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Structural active vibration control using active mass damper by block pulse functions

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Abstract

Active vibration control is one of the most efficient systems to mitigate excessive vibrations by providing significantly superior supplemental damping in civil engineering structures. One of the most challenging components of active control is development of an accurate analytical approach with minor computational expenses. Active mass damper (AMD) is one of the most commonly used active control devices, including a mass-spring-damper system with an actuator to increase the amount of damping in structures. Block pulse functions have been studied and applied frequently in recent years as a basic set of functions for signal characterizations in systems science and control. The purpose of this study is to establish an innovative method by using block pulse functions to minimize expenses of computations. Numerical simulations of earthquake-excited 10-story shear buildings equipped with an active mass damper are provided to verify the validity and feasibility of the proposed method. The proposed method's uncontrolled and controlled responses of structural system are compared with linear quadratic regulator method's results. The results reveal the proposed method can be beneficial in reducing seismic responses of structures with less computational expenses and high accuracy.

Keywords

Active control, active mass damper, block pulse function, linear quadratic regulator, seismic response

1. Introduction

Recent trends toward tall and more flexible structure designs have resulted in choosing appropriate lateral-load resisting systems to withstand severe dynamic loading such as strong earthquakes and high winds. At some point it may no longer be prudent to rely entirely on the strength of the structure and its ability to dissipate energy during extreme loading (Soong, 1990). In recent decades, the attention of civil engineering communities has focused on reducing forces and deformation in structures through the methods of structural control. It is well recognized that, the implementation of control strategies in structural design has significantly reduced damage and loss of life. Structural control strategies are materialized by special devices which are added to the structure to reduce the structural response and fulfill multiple objectives. Depending on the mode of operation of these special devices, the structural control methods can be broadly classified as passive, semi-active and active control methods (Fisco and Adeli, 2011). In passive control schemes, the idea is to diminish the seismic input energy from reaching to structural elements by using special devices. This goal is achieved by dissipation of the vibration energy in elements called energy-absorbing devices. Passive control does not require external power sources, cannot increase the energy of the system through this control scheme and have the advantage of requiring little maintenance (Soong and Constantinou, 1994). However, passive control system has limited ability because it is not able to adapt to structural changes or varying usage patterns and loading conditions. To overcome these deficiencies, active and semi-active controls can be implemented. They can adapt to various operating conditions and apply

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external powers to generate control forces. Semi-active control devices provide some of the best features of both the passive and active control systems. Many of them can be operated by a battery when the main power system fails during the seismic events. Semi-active approach is limited in certain applications and does not have the potential to destabilize the structure system under a variety of dynamics loading conditions. Developing much more adaptive systems to protect structures from damages caused by natural hazards such as severe excitations can be utilized by active control systems. Active control schemes promise to effectively minimize structural responses. They rely on external power sources to operate actuators generating control forces and require routine maintenance (Cheng et al., 2008). In active control schemes, sensing devices are used to measure the external disturbance and/or the structural response. This information is fed into the controller which synthesizes the measured quantities and structural parameter information to generate the corresponding control signal which input to actuators to produce the desired control action. The idea of active control for civil engineering structures started to emerge around 1970. For instance, the possible use of active systems for elastic-resistant structures was suggested by Nordell (1969). Zuk and Clark (1970) discussed the concept of kinetic structures. The concept of active control for civil engineering structures was advocated by Yao (1972). The high control authority property of the active control approaches can provide important response reductions even under severe dynamic loads. The active mass damper (AMD) is one of the commonly used active control device. The device is similar to the tuned mass damper (TMD), including an actuator that is used to position the mass at each instant, to increase the amount of damping achieved and the operational frequency range of the device (Mitchell et al., 2012). Approaches such as linear quadratic regulator (LQR), linear quadratic Gaussian (LQG), sliding mode control (SMC) and H_∞ control are commonly found in research works focused on cancelling excessive vibrations due to earthquakes. Analytical methods are much more reliable schemes to verify the control forces that are generated to reduce seismic response. Various semi-active and active control algorithms have been proposed for control devices such as the clipped-optimal algorithm (Dyke et al., 1996), optimal controllers (Zhang and Roschke, 1999), Lyapunov stability theory (Dyke and Spencer, 1997) and decentralized bang-bang, maximum energy dissipation (Jansen and Dyke, 2000). Yang et al. (1987) proposed open-loop and close-loop instantaneous control algorithms for active control against seismic excitations. Adaptive algorithms estimate the parameters of the disturbance and then design controllers that have a set of poles varying according to the disturbance estimation (Landau et al., 2005). Model-based control algorithms are commonly used in structural control systems (Li et al., 2010). Ghaffarzadeh et al. (2013) and Ghaffarzadeh (2013) used a fuzzy-rule-based semi-active control of building frames and Chung et al. (1989) concluded that computational advantages exist in the use of instantaneous optimal control since instantaneous optimal control algorithms do not require solving the Riccati matrix equation, as required in classical optimal control. These techniques usually require complex design methodologies based on a system model and these methods are time consuming and impose computational expenses (Orivuori and Zenger, 2012). Block pulse functions (BPFs) have been widely studied and used as a rudimentary set of functions for signal characterizations in controlled systems. The BPFs set proved to be the most fundamental and it enjoyed prolific popularity in different applications in the area of control systems. In comparison with other basic functions or polynomials, the BPFs can result more readily to recursive computation in order to solve concrete problems (Wang, 2007, Deb et al., 1994).

The present study aims at proposing an innovative active control methodology for AMDs that intend to minimize the computational expenses by using BPFs. Proposed analytical approach for calculating state space parameters and feedback gain matrix by BPFs is presented. The feasibility and effectiveness of the proposed method is investigated by a numerical example. One effective way of designing a full state feedback is to use the LQR approach (in many papers this approach has been used for comparing and verifying the feasibility of proposed methods). The uncontrolled and controlled dynamic responses of structural system are obtained by proposed method and compared to LQR method. An optimal controller for a deterministically excited system is generated by minimizing a quadratic performance index. Additionally, the effectiveness of proposed method on response reduction in earthquake excitation is further evaluated by comparing the controlled response against the results obtained from the uncontrolled case.

2. Active mass damper (AMD)

AMD is one of the active control devices which consists of an auxiliary mass connected to the reaction wall of the building by the actuation system. It was evolved from tuned mass dampers with the introduction of an active control mechanism. TMD has been proved effective and beneficial to dissipating structural response when the first mode of structure is dominant. Development of AMD focuses on seeking control of

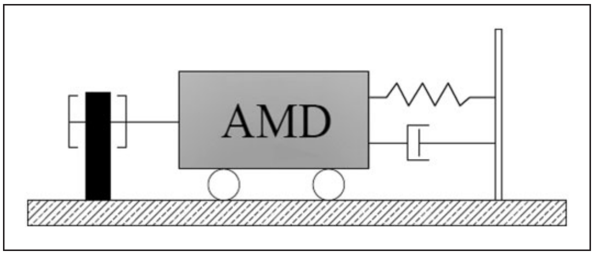

Figure 1. Schematic view of an AMD.

structural response against earthquake-excitation with a wide frequency band (Pourzeynali et al., 2007). It is expected that structures with AMD will present enhanced effectiveness over structures with TMD. An actuator is installed between the structure and the TMD systems. The motion of the auxiliary system can be controlled by the actuator. A schematic view of an AMD system is illustrated in Figure 1.

The dynamic equation of AMD is

$$m_d\ddot{x}_d(t) + c_d(\dot{x}_d(t) - \dot{x}_n(t)) + k_d(x_d(t) - x_n(t)) = u_d(t) + m_d\ddot{x}_g(t) \tag{1}$$

where m_d, c_d and k_d are the mass, damping and stiffness of AMD. $x_d(t)$ is AMD displacement and $u_d(t)$ is AMD control force. $x_n(t)$ is the top floor displacement.

AMDs were proposed in the early 1980s and have been studied analytically (Nishimura et al., 1992). The first implementation of this control method and of active control in general was performed in 1989, in Tokyo, Japan, by the Kajima Corporation (Kobori et al., 1991, Soong and Spencer, 2002). In general view AMD has an economical advantage, this active control device's actuator by comparing other devices is smaller and consequently need little control force. Beside actuator of AMD apply control force to the mass of AMD included; however, the other devices apply the control forces to structure straightly. AMD allows arbitrary forces to be applied to the structure. The damping capacity of an AMD is theoretically unbounded, but it is obtained at the expense of the actuation force and power. The latter quantities can easily increase beyond technical ranges, making the design unpractical (Ricciardelli et al., 2003). Also for active devices a trade-off solution between performance and design constrains has to be found.

3. Block pulse functions (BPFs)

BPFs are a set of orthogonal functions with piecewise constant values and are usually applied as a useful tool within the analysis, identification and other problems of systems science. Also BPFs can be used in control system engineering for analysis and synthesis of dynamic systems. Studies show that BPFs may have definite advantages for problems due to their explicit expression and their simple formulations. By using BPFs after original problems are transformed into their corresponding algebraic expressions, piecewise constant approximate solutions can be computed efficiently to show the tendency of the exact solutions under flexible weighting of the accuracy of the results and the size of computations. In comparison with other basis functions or polynomials, the block pulse functions can lead more easily to recursive computations to solve concrete problems. Sannuti (1977) showed that the application of BPFs results in an enormous reduction of computational effort over Walsh functions in control system applications.

A set of BPF on a unit time interval [0, 1) is defined as (Babolian and Masouri, 2008)

$$\varphi_{\mathbf{i}}(t) = \begin{cases} 1 & \frac{i}{m} \le t \le \frac{i+1}{m} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where $i = 0, 1, 2, \ldots, m-1$ with a positive integer value for m. Also, consider $h = 1/m$, and φ_i is the i-th BPF.

BPFs possess disparate properties, the most salient characteristics are disjointness, orthogonality and completeness. The disjointness property can be clearly obtained from the definition of BPFs

$$\varphi_{\mathbf{i}}(t)\varphi_{\mathbf{j}}(t) = \begin{cases} \varphi_{\mathbf{i}}(t), & i = j \\ 0, & i \neq j \end{cases} \tag{3}$$

where $i, j = 0, 1, \ldots, m-1$. This property will simplify the calculations in the problems.

The second property is orthogonality, which can be stated as follows

$$\int_0^T \varphi_{\mathbf{i}}(t)\varphi_{\mathbf{j}}(t)\mathrm{d}t = \begin{cases} h & i = j \\ 0 & i \neq j \end{cases} \tag{4}$$

The orthogonal property of block pulse functions is the basis of expanding functions into their block pulse series. An arbitrary real bounded function $\mathbf{f}(t)$, that is square integrable in the interval $t \in [0, T)$, can be expanded into a block pulse series in the sense of minimizing the mean square error between $\mathbf{f}(t)$ and its approximation

$$\mathbf{f}(t) = \sum_{i=1}^{m} \mathbf{f}_{\mathbf{i}}\varphi_{\mathbf{i}}(t) \tag{5}$$

where f_i is the block pulse coefficient with respect to the ith block pulse function $\varphi_{\mathbf{i}}(t)$.

The last property presented in this paper is completeness. For every $\mathbf{f} \in L^2([0,1))$ Parseval's identity holds

$$\int_0^1 \mathbf{f}^2(t)\mathrm{d}t = \sum_{i=1}^{m} \mathbf{f}_\mathbf{i}^2(t)||\varphi_\mathbf{i}(t)||^2 \tag{6}$$

where

$$\mathbf{f}_\mathbf{i} = \frac{1}{h}\int_0^1 \mathbf{f}(t)\varphi_\mathbf{i}(t)\mathrm{d}t \tag{7}$$

when BPFs are used to solve problems, original functions are typically approximated by their block pulse series. Therefore it is necessary to know, whether the mean square error between a given function and its approximate block pulse series can become arbitrary small under certain conditions. The completeness of block pulse functions guarantees that an arbitrarily small mean square error can be obtained for a real bounded function.

As mentioned previously, the aim of this study is to utilize BPFs to recalculate state space parameters and feedback gain matrix. The above formulation shows us how to easily model these parameters as function **f** using BPFs.

4. BPFs formulation of control problem

The equations of motion for structural control purposes can be written as follows

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathrm{x}(t) = \boldsymbol{\gamma}\mathbf{u}(t) + \boldsymbol{\delta}\ddot{\mathbf{x}}_\mathbf{g}(t) \tag{8}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are $(n+1)\times(n+1)$ matrices of mass, damping and stiffness of a given structure,n is number of building stories. Respectively, $\mathbf{x}(t)$ of $(n+1)\times 1$ is vector of displacements at each degree of freedom and $\mathbf{u}(t)$ is the control force, respectively, $\boldsymbol{\gamma}$ of $(n+1)\times 1$ is the location matrix of control force, and $\boldsymbol{\delta}$ of $(n+1)\times 1$ is the coefficient vector for earthquake ground acceleration $\ddot{\mathbf{x}}_\mathbf{g}(t)$. This equation can be written in state space form, this equation is written as

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{B}_\mathbf{u}\mathbf{u}(t) + \mathbf{B}_\mathbf{r}\ddot{\mathbf{x}}_\mathbf{g}(t) \tag{9}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \tag{10}$$

$$\mathbf{B}_\mathbf{u} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\boldsymbol{\gamma} \end{bmatrix} \tag{11}$$

$$\mathbf{B}_\mathbf{r} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\boldsymbol{\delta} \end{bmatrix} \tag{12}$$

Matrix $\mathbf{A}$ is called plant matrix of the system. $\mathbf{B}_\mathbf{u}$ and $\mathbf{B}_\mathbf{r}$ are vector location matrix and excitation influence matrix. Respectively $\mathbf{z}(t)$ is the $2(n+1)$ order state variable contains displacement and velocity vectors. Equation (8) is not solvable due to the one additional unknown variables related to the control force. Consequently, we need one more equation to solve the active control problem. This one equation referred as the feedback control law makes equation (8) solvable. In control theory the additional equation is written based on the feedback law as follow

$$\mathbf{u}(t) = -\mathbf{G}(t)\mathbf{z}(t) \tag{13}$$

where $\mathbf{G}(t)$ is $1\times 2(n+1)$ feedback gain matrix. In order to design a control system, the control law should achieve the control purpose, such as maximizing the reduction of structural response by minimizing the state parameters (i.e. displacement and velocity).

Define the Hamiltonian H as (Bryson and Ho, 1975)

$$H = \frac{1}{2}\left(\mathbf{z}(t)^T\mathbf{Q}\mathbf{z}(t) + \mathbf{u}(t)^T\mathbf{R}\mathbf{u}(t)\right) + \boldsymbol{\eta}(t)^T(\mathbf{A}\mathbf{z}(t) + \mathbf{B}_\mathbf{u}\mathbf{u}(t)) \tag{14}$$

From the theory of functional, the following equations are the necessary conditions for optimality

$$\frac{\partial H}{\partial \mathbf{u}(t)} = 0 \tag{15}$$

and

$$-\frac{\partial H}{\partial \mathbf{z}(t)} = \dot{\boldsymbol{\eta}}(t)^T \tag{16}$$

Performing the differentiations indicated in equation (14) and (15), we have

$$\mathbf{R}\mathbf{u}(t) + \mathbf{B}_\mathbf{u}^T\boldsymbol{\eta}(t) = 0 \Rightarrow \mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{B}_\mathbf{u}\boldsymbol{\eta}(t) \tag{17}$$

and

$$\dot{\boldsymbol{\eta}}^T(t) = -\mathbf{Q}\mathbf{z}(t) - \mathbf{A}^T\boldsymbol{\eta}(t) \tag{18}$$

By assuming that $\boldsymbol{\eta}(t) = \mathbf{P}(t)\mathbf{z}(t)$ and substituting equation (16) into equation (8) and repeating equation (17) we have a linear two-point boundary value problem

$$\begin{pmatrix} \dot{\mathbf{z}}(t) \\ \dot{\mathbf{P}}(t) \end{pmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B}_\mathbf{u}\mathbf{R}^{-1}\mathbf{B}_\mathbf{u}^T \\ \mathbf{Q} & -\mathbf{A}^T \end{bmatrix} \begin{pmatrix} \mathbf{z}(t) \\ \mathbf{P}(t) \end{pmatrix} \tag{19}$$

Under the two-point boundary values $\mathbf{z}(0) = \mathbf{z}_0$ and $\mathbf{P}(T) = 0$. Respectively, T is the final time-instants in simulation.

Where, $\mathbf{Q}$ and $\mathbf{R}$ are positive semi definite matrices and positive scalar, respectively, which are defined in section five.

To avoid this two-point boundary value problem in solving $\mathbf{P}(t)$. We set the 2 n-dimensional transition matrix of equation (18) as

$$\boldsymbol{\psi}(T,t) = \begin{bmatrix} \boldsymbol{\psi}_{\mathbf{11}}(T,t) & \boldsymbol{\psi}_{\mathbf{12}}(T,t) \\ \boldsymbol{\psi}_{\mathbf{21}}(T,t) & \boldsymbol{\psi}_{\mathbf{22}}(T,t) \end{bmatrix} \tag{20}$$

where all the sub-matrices $\boldsymbol{\psi}_{\mathbf{11}}(T,t)$, $\boldsymbol{\psi}_{\mathbf{12}}(T,t)$, $\boldsymbol{\psi}_{\mathbf{21}}(T,t)$ and $\boldsymbol{\psi}_{\mathbf{22}}(T,t)$ are n-dimensional. Noticing that

$$\boldsymbol{\psi}(T,t)\begin{bmatrix} \mathbf{z}(t) \\ \mathbf{P}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{z}(T) \\ \mathbf{P}(T) \end{bmatrix} \tag{21}$$

and $\mathbf{P}(T) = 0$.

Using equations (12) and (16), the gain matrix is expressed as follows

$$\mathbf{G}(t) = \mathbf{R}^{-1}\mathbf{B}_{\mathbf{u}}^{T}\mathbf{P}(t) \tag{22}$$

In which $\mathbf{P}(t)$ is expressed as

$$\mathbf{P}(t) = \boldsymbol{\psi}_{\mathbf{22}}^{-1}(T,t)\boldsymbol{\psi}_{\mathbf{21}}(T,t) \tag{23}$$

In applying BPFs in this problem, a suboptimal solution with piecewise constant feedback gains can be obtained (Jiang and Schaufelberger, 1992)

$$\mathbf{G}(t) = \sum_{i=1}^{m} \mathbf{G}_{\mathbf{i}}(t)\varphi_{\mathbf{i}}(t) \tag{24}$$

Since the block pulse coefficients of the transition matrix $\boldsymbol{\psi}_{\mathbf{i}}(t)$, $(i = 1, 2, \ldots, m)$ can be computed iteratively from this series

$$\boldsymbol{\psi}_{\mathbf{m-i}}(t) = \sum_{i=0}^{m} \boldsymbol{\alpha}^{i}\boldsymbol{\lambda}\varphi_{\mathbf{i}}(t) \tag{25}$$

And the matrices $\boldsymbol{\lambda}$ and $\boldsymbol{\alpha}$ are

$$\boldsymbol{\lambda} = \left[\mathbf{I} - \frac{h}{2}\mathbf{F}\right]^{-1} \tag{26}$$

$$\boldsymbol{\alpha} = \boldsymbol{\lambda}\left[\mathbf{I} + \frac{h}{2}\mathbf{F}\right] \tag{27}$$

5. Numerical study

A 10-story shear building equipped with an AMD on the top floor is considered. Figure 2 illustrates the structural model for this study. Mass and stiffness parameters are listed in Table 1. The mass matrix of example shear building structure is a diagonal matrix in which the mass of each story is sorted on its diagonal, as given in the following

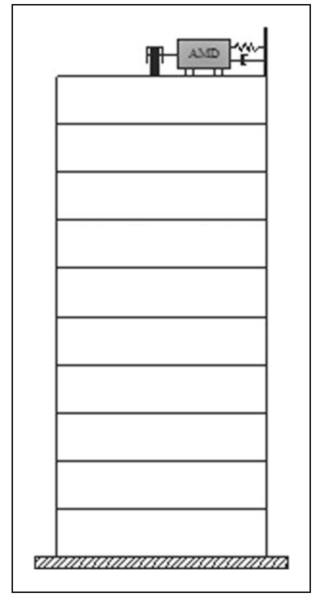

Figure 2. Schematic view of a test structure equipped with an AMD on its roof.

Table 1. Mass and stiffness values of test structure.

Story	Mass	Stiffness
1–3	105,000 kg	1700×10^5 N/m
4–6	95,000 kg	1600×10^5 N/m
7–9	90,000 kg	1400×10^5 N/m
10	85,000 kg	1100×10^5 N/m

$$\mathbf{M} = \begin{bmatrix} m_1 & & & & 0 \\ & m_2 & & & \\ & & \ddots & & \\ & & & m_{10} & \\ 0 & & & & m_d \end{bmatrix} \tag{28}$$

The structural stiffness and damping matrices, $\mathbf{C}$ and $\mathbf{K}$ can be described based on the individual stiffness and damping coefficients of each story, k_i and c_i as follow

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & & & & 0 \\ -k_2 & k_2 + k_3 & -k_3 & & & \\ & & \cdots & & -k_{10} & \\ & & & -k_{10} & k_{10} & -k_d \\ 0 & & & & -k_d & k_d \end{bmatrix} \tag{29}$$

$$\mathbf{C} = \begin{bmatrix} c_1 + c_2 & -c_2 & & & & 0 \\ -c_2 & c_2 + c_3 & -c_3 & & & \\ & & \cdots & & -c_{10} & \\ & & & -c_{10} & c_{10} & -c_d \\ 0 & & & & -c_d & c_d \end{bmatrix} \tag{30}$$

where subscript i indicates to the i-th story. k_d and c_d are stiffness and damping coefficients of AMD, respectively.

The mass of the AMD system was chosen to be 2% of total mass of the building and its damping ratio was considered to be 5% of the critical value. The structural system simulate against the seismic motions.

As mentioned in section four Q and R are positive semi definite matrices and positive scalar, respectively, which are defined as follow

$$\mathbf{Q} = \mathbf{I}_{\mathbf{2(n+1)\times 2(n+1)}}, \quad \mathbf{R} = 10^{-12}\mathbf{I}_{\mathbf{(n+1)\times(n+1)}}$$

Three pairs of earthquake records related to different earthquakes cited in Table 2 have been used to evaluate the performance of the proposed analytical method in reducing the structural responses under earthquake loading. The sampling rate for Imperial Valley, Landers and Chi-Chi earthquakes are 0.005, 0.0025 and 0.004. Figure 3 shows frequency spectra of selected earthquakes.

The results of the proposed method are compared with these of LQR method to verify the practicality and effectiveness of described new active control scheme. As illustrated in Figures 4 and 5, the time history responses of top floor displacement and profiles of drift for Imperial Valley and Landers revealed both approaches as expected are very close.

Furthermore, to evaluate the control system performance the following indices must be considered. These set of performance indices comparing the controlled response against the results obtained from the uncontrolled cases. There are different sets of evaluation criteria which are used in structural control to evaluate the performance of the buildings. The set of evaluation criteria used in this study to compare the performance of the structure are defined based on maximum responses (Ohtori et al., 2004). The first evaluation criteria for the proposed method pertain to its ability to reduce inter-story drift. The second and third evaluation criteria relate to the ability of the proposed method to reduce the maximum displacement and acceleration of floor. The fourth one relates to the ability of the new method to reduce the maximum force of floor.

$$\mathbf{J_1} = \frac{\max\limits_{t,i} \frac{|d_i^c(t)|}{h_i}}{\max\limits_{t,i} \frac{|d_i^u(t)|}{h_i}}, \tag{31}$$

$$\mathbf{J_2} = \frac{\max\limits_{t,i} |x_i^c(t)|}{\max\limits_{t,i} |x_i^u(t)|}, \tag{32}$$

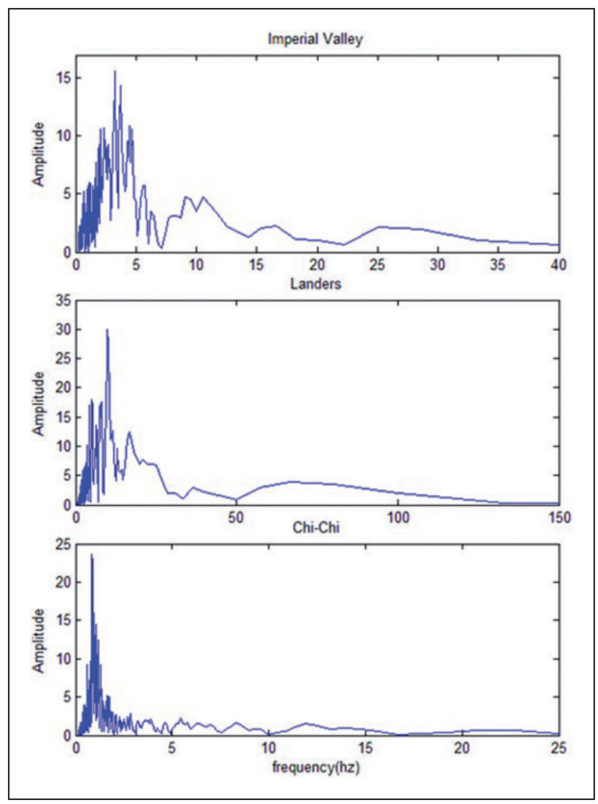

Figure 3. Frequency spectra of the selected earthquakes.

Table 2. Properties of selected ground motions.

Earthquake	Station	d(km)	Peak ground acceleration (g)	Peak ground velocity (cm/s)	Peak ground displacement (cm)
Imperial Valley	5054 Bonds Comer	2.5	0.588	45.2	16.78
Landers	23 Coolwater	21.2	0.417	42.3	13.76
Chi-Chi	Chay 014	41.49	0.263	21.9	6.57

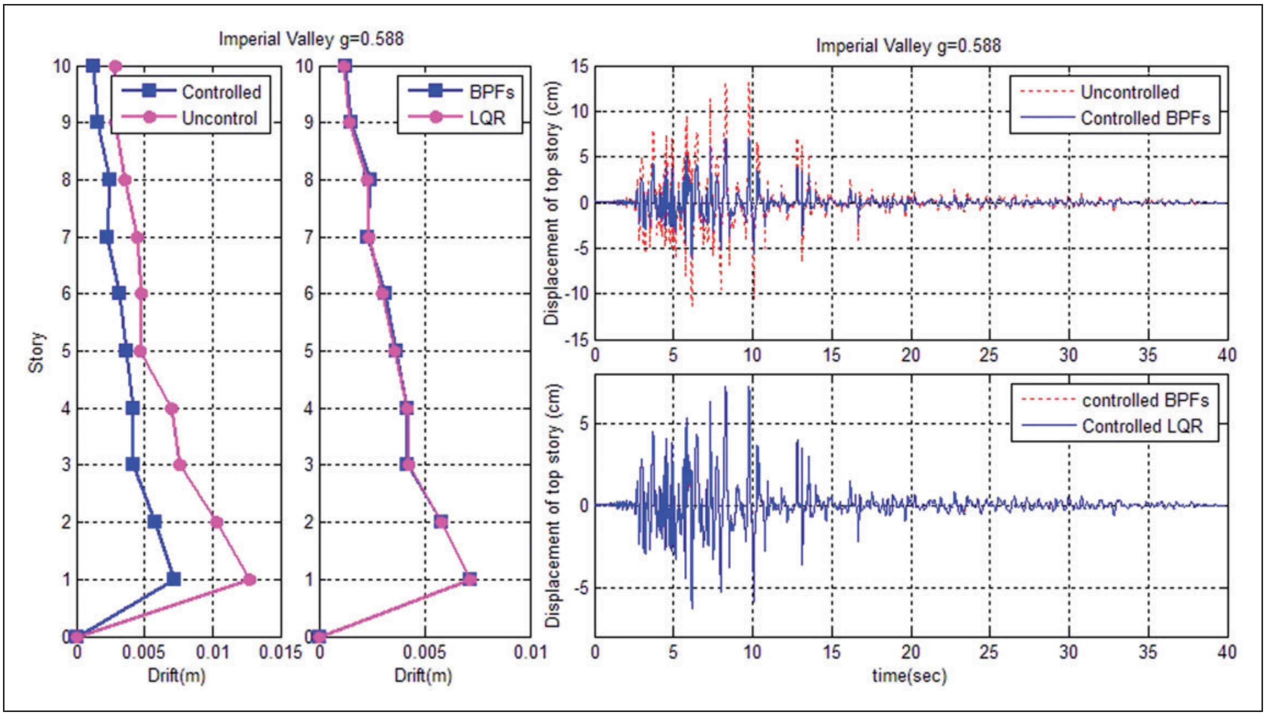

Figure 4. Results of time history displacement of top story and drift in controlled and uncontrolled structure.

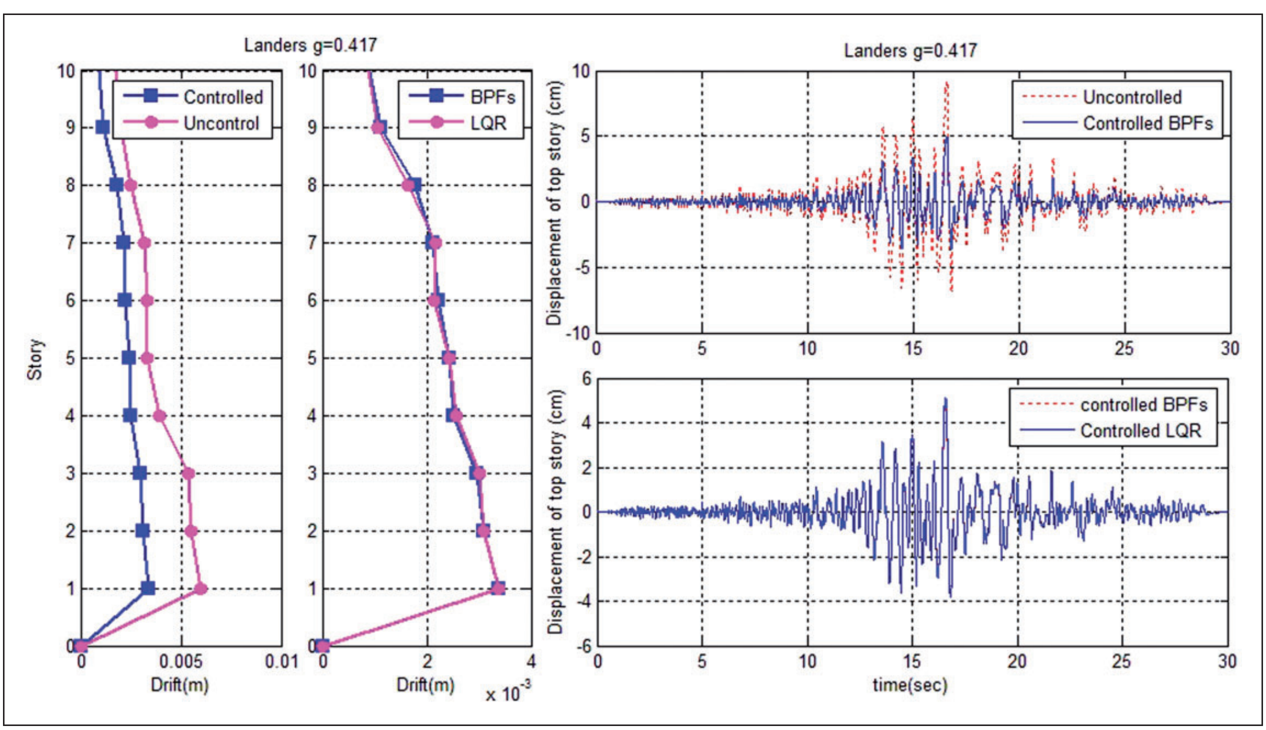

Figure 5. Results of time history displacement of top story and drift in controlled and uncontrolled structure.

Table 3. Evaluation criteria of ground motions.

	Imperial Valley		Landers		Chi-Chi	
	BPFs	LQR	BPFs	LQR	BPFs	LQR
J₁	0.5721	0.5688	0.5528	0.5524	0.5625	0.5625
J₂	0.6002	0.5905	0.4826	0.4793	0.5685	0.5648
J₃	0.8201	0.8026	0.7521	0.7420	0.7078	0.7035
J₄	0.7042	0.7030	0.7925	0.7888	0.7041	0.7037
J₅	0.5015	0.5086	0.3062	0.3104	0.2325	0.2357

BPFs: Block pulse functions, LQR: linear quadratic regulator.

$$\mathbf{J_3} = \frac{\max\limits_{t,i}\left|\ddot{x}^{c}_{ai}(t)\right|}{\max\limits_{t,i}\left|\ddot{x}^{u}_{ai}(t)\right|}, \tag{33}$$

$$\mathbf{J_4} = \frac{\max\limits_{t}\left|\sum\limits_{i} m_i \ddot{x}^{c}_{ai}(t)\right|}{\max\limits_{t}\left|\sum\limits_{i} m_i \ddot{x}^{u}_{ai}(t)\right|} \tag{34}$$

The last one is related to the control devices

$$\mathbf{J_5} = \frac{\max\limits_{t,l}\left|f_l(t)\right|}{W} \tag{35}$$

where $x_i(t)$ is displacement of i-th story, $d_i(t)$ is drift of i-th story, $\ddot{x}_i(t)$ is acceleration of i-th story, $f_l(t)$ is control force produced by l-th device, m_i is mass of i-th story, h_i is height of i-th story and W is seismic weight of building. The term 'c' and 'u' refer to the controlled system and uncontrolled system.

The performance of the system according to set of evaluation criteria for seismic records is tabulated in Table 3 for both proposed and LQR control methods. By comparison between the results of evaluation criteria for ground motions it can be conclude the proposed control method based on BPFs is very close to LQR control method.

6. Conclusion

In this paper, an innovative analytical method based on BPFs for AMD is proposed and successfully verified through the time-history analyses of a 10-story model structure under various seismic motion records with different properties. Controlled and uncontrolled top floor displacement responses and profiles of drift for stories based on proposed method and LQR method were compared. Results showed that the proposed method had reasonable effect on dissipating the responses of the structure as satisfactorily as LQR control algorithm. Also the proposed method exhibited satisfactory control performance for the evaluation criteria and the values of evaluation criteria revealed very competent control performance. The results of this investigation indicated the proposed method had acceptable accuracy with minor computational expenses.

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