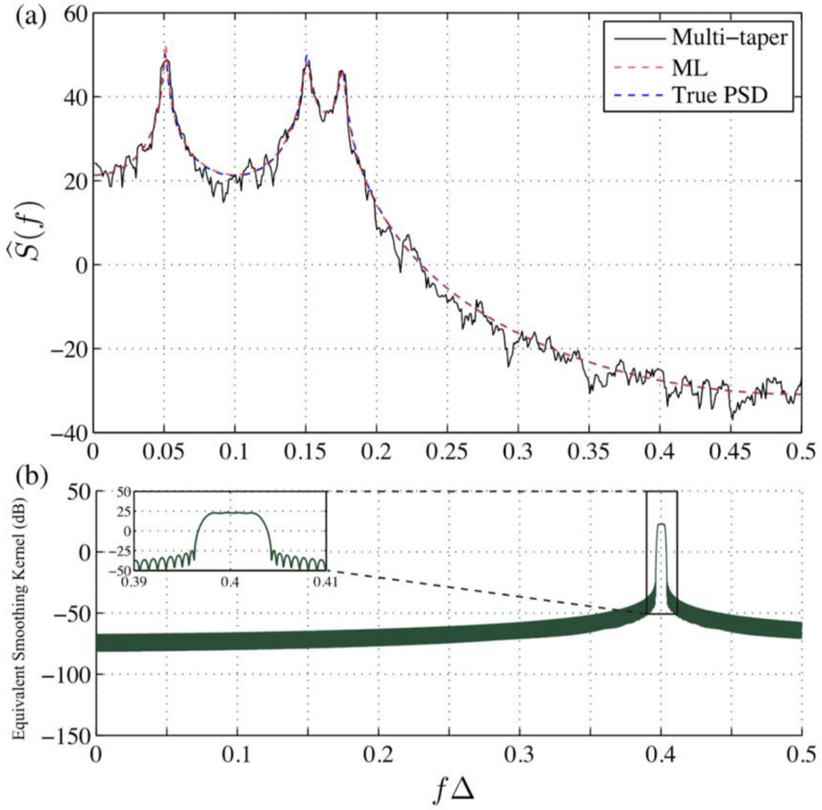

Fig. 6. Multitaper and ML estimates of the PSD of the AR(6) process with $R = 8/N\Delta$ and $L = 4$. (a) the multitaper and ML estimates, (b) the equivalent smoothing kernel centered at $f\Delta = 0.4$. The zoomed-in view of the main lobe is shown on the top left.

We have introduced a concept, multitaper frequency analysis which is complex, but quite straightforward to implement.

Interactive Tutorial

Resources Used

1. Buzsáki, György; Anastassiou, Costas A.; Koch, Christof (2012). The origin of extracellular fields and currents — EEG, ECoG, LFP and spikes. Nature Reviews Neuroscience, 13(6), 407–420. doi:10.1038/nrn3241
2. https://www.mayoclinic.org/tests-procedures/eeg/about/pac-20393875
3. https://battlebornbatteries.com/amps-volts-watts/
4. Britton JW, Frey LC, Hopp JLet al., authors; St. Louis EK, Frey LC, editors. Electroencephalography (EEG): An Introductory Text and Atlas of Normal and Abnormal Findings in Adults, Children, and Infants [Internet]. Chicago: American Epilepsy Society; 2016. Available from: https://www.ncbi.nlm.nih.gov/books/NBK390354/