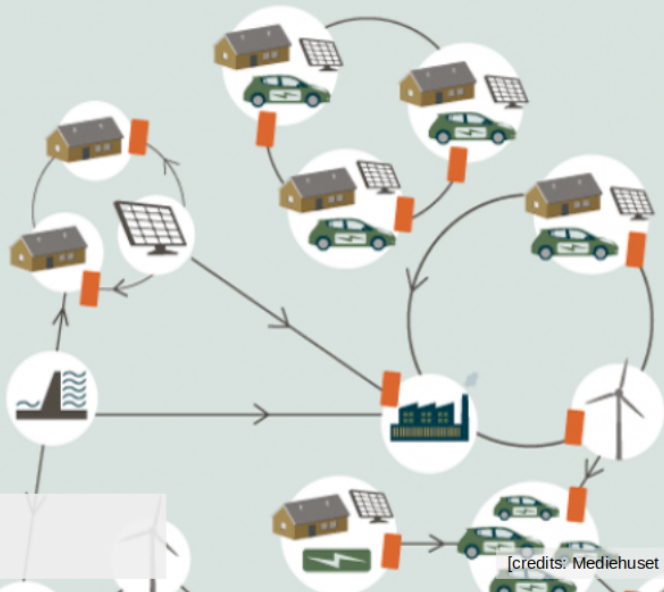


# Module 2 – Electricity Spot Markets (e.g. day-ahead)

## 2.4 Zonal and network aspects

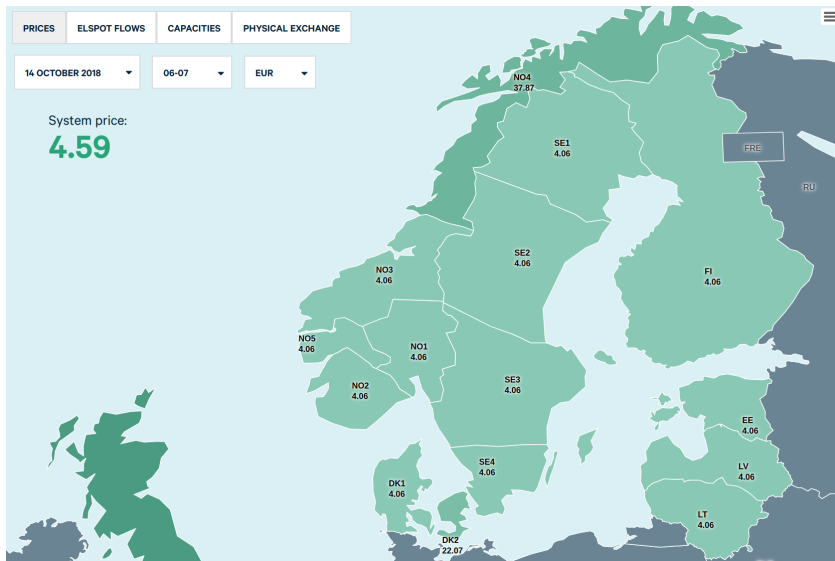


**Pierre Pinson**  
Technical University of Denmark

[credits: Mediehuset Ingeniøren]

# Prices may vary geographically

- Remember there is a network involved, and power has to flow...
- This was not accounted for so far!



- PRICES   ELSPOT FLOWS   CAPACITIES   PHYSICAL EXCHANGE

14 OCTOBER 2018   06-07   EUR

System price:  
**4.59**

The map illustrates the physical exchange of electricity across Europe. Arrows indicate the direction and volume of power flows between different regions. For example, significant flows are shown from SE1 to SE2, from SE2 to SE3, and from SE3 to SE4. Other flows are shown from NO4 to SE1, from NO3 to NO2, and from NO2 to DK1. The system price is 4.59.

## Approaches to handling exchange capacity limitations

- There are basically two philosophies, developed on both sides of the Atlantic Ocean, i.e., in Europe and the USA

	Europe	US
System Operator	TSO	ISO
Market Operator	Ind. Market Operator	ISO
Offers	Market products	Unit capabilities
Clearing	Supply-demand equilibrium	UCED problem
Network representation	Highly simplified	Fairly detailed
Prices	<b>Zonal</b>	<b>Nodal</b>

TSO: Transmission System Operator

ISO: Independent System Operator

UCED: Unit Commitment and Economic Dispatch

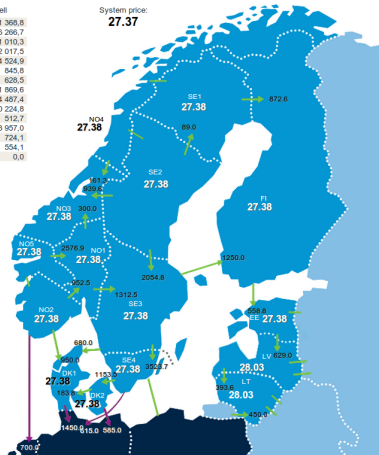
# Illustration of zonal and nodal pricing

## Scandinavia (Zonal):

### Elspot volumes

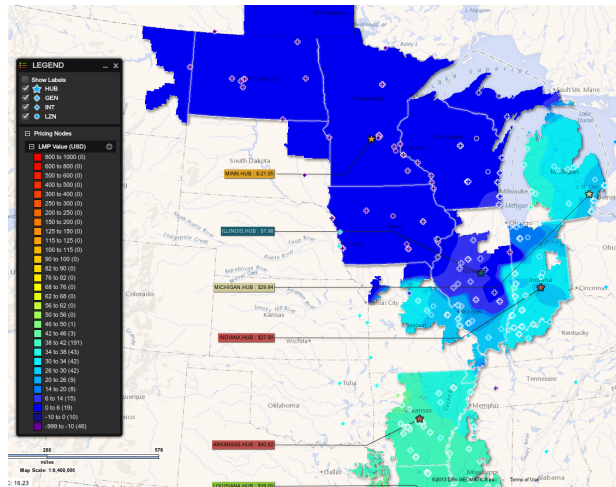
	Buy	Sell
NO1	3 285,7	1 368,8
NO2	4 364,2	6 266,7
NO3	2 411,2	1 010,3
NO4	1 856,2	2 017,5
NO5	1 948,0	4 524,9
DK1	2 659,6	845,8
DK2	1 596,2	628,5
SE1	1 066,0	1 869,6
SE2	1 404,0	4 487,4
SE3	8 136,4	10 224,8
SE4	2 882,9	512,7
FI	5 520,8	3 957,0
EE	853,9	724,1
LT	947,7	554,1
LV	235,4	0,0

System price:  
**27.37**



Go visit: <http://nordpoolgroup.com>

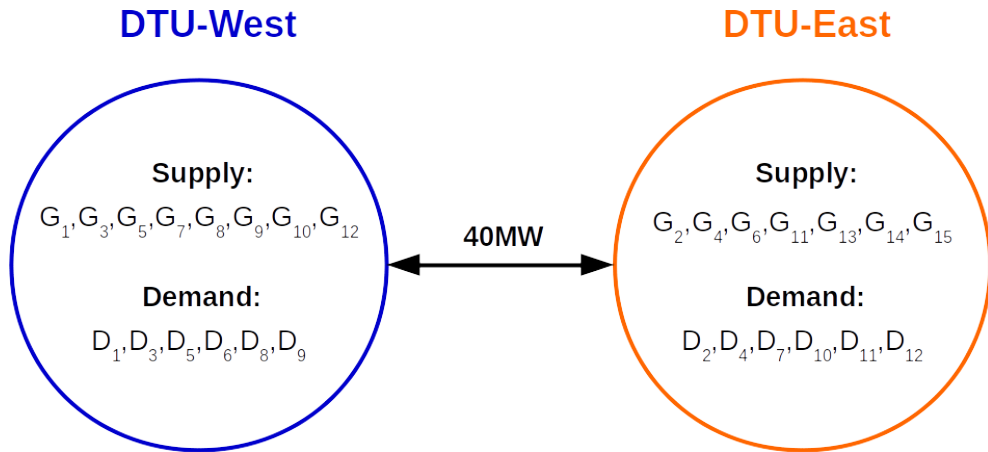
## Midwest US (Nodal):



Go visit: <https://www.misoenergy.org>

## From system price to area prices

- Let us revisit our previous market clearing example,
  - considering two areas **DTU-West** and **DTU-East**, and
  - with a transmission capacity of **40 MW** (so, only 40MWh can flow)



*Demand:* (for a total of 1065 MWh)

Company	id	Amount (MWh)	Price (€/MWh)	Area
CleanRetail	D <sub>1</sub>	250	200	DTU-West
El4You	D <sub>2</sub>	300	110	DTU-East
EVcharge	D <sub>3</sub>	120	100	DTU-West
QualiWatt	D <sub>4</sub>	80	90	DTU-East
IntelliWatt	D <sub>5</sub>	40	85	DTU-West
El4You	D <sub>6</sub>	70	75	DTU-West
CleanRetail	D <sub>7</sub>	60	65	DTU-East
IntelliWatt	D <sub>8</sub>	45	40	DTU-West
QualiWatt	D <sub>9</sub>	30	38	DTU-West
IntelliWatt	D <sub>10</sub>	35	31	DTU-East
CleanRetail	D <sub>11</sub>	25	24	DTU-East
El4You	D <sub>12</sub>	10	16	DTU-East

## And on the supply side

*Supply:* (for a total of 1435 MWh)

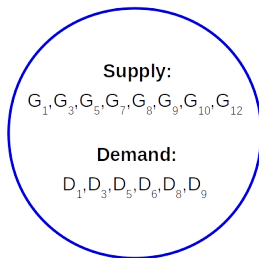
Company	id	Amount (MWh)	Price (€/MWh)	Area
RT <sup>®</sup>	G <sub>1</sub>	120	0	DTU-West
WeTrustInWind	G <sub>2</sub>	50	0	DTU-East
BlueHydro	G <sub>3</sub>	200	15	DTU-West
RT <sup>®</sup>	G <sub>4</sub>	400	30	DTU-East
KøbenhavnCHP	G <sub>5</sub>	60	32.5	DTU-West
KøbenhavnCHP	G <sub>6</sub>	50	34	DTU-East
KøbenhavnCHP	G <sub>7</sub>	60	36	DTU-West
DirtyPower	G <sub>8</sub>	100	37.5	DTU-West
DirtyPower	G <sub>9</sub>	70	39	DTU-West
DirtyPower	G <sub>10</sub>	50	40	DTU-West
RT <sup>®</sup>	G <sub>11</sub>	70	60	DTU-East
RT <sup>®</sup>	G <sub>12</sub>	45	70	DTU-West
SafePeak	G <sub>13</sub>	50	100	DTU-East
SafePeak	G <sub>14</sub>	60	150	DTU-East
SafePeak	G <sub>15</sub>	50	200	DTU-East



# Localizing the previous market-clearing results

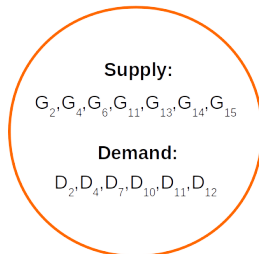
- Following previous market clearing results, one obtains

## DTU-West



- Supply side:  $\{G_1, G_3, G_5, G_7, G_8\}$  (but only 55 MWh for  $G_8$ ) - Total: 495 MWh
- Demand side:  $\{D_1, D_3, D_5, D_6, D_8, D_9\}$  - Total: 555 MWh  
 → Deficit of **60 MWh**

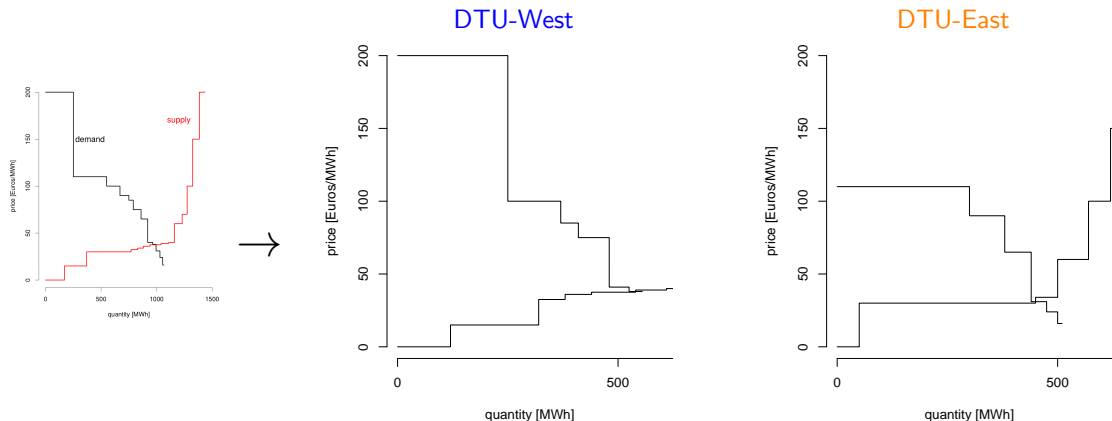
## DTU-East



- Supply side:  $\{G_2, G_4, G_6\}$  - Total: 500 MWh
- Demand side:  $\{D_2, D_4, D_7\}$  - Total: 440 MWh  
 → Surplus of **60 MWh**

**BUT**, only **40 MWh** can flow through the interconnection!

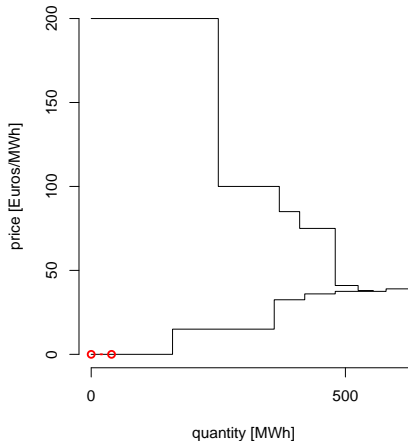
- Due to transmission constraints, the market has to split and becomes two markets



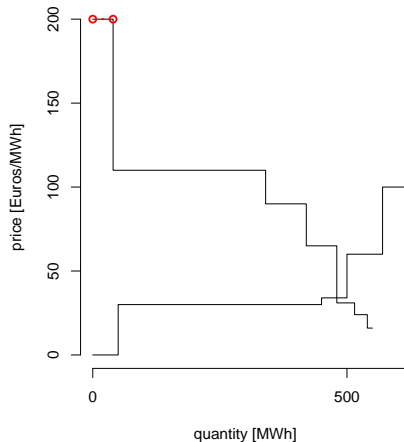
- In practice:
  - 2 market zones with their own supply-demand equilibrium
  - extra (price-independent) consumption/generation offers representing the transmission from one zone to the next to be added

## Adding transmission-related offers

- Extra supply in the high price area, i.e., **DTU-West** (40 MWh coming from **DTU-East**)



- Extra consumption in the low price area, i.e., **DTU-East** (40 MWh for **DTU-West**)

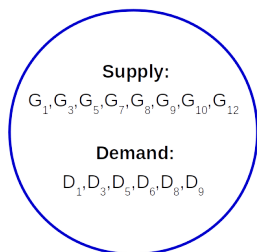


- Power ought to flow from the low price area to the high price area

# Market clearing results for both zones

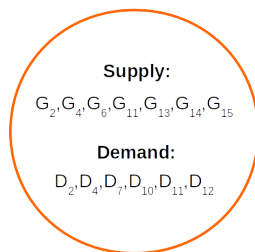
- The same type of LP problems as introduced before is solved
  - for each zone individually,
  - with the extra consumption/generation offers representing the amount of energy transmitted

## DTU-West



- Supply side:  $\{G_1, G_3, G_5, G_7, G_8\}$  (but only 75 MWh for  $G_8$ ) - Total: 515 MWh
  - Demand side:  $\{D_1, D_3, D_5, D_6, D_8, D_9\}$  - Total: 555 MWh
- **Zonal price: 37.5 €**

## DTU-East

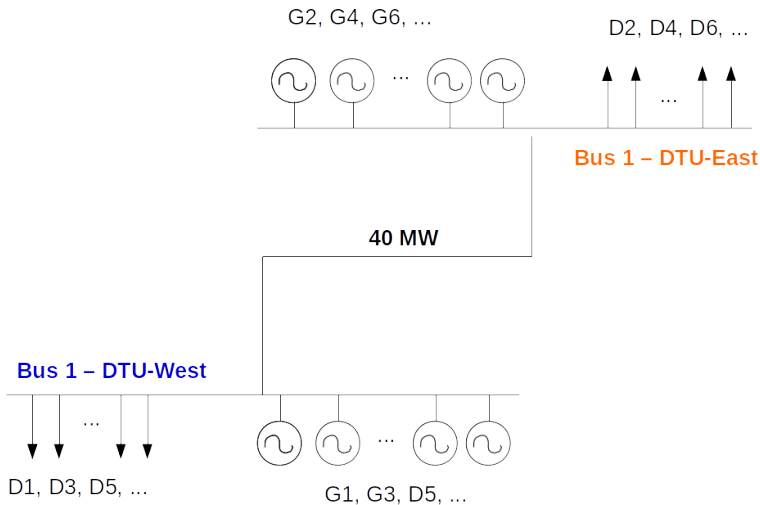


- Supply side:  $\{G_2, G_4, G_6\}$  (but only 30 MWh for  $G_6$ ) - Total: 480 MWh
  - Demand side:  $\{D_2, D_4, D_7\}$  - Total: 440 MWh
- **Zonal price: 34 €**

## More elegantly with flow-based coupling

- Instead of boldly splitting the market, one could instead acknowledge how power flows...
- This allows clearing a single market with geographically differentiated prices

- our DTU system with 2 zones can be modelled as a 2-bus system,
- loads and generators are associated to the relevant bus
- DC power flow is assumed as commonly done at transmission level



## Formulating the market clearing

- The network-constrained social welfare maximization problem can be written as:

$$\begin{aligned}
 & \max_{\{y_i^D\}, \{y_j^G\}} \quad \sum_i \lambda_i^D y_i^D - \sum_j \lambda_j^G y_j^G \\
 & \text{subject to} \quad \sum_i y_i^{D, West} - \sum_j y_j^{G, West} = B \Delta \delta \\
 & \quad \quad \quad \sum_i y_i^{D, East} - \sum_j y_j^{G, East} = -B \Delta \delta \\
 & \quad \quad \quad 0 \leq y_i^D \leq P_i^D, \quad i = 1, \dots, N_D \\
 & \quad \quad \quad 0 \leq y_j^G \leq P_j^G, \quad j = 1, \dots, N_G \\
 & \quad \quad \quad -40 \leq B \Delta \delta \leq 40
 \end{aligned}$$

where:

- $B$  is the absolute value of susceptance (physical constant) of the interconnection between DTU-West and DTU-East
- $\Delta \delta$  is the difference of voltage angles between the 2 buses  
 $\rightarrow B \Delta \delta$  represents the signed power flow from DTU-West to DTU-East

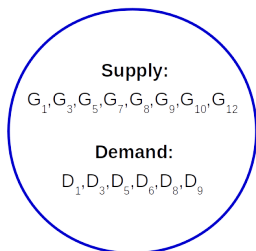
## Obtaining the zonal prices

- As for the case of a single zone, the dual LP allows obtaining **market-clearing prices**
- These **2 prices** corresponds to the Lagrange multipliers for the **2 equality constraints** (i.e., balance equations):

$$\begin{aligned}
 & \max_{\{y_i^D\}, \{y_j^G\}} \quad \sum_i \lambda_i^D y_i^D - \sum_j \lambda_j^G y_j^G \\
 & \text{subject to} \quad \sum_i y_i^{D, West} - \sum_j y_j^{G, West} = B\Delta\delta : \lambda^{S, West} \\
 & \quad \quad \quad \sum_i y_i^{D, East} - \sum_j y_j^{G, East} = -B\Delta\delta : \lambda^{S, East} \\
 & \quad \quad \quad 0 \leq y_i^D \leq P_i^D, \quad i = 1, \dots, N_D \\
 & \quad \quad \quad 0 \leq y_j^G \leq P_j^G, \quad j = 1, \dots, N_G \\
 & \quad \quad \quad -40 \leq B\Delta\delta \leq 40
 \end{aligned}$$

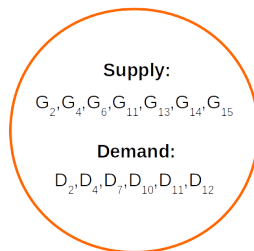
- Results are the same than those based on the import-export approach

## DTU-West



- Supply side:  $\{G_1, G_3, G_5, G_7, G_8\}$  (but only 75 MWh for  $G_8$ ) - Total: 515 MWh
- Demand side:  $\{D_1, D_3, D_5, D_6, D_8, D_9\}$  - Total: 555 MWh  
→ **Zonal price: 37.5 €**

## DTU-East



- Supply side:  $\{G_2, G_4, G_6\}$  (but only 30 MWh for  $G_6$ ) - Total: 480 MWh
  - Demand side:  $\{D_2, D_4, D_7\}$  - Total: 440 MWh  
→ **Zonal price: 34 €**
- However, all zones are modeled at once, and the approach can scale readily



## Final extension to nodal pricing

- In a US-like setup, each node of the power system is to be seen as an area
- For a system with  $K$  nodes, the network-constrained social welfare maximization market-clearing writes:

$$\begin{aligned}
 & \max_{\{y_i^D\}, \{y_j^G\}} \quad \sum_i \lambda_i^D y_i^D - \sum_j \lambda_j^G y_j^G \\
 & \text{subject to} \quad \sum_i y_i^{D,k} - \sum_j y_j^{G,k} = \sum_{l \in \mathcal{L}_k} B_{kl} (\delta_k - \delta_l), \quad k = 1, \dots, K : \lambda^{S,k} \\
 & \quad 0 \leq y_i^D \leq P_i^D, \quad i = 1, \dots, N_D \\
 & \quad 0 \leq y_j^G \leq P_j^G, \quad j = 1, \dots, N_G \\
 & \quad -C_{kl} \leq B_{kl} (\delta_k - \delta_l) \leq C_{kl}, \quad k, l \in \mathcal{L}_N
 \end{aligned}$$

where

- $\mathcal{L}_N$  is the set of nodes,  $\mathcal{L}_k$  the set of nodes connected to node  $k$
- $B_{kl}$  are the line susceptances,  $(\delta_k - \delta_l)$  the phase angle differences
- $\lambda^{S,k}$  are the  $K$  nodal prices

[Extra: Enerdynamics (2012). Locational Marginal Pricing. *Electricity Markets Dynamics online course* ([video](#))]

## Settlement under zonal and nodal pricing

- Market participants are subject to the price where they are physically located, i.e.,
  - *Consumption* side:  $R_i^{DA,D} = -\lambda^{S,location} y_i^D$ ,  $R_i^{DA,D} \leq 0$ , (since being a payment)
  - *Supply* side:  $R_j^{DA,G} = \lambda^{S,location} y_j^G$ ,  $R_j^{DA,G} \geq 0$  (since being a revenue)

### Payment and revenues for our example market clearing

- *Consumption* side (payments):
  - $D_1$  pays  $250 \times 37.5 = 9375$  €, ( $R_9^{DA,D} = -9375$ )
  - $D_2$  pays  $300 \times 34 = 10200$  €, ( $R_9^{DA,D} = -10200$ ), etc.
  - $D_9$  pays  $30 \times 37.5 = 1125$  €, ( $R_9^{DA,D} = -1125$ )
- *Supply* side (revenues):
  - $G_1$  receives  $120 \times 37.5 = 4500$  €, ( $R_8^{DA,G} = 4500$ )
  - $G_2$  receives  $50 \times 34 = 1700$  €, ( $R_2^{DA,G} = 1700$ ), etc.
  - $G_8$  receives  $55 \times 37.5 = 2062.5$  €, ( $R_8^{DA,G} = 2062.5$ )

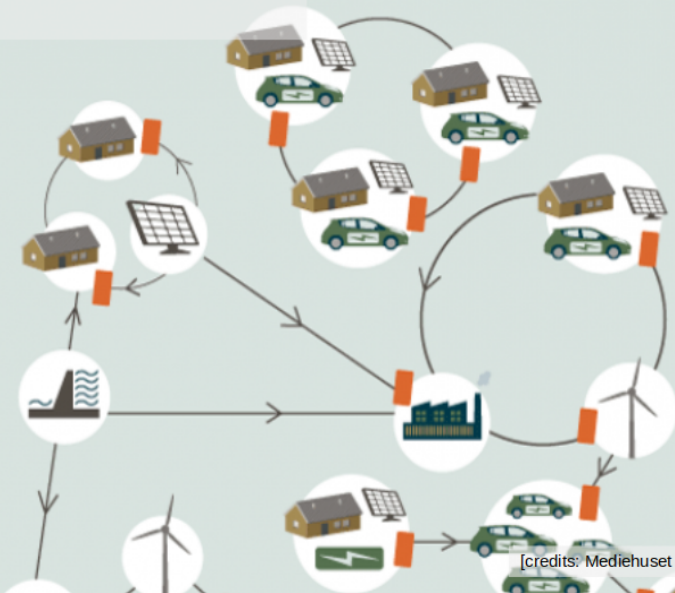
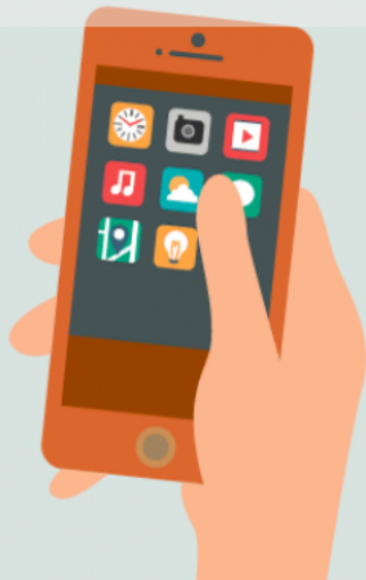
- Market participants are subject to the price where they are physically located, i.e.,
  - *Consumption* side:  $R_i^{DA,D} = -\lambda^{S,location} y_i^D$ ,  $R_i^{DA,D} \leq 0$ , (since being a payment)
  - *Supply* side:  $R_j^{DA,G} = \lambda^{S,location} y_j^G$ ,  $R_j^{DA,G} \geq 0$  (since being a revenue)

## Payment and revenues for our example market clearing

- *Consumption* side (payments):
  - $D_1$  pays  $250 \times 37.5 = 9375$  €, ( $R_9^{DA,D} = -9375$ )
  - $D_2$  pays  $300 \times 34 = 10200$  €, ( $R_9^{DA,D} = -10200$ ), etc.
  - $D_9$  pays  $30 \times 37.5 = 1125$  €, ( $R_9^{DA,D} = -1125$ )
- *Supply* side (revenues):
  - $G_1$  receives  $120 \times 37.5 = 4500$  €, ( $R_8^{DA,G} = 4500$ )
  - $G_2$  receives  $50 \times 34 = 1700$  €, ( $R_2^{DA,G} = 1700$ ), etc.
  - $G_8$  receives  $55 \times 37.5 = 2062.5$  €, ( $R_8^{DA,G} = 2062.5$ )

- The market is **not budget balanced anymore**, since the sum of consumer payments is greater than the sum of supplier revenues
- The difference defines a **congestion rent** to be collected by the system operator(s)

**Use the self-assessment quizz to check your understanding!**



[credits: Mediehuset Ingeniøren]