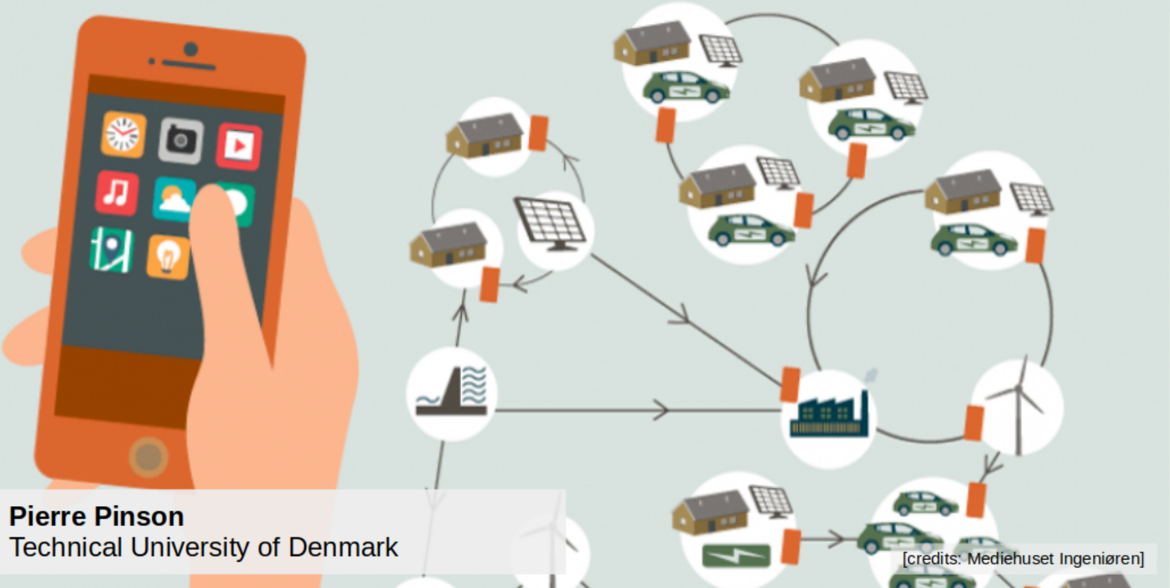
Module 2 – Electricity Spot Markets (e.g. day-ahead)

2.2 Market clearing as an optimization problem



[credits: Mediehuset Ingeniøren]

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Introducing notations first

Inputs:

- All offers in the market are formulated in terms of a *quantity* P and a *price* λ
- On the *supply* side (N_G supply offers):
  - set of offers: $\mathcal{L}_G = \{G_j,\ j = 1, \ldots, N_G\}$
  - maximum quantity for offer G_j: P_j^G
  - price for offer G_j: λ_j^G
- On the *demand* side (N_D demand offers):
  - set of offers: $\mathcal{L}_D = \{D_i,\ i = 1, \ldots, N_D\}$
  - maximum quantity for offer D_i: P_i^D
  - price for offer D_i: λ_i^D

Decision variables:

- *Generation* schedule: $\mathbf{y}^G = \left[y_j^G, \ldots, y_{N_G}^G\right]^\top$, $0 \leq y_j^G \leq P_j^G$
- *Consumption* schedule: $\mathbf{y}^D = \left[y_1^D, \ldots, y_{N_D}^D\right]^\top$, $0 \leq y_i^D \leq P_i^D$

Our example auction setup

Supply: (for a total of 1435 MWh)

Company	Supply/Demand	id	P_j^G (MWh)	λ_j^G (€/MWh)
RT®	Supply	G_1	120	0
WeTrustInWind	Supply	G_2	50	0
BlueHydro	Supply	G_3	200	15
RT®	Supply	G_4	400	30
KøbenhavnCHP	Supply	G_5	60	32.5
KøbenhavnCHP	Supply	G_6	50	34
KøbenhavnCHP	Supply	G_7	60	36
DirtyPower	Supply	G_8	100	37.5
DirtyPower	Supply	G_9	70	39
DirtyPower	Supply	G_{10}	50	40
RT®	Supply	G_{11}	70	60
RT®	Supply	G_{12}	45	70
SafePeak	Supply	G_{13}	50	100
SafePeak	Supply	G_{14}	60	150
SafePeak	Supply	G_{15}	50	200

Our example auction setup

Demand: (for a total of 1065 MWh)

Company	Supply/Demand	id	P_i^D (MWh)	λ_i^D (€/MWh)
CleanRetail	Demand	D_1	250	200
El4You	Demand	D_2	300	110
EVcharge	Demand	D_3	120	100
QualiWatt	Demand	D_4	80	90
IntelliWatt	Demand	D_5	40	85
El4You	Demand	D_6	70	75
CleanRetail	Demand	D_7	60	65
IntelliWatt	Demand	D_8	45	40
QualiWatt	Demand	D_9	30	38
IntelliWatt	Demand	D_{10}	35	31
CleanRetail	Demand	D_{11}	25	24
El4You	Demand	D_{12}	10	16

That is a lot of offers to match... Could an optimization problem readily give us the solution?

Centralized social welfare optimization

- The *social welfare maximization* problem can be written as

$$\max_{\mathbf{y}^G,\mathbf{y}^D} \quad \sum_{i=1}^{N_D} \lambda_i^D y_i^D - \sum_{j=1}^{N_G} \lambda_j^G y_j^G \tag{1a}$$

$$\text{subject to} \quad \sum_{j=1}^{N_G} y_j^G - \sum_{i=1}^{N_D} y_i^D = 0 \tag{1b}$$

$$0 \leq y_i^D \leq P_i^D, \;\; i = 1, \ldots, N_D \tag{1c}$$

$$0 \leq y_j^G \leq P_j^G, \;\; j = 1, \ldots, N_G \tag{1d}$$

- And equivalently as a *minimization problem* by minimizing the opposite objective function, i.e.

$$\min_{\mathbf{y}^G,\mathbf{y}^D} \quad \sum_{j=1}^{N_G} \lambda_j^G y_j^G - \sum_{i=1}^{N_D} \lambda_i^D y_i^D \tag{2a}$$

$$\text{subject to} \quad \text{(1b)-(1d)} \tag{2b}$$

It is a simple linear program!

- One recognize a so-called **Linear Program** (**LP**, here in a compact form):

$$\min_{\mathbf{y}} \quad \mathbf{c}^\top \mathbf{y} \tag{3a}$$
$$\text{subject to} \quad \mathbf{A}\mathbf{y} \leq \mathbf{b} \tag{3b}$$
$$\mathbf{A}_{\text{eq}}\mathbf{y} = \mathbf{b}_{\text{eq}} \tag{3c}$$
$$\mathbf{y} \geq 0 \tag{3d}$$

- LP problems can be readily solved in
  - **Matlab**, for instance with the function linprog,
  - **R**, with the library/function lp_solve,
  - and also obviously with **GAMS**, **Gurobi**, etc.
- However, for e.g. R and Matlab, you need to know how to build relevant vectors and matrices
- And, the solution will *only* give you the energy schedules in terms of supply and demand

Vector and matrices in the objective function

- The vector $\mathbf{y}$ of optimization variables $\mathbf{c}$ of weights in the objective function are constructed as

$$\mathbf{y} = \begin{bmatrix} y_1^G \\ y_2^G \\ \vdots \\ y_{N_G}^G \\ \\ y_1^D \\ y_2^D \\ \vdots \\ y_{N_D}^D \end{bmatrix}, \quad \mathbf{y} \in \mathbb{R}^{(N_G+N_D)} \qquad \mathbf{c} = \begin{bmatrix} \lambda_1^G \\ \lambda_2^G \\ \vdots \\ \lambda_{N_G}^G \\ \\ -\lambda_1^D \\ -\lambda_2^D \\ \vdots \\ -\lambda_{N_D}^D \end{bmatrix}, \quad \mathbf{c} \in \mathbb{R}^{(N_G+N_D)}$$

Vector and matrices defining constraints

- For the equality constraint (balance of generation and consumption):

$$\mathbf{A}_{eq} = [1 \; \ldots \; 1 \;\; -1 \; \ldots \; -1] \,, \;\; \mathbf{A}_{eq} \in \mathbb{R}^{(N_G+N_D)} \;, \qquad \mathbf{b}_{eq} = 0$$

- For the inequality constraint (i.e., generation and consumption levels within limits):

$$\mathbf{A} = \begin{bmatrix} 1 & & & & & \\ & \ddots & & & \mathbf{0} & \\ & & 1 & & & \\ & & & 1 & & \\ & \mathbf{0} & & & \ddots & \\ & & & & & 1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} \overline{P}^G_1 \\ \overline{P}^G_2 \\ \vdots \\ \overline{P}^G_{N_G} \\ \overline{P}^D_1 \\ \overline{P}^D_2 \\ \vdots \\ \overline{P}^D_{N_D} \end{bmatrix},$$

with $\mathbf{A} \in \mathbb{R}^{(N_G+N_D)\times(N_G+N_D)}$ and $\mathbf{b} \in \mathbb{R}^{(N_G+N_D)}$

- Do not forget the non-negativity constraints for the elements of $\mathbf{y}$...

Getting the complete market-clearing

- By *complete* market-clearing is meant obtaining
  - **the schedule** for all supply and demand offers, as well as
  - **the price** at which the market is cleared, i.e., the so-called *market-clearing* or *system* price (in, e.g., Nord Pool)

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- The system price is obtained through the *dual of the LP* previously defined, i.e.,

$$\begin{aligned} \max_{\boldsymbol{\lambda},\boldsymbol{\nu}} \quad & -\mathbf{b}^\top \boldsymbol{\nu} \\ \text{subject to} \quad & \mathbf{A}_{\text{eq}}^\top \boldsymbol{\lambda} - \mathbf{A}^\top \boldsymbol{\nu} \leq \mathbf{c} \\ & \boldsymbol{\nu} \geq 0 \end{aligned}$$

- This is also an LP: it can be solved with Matlab, R, GAMS, etc.
- $\boldsymbol{\lambda}$ and $\boldsymbol{\nu}$ are sets of *Lagrange multipliers* associated to all **equality** and **inequality** constraints:

$$\boldsymbol{\lambda} = \lambda^S$$
$$\boldsymbol{\nu} = [\nu_1^G \; \ldots \; \nu_{N_G}^G \;\; \nu_1^D \; \ldots \; \nu_{N_D}^D]^\top$$

[Note: basics of optimization for application in electricity markets are given in: JM Morales, A Conejo, H Madsen, P Pinson, M Zugno (2014). *Integration Renewables in Electricity Markets: Operational Problems*. Springer (link)]

More specifically for the market-clearing problem

- Only one **equality** constraint, i.e.,

$$\sum_i y_i^D - \sum_j y_j^G = 0$$

for which the associated Lagrange multiplier λ^S represents the system price.

More specifically for the market-clearing problem

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$$\sum_i y_i^D - \sum_j y_j^G = 0$$

for which the associated Lagrange multiplier λ^S represents the system price.

- And $N_D + N_G$ **inequality** constraints:

$$0 \leq y_i^D \leq P_i^D, \; i = 1, \ldots, N_D \; , \qquad 0 \leq y_j^G \leq P_j^G, \; j = 1, \ldots, N_G$$

for which the associated Lagrange multipliers ν_i^D and ν_j^G represents the unitary benefits for the various demand and supply offers if the market is cleared at λ^S.

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- The dual of the market clearing LP is also an LP which writes

$$\begin{aligned}
\max_{\lambda^S, \{\nu_i^D\}, \{\nu_j^G\}} \quad & - \sum_j \nu_j^G P_j^G - \sum_i \nu_i^D P_i^D \\
\text{subject to} \quad & \lambda^S - \nu_j^G \leq \lambda_j^G, \;\; j = 1, \ldots, N_G \\
& - \lambda^S - \nu_i^D \leq -\lambda_i^D, \;\; i = 1, \ldots, N_D \\
& \nu_j^G \geq 0, \;\; j = 1, \ldots, N_G, \quad \nu_i^D \geq 0, \;\; i = 1, \ldots, N_D
\end{aligned}$$

[To retrieve the dual LP, follow: Lahaie S (2008). How to take the dual of a Linear Program. (link)]

Let's also write it as a compact linear program!

- As for the **primal LP** allowing to obtain the dispatch for market participants on both supply and demand side, we write here the **dual LP** in a compact form:

$$\begin{aligned} \max_{\tilde{\mathbf{y}}} \quad & \tilde{\mathbf{c}}^\top \tilde{\mathbf{y}} \\ \text{subject to} \quad & \tilde{\mathbf{A}}\tilde{\mathbf{y}} \leq \tilde{\mathbf{b}} \\ & \tilde{\mathbf{y}} \geq 0 \end{aligned}$$

- The next 2 slides describe how to build the assemble the relevant vectors and matrices in the above LP...
- Then, it can be solved with **Matlab**, **R**, **GAMS**, etc.
- And, the solution will give you the equilibrium price, as well as the unit benefits for each and every market participant

[NB: Most optimization functions and tools readily give you the solution of dual problems when solving the primal ones! E.g., see documentation of linprog in Matlab]

Vector and matrices in the objective function

- The vector $\mathbf{y}$ of optimization variables $\mathbf{c}$ of weights in the objective function are constructed as

$$\tilde{\mathbf{y}} = \begin{bmatrix} \nu_1^G \\ \nu_2^G \\ \vdots \\ \nu_{N_G}^G \\ \\ \nu_1^D \\ \nu_2^D \\ \vdots \\ \nu_{N_D}^D \\ \lambda_S \end{bmatrix}, \quad \tilde{\mathbf{y}} \in \mathbb{R}^{(N_G+N_D+1)} \qquad \tilde{\mathbf{c}} = \begin{bmatrix} -P_1^G \\ -P_2^G \\ \vdots \\ -P_{N_G}^G \\ \\ -P_1^D \\ -P_2^D \\ \vdots \\ -P_{N_D}^D \\ 0 \end{bmatrix}, \quad \tilde{\mathbf{c}} \in \mathbb{R}^{(N_G+N_D+1)}$$

Vector and matrices defining constraints

- No equality constraint!
- For the inequality constraint:

$$\tilde{\mathbf{A}} = \begin{bmatrix} -1 & & & & & & 1 \\ & \ddots & & & \mathbf{0} & & \vdots \\ & & -1 & & & & 1 \\ & & & -1 & & & -1 \\ & \mathbf{0} & & & \ddots & & \vdots \\ & & & & & -1 & -1 \end{bmatrix}, \qquad \tilde{\mathbf{b}} = \begin{bmatrix} \lambda_1^G \\ \lambda_2^G \\ \vdots \\ \lambda_{N_G}^G \\ -\lambda_1^D \\ -\lambda_2^D \\ \vdots \\ -\lambda_{N_D}^D \end{bmatrix},$$

with $\tilde{\mathbf{A}} \in \mathbb{R}^{(N_G+N_D)\times(N_G+N_D)}$ and $\tilde{\mathbf{b}} \in \mathbb{R}^{(N_G+N_D)}$

Application to our simple auction example

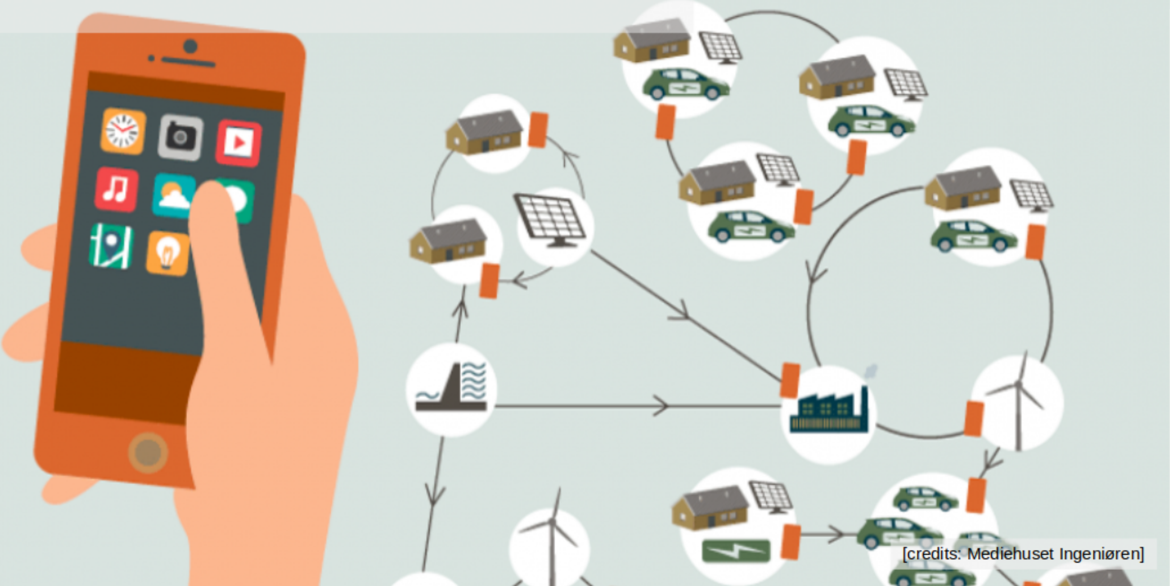
- Solving the **primal LP** for obtaining the supply and demand schedules yields:

Supply id.	Schedule (MWh)	Demand id.	Schedule (MWh)
G_1	120	D_1	250
G_2	50	D_2	300
G_3	200	D_3	120
G_4	400	D_4	80
G_5	60	D_5	40
G_6	50	D_6	70
G_7	60	D_7	60
G_8	55	D_8	45
G_9-G_{15}	0	D_9	30
		D_{10}-D_{12}	0

for a total amount of energy scheduled of 995 MWh

- Solving the **dual LP** gives a system price of 37.5 €/MWh which corresponds to the price offer of G_8

Use the self-assessment quizz to check your understanding!



[credits: Mediehuset Ingeniøren]