In [90]:
```
%matplotlib inline
import matplotlib.pyplot as plt
plt.rcParams["figure.figsize"] = (11, 5)  #set default figure size
import numpy as np
from scipy.stats import norm
import copy
import pandas as pd
```

Q1 (a)

In [91]:
```
N = 9
m = 3
mu = 3
beta = 0.96
phi = 1
rho = 0.3
sigma_sq_er = 0.01 * (1-(rho)**2)
sigma_sq_z = sigma_sq_er/(1-rho**2)
zn = m*(sigma_sq_z)**(1/2)
z0 = -zn
s = (zn-z0)/(N-1)
z_grid = np.arange(z0,zn+s,s)
print(z_grid)
b = np.zeros((9,9))
for j in range(N):
    for i in range(N):
        if j==0:
            b[i,j] = norm.cdf((z0-rho*z_grid[i]+s/2)/sigma_sq_er**(1/2))
        elif j==N-1:
            b[i,j] = 1-norm.cdf((zn-rho*z_grid[i]-s/2)/sigma_sq_er**(1/2))
        else:
            b[i,j] = norm.cdf((z_grid[j] - rho*z_grid[i] + s/2)/sigma_sq_er**(1/2))



print(b)
print(np.exp(z_grid))
```

```
[-0.3   -0.225 -0.15  -0.075  0.     0.075  0.15   0.225  0.3  ]
[[3.52805842e-02 1.18091475e-01 2.53397008e-01 3.02191106e-01
  2.00357192e-01 7.37963542e-02 1.50732903e-02 1.70313471e-03
  1.09855133e-04]
 [2.04690973e-02 8.37373046e-02 2.14354009e-01 3.04863406e-01
  2.41062359e-01 1.05927790e-01 2.58286367e-02 3.48664672e-03
  2.70750483e-04]
 [1.13032654e-02 5.63099151e-02 1.71986068e-01 2.91733854e-01
  2.75102216e-01 1.44200358e-01 4.19648527e-02 6.76624640e-03
  6.33224834e-04]
 [5.93671933e-03 3.59079082e-02 1.30879138e-01 2.64803013e-01
  2.97789168e-01 1.86176853e-01 6.46532749e-02 1.24480924e-02
  1.40583226e-03]
 [2.96390859e-03 2.17123197e-02 9.44590725e-02 2.27984995e-01
  3.05759408e-01 2.27984995e-01 9.44590725e-02 2.17123197e-02
  2.96390859e-03]
 [1.40583226e-03 1.24480924e-02 6.46532749e-02 1.86176853e-01
  2.97789168e-01 2.64803013e-01 1.30879138e-01 3.59079082e-02
  5.93671933e-03]
 [6.33224834e-04 6.76624640e-03 4.19648527e-02 1.44200358e-01
  2.75102216e-01 2.91733854e-01 1.71986068e-01 5.63099151e-02
  1.13032654e-02]
```

```
 [2.70750483e-04 3.48664672e-03 2.58286367e-02 1.05927790e-01
  2.41062359e-01 3.04863406e-01 2.14354009e-01 8.37373046e-02
  2.04690973e-02]
 [1.09855133e-04 1.70313471e-03 1.50732903e-02 7.37963542e-02
  2.00357192e-01 3.02191106e-01 2.53397008e-01 1.18091475e-01
  3.52805842e-02]]
[0.74081822 0.79851622 0.86070798 0.92774349 1.         1.07788415
 1.16183424 1.25232272 1.34985881]
```

Q1 (b):

Primeira parte é uma tentativa mais elegante, o segundo bloco é mais 'braçal'

In [92]:

```
a_size=201
valor = np.zeros((1,a_size))
print(valor[:,1])
```

```
[0.]
```

In [93]:

```
# ORDEM DA FUNÇÃO VALOR: LINHA É a, COLUNA É Z
a_size=201
a_max=4
tol = 10**(-6)
r=0.04
V_0 = np.zeros((a_size, N))
grid_a = np.linspace(-phi, a_max, a_size)
print(grid_a)
V_new = copy.deepcopy(V_0)
erro = 1
g = np.zeros((a_size, N))
t= 1
while(erro > tol):
    V_old = copy.deepcopy(V_new)
    t += 1
    for i in range(N):
        probabilidades = b[i,:]
        for j in range(a_size):
            estoque = np.exp(z_grid[i]) + (1+r)*grid_a[j]
            if j == 0:
                start = 0
            valor = np.zeros((1,a_size))
            t = copy.deepcopy(start)
            c_t = (estoque - grid_a[t])
            if c_t >= 0:
                u = ((c_t**(1-mu) - 1)/(1-mu))
            elif c_t < 0:
                u = -1/(1-mu)
            esperanca = 0
            for w in range(N):
                esperanca = (esperanca + probabilidades[w]*V_old[t,w])
            valor[:,t] = (u + beta*esperanca)
            for t in range((start+1), a_size):
                c_t = (estoque - grid_a[t])
                if c_t >= 0:
                    u = ((c_t**(1-mu) - 1)/(1-mu))
                elif c_t < 0:
                    u = -1/(1-mu)
                esperanca = 0
                for w in range(N):
                    esperanca = esperanca + probabilidades[w]*V_old[t,w]
                valor[:,t] = (u + beta*esperanca)
```

```
                if valor[:,t]< valor[:,t-1]:
                    break
            if valor[:,t]< valor[:,t-1]:
                V_new[j,i] = copy.deepcopy(valor[:,t-1])
                indice_a = (t-1)
            elif valor[:,t] >= valor[:,t-1]:
                V_new[j,i] = copy.deepcopy(valor[:,t])
                indice_a = (t)
            start = copy.deepcopy(indice_a)
            g[j,i] = copy.deepcopy(grid_a[indice_a])

    dif = np.abs(V_new - V_old)
    erro = np.max(np.max(dif))
    print(erro)
fig, ax = plt.subplots()
for i in range(N):
    ax.plot(grid_a, V_new[:,i], color=plt.cm.jet(i / N), lw=2, alpha=0.6, label=f'Fu
ax.legend()
ax.set(xlim=(np.min(grid_a), np.max(grid_a)))
plt.show()
fig, ax = plt.subplots()
for i in range(N):
    ax.plot(grid_a, g[:,i], color=plt.cm.tab20c(i / N), lw=2, alpha=0.6, label=f'Fun
ax.legend()
ax.set(xlim=(np.min(grid_a), np.max(grid_a)))
plt.show()
fig, ax = plt.subplots()
for i in range(N):
    ax.plot(grid_a, -g[:,i]+(1+r)*grid_a+np.exp(z_grid[i]), color=plt.cm.tab20b(i /
ax.legend()
ax.set(xlim=(np.min(grid_a), np.max(grid_a)))
plt.show()
```

```
[-1.    -0.975 -0.95  -0.925 -0.9   -0.875 -0.85  -0.825 -0.8   -0.775
 -0.75  -0.725 -0.7   -0.675 -0.65  -0.625 -0.6   -0.575 -0.55  -0.525
 -0.5   -0.475 -0.45  -0.425 -0.4   -0.375 -0.35  -0.325 -0.3   -0.275
 -0.25  -0.225 -0.2   -0.175 -0.15  -0.125 -0.1   -0.075 -0.05  -0.025
  0.     0.025  0.05   0.075  0.1    0.125  0.15   0.175  0.2    0.225
  0.25   0.275  0.3    0.325  0.35   0.375  0.4    0.425  0.45   0.475
  0.5    0.525  0.55   0.575  0.6    0.625  0.65   0.675  0.7    0.725
  0.75   0.775  0.8    0.825  0.85   0.875  0.9    0.925  0.95   0.975
  1.     1.025  1.05   1.075  1.1    1.125  1.15   1.175  1.2    1.225
  1.25   1.275  1.3    1.325  1.35   1.375  1.4    1.425  1.45   1.475
  1.5    1.525  1.55   1.575  1.6    1.625  1.65   1.675  1.7    1.725
  1.75   1.775  1.8    1.825  1.85   1.875  1.9    1.925  1.95   1.975
  2.     2.025  2.05   2.075  2.1    2.125  2.15   2.175  2.2    2.225
  2.25   2.275  2.3    2.325  2.35   2.375  2.4    2.425  2.45   2.475
  2.5    2.525  2.55   2.575  2.6    2.625  2.65   2.675  2.7    2.725
  2.75   2.775  2.8    2.825  2.85   2.875  2.9    2.925  2.95   2.975
  3.     3.025  3.05   3.075  3.1    3.125  3.15   3.175  3.2    3.225
  3.25   3.275  3.3    3.325  3.35   3.375  3.4    3.425  3.45   3.475
  3.5    3.525  3.55   3.575  3.6    3.625  3.65   3.675  3.7    3.725
  3.75   3.775  3.8    3.825  3.85   3.875  3.9    3.925  3.95   3.975
  4.   ]
0.5180268562582369
0.42518755100743716
0.35878265006491916
0.3004793667388097
0.2521028737762654
0.21264477299240392
0.18050947510148374
0.15424131575757105
0.13260982307184577
0.11471134019888618
```

```
0.09970241980523076
0.08706233747664704
0.07645996127429733
0.0670522327599361
0.058802633304807284
0.05172791833855772
0.045674495390779946
0.04046591182639592
0.035896950576258746
0.03193161301771763
0.028489719702883498
0.025448354349642877
0.022765377857134617
0.020454235486778494
0.01833566918116425
0.016472256394512108
0.014840333051740373
0.013358075207032805
0.012145485501385167
0.011615977881410222
0.011111633005694843
0.010630653863937134
0.010172024397829382
0.009734471462357508
0.00931682805081202
0.008918154268453549
0.00853750350596871 3
0.00817394127928317
0.007826641210338714
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0.007177700519972685
0.006874567124824926
0.006584728890987046
0.006307544125082387
0.006042276024054827
0.00578849487682076
0.005545650227058108
0.005313302566031464
0.005091014878799038
0.004878314755881075
0.004674735688236176
0.004479846760780504
0.004293259584241982
0.004114624468190886
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0.0037798209827437645
0.003622993280412601
0.0034728006805606615
0.00332895177452785 5
0.0031911697836481956
0.0030591882859707376
0.002932754302743090 2
0.00281163058507427 6
0.0026955901949137218
0.00258441431095501 9
0.0024778931271067783
0.0023758264530238105
0.0022780232840848846
0.0021843011064341944
0.0020944854804902935
0.0020084098387709304
0.0019259151093542481
0.0018468492620677956
0.001771066897638729
```

```
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0.0015620612068270034
0.0014980826461106211
0.001436750907282569
0.0013779547649752644
0.0013215877634056294
0.001267547997741314
0.0012157379458788142
0.0011660642997102233
0.0011184377746582541
0.0010727729188992896
0.0010289879283389425
0.00098700446747646
0.0009467475066891051
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0.0008711285869642538
0.00083563136615461
0.0008015903768505694
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0.0005516021295073337
0.0005291928636692589
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0.0003497226313153856
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0.0003219533523113327
0.0003089107087310694
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0.0002843959578322749
0.0002728813151187115
0.00026183486382347354
0.00025123747283917197
0.0002410707807808033763
0.0002313171833883132
0.00022195980164796403
0.00021298245171319508
0.000204369615786959
0.00019610641440781507
0.00018817857987052378
0.0001805724307828438
0.0001732748477540902
0.0001662732501177544
0.00015955557359870198
0.00015311024894626435
0.00014692618144751535
0.00014099273129741796
0.00013529969479098014
0.00012983728628546132
0.00012459612090776062
```

```
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0.00010566931205624464
0.00010140649523249223
9.731614832597657e-05
9.339126519414265e-05
8.962512588039928e-05
8.60112848410921e-05
8.25435596614188e-05
7.921602023053254e-05
7.60229783702826e-05
7.295897789916239e-05
7.001878509660386e-05
6.719737957228133e-05
6.448994550511955e-05
6.189186325800122e-05
5.939870132509739e-05
5.700620862292283e-05
5.4710307097138866e-05
5.2507084630670775e-05
5.039278825735849e-05
4.836381763473341e-05
4.641671879990206e-05
4.454817818944079e-05
4.2755016884665764e-05
4.1034185122246214e-05
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3.7797925487748074e-05
3.627699745312185e-05
3.4817389132690835e-05
3.3416621617154973e-05
3.207231658297616e-05
3.078219218854983e-05
2.9544059145347745e-05
2.835581694338174e-05
2.721545023187666e-05
2.6121025353376126e-05
2.507068701218529e-05
2.4062655088030738e-05
2.309522157006 8603e-05
2.216674762900439e-05
2.127566079579 4238e-05
2.0420452267577716e-05
1.9599674315973203e-05
1.8811937805063295e-05
1.805590980907823e-05
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9.369955831228793e-06
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```

```
8.632661987029167e-06
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7.634174114201642e-06
7.327731169182172e-06
7.033602511086201e-06
6.751292572326761e-06
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5.970614338490776e-06
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1.6784432714800346e-06
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1.070227277466529e-06
1.0273353798773144e-06
9.86163428029485e-07
```
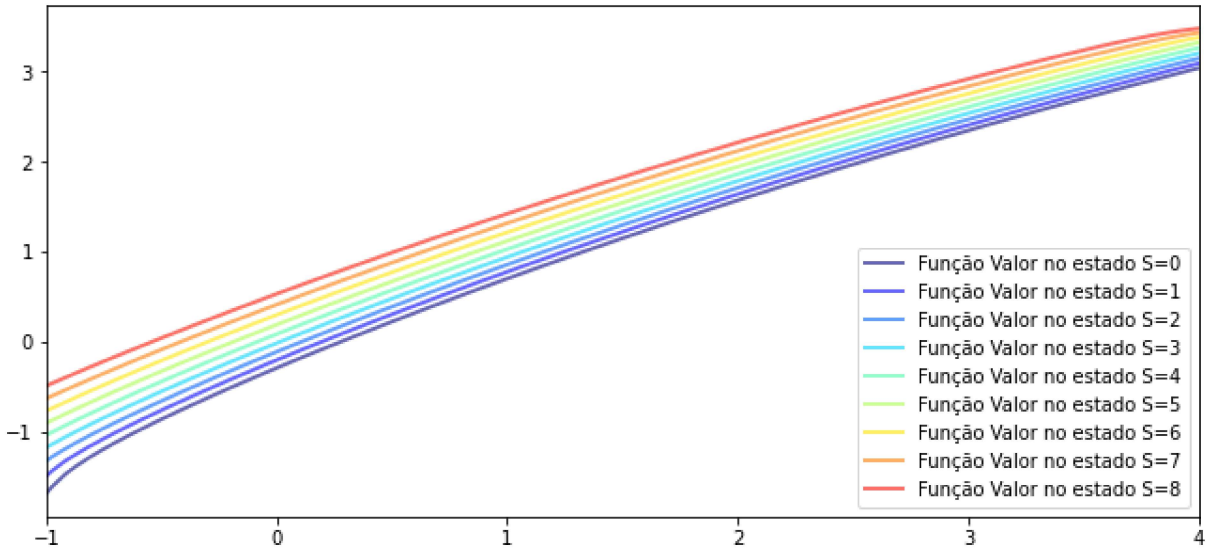
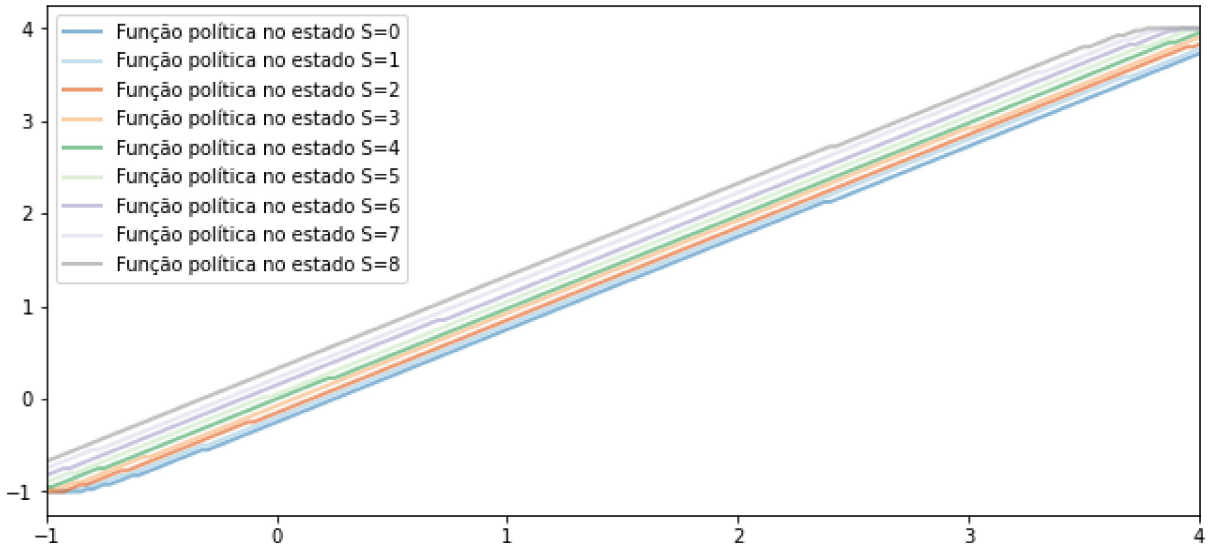
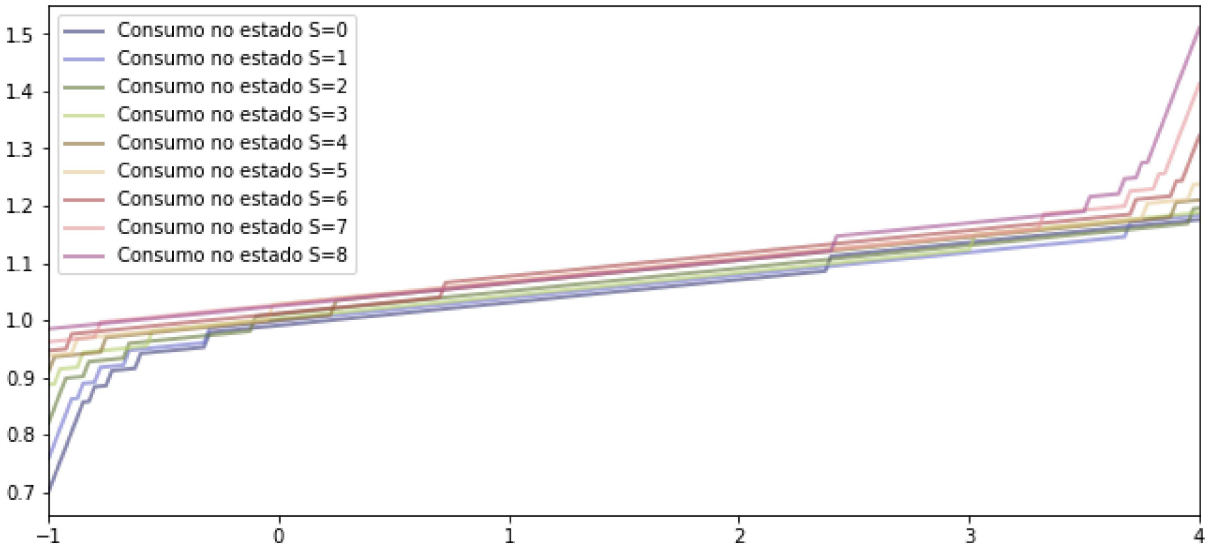







Q1(c)

In [94]:

```
pi_0 = np.full((a_size, N),(1/(N*a_size)))

tol_distrb = 10**(-6)
pi_old = copy.deepcopy(pi_0)
pi_new = copy.deepcopy(pi_0)
pi_dif = np.ones((a_size,N),dtype=float)
```

```
erro = 1
t=0
pi_iter = [pi_0]
while(erro>tol_distrb):
    t += 1
    for i in range(N):
        for j in range(a_size):
            aux = pi_old*(g == grid_a[j])
            pi_new[j,i] = sum(aux)@b[:,i]
            pi_dif[j,i] = (pi_new[j,i]-pi_old[j,i])
            pi_dif[j,i] = np.abs(pi_dif[j,i])
    erro = np.max(pi_dif)
    pi_old = copy.deepcopy(pi_new)

print(f'Convergência após t={t}')
```

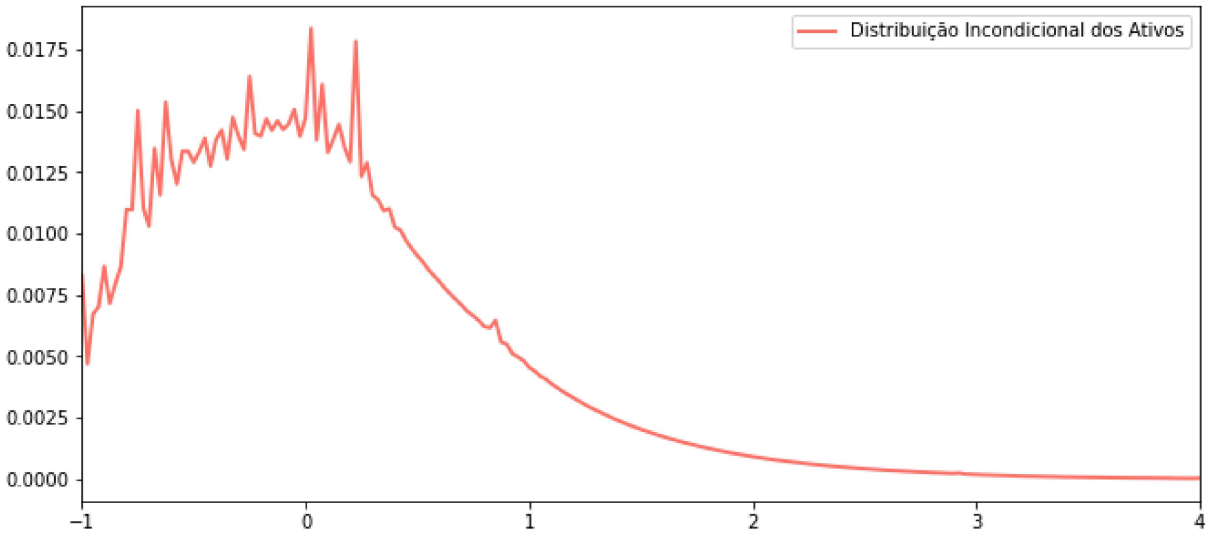
```
Convergência após t=412
```

In [95]:

```
def asset_marginal(pi,a_size,N):
    asset_dist = np.zeros(a_size)
    for i in range(a_size):
        for j in range(N):
            asset_dist[i] += pi[i,j]
    return(asset_dist)
```

In [96]:

```
dist_a = asset_marginal(pi_new,a_size,N)

fig, ax = plt.subplots()
ax.plot(grid_a, dist_a, color=plt.cm.jet(i / N), lw=2, alpha=0.6, label=f'Distribuiç
ax.legend()
ax.set(xlim=(np.min(grid_a), np.max(grid_a)))
plt.show()
```

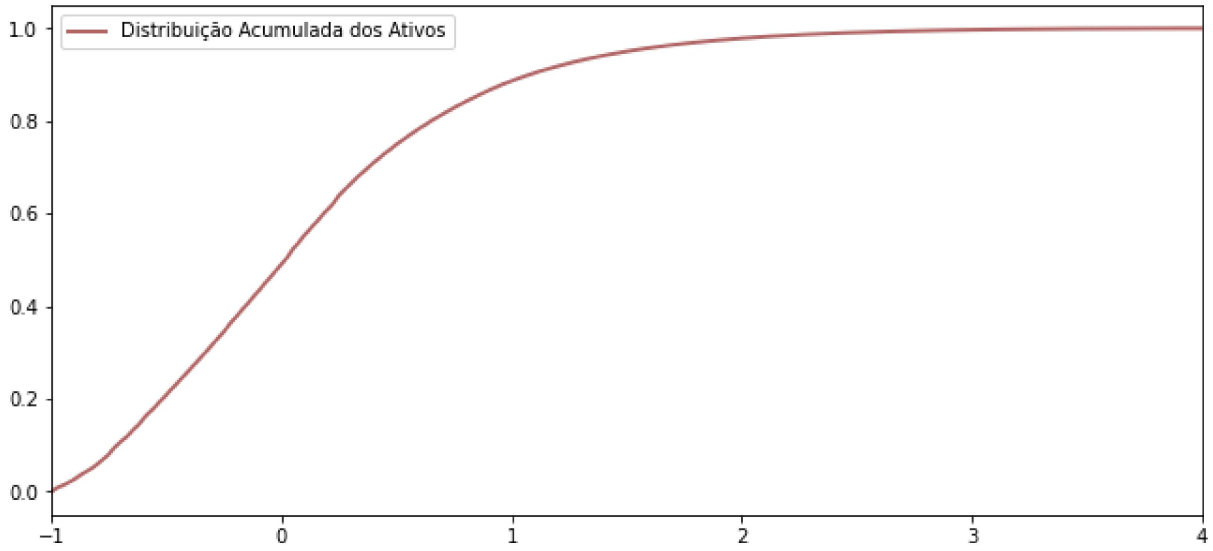


In [97]:

```
cdf_a = np.zeros(a_size)
for i in range(a_size):
    for j in range(i):
        cdf_a[i] += dist_a[j]

print(cdf_a)

fig, ax = plt.subplots()
ax.plot(grid_a, cdf_a, color=plt.cm.jet(i / N), lw=2, alpha=0.6, label=f'Distribuiçã
ax.legend()
```

```
ax.set(xlim=(np.min(grid_a), np.max(grid_a)))
plt.show()

a_inf_0 = cdf_a[np.where(grid_a == 0)]
a_inf_1 = cdf_a[np.where(grid_a == 1)]
a_inf_2 = cdf_a[np.where(grid_a == 2)]
a_inf_3 = cdf_a[np.where(grid_a == 3)]

a_inf_0 = "{:.2%}".format(a_inf_0[0])
a_inf_1 = "{:.2%}".format(a_inf_1[0])
a_sup_2 = "{:.2%}".format(1-a_inf_2[0])
a_sup_3 = "{:.2%}".format(1-a_inf_3[0])


print(f'{a_inf_0} das famílias tem ativos negativos')
print(f'{a_inf_1} das famílias tem ativos entre -1 e 1')
print(f'Apenas {a_sup_2} das famílias tem ativos acima de 2')
print(f'Apenas {a_sup_3} das famílias tem ativos acima de 3')
```

```
[0.         0.00828588 0.01297424 0.01968302 0.02669642 0.0353504
 0.04250101 0.05044726 0.05911495 0.07009476 0.0810659  0.09607398
 0.10711152 0.117413   0.13088067 0.1424515  0.15780271 0.17084835
 0.18286919 0.19622184 0.20958146 0.22248248 0.23581501 0.2497008
 0.26244399 0.27625725 0.29046966 0.30350436 0.3182369  0.33222595
 0.34564855 0.36204287 0.37611225 0.39007476 0.40475052 0.41895041
 0.43354413 0.44778196 0.46224587 0.47730633 0.49126634 0.50597482
 0.52433859 0.53814388 0.55421571 0.5675207  0.58136357 0.59581141
 0.60931295 0.62223131 0.64006355 0.65238672 0.66527111 0.67683697
 0.68820525 0.69912692 0.71013705 0.72039394 0.7305303  0.74024348
 0.74962258 0.75871759 0.767554   0.7760765  0.78434932 0.79237734
 0.80012683 0.80763827 0.81492885 0.82199225 0.82880563 0.83544985
 0.84189696 0.84810065 0.85425189 0.8607086  0.86628543 0.87177388
 0.87687655 0.88184669 0.88666323 0.89120634 0.89559424 0.89977768
 0.90384099 0.9077091  0.9114256  0.91498441 0.91840869 0.92169835
 0.92484845 0.92787355 0.93077469 0.93356622 0.9362422  0.93881195
 0.94127796 0.94364735 0.94592257 0.9481065  0.95020375 0.95221764
 0.9541525  0.95601028 0.95779464 0.95950835 0.96115476 0.96273626
 0.96425543 0.96571484 0.96711695 0.96846414 0.96975848 0.9710021
 0.97219712 0.97334542 0.97444893 0.97550945 0.97652863 0.97750807
 0.97844973 0.97935428 0.98022383 0.98105986 0.98186306 0.98263536
 0.98337905 0.98409225 0.98477783 0.98543677 0.98607023 0.98667916
 0.98726456 0.98782731 0.98836834 0.98888849 0.98938853 0.98986933
 0.99033159 0.99077594 0.99120332 0.99161412 0.99200901 0.99238899
 0.9927541  0.993105   0.99344306 0.99376723 0.99407914 0.99438027
 0.99467044 0.99494788 0.99521743 0.99547047 0.99571648 0.99595619
 0.99617967 0.99640124 0.99663578 0.99682828 0.99701398 0.99718557
 0.99735034 0.99750669 0.99765421 0.99779461 0.99792742 0.99805406
 0.99817404 0.99828801 0.99839612 0.99849874 0.99859638 0.99868883
 0.99877684 0.99886017 0.99893951 0.99901459 0.99908597 0.9991536
 0.9992185  0.99927962 0.99933974 0.99939429 0.99944629 0.99949493
 0.99954082 0.99958498 0.99962612 0.99966631 0.99970272 0.99973736
 0.99977033 0.99981001 0.99984367 0.99987391 0.99989586 0.99991729
 0.99993373 0.99995126 0.99996953]
```




```
49.13% das famílias tem ativos negativos
88.67% das famílias tem ativos entre -1 e 1
Apenas 2.16% das famílias tem ativos acima de 2
Apenas 0.30% das famílias tem ativos acima de 3
```

Q1 (d)

In [98]:

```
# CALCULAR DEMANDA POR CAPITAL

demanda = 0

for i in range(N):
    for j in range(a_size):
        demanda += g[j,i]*pi_new[j,i]


print(demanda)
```

```
0.10567905266217047
```

Q1 (e)

i.

In [99]:

```
N = 9
m = 3
mu = 5
beta = 0.96
phi = 1
rho = 0.3
sigma_sq_er = 0.01 * (1-(rho)**2)
sigma_sq_z = sigma_sq_er/(1-rho**2)
zn = m*(sigma_sq_z)**(1/2)
z0 = -zn
s = (zn-z0)/(N-1)
z_grid = np.arange(z0,zn+s,s)
b = np.zeros((9,9))
for j in range(N):
    for i in range(N):
        if j==0:
            b[i,j] = norm.cdf((z0-rho*z_grid[i]+s/2)/sigma_sq_er**(1/2))
        elif j==N-1:
```

```
            b[i,j] = 1-norm.cdf((zn-rho*z_grid[i]-s/2)/sigma_sq_er**(1/2))
        else:
            b[i,j] = norm.cdf((z_grid[j] - rho*z_grid[i] + s/2)/sigma_sq_er**(1/2))

a_size=201
valor = np.zeros((1,a_size))
# ORDEM DA FUNÇÃO VALOR: LINHA É a, COLUNA É Z
a_size=201
a_max=4
tol = 10**(-6)
r=0.04
V_0 = np.zeros((a_size, N))
grid_a = np.linspace(-phi, a_max, a_size)
V_new = copy.deepcopy(V_0)
erro = 1
g = np.zeros((a_size, N))
t= 1
while(erro > tol):
    V_old = copy.deepcopy(V_new)
    t += 1
    for i in range(N):
        probabilidades = b[i,:]
        for j in range(a_size):
            estoque = np.exp(z_grid[i]) + (1+r)*grid_a[j]
            if j == 0:
                start = 0
            valor = np.zeros((1,a_size))
            t = copy.deepcopy(start)
            c_t = (estoque - grid_a[t])
            if c_t >= 0:
                u = ((c_t**(1-mu) - 1)/(1-mu))
            elif c_t < 0:
                u = -1/(1-mu)
            esperanca = 0
            for w in range(N):
                esperanca = (esperanca + probabilidades[w]*V_old[t,w])
            valor[:,t] = (u + beta*esperanca)
            for t in range((start+1), a_size):
                c_t = (estoque - grid_a[t])
                if c_t >= 0:
                    u = ((c_t**(1-mu) - 1)/(1-mu))
                elif c_t < 0:
                    u = -1/(1-mu)
                esperanca = 0
                for w in range(N):
                    esperanca = esperanca + probabilidades[w]*V_old[t,w]
                valor[:,t] = (u + beta*esperanca)
                if valor[:,t]< valor[:,t-1]:
                    break
            if valor[:,t]< valor[:,t-1]:
                V_new[j,i] = copy.deepcopy(valor[:,t-1])
                indice_a = (t-1)
            elif valor[:,t] >= valor[:,t-1]:
                V_new[j,i] = copy.deepcopy(valor[:,t])
                indice_a = (t)
            start = copy.deepcopy(indice_a)
            g[j,i] = copy.deepcopy(grid_a[indice_a])

    dif = np.abs(V_new - V_old)
    erro = np.max(np.max(dif))
pi_0 = np.full((a_size, N),(1/(N*a_size)))

tol_distrb = 10**(-6)
pi_old = copy.deepcopy(pi_0)
```

```
pi_new = copy.deepcopy(pi_0)
pi_dif = np.ones((a_size,N),dtype=float)
erro = 1
t=0
pi_iter = [pi_0]
while(erro>tol_distrb):
    t += 1
    for i in range(N):
        for j in range(a_size):
            aux = pi_old*(g == grid_a[j])
            pi_new[j,i] = sum(aux)@b[:,i]
            pi_dif[j,i] = (pi_new[j,i]-pi_old[j,i])
            pi_dif[j,i] = np.abs(pi_dif[j,i])
    erro = np.max(pi_dif)
    pi_old = copy.deepcopy(pi_new)
# CALCULAR DEMANDA POR CAPITAL

demanda2 = 0

for i in range(N):
    for j in range(a_size):
        demanda2 += g[j,i]*pi_new[j,i]

if demanda2>demanda:
    sinal = 'um aumento'
else:
    sinal = 'uma diminuição'

print(f'Nosso excesso de demanda é de {demanda2}, {sinal} de {abs(demanda2-demanda)}
```

```
Nosso excesso de demanda é de 1.193686529578574, um aumento de 1.0880074769164034. O
riginalmente tínhamos 0.10567905266217047.
```

ii.

In [ ]:

```
N = 9
m = 3
mu = 3
beta = 0.96
phi = 1
rho = 0.6
sigma_sq_er = 0.01 * (1-(rho)**2)
sigma_sq_z = sigma_sq_er/(1-rho**2)
zn = m*(sigma_sq_z)**(1/2)
z0 = -zn
s = (zn-z0)/(N-1)
z_grid = np.arange(z0,zn+s,s)
b = np.zeros((9,9))
for j in range(N):
    for i in range(N):
        if j==0:
            b[i,j] = norm.cdf((z0-rho*z_grid[i]+s/2)/sigma_sq_er**(1/2))
        elif j==N-1:
            b[i,j] = 1-norm.cdf((zn-rho*z_grid[i]-s/2)/sigma_sq_er**(1/2))
        else:
            b[i,j] = norm.cdf((z_grid[j] - rho*z_grid[i] + s/2)/sigma_sq_er**(1/2))

a_size=201
valor = np.zeros((1,a_size))
# ORDEM DA FUNÇÃO VALOR: LINHA É a, COLUNA É Z
a_size=201
a_max=4
tol = 10**(-6)
```

```
r=0.04
V_0 = np.zeros((a_size, N))
grid_a = np.linspace(-phi, a_max, a_size)
V_new = copy.deepcopy(V_0)
erro = 1
g = np.zeros((a_size, N))
t= 1
while(erro > tol):
    V_old = copy.deepcopy(V_new)
    t += 1
    for i in range(N):
        probabilidades = b[i,:]
        for j in range(a_size):
            estoque = np.exp(z_grid[i]) + (1+r)*grid_a[j]
            if j == 0:
                start = 0
            valor = np.zeros((1,a_size))
            t = copy.deepcopy(start)
            c_t = (estoque - grid_a[t])
            if c_t >= 0:
                u = ((c_t**(1-mu) - 1)/(1-mu))
            elif c_t < 0:
                u = -1/(1-mu)
            esperanca = 0
            for w in range(N):
                esperanca = (esperanca + probabilidades[w]*V_old[t,w])
            valor[:,t] = (u + beta*esperanca)
            for t in range((start+1), a_size):
                c_t = (estoque - grid_a[t])
                if c_t >= 0:
                    u = ((c_t**(1-mu) - 1)/(1-mu))
                elif c_t < 0:
                    u = -1/(1-mu)
                esperanca = 0
                for w in range(N):
                    esperanca = esperanca + probabilidades[w]*V_old[t,w]
                valor[:,t] = (u + beta*esperanca)
                if valor[:,t]< valor[:,t-1]:
                    break
            if valor[:,t]< valor[:,t-1]:
                V_new[j,i] = copy.deepcopy(valor[:,t-1])
                indice_a = (t-1)
            elif valor[:,t] >= valor[:,t-1]:
                V_new[j,i] = copy.deepcopy(valor[:,t])
                indice_a = (t)
            start = copy.deepcopy(indice_a)
            g[j,i] = copy.deepcopy(grid_a[indice_a])

    dif = np.abs(V_new - V_old)
    erro = np.max(np.max(dif))
pi_0 = np.full((a_size, N),(1/(N*a_size)))

tol_distrb = 10**(-6)
pi_old = copy.deepcopy(pi_0)
pi_new = copy.deepcopy(pi_0)
pi_dif = np.ones((a_size,N),dtype=float)
erro = 1
t=0
pi_iter = [pi_0]
while(erro>tol_distrb):
    t += 1
    for i in range(N):
        for j in range(a_size):
            aux = pi_old*(g == grid_a[j])
```

```
            pi_new[j,i] = sum(aux)@b[:,i]
            pi_dif[j,i] = (pi_new[j,i]-pi_old[j,i])
            pi_dif[j,i] = np.abs(pi_dif[j,i])
    erro = np.max(pi_dif)
    pi_old = copy.deepcopy(pi_new)
# CALCULAR DEMANDA POR CAPITAL

demanda3 = 0

for i in range(N):
    for j in range(a_size):
        demanda3 += g[j,i]*pi_new[j,i]

if demanda3>demanda:
    sinal = 'um aumento'
else:
    sinal = 'uma diminuição'

print(f'Nosso excesso de demanda é de {demanda3}, {sinal} de {abs(demanda3-demanda)}
```

iii.

In [ ]:

```
N = 9
m = 3
mu = 3
beta = 0.96
phi = 1
rho = 0.3
sigma_sq_er = 0.04 * (1-(rho)**2)
sigma_sq_z = sigma_sq_er/(1-rho**2)
zn = m*(sigma_sq_z)**(1/2)
z0 = -zn
s = (zn-z0)/(N-1)
z_grid = np.arange(z0,zn+s,s)
b = np.zeros((9,9))
for j in range(N):
    for i in range(N):
        if j==0:
            b[i,j] = norm.cdf((z0-rho*z_grid[i]+s/2)/sigma_sq_er**(1/2))
        elif j==N-1:
            b[i,j] = 1-norm.cdf((zn-rho*z_grid[i]-s/2)/sigma_sq_er**(1/2))
        else:
            b[i,j] = norm.cdf((z_grid[j] - rho*z_grid[i] + s/2)/sigma_sq_er**(1/2))

a_size=201
valor = np.zeros((1,a_size))
# ORDEM DA FUNÇÃO VALOR: LINHA É a, COLUNA É Z
a_size=201
a_max=4
tol = 10**(-6)
r=0.04
V_0 = np.zeros((a_size, N))
grid_a = np.linspace(-phi, a_max, a_size)
V_new = copy.deepcopy(V_0)
erro = 1
g = np.zeros((a_size, N))
t= 1
while(erro > tol):
    V_old = copy.deepcopy(V_new)
    t += 1
    for i in range(N):
        probabilidades = b[i,:]
```

```
        for j in range(a_size):
            estoque = np.exp(z_grid[i]) + (1+r)*grid_a[j]
            if j == 0:
                start = 0
            valor = np.zeros((1,a_size))
            t = copy.deepcopy(start)
            c_t = (estoque - grid_a[t])
            if c_t >= 0:
                u = ((c_t**(1-mu) - 1)/(1-mu))
            elif c_t < 0:
                u = -1/(1-mu)
            esperanca = 0
            for w in range(N):
                esperanca = (esperanca + probabilidades[w]*V_old[t,w])
            valor[:,t] = (u + beta*esperanca)
            for t in range((start+1), a_size):
                c_t = (estoque - grid_a[t])
                if c_t >= 0:
                    u = ((c_t**(1-mu) - 1)/(1-mu))
                elif c_t < 0:
                    u = -1/(1-mu)
                esperanca = 0
                for w in range(N):
                    esperanca = esperanca + probabilidades[w]*V_old[t,w]
                valor[:,t] = (u + beta*esperanca)
                if valor[:,t]< valor[:,t-1]:
                    break
            if valor[:,t]< valor[:,t-1]:
                V_new[j,i] = copy.deepcopy(valor[:,t-1])
                indice_a = (t-1)
            elif valor[:,t] >= valor[:,t-1]:
                V_new[j,i] = copy.deepcopy(valor[:,t])
                indice_a = (t)
            start = copy.deepcopy(indice_a)
            g[j,i] = copy.deepcopy(grid_a[indice_a])

    dif = np.abs(V_new - V_old)
    erro = np.max(np.max(dif))
pi_0 = np.full((a_size, N),(1/(N*a_size)))

tol_distrb = 10**(-6)
pi_old = copy.deepcopy(pi_0)
pi_new = copy.deepcopy(pi_0)
pi_dif = np.ones((a_size,N),dtype=float)
erro = 1
t=0
pi_iter = [pi_0]
while(erro>tol_distrb):
    t += 1
    for i in range(N):
        for j in range(a_size):
            aux = pi_old*(g == grid_a[j])
            pi_new[j,i] = sum(aux)@b[:,i]
            pi_dif[j,i] = (pi_new[j,i]-pi_old[j,i])
            pi_dif[j,i] = np.abs(pi_dif[j,i])
    erro = np.max(pi_dif)
    pi_old = copy.deepcopy(pi_new)
# CALCULAR DEMANDA POR CAPITAL

demanda4 = 0

for i in range(N):
    for j in range(a_size):
        demanda4 += g[j,i]*pi_new[j,i]
```

```
if demanda4>demanda:
    sinal = 'um aumento'
else:
    sinal = 'uma diminuição'

print(f'Nosso excesso de demanda é de {demanda4}, {sinal} de {abs(demanda4-demanda)}
```

Q1 (bonus)

In [ ]:

```
N = 9
m = 3
mu = 3
beta = 0.96
phi = 1
rho = 0.3
sigma_sq_er = 0.01 * (1-(rho)**2)
sigma_sq_z = sigma_sq_er/(1-rho**2)
```

In [ ]:

```
# Repetindo o código da questão 1 (pois ele não vai mudar a cada loop)
zn = m*(sigma_sq_z)**(1/2)
z0 = -zn
s = (zn-z0)/(N-1)
z_grid = np.arange(z0,zn+s,s)
b = np.zeros((9,9))
for j in range(N):
    for i in range(N):
        if j==0:
            b[i,j] = norm.cdf((z0-rho*z_grid[i]+s/2)/sigma_sq_er**(1/2))
        elif j==N-1:
            b[i,j] = 1-norm.cdf((zn-rho*z_grid[i]-s/2)/sigma_sq_er**(1/2))
        else:
            b[i,j] = norm.cdf((z_grid[j] - rho*z_grid[i] + s/2)/sigma_sq_er**(1/2))

a_size=201
a_max=4
tol = 10**(-6)
grid_a = np.linspace(-phi, a_max, a_size)
```

In [ ]:

```
R_inf = 0
R_sup = 1.04 #1+r = R

demanda_inf = -1
demanda_sup = copy.deepcopy(demanda)

# Vamos agora fazer o loop para o método da bissecção
cont = 0
demanda_abs=1
while (demanda_abs>10**(-3)):
    r = (-demanda_inf*R_sup + demanda_sup*R_inf)/(demanda_sup-demanda_inf) - 1

    V_0 = np.zeros((a_size, N))
    V_new = copy.deepcopy(V_0)
    erro = 1
    g = np.zeros((a_size, N))
```

```
t= 1
while(erro > tol):
    V_old = copy.deepcopy(V_new)
    t += 1
    for i in range(N):
        probabilidades = b[i,:]
        for j in range(a_size):
            estoque = np.exp(z_grid[i]) + (1+r)*grid_a[j]
            if j == 0:
                start = 0
            valor = np.zeros((1,a_size))
            t = copy.deepcopy(start)
            c_t = (estoque - grid_a[t])
            if c_t >= 0:
                u = ((c_t**(1-mu) - 1)/(1-mu))
            elif c_t < 0:
                u = -1/(1-mu)
            esperanca = 0
            for w in range(N):
                esperanca = (esperanca + probabilidades[w]*V_old[t,w])
            valor[:,t] = (u + beta*esperanca)
            for t in range((start+1), a_size):
                c_t = (estoque - grid_a[t])
                if c_t >= 0:
                    u = ((c_t**(1-mu) - 1)/(1-mu))
                elif c_t < 0:
                    u = -1/(1-mu)
                esperanca = 0
                for w in range(N):
                    esperanca = esperanca + probabilidades[w]*V_old[t,w]
                valor[:,t] = (u + beta*esperanca)
                if valor[:,t]< valor[:,t-1]:
                    break
            if valor[:,t]< valor[:,t-1]:
                V_new[j,i] = copy.deepcopy(valor[:,t-1])
                indice_a = (t-1)
            elif valor[:,t] >= valor[:,t-1]:
                V_new[j,i] = copy.deepcopy(valor[:,t])
                indice_a = (t)
            start = copy.deepcopy(indice_a)
            g[j,i] = copy.deepcopy(grid_a[indice_a])

    dif = np.abs(V_new - V_old)
    erro = np.max(np.max(dif))
pi_0 = np.full((a_size, N),(1/(N*a_size)))

tol_distrb = 10**(-6)
pi_old = copy.deepcopy(pi_0)
pi_new = copy.deepcopy(pi_0)
pi_dif = np.ones((a_size,N),dtype=float)
erro = 1
t=0
pi_iter = [pi_0]
while(erro>tol_distrb):
    t += 1
    for i in range(N):
        for j in range(a_size):
            aux = pi_old*(g == grid_a[j])
            pi_new[j,i] = sum(aux)@b[:,i]
            pi_dif[j,i] = (pi_new[j,i]-pi_old[j,i])
            pi_dif[j,i] = np.abs(pi_dif[j,i])
    erro = np.max(pi_dif)
    pi_old = copy.deepcopy(pi_new)
# CALCULAR DEMANDA POR CAPITAL
```

```
    demanda_new = 0
    cont += 1

    for i in range(N):
        for j in range(a_size):
            demanda_new += g[j,i]*pi_new[j,i]
    demanda_abs=abs(copy.deepcopy(demanda_new))
    if demanda_new>0:
        R_sup = copy.deepcopy(1+r)
        demanda_sup = copy.deepcopy(demanda_new)
        print(f'{cont})r = {r}, o excesso de de demanda é de {demanda_sup}')
    elif demanda_new<0:
        R_inf = copy.deepcopy(1+r)
        demanda_inf = copy.deepcopy(demanda_new)
        print(f'{cont})r = {r}, o excesso de de demanda é de {demanda_inf}')

print(f'A taxa de juros que equilibra o mercado é {r}')
print(demanda_new)
```

Q2

In [100…

```
N = 9
m = 3
mu = 3
beta = 0.96
phi = 1
rho = 0.3
sigma_sq_er = 0.01 * (1-(rho)**2)
sigma_sq_z = sigma_sq_er/(1-rho**2)
zn = m*(sigma_sq_z)**(1/2)
z0 = -zn
s = (zn-z0)/(N-1)
z_grid = np.arange(z0,zn+s,s)
print(z_grid)
b = np.zeros((9,9))
for j in range(N):
    for i in range(N):
        if j==0:
            b[i,j] = norm.cdf((z0-rho*z_grid[i]+s/2)/sigma_sq_er**(1/2))
        elif j==N-1:
            b[i,j] = 1-norm.cdf((zn-rho*z_grid[i]-s/2)/sigma_sq_er**(1/2))
        else:
            b[i,j] = norm.cdf((z_grid[j] - rho*z_grid[i] + s/2)/sigma_sq_er**(1/2))
```

```
[-0.3   -0.225 -0.15  -0.075  0.     0.075  0.15   0.225  0.3  ]
```

In [101…

```
# Fazendo a calibração da demografia
data = pd.read_excel('LifeTable.xlsx')

# Vamos eliminar as variáveis Death probability e Life expatancy
data.drop(['Death probability - male', 'Life expectancy - male','Death  probability

# Retirando os individuos com mais de 87 anos e menos de 18
data.drop(range(88,120),axis = 0, inplace = True)
data.drop(range(18),axis = 0, inplace = True)


# Combinando o numero de homens e mulheres vivos
```

```
sum_col = data["Number of lives - male"]+data["Number of lives - male"]
data["Number"] = sum_col


data.drop(["Number of lives - male","Number of lives - female"], axis = 1, inplace =


# Vamos definir nossa medida
measure = data.iloc[0:45,1].sum()/45
print(measure)

# Vamos agora preencher o vetor surv, que no dá a probabilidade de continuar vivo ao
# por definição, para todo t menor ou igual a 8, st = 1
surv = np.ones(14)
s = np.ones(14)
for i in range(9,13):
    s[i] = data.iloc[(i*5+6):(5*i+10),1].sum()/(5*measure)
for j in range(9,13):
    surv[j] = s[j]/s[j-1]
surv[13] = 0

print(surv)
```

```
188.22853333333333
[1.         1.         1.         1.         1.         1.
 1.         1.         1.         0.61257875 0.8681616  0.79595457
 0.68210388 0.        ]
```

In [102…

```
a_size=201
a_max=4
a_min=-1
phi1 = -9.5
phi2 = 11.6
r = 0.04
grid_a = np.linspace(-phi, a_max, a_size)

psi = np.zeros(a_size)
def psi(i):
    psi = phi1 *(1 + grid_a[i]/phi2)**(1-mu)
    return(psi)
```

In [103…

```
tol = 10**(-6)
previdencia =0.4
# Gerando a função valor e a função política dos aposentados
V_ret = np.zeros((a_size, 5))
a_ret = np.zeros((a_size, 5))

# Preenchendo a ultima coluna de V_ret

for i in range(a_size):
    estoque = copy.deepcopy((1+r)*grid_a[i]+previdencia)
    if i==0:
        start = copy.deepcopy(0)
    valor = np.zeros((1,a_size))
    c = (estoque - grid_a[start])
    if c>=0:
        u_c = ((c**(1-mu) - 1)/(1-mu))
    elif c < 0:
        u_c = -1/(1-mu)
    valor[:,(start)] = u_c + beta*psi(start)
    for j in range((start+1),a_size):
```

```
        c = (estoque - grid_a[j])
        if c >= 0:
            u_c = ((c**(1-mu) - 1)/(1-mu))
        elif c < 0:
            u_c = -1/(1-mu)
        valor[:,j] = u_c + beta*psi(j)
        if valor[:,j]<valor[:,(j-1)]:
            break
    if valor[:,j]<valor[:,(j-1)]:
        V_ret[i,4] = copy.deepcopy(valor[:,(j-1)])
        indice_a = copy.deepcopy(j-1)
    elif valor[:,j]>=valor[:,(j-1)]:
        V_ret[i,4] = copy.deepcopy(valor[:,j])
        indice_a = copy.deepcopy(j)
    start=copy.deepcopy(indice_a)
    a_ret[i,4] = copy.deepcopy(grid_a[indice_a])
print(a_ret[:,4])
```

```
[-1.    -1.    -1.    -1.    -1.    -1.    -1.    -1.    -1.    -1.
 -1.    -1.    -1.    -1.    -1.    -1.    -1.    -0.975 -0.95  -0.925
 -0.9   -0.9   -0.875 -0.85  -0.825 -0.8   -0.775 -0.75  -0.725 -0.7
 -0.675 -0.65  -0.625 -0.6   -0.575 -0.55  -0.525 -0.5   -0.475 -0.45
 -0.425 -0.4   -0.375 -0.35  -0.325 -0.3   -0.275 -0.25  -0.225 -0.2
 -0.175 -0.15  -0.15  -0.125 -0.1   -0.075 -0.05  -0.025  0.     0.025
  0.05   0.075  0.1    0.125  0.15   0.175  0.2    0.225  0.25   0.275
  0.3    0.325  0.35   0.375  0.4    0.425  0.45   0.475  0.5    0.525
  0.55   0.575  0.6    0.625  0.625  0.65   0.675  0.7    0.725  0.75
  0.775  0.8    0.825  0.85   0.875  0.9    0.925  0.95   0.975  1.
  1.025  1.05   1.075  1.1    1.125  1.15   1.175  1.2    1.225  1.25
  1.275  1.3    1.325  1.35   1.375  1.375  1.4    1.425  1.45   1.475
  1.5    1.525  1.55   1.575  1.6    1.625  1.65   1.675  1.7    1.725
  1.75   1.775  1.8    1.825  1.85   1.875  1.9    1.925  1.95   1.975
  2.     2.025  2.05   2.075  2.1    2.125  2.125  2.15   2.175  2.2
  2.225  2.25   2.275  2.3    2.325  2.35   2.375  2.4    2.425  2.45
  2.475  2.5    2.525  2.55   2.575  2.6    2.625  2.65   2.675  2.7
  2.725  2.75   2.775  2.8    2.825  2.85   2.875  2.9    2.9    2.925
  2.95   2.975  3.     3.025  3.05   3.075  3.1    3.125  3.15   3.175
  3.2    3.225  3.25   3.275  3.3    3.325  3.35   3.375  3.4    3.425
  3.45 ]
```

In [104…

```
# Preenchendo os demais valores
for t in range(1,5):
    for i in range(a_size):
        estoque = copy.deepcopy((1+r)*grid_a[i]+previdencia)
        if i==0:
            start = copy.deepcopy(0)
        valor = np.zeros((1,a_size))
        c = (estoque - grid_a[start])
        if c>=0:
            u_c = ((c**(1-mu) - 1)/(1-mu))
        elif c < 0:
            u_c = -1/(1-mu)
        valor[:,(start)] = (u_c + beta*(surv[13-t]*V_ret[(start),(5-t)]+(1-surv[13-t
        for j in range((start+1),a_size):
            c = (estoque - grid_a[j])
            if c >= 0:
                u_c = ((c**(1-mu) - 1)/(1-mu))
            elif c < 0:
                u_c = -1/(1-mu)
            valor[:,j] = u_c + beta*(surv[13-t]*V_ret[j,(5-t)]+(1-surv[13-t])*psi(j)
            if valor[:,j]<valor[:,(j-1)]:
                break
        if valor[:,j]<valor[:,(j-1)]:
```

```
            V_ret[i,(4-t)] = copy.deepcopy(valor[:,(j-1)])
            indice_a = copy.deepcopy(j-1)
        elif valor[:,j]>=valor[:,(j-1)]:
            V_ret[i,(4-t)] = copy.deepcopy(valor[:,j])
            indice_a = copy.deepcopy(j)
        start=copy.deepcopy(indice_a)
        a_ret[i,(4-t)] = copy.deepcopy(grid_a[indice_a])

print(a_ret[:,0])

print(V_ret[:,2])
```

```
[-1.    -1.    -1.    -1.    -0.975 -0.95  -0.95  -0.925 -0.9   -0.875
 -0.875 -0.85  -0.825 -0.8   -0.8   -0.775 -0.75  -0.725 -0.7   -0.7
 -0.675 -0.65  -0.625 -0.6   -0.6   -0.575 -0.55  -0.525 -0.5   -0.5
 -0.475 -0.45  -0.425 -0.4   -0.4   -0.375 -0.35  -0.325 -0.3   -0.275
 -0.25  -0.25  -0.225 -0.2   -0.175 -0.15  -0.125 -0.125 -0.1   -0.075
 -0.05  -0.025  0.     0.025  0.025  0.05   0.075  0.1    0.125  0.15
  0.15   0.175  0.2    0.225  0.25   0.275  0.275  0.3    0.325  0.35
  0.375  0.4    0.425  0.45   0.475  0.5    0.5    0.525  0.55   0.575
  0.6    0.625  0.65   0.675  0.7    0.725  0.75   0.775  0.8    0.825
  0.85   0.875  0.875  0.9    0.925  0.95   0.975  1.     1.025  1.05
  1.075  1.1    1.125  1.15   1.175  1.2    1.225  1.25   1.275  1.3
  1.325  1.35   1.375  1.4    1.425  1.45   1.475  1.5    1.525  1.55
  1.575  1.6    1.625  1.65   1.675  1.7    1.725  1.75   1.75   1.775
  1.8    1.825  1.85   1.875  1.9    1.925  1.95   1.975  2.     2.025
  2.05   2.075  2.1    2.125  2.15   2.175  2.2    2.225  2.25   2.275
  2.3    2.325  2.35   2.375  2.4    2.425  2.45   2.475  2.5    2.525
  2.55   2.575  2.6    2.6    2.625  2.65   2.675  2.7    2.725  2.75
  2.775  2.8    2.825  2.85   2.875  2.9    2.925  2.95   2.975  3.
  3.025  3.05   3.075  3.1    3.125  3.15   3.175  3.2    3.225  3.25
  3.275  3.3    3.325  3.35   3.375  3.4    3.425  3.45   3.475  3.475
  3.5  ]
[-17.95064759 -17.44841768 -17.03686034 -16.62975077 -16.2917781
 -15.95720179 -15.66052231 -15.38118407 -15.10471376 -14.85818305
 -14.62433794 -14.39325845 -14.18543699 -13.98743593 -13.79232887
 -13.61487485 -13.4455036  -13.27927242 -13.12598662 -12.9797639
 -12.83697946 -12.7031769  -12.57587774 -12.45232932 -12.33441594
 -12.2227412  -12.11512256 -12.01030374 -11.91164683 -11.81728945
 -11.72340883 -11.63568822 -11.55054246 -11.4677932  -11.38933034
 -11.31197639 -11.23866585 -11.16762775 -11.09738114 -11.031814
 -10.9668248  -10.90331129 -10.84273398 -10.78462545 -10.72692518
 -10.67068107 -10.61779886 -10.56517548 -10.51340237 -10.46363638
 -10.41419877 -10.36508659 -10.31629694 -10.26775178 -10.21951908
 -10.17160152 -10.12399633 -10.07670073 -10.02971201  -9.98302745
  -9.93664439  -9.89056019  -9.8446984   -9.79909918  -9.75379254
  -9.70877592  -9.66404681  -9.61960272  -9.57544118  -9.53155976
  -9.48795605  -9.44453531  -9.40136113  -9.35845919  -9.31582716
  -9.27346274  -9.23136366  -9.18952765  -9.14795249  -9.10663598
  -9.06557593  -9.02477019  -8.98421661  -8.94391309  -8.90385753
  -8.86404787  -8.82445415  -8.78506931  -8.74592509  -8.70701951
  -8.6683506   -8.62991641  -8.591715    -8.55374445  -8.51600289
  -8.47848843  -8.44114111  -8.40399549  -8.36707252  -8.33037037
  -8.29388727  -8.25762143  -8.2215711   -8.18573454  -8.15011004
  -8.11466674  -8.07937025  -8.04428191  -8.00940005  -7.97472303
  -7.94024922  -7.90597699  -7.87190476  -7.83803094  -7.80435396
  -7.77087228  -7.73758435  -7.70448865  -7.67158369  -7.63886796
  -7.60629372  -7.57390529  -7.5417023   -7.50968332  -7.47784691
  -7.44619166  -7.41471619  -7.38341909  -7.352299    -7.32135456
  -7.29053786  -7.2598789   -7.22939235  -7.19907689  -7.16893123
  -7.13895407  -7.10914413  -7.07950016  -7.05002089  -7.02070509
  -6.99149003  -6.9624274   -6.93352542  -6.90478287  -6.87619856
  -6.84777132  -6.81949998  -6.79138336  -6.76342033  -6.73560975
  -6.70795048  -6.68044142  -6.65308145  -6.62585078  -6.59874877
```

```
-6.57179303  -6.5449825   -6.51831611  -6.49179281  -6.46541156
-6.43917134  -6.41307111  -6.38710986  -6.36128659  -6.33556241
-6.30996353  -6.28450023  -6.25917154  -6.23397649  -6.20891412
-6.18398348  -6.15918365  -6.13451368  -6.10997266  -6.08553627
-6.06119495  -6.03698048  -6.01289195  -5.98892848  -5.96508919
-5.9413732   -5.91777966  -5.89430769  -5.87095646  -5.84772512
-5.82461284  -5.80161798  -5.77871174  -5.75592242  -5.73324921
-5.71069132  -5.68824794  -5.6659183   -5.6437016   -5.62159707
-5.59960394  -5.57772146  -5.55594886  -5.53425415  -5.51266035
-5.49117463]
```

In [105…

```
# Gerando a função valor e a função política dos agentes que já receberam herança
V_after = np.zeros((a_size, N, 9))
g_after = np.zeros((a_size, N, 9))

# Preenchendo o ultimo período (que é dado em função de V_ret)
for i in range(N):
    for j in range(a_size):
        estoque = np.exp(z_grid[i]) + (1+r)*grid_a[j]
        if j == 0:
            start = copy.deepcopy(0)
        valor = np.zeros((1,a_size))
        c_t = (estoque - grid_a[start])
        if c_t >= 0:
            u = ((c_t**(1-mu) - 1)/(1-mu))
        elif c_t < 0:
            u = -1/(1-mu)
        valor[:,(start)] = u + beta*V_ret[(start),0]
        for k in range((start+1), a_size):
            c_k = (estoque - grid_a[k])
            if c_k >= 0:
                u = ((c_k**(1-mu) - 1)/(1-mu))
            elif c_k < 0:
                u = -1/(1-mu)
            valor[:,k] = u + beta*(surv[9]*V_ret[k,0]+(1-surv[9])*psi(k))
            if valor[:,k]< valor[:,k-1]:
                break
        if valor[:,k]< valor[:,k-1]:
            V_after[j,i,8] = copy.deepcopy(valor[:,k-1])
            indice_a = copy.deepcopy(k-1)
        elif valor[:,k] >= valor[:,k-1]:
            V_after[j,i,8] = copy.deepcopy(valor[:,k])
            indice_a = copy.deepcopy(k)
        start = copy.deepcopy(indice_a)
        g_after[j,i,8] = copy.deepcopy(grid_a[indice_a])

for t in range(1,9):
    for i in range(N):
        probabilidades = b[i,:]
        for j in range(a_size):
            estoque = np.exp(z_grid[i]) + (1+r)*grid_a[j]
            if j == 0:
                start = copy.deepcopy(0)
            valor = np.zeros((1,a_size))
            c_t = (estoque - grid_a[start])
            if c_t >= 0:
                u = ((c_t**(1-mu) - 1)/(1-mu))
            elif c_t < 0:
                u = -1/(1-mu)
            esperanca = 0
            for w in range(N):
                esperanca += probabilidades[w]*V_after[(start),w,(9-t)]
            valor[:,(start)] = u + beta*esperanca
```

```
            for k in range((start+1), a_size):
                c_k = (estoque - grid_a[k])
                if c_k >= 0:
                    u = ((c_k**(1-mu) - 1)/(1-mu))
                elif c_k < 0:
                    u = -1/(1-mu)
                esperanca = 0
                for w in range(N):
                    esperanca += probabilidades[w]*V_after[k,w,(9-t)]
                valor[:,k] = u + beta*esperanca
                if valor[:,k]< valor[:,k-1]:
                    break
            if valor[:,k]< valor[:,k-1]:
                V_after[j,i,(8-t)] = copy.deepcopy(valor[:,k-1])
                indice_a = copy.deepcopy(k-1)
            elif valor[:,k] >= valor[:,k-1]:
                V_after[j,i,(8-t)] = copy.deepcopy(valor[:,k])
                indice_a = copy.deepcopy(k)
            start = copy.deepcopy(indice_a)
            g_after[j,i,(8-t)] = copy.deepcopy(grid_a[indice_a])
```

In [106…

```
# Desejamos a distribuição de ativos de acordo com a idade
dist_et_a = np.zeros((a_size,14))

# todo agente nasce com zero ativos, portanto
dist_et_a[40,0] = 1

# Desse ponto em diante, precisaremos estimar uma distribuição inical dos ativos, o
# distribuição uniforme nos ativos para os demais perídos até sua aposentadoria.
# depois, utilizaremos a função política dos aposentados para obter uma estimativa i
for t in range(1,9):
    for j in range(a_size):
        dist_et_a[j,t]=dist_a[j]

for t in range(9,14):
    aux = np.zeros(a_size)
    for k in range(a_size):
        for j in range(a_size):
            if a_ret[k,(t-9)]==grid_a[j]:
                aux[j]+=dist_et_a[k,(t-1)]
    for j in range(a_size):
        dist_et_a[j,t] = aux[j]
```

In [107…

```
# Primeiro, vamos criar uma distribuição invariante nos estados, pois isso será impo
# conjunta dos estados e ativos para os agentes que acabaram de nascer
dist_0 = np.full( N,(1/N))
tol_distrb = 10**(-6)
dist_old = copy.deepcopy(dist_0)
dist_s = copy.deepcopy(dist_0)
dist_diff = np.ones(N)
erro=1
while(erro>tol_distrb):
    t += 1
    for i in range(N):
        dist_s[i] = dist_old@b[:,i]
        dist_diff[i] = (dist_s[i]-dist_old[i])
        dist_diff[i] = np.abs(dist_diff[i])
    erro = np.max(dist_diff)
    dist_old = copy.deepcopy(dist_s)
```

```
# Criando a distribuição conjunta inicial
pi_0 = np.zeros((a_size, N))
a_zero = np.where(grid_a==0)
for i in range(N):
    pi_0[a_zero, i] = dist_s[i]
```

In [108…

```
# Gerando a função valor e a função política dos agentes que ainda não receberam her
V_before = np.zeros((a_size, N, 9))
g_before = np.zeros((a_size, N, 9))
pi_old = copy.deepcopy(pi_0)
pi_new = copy.deepcopy(pi_0)
dist_et_a2 = copy.deepcopy(dist_et_a)
tol = 10**(-2)
erro = 1
cont = 0
# Preenchendo o ultimo período (que é dado em função de V_ret)
while erro>=tol:
    print(cont)
    cont += 1
    dist_et_a = copy.deepcopy(dist_et_a2)
    for i in range(N):
        for j in range(a_size):
            estoque = np.exp(z_grid[i]) + (1+r)*grid_a[j]
            if j == 0:
                start = copy.deepcopy(0)
            valor = np.zeros((1,a_size))
            c_t = (estoque - grid_a[start])
            if c_t >= 0:
                u = ((c_t**(1-mu) - 1)/(1-mu))
            elif c_t < 0:
                u = -1/(1-mu)
            esperanca2 = 0
            for w in range(a_size):
                esperanca2 += dist_et_a[w,13]*V_ret[max(0,min((start+w),(a_size-1)))
            valor[:,(start)] = u + beta*(surv[13]*V_ret[(start),0]+(1-surv[13])*espe
            for k in range((start+1), a_size):
                c_k = (estoque - grid_a[k])
                if c_k >= 0:
                    u = ((c_k**(1-mu) - 1)/(1-mu))
                elif c_k < 0:
                    u = -1/(1-mu)
                esperanca2 = 0
                for w in range(a_size):
                    esperanca2 += dist_et_a[w,13]*V_ret[max(0,min((k+w),(a_size-1)))
                valor[:,k] = u + beta*(surv[13]*V_ret[k,0]+(1-surv[13])*esperanca2)
                if valor[:,k]< valor[:,k-1]:
                    break
            if valor[:,k]< valor[:,k-1]:
                V_before[j,i,8] = copy.deepcopy(valor[:,k-1])
                indice_a = copy.deepcopy(k-1)
            elif valor[:,k] >= valor[:,k-1]:
                V_before[j,i,8] = copy.deepcopy(valor[:,k])
                indice_a = copy.deepcopy(k)
            start = copy.deepcopy(indice_a)
            g_before[j,i,8] = copy.deepcopy(grid_a[indice_a])


    for t in range(1,9):
        for i in range(N):
            probabilidades = b[i,:]
            for j in range(a_size):
                estoque = np.exp(z_grid[i]) + (1+r)*grid_a[j]
```

```
                if j == 0:
                    start = copy.deepcopy(0)
                valor = np.zeros((1,a_size))
                c_t = (estoque - grid_a[start])
                if c_t >= 0:
                    u = ((c_t**(1-mu) - 1)/(1-mu))
                elif c_t < 0:
                    u = -1/(1-mu)
                esperanca1 = 0
                for w in range(N):
                    esperanca1 += probabilidades[w]*V_after[(start),w,(9-t)]
                esperanca2 = 0
                for w in range(N):
                    for v in range(a_size):
                        esperanca2 += probabilidades[w]*dist_et_a[v,13-t]*V_after[ma
                valor[:,(start)] = u + beta*(surv[13-t]*esperanca1+(1-surv[13-t])*es
                for k in range((start+1), a_size):
                    c_k = (estoque - grid_a[k])
                    if c_k >= 0:
                        u = ((c_k**(1-mu) - 1)/(1-mu))
                    elif c_k < 0:
                        u = -1/(1-mu)
                    esperanca1 = 0
                    for w in range(N):
                        esperanca1 += probabilidades[w]*V_after[k,w,(9-t)]
                    esperanca2 = 0
                    for w in range(N):
                        for v in range(a_size):
                            esperanca2 += probabilidades[w]*dist_et_a[v,13-t]*V_afte
                    valor[:,k] = u + beta*(surv[13-t]*esperanca1+(1-surv[13-t])*espe
                    if valor[:,k]< valor[:,k-1]:
                        break
                if valor[:,k]< valor[:,k-1]:
                    V_before[j,i,(8-t)] = copy.deepcopy(valor[:,k-1])
                    indice_a = copy.deepcopy(k-1)
                elif valor[:,k] >= valor[:,k-1]:
                    V_before[j,i,(8-t)] = copy.deepcopy(valor[:,k])
                    indice_a = copy.deepcopy(k)
                start = copy.deepcopy(indice_a)
                g_before[j,i,(8-t)] = copy.deepcopy(grid_a[indice_a])

    for t in range(9):
        g_beforeiter = copy.deepcopy(g_before[:,:,t])
        g_afteriter = copy.deepcopy(g_after[:,:,t])
        for i in range(N):
            for j in range(a_size):
                aux1 = pi_old*(g_beforeiter == grid_a[j])
                aux2 = pi_old*(g_afteriter == grid_a[j])
                pi_new[j,i] = surv[t+5]*sum(aux1)@b[:,i] + (1 -surv[t+5])*sum(aux2)@
        d = np.zeros(a_size)

        for j in range(a_size):
            for i in range(N):
                d[j]+=pi_new[j,i]
        dist_et_a2[:,t+1] =copy.deepcopy(d)
        pi_old = copy.deepcopy(pi_new)
    for t in range(9,14):
        aux = np.zeros(a_size)
        for k in range(a_size):
            for j in range(a_size):
                if a_ret[k,(t-9)]==grid_a[j]:
                    aux[j]+=dist_et_a2[k,(t-1)]
        for j in range(a_size):
            dist_et_a2[j,t] = aux[j]
```

```
    dif = np.abs(dist_et_a2 - dist_et_a)
    erro = np.max(np.max(dif))
    print(erro)
```

```
0
0.4153146301144736
1
0.4741148565611896
2
0.23043302520902806
3
0.6587135118469519
4
0.10249090346034724
5
0.3520356425389905
6
0.27072205532069044
7
0.5854950793234475
8
0.5940337901668192
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0.3323163462722032
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0.25843652166494313
11
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12
0.5131068383380989
13
0.26367873934662367
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0.5753769136129688
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0.5756896714466606
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0.2637053275597126
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0.35536068465986465
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0.21571158772643623
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---------------------------------------------------------------------------
KeyboardInterrupt                         Traceback (most recent call last)
~\AppData\Local\Temp/ipykernel_2776/2972832650.py in <module>
     57                 if j == 0:
     58                     start = copy.deepcopy(0)
---> 59                 valor = np.zeros((1,a_size))
     60                 c_t = (estoque - grid_a[start])
     61                 if c_t >= 0:

KeyboardInterrupt:
```

In [ ]:

```
    for t in range(9):
        g_beforeiter = copy.deepcopy(g_before[:,:,t])
        for i in range(N):
            for j in range(a_size):
                aux = pi_old*(g_beforeiter == grid_a[j])
                pi_new[j,i] = sum(aux)@b[:,i]
        d = np.zeros(a_size)

        for j in range(a_size):
            for i in range(N):
                d[j]+=pi_new[j,i]
        dist_et_a[:,t+1] =copy.deepcopy(d)
        pi_old = copy.deepcopy(pi_new)
    for t in range(9,14):
        aux = np.zeros(a_size)
        for k in range(a_size):
            for j in range(a_size):
                if a_ret[k,(t-9)]==grid_a[j]:
                    aux[j]+=dist_et_a[k,(t-1)]
        for j in range(a_size):
            dist_et_a[j,t] = aux[j]
```

In [ ]:

```
print(g_before[:,4,8])
print(g_after[:,4,8])
```

In [ ]:

```
a_size = 201
test_a = np.full((a_size,14),1/a_size)
test_b = copy.deepcopy(test_a)

test_b[87,9] = 1.28/a_size
test_b[32,9] += -0.8*test_b[87,9]
test_b[91,9] += -0.2*test_b[87,9]

test_dif = np.abs(test_b - test_a)
print(test_dif[:,9])
```

```
test_erro = np.max(np.max(test_dif))
print(test_erro)
```

In [ ]: