# Control of Buildings Using Active Tuned Mass Dampers

By C. C. Chang[1] and Henry T. Y. Yang[2]

**Abstract:** A closed-loop complete-feedback control algorithm is proposed for the control of a building modeled as a single-degree-of-freedom (SDOF) system using an active tuned mass damper (ATMD). The control force is calculated from the acceleration, velocity, and displacement feedbacks of the SDOF system and the active mass damper. The passive-control-device properties, including the stiffness and damping constants of the tuned mass damper and the three gain coefficients of the actuator, are derived by minimizing the displacement variance of the SDOF system. Simulations are performed to evaluate the performance of the active-tuned-mass-damper design on the examples of a SDOF system and a 10-story three-bay building frame. The results show that the control efficiency of the ATMD based on velocity feedback depends on the passive-control-device properties assumed. Also, the ATMD with optimal passive properties using complete feedback results in further reduction in the structural displacement. The results also show that for the same level of reduction in structural displacements, the control force required is smaller using complete feedback.

## INTRODUCTION

The response of a high-rise building to dynamic forces such as earthquakes and wind loads has been of primary interest to civil engineers. In the last two to three decades, control devices, passive as well as active, have been developed to suppress structural vibration due to these environmental disturbances. Among the concepts behind the development of these devices, the one based on the use of a mass as an added energy-absorbing system has been under rather intensive study, and the results have been fruitful (McNamara 1977). Luft (1979), Ayorinde and Warburton (1980), and Warburton and Ayorinde (1980) presented approximated formulas for the optimal parametric design of a tuned-mass-damper (TMD) system. Examples of the application of such technology include the installations of one 400-ton and two 300-ton TMD systems on the tops of the Citicorp Center in New York and the John Hancock Tower in Boston, respectively, to reduce motion induced by wind (Isyumov et al. 1975). More recently, a highly efficient and compact TMD system was developed and used for reducing the expected vibration of the pylons for the Iwakurojima-Bridge, a part of the Honshu-Shikoku Bridge Project, constructed in Japan (Naruse and Hirashima 1987).

In an attempt to increase the effectiveness of a TMD system, Chang and Soong (1980) introduced an active force to act between the structure and the TMD system. Optimization procedures were proposed to compute the required control forces. The results demonstrated that further reduction can be achieved when the TMD system was operated in an active mode. A process for designing an effective active-tuned-mass-damper (ATMD) system to control a tall building subjected to stationary random wind forces was proposed by Abdel-Rohman (1984) using the pole-assignment method. The results suggested that the design of an optimal ATMD required at least a parametric study to select the ATMD parameters.

In this study, a form of a closed-loop complete-feedback control algorithm is proposed for the control of a building modeled as a single-degree-of-freedom (SDOF) system using an active tuned mass damper. The SDOF system is assumed to be under stationary Gaussian white noise ground excitation. The control force is calculated from the acceleration, velocity, and displacement feedbacks of the SDOF system and the active mass damper. The stiffness and damping constants of the tuned mass damper and the gain coefficients of the actuator are derived by minimizing the displacement variance of the SDOF system. The stability of the proposed algorithm is also discussed using the Routh-Hurwitz criterion. Monte Carlo simulations are performed to evaluate the performance of the active-tuned-mass-damper design on the examples of a SDOF system and a 10-story three-bay building frame. The control effects of the ATMD with optimal and nonoptimal passive-control-device properties using velocity feedback are calculated and discussed. Comparisons are also made on the control efficiency between ATMD using velocity feedback and complete feedback.

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Note. Associate Editor: Paul Christiano. Discussion open until August 1, 1995. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on May 19, 1993. This paper is part of the ***Journal of Engineering Mechanics***, Vol. 121, No. 3, March, 1995. ©ASCE, ISSN 0733-9399/95/0003-0355–0366/$2.00 + $.25 per page. Paper No. 6203.



J. Eng. Mech., 1995, 121(3): 355-366

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## FORMULATION

### Equations of Motion

Assume that the equations of motion for a building structure controlled by one actuator and subjected to the ground excitation $\ddot{x}_g$ can be written as follows:

$$\mathbf{M}\ddot{\mathbf{d}} + \mathbf{C}\dot{\mathbf{d}} + \mathbf{K}\mathbf{d} = -\mathbf{M}\mathbf{r}\ddot{x}_g - \mathbf{B}u \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ = mass, damping, and stiffness matrices, respectively; $\mathbf{d}$ = displacement vector (relative to the ground); $\mathbf{r}$ and $\mathbf{B}$ = location matrices, which define the locations of the excitation and applied control force, respectively; and $u$ = control force generated by the actuator. For a linear system, the equations of motion can be decoupled in the normal coordinates. It has been pointed out by Clough and Penzien (1975) that the base motions tend to excite only the lowest modes of vibration. The equation of motion of the dominant mode can be written as follows:

$$m_0\ddot{x}_0 + c_0\dot{x}_0 + k_0x_0 = -\beta_0m_0\ddot{x}_g - u \tag{2}$$

where $x_0$ represents the modal amplitude. The definitions of the modal mass, damping, and stiffness constants, $m_0$, $c_0$, and $k_0$, and the modal participation factor, $\beta_0$, can be found, for example, in the book by Clough and Penzien (1975).

It is noted that in (2), the eigenvector of the dominant mode $\Phi_0$ is scaled proportionally according to the following relationship:

$$\Phi_0'\mathbf{B} = 1 \tag{3}$$

The modal amplitude $x_0$ is related to the displacement vector $\mathbf{d}$ by the following equation:

$$x_0 = \frac{\Phi_0'\mathbf{M}}{\Phi_0'\mathbf{M}\Phi_0}\mathbf{d} \tag{4}$$

Assuming that this single-degree-of-freedom (SDOF) system is controlled by an auxiliary mass damper and a control force acting between the SDOF system and the auxiliary mass (see Fig. 1), the equations of motion can be written as follows:

$$\begin{bmatrix} m_0 & 0 \\ 0 & m_1 \end{bmatrix}\begin{Bmatrix} \ddot{x}_0 \\ \ddot{x}_1 \end{Bmatrix} + \begin{bmatrix} c_0 + c_1 & -c_1 \\ -c_1 & c_1 \end{bmatrix}\begin{Bmatrix} \dot{x}_0 \\ \dot{x}_1 \end{Bmatrix} + \begin{bmatrix} k_0 + k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix}\begin{Bmatrix} x_0 \\ x_1 \end{Bmatrix} = -\begin{Bmatrix} \beta_0m_0 \\ m_1 \end{Bmatrix}\ddot{x}_g + \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}u \tag{5}$$

where $m_1$, $c_1$, and $k_1$ = mass, damping, and stiffness constants of the auxiliary mass damper; and $x_0$ and $x_1$ = displacements of the SDOF system and the mass damper relative to the ground, respectively.

### Tuned Mass Damper (TMD)

When the control force is inactive, the auxiliary mass damper is of passive type, and the response of the SDOF system is attenuated by $m_1$, $c_1$, and $k_1$ only. By properly adjusting these parameters, the response of the SDOF system can be reduced to various degrees. Ayorinde and Warburton (1980) and Warburton and Ayorinde (1980) proposed a method of determining $c_1$ and $k_1$ as functions of a predetermined $m_1$ value by minimizing the response of the SDOF system with the participation factor equal to 1. The derivation is extended in the following for the case with an arbitrary $\beta_0$ value. Assuming that the ground excitation $\ddot{x}_g$ is a stationary

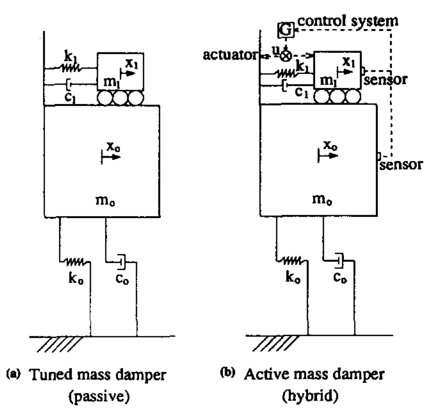


**FIG. 1. Modelings of Passive and Hybrid Types of Control System Using Auxiliary Mass**



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Gaussian white noise random process with spectral density $S_0$, the variance of the displacement of the SDOF system, $\sigma^2_{x_0}$, can be derived by assuming $\xi_0 = 0$ as follows (Warburton and Ayorinde 1980):

$$\sigma^2_{x_0} = \frac{\pi S_0}{2\omega_0^3}\left\{\frac{1}{\mu_1 f_1 \xi_1}\left[\beta_0^2 - (1+\mu_1)^2(2\beta_0 - \mu_1)f_1^2 + (1+\mu_1)^4 f_1^4\right] + \frac{4\xi_1 f_1}{\mu_1}(1+\mu_1)^3\right\} \quad (6)$$

with

$$\omega_i = \left(\frac{k_i}{m_i}\right)^{1/2}, \quad \text{for } i = 0, 1; \quad \xi_i = \frac{c_i}{2\sqrt{m_i k_i}}, \quad \text{for } i = 0, 1 \quad (7, 8)$$

$$\mu_1 = \frac{m_1}{m_0}; \quad f_1 = \frac{\omega_1}{\omega_0} \quad (9, 10)$$

The optimal values of $f_1$ and $\xi_1$ can be found by taking the derivative of $\sigma^2_{x_0}$ with respect to $f_1$ and $\xi_1$ and setting them equal to zero, respectively. Solving these two simultaneous equations gives the following:

$$f_{1_{opt}} = \frac{\left(\alpha - \frac{1}{2}\mu_1\right)^{1/2}}{1+\mu_1}; \quad \xi_{1_{opt}} = \left[\frac{\mu_1\left(\alpha - \frac{1}{4}\mu_1\right)}{4(1+\mu_1)\left(\alpha - \frac{1}{2}\mu_1\right)}\right]^{1/2} \quad (11, 12)$$

and

$$k_{1_{opt}} = \frac{\mu_1\left(\alpha - \frac{1}{2}\mu_1\right)}{(1+\mu_1)^2}k_0; \quad c_{1_{opt}} = \frac{\mu_1^{3/2}\left(\alpha - \frac{1}{4}\mu_1\right)^{1/2}}{2\xi_0(1+\mu_1)^{3/2}}c_0 \quad (13, 14)$$

with

$$\alpha = \frac{\beta_0 + \beta_0\mu_1}{\beta_0 + \mu_1} \quad (15)$$

## Active Tuned Mass Damper (ATMD) Using Velocity Feedback

In addition to adjusting the passive parameters $m_1$, $c_1$, and $k_1$, the response of the SDOF system can be further reduced by activating the actuator installed between the system and the auxiliary mass. The control force generated by the actuator can be designed by using either the optimal control theory (Chang and Soong 1980; Yang 1982) or the pole-assignment method (Abdel-Rohman 1984). One of the optimal control algorithms that has been designed and implemented on an 11-story office building in Tokyo by the Kajima Corporation of Japan involves the use of velocity feedback of the SDOF system and the auxiliary mass to calculate the control force (Kobori et al. 1991a, b):

$$u = g_0\dot{x}_0 - g_1\dot{x}_1 \quad (16)$$

where $g_0$ and $g_1$ = velocity-feedback gains.

Substituting (16) into (5) and moving the terms associated with the control force to the left-hand side of the equations gives the following:

$$\begin{bmatrix} m_0 & 0 \\ 0 & m_1 \end{bmatrix}\begin{Bmatrix} \ddot{x}_0 \\ \ddot{x}_1 \end{Bmatrix} + \begin{bmatrix} c_0 + c_1 + g_0 & -c_1 - g_1 \\ -c_1 - g_0 & c_1 + g_1 \end{bmatrix}\begin{Bmatrix} \dot{x}_0 \\ \dot{x}_1 \end{Bmatrix} + \begin{bmatrix} k_0 + k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix}\begin{Bmatrix} x_0 \\ x_1 \end{Bmatrix} = -\begin{Bmatrix} \beta_0 m_0 \\ m_1 \end{Bmatrix}\ddot{x}_g \quad (17)$$

Two interesting observations can be made. First, the damping matrix becomes unsymmetrical if $g_0$ is not equal to $g_1$, and thereby the system becomes potentially unstable. The stability of (17) can be verified using the Routh-Hurwitz stability criterion (Wylie and Barret 1982). The characteristic equation of (17) can be written as follows:

$$a_4S^4 + a_3S^3 + a_2S^2 + a_1S + a_0 = 0 \quad (18)$$

with

$$a_4 = m_0m_1; \quad a_3 = m_0c_1 + m_0g_1 + m_1c_0 + m_1c_1 + m_1g_0 \quad (19, 20)$$

$$a_2 = m_0k_1 + m_1k_0 + m_1k_1 + c_0c_1 + c_0g_1; \quad a_1 = k_0c_1 + k_0g_1 + k_1c_0; \quad a_0 = k_0k_1 \quad (21\text{–}23)$$



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The Routh-Hurwitz stability criterion states that the roots of the characteristic equation [(18)] have negative real parts if and only if the following conditions are satisfied:

$$a_0, a_1, a_2, a_3, a_4 > 0; \quad a_2a_3 - a_1a_4 > 0; \quad a_1a_2a_3 - a_1^2a_4 - a_0a_3^2 > 0 \qquad (24\text{–}26)$$

Neglecting the terms associating with $c_0$ and $c_1$ (Koizumi et al. 1989; Tsijiuchi et al. 1991) and substituting (19)–(23) into (24)–(26) gives the following:

$$g_0 > 0; \quad g_1 > 0; \quad g_1 > \frac{\mu_1 f_1^2}{1 - f_1^2} g_0 \qquad (27\text{–}29)$$

For the case of an optimally tuned mass damper where $k_1 = k_{1_{opt}}$ and $c_1 = c_{1_{opt}}$, the third stability condition can be further simplified by substituting (11) into (29) as follows:

$$g_1 > \frac{2\alpha\mu_1 - \mu_1^2}{2(1-\alpha) + 5\mu_1 + 2\mu_1^2} g_0 \qquad (30)$$

As regards the second observation, if $g_0 = g_1$ in (17), the damping matrix remains symmetrical and the equations of motion become the same as those of a passive-tuned-mass-damper system as discussed in the previous section with $c_1 + g_0$ replacing $c_1$ in (5). Since the optimal values $k_{1_{opt}}$ and $c_{1_{opt}}$ derived in (13) and (14) minimize $\sigma_{x0}^2$, the implication is that under the optimally tuned condition of $k_1 = k_{1_{opt}}$ and $c_1 = c_{1_{opt}}$, the addition of control force using either velocity or displacement feedback would not decrease the displacement variance of the SDOF system. To further reduce the response of this SDOF system, one of the choices would be to include feedbacks of all three quantities, namely, acceleration, velocity, and displacement (henceforth referred to, for lack of a better term, as complete feedback).

## Active Tuned Mass Damper Using Complete Feedback

Assume that the control force $u$ is calculated from the feedbacks of acceleration, velocity, and displacement of the SDOF system and the auxiliary mass in the following form:

$$u = m_2(\ddot{x}_0 - \ddot{x}_1) + c_2(\dot{x}_0 - \dot{x}_1) + k_2(x_0 - x_1) \qquad (31)$$

where the feedback gains $m_2$, $c_2$, and $k_2$ = equivalent mass, damping, and stiffness constants, respectively, of the active force. Substituting (31) into (5) and rearranging the terms gives the following:

$$\begin{bmatrix} m_0 + m_2 & -m_2 \\ -m_2 & m_1 + m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_0 \\ \ddot{x}_1 \end{Bmatrix} + \begin{bmatrix} c_0 + c_1 + c_2 & -c_1 - c_2 \\ -c_1 - c_2 & c_1 + c_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_0 \\ \dot{x}_1 \end{Bmatrix} + \begin{bmatrix} k_0 + k_1 + k_2 & -k_1 - k_2 \\ -k_1 - k_2 & k_1 + k_2 \end{bmatrix} \begin{Bmatrix} x_0 \\ x_1 \end{Bmatrix} = -\begin{Bmatrix} \beta_0 m_0 \\ m_1 \end{Bmatrix} \ddot{x}_g \qquad (32)$$

To simplify the derivation, let $x_1$ be expressed as follows:

$$x_1 = z_1 + x_0 \qquad (33)$$

with $z_1$ = relative displacement between the SDOF system and the auxiliary mass. Substituting (33) into (32) gives the following:

$$\begin{bmatrix} m_0 + m_1 & m_1 \\ m_1 & m_1 + m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_0 \\ \ddot{z}_1 \end{Bmatrix} + \begin{bmatrix} c_0 & 0 \\ 0 & c_1 + c_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_0 \\ \dot{z}_1 \end{Bmatrix} + \begin{bmatrix} k_0 & 0 \\ 0 & k_1 + k_2 \end{bmatrix} \begin{Bmatrix} x_0 \\ z_1 \end{Bmatrix} = -\begin{Bmatrix} \beta_0 m_0 + m_1 \\ m_1 \end{Bmatrix} \ddot{x}_g \qquad (34)$$

The displacement variance of the SDOF system $\sigma_{x_0}^2$ can then be found using the results by Crandall and Mark (1963) as follows:

$$\sigma_{x_0}^2 = \pi S_0 \left[ \frac{B_0^2}{A_0}(A_2A_3 - A_1A_4) + A_3(B_1^2 - 2B_0B_2) + A_1(B_2^2 - 2B_1B_3) + \frac{B_3^2}{A_4}(A_1A_2 - A_0A_3) \right] \Big/ [A_1(A_2A_3 - A_1A_4) - A_0A_3^2] \qquad (35)$$

with

$$A_4 = m_0m_1 + m_0m_2 + m_1m_2; \quad A_3 = (m_0 + m_1)(c_1 + c_2) + (m_1 + m_2)c_0 \qquad (36, 37)$$

$$A_2 = (m_0 + m_1)(k_1 + k_2) + (m_1 + m_2)k_0 \qquad (38)$$

$$A_1 = c_0(k_1 + k_2) + (c_1 + c_2)k_0; \quad A_0 = k_0(k_1 + k_2) \qquad (39, 40)$$

$$B_3 = 0; \quad B_2 = -(\beta_0 m_0 m_1 + \beta_0 m_0 m_2 + m_1 m_2) \qquad (41, 42)$$



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$$B_1 = -(\beta_0 m_0 + m_1)(c_1 + c_2); \quad B_0 = -(\beta_0 m_0 + m_1)(k_1 + k_2) \tag{43, 44}$$

Again, the optimal values of $c_2$ and $k_2$ can be found by taking the derivative of $\sigma^2_{x_0}$ with respect to $c_1 + c_2$ and $k_1 + k_2$ and setting them equal to zero, respectively. Neglecting the terms associating with $c_0$ and solving the two simultaneous equations gives the following:

$$(k_1 + k_2)_{opt} = \frac{\mu_1\left(\alpha + \mu_2 + \dfrac{\mu_2}{\mu_1} - \dfrac{1}{2}\mu_1\right)}{(1 + \mu_1)^2} k_0 \tag{45}$$

$$(c_1 + c_2)_{opt} = \frac{\mu_1^{3/2}\left[\alpha\left(1 + \mu_2 + \dfrac{\mu_2}{\mu_1}\right) - \dfrac{1}{4}\mu_1\right]^{1/2}}{2\xi_0(1 + \mu_1)^{3/2}} c_0 \tag{46}$$

where the equivalent mass ratio $\mu_2$ is defined as follows:

$$\mu_2 = \frac{m_2}{m_0} \tag{47}$$

If $k_1$ and $c_1$ are equal to $k_{1_{opt}}$ and $c_{1_{opt}}$, respectively, then

$$k_{2_{opt}} = \frac{\mu_2}{1 + \mu_1} k_0; \quad c_{2_{opt}} = \frac{\mu_1^{3/2}\left\{\left[\alpha\left(1 + \mu_2 + \dfrac{\mu_2}{\mu_1}\right) - \dfrac{1}{4}\mu_1\right]^{1/2} - \left(\alpha - \dfrac{1}{4}\mu_1\right)^{1/2}\right\}}{2\xi_0(1 + \mu_1)^{3/2}} c_0 \tag{48, 49}$$

Substituting (45) and (46) into (35) gives the following:

$$\sigma^2_{x_0} = \frac{2\pi S_0}{\omega_0^3}\frac{(1 + \mu_1)^{3/2}}{\mu_1^{1/2}}\left[\alpha\left(1 + \mu_2 + \frac{\mu_2}{\mu_1}\right) - \frac{1}{4}\mu_1\right]^{1/2} \tag{50}$$

Eq. (50) suggests that $\sigma^2_{x_0}$ can be reduced for values of $\mu_2$ smaller than zero. The stability condition of this complete-feedback control algorithm can be determined using the Routh-Hurwitz stability criterion as shown in (24)–(26) but with $A_4$, $A_3$, $A_2$, $A_1$, and $A_0$ in (36)–(40) replacing $a_4$, $a_3$, $a_2$, $a_1$, and $a_0$ in (19)–(23), respectively. The algorithm is found to be stable when

$$\mu_2 > -\frac{\mu_1\left(\alpha - \dfrac{\mu_1}{2}\right)}{(1 + \mu_1)} \tag{51}$$

## NUMERICAL RESULTS

To evaluate the designs of tuned mass damper (TMD) and active tuned mass damper (ATMD) discussed in this study, the control performance of a single-degree-of-freedom system and a 10-story, three-bay building frame were analyzed and discussed.

### Single-Degree-of-Freedom System

The mass, stiffness, and damping constants of the SDOF system were assumed as $4.61 \times 10^7$ N-s$^2$/m, $5.83 \times 10^7$ N/m, and $1.04 \times 10^6$ N-s/m (1% damping ratio), respectively. These properties represented the first mode of a 75-story flexible skyscraper (Tsijiuchi et al. 1991). The participation factor was assumed to be 1. The frequency was calculated as 0.179 Hz. This SDOF system was assumed to be subjected to a stationary Gaussian white noise ground excitation with spectral density $S_0 = 3.62 \times 10^{-4}$ m$^2$/s$^3$. The standard deviation of the SDOF displacement relative to the ground was found to be 0.2 m.

In the following analysis, Monte Carlo simulations incorporating the Newmark integration scheme were used. The Newmark parameters were assumed as $\beta = 0.25$ and $\gamma = 0.5$, which guaranteed unconditional stability. The time increment was assumed as 0.1 sec. Two hundred samples were used for every case studied.

#### *Tuned Mass Damper*

The displacement standard deviations of the SDOF system controlled by a passive mass damper with optimal and nonoptimal parameters were first calculated for mass ratio $\mu_1$ between $10^{-3}$ and $10^{-1}$. The results using the optimal parameters calculated using (13)–(15) and two sets of nonoptimal frequency ratio $f_1 = 0.99$ and 0.95 with $\xi_1 = 1, 3, 5, 7$, and 9%, respectively, are plotted in Fig. 2. It shows that the tuned mass damper with optimal parameters gives the lowest-displacement standard deviations as compared to those of the nonoptimal cases for the



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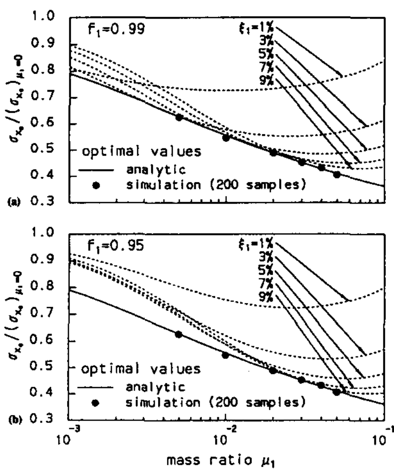


FIG. 2. Displacement Standard Deviations of SDOF System Using Passive Mass Damper, $(\sigma_{x_0})_{\mu_1=0} = 0.2$ m

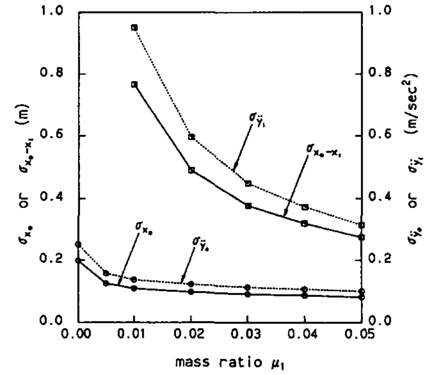


FIG. 3. Effects of Mass Ratio $\mu_1$ of Optimally Tuned Mass Damper on Responses of SDOF System

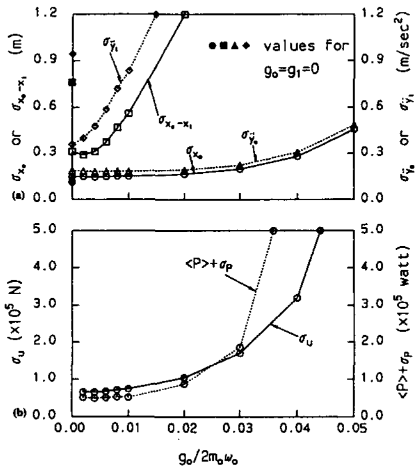


FIG. 4. Effects of Applying Control Force Using Velocity Feedback on Optimally Tuned System ($\mu_1$ = 0.01; $f_1$ = 0.9876; $\xi_1$ = 0.0498; $g_1/2m_1\omega_1$ = 0.2)

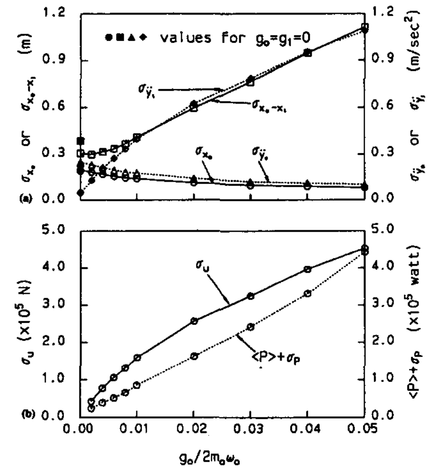


FIG. 5. Effects of Applying Control Force Using Velocity Feedback on Nonoptimally Tuned System ($\mu_1$ = 0.01; $f_1$ = 0.3; $\xi_1$ = 0.0498; $g_1/2m_1\omega_1$ = 0.2)

ranges of mass ratios considered. Fairly good agreement between the analytical solutions and the Monte Carlo simulation results is seen for the case of mass damper with optimally tuned parameters.

Fig. 3 shows the standard deviations of the relative displacement, $x_0$, and the absolute acceleration, $\ddot{y}_0$, of the SDOF system; the stroke, $x_0 - x_1$, between the SDOF and TMD systems; and the absolute acceleration, $\ddot{y}_1$, of the TMD system for the mass damper with optimal parameters for mass ratio $\mu_1$ ranging from 0 to 0.05. All four standard deviations $\sigma_{x_0}$, $\sigma_{x_0-x_1}$, $\sigma_{\ddot{y}0}$, and $\sigma_{\ddot{y}_1}$ decrease as the mass ratio $\mu_1$ increases. Also, the largest reduction rates for $\sigma_{x_0}$ and $\sigma_{\ddot{y}_0}$ are seen for the mass ratio $\mu_1$ between 0 and 0.01.

### *Active Tuned Mass Damper Using Velocity Feedback*

It was assumed that the SDOF system could be controlled using TMD and active control forces based on velocity feedback. The mass ratio $\mu_1$ was assumed to be 0.01. Two types of TMD systems, optimally and nonoptimally tuned, were considered. The frequency ratio $f_1$ and damping ratio $\xi_1$ were calculated to be 0.9876 and 0.0498, respectively, for the optimal TMD system, and assumed to be 0.3 and 0.0498, respectively, for the optimal TMD system. To simplify the discussion, one of the gain coefficients, $g_1$, was assumed to be constant with a value of $0.4m_1\omega_1$, while the other, $g_0$, varied from 0 to $0.1m_0\omega_0$.

Fig. 4 shows the one-standard-deviation bounds for the relative displacement, $x_0$, and the absolute acceleration, $\ddot{y}_0$, of the SDOF system; the stroke, $x_0 - x_1$; the absolute acceleration, $\ddot{y}_1$, of the TMD; the control force, $u$; and control power, $P$, with different values of the velocity gain coefficient $g_0$ for the case of TMD with optimal parameters. It is seen that all six curves



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follow an increasing trend as $g_0$ increases. The results suggest that by placing an actuator that generates control forces using velocity feedback between the SDOF system and the TMD with optimal parameters not only fails to reduce the response of the SDOF system, but increases the responses instead.

Fig. 5 shows the six-standard-deviation bounds for the case of TMD with nonoptimally tuned parameters. Although the values of $x_0 - x_1$, $\ddot{y}_1$, $u$, and $P$ increase as the velocity gain coefficient $g_0$ increases, the values of $x_0$ and $\ddot{y}_0$ decrease slightly.

At this point, it is worth mentioning that in actual design of tall buildings or large-scale structures with high degrees of freedom, an exact optimally tuned mass damping system can not be easily achieved. The efficiency of the TMD system could reduce drastically if it becomes unsynchronized. Furthermore, an enormously auxiliary mass might be needed for the TMD system to be effective (Kobori 1990).

### *Active Tuned Mass Damper Using Complete Feedback*

It was then assumed that the SDOF system be controlled by an optimally tuned mass damper and active control forces based on the complete feedback. Fig. 6 shows the one-standard-deviation bounds of $x_0$, $\ddot{y}_0$, $x_0 - x_1$, $\ddot{y}_1$, $u$, and $P$ as functions of the equivalent mass ratio $\mu_2$. The values of $\mu_2$ were assumed to be between 0 and $-0.01$. The results demonstrate similar trends to those seen in Fig. 5, i.e., the values of $x_0 - x_1$, $\ddot{y}_1$, $u$, and $P$ follow an increasing trend as $-\mu_2$ moves from 0 to 0.01, and the values of $x_0$ and $\ddot{y}_0$, on the other hand, decrease slightly.

### *Comparisons*

Fig. 7 shows the comparisons between the results of hybrid control using the active tuned mass damper with nonoptimal parameters and velocity feedback and the active tuned mass damper with optimal parameters and complete feedback. The results of the standard deviations $\sigma_{x0}$, $\sigma_{\ddot{y}0}$, $\sigma_{x0=x1}$, and $\sigma_{\ddot{y}1}$ are plotted against $\sigma_u$ for these two cases. It is seen that for the same level of reduction in SDOF responses ($\sigma_{x0}$ and $\sigma_{\ddot{y}0}$), the control force $\sigma_u$ required is smaller for the case of complete feedback. This observation seems to suggest that the ATMD control using the optimal active tuned mass damper with complete feedback is more efficient than that of nonoptimal active tuned mass damper with velocity feedback. It should be noted, however, that for both cases the reduction in the responses of $\sigma_{x_0}$ and and $\sigma_{\ddot{y}_0}$ is accompanied by the increase of TMD response characteristics, such as, the stroke $x_0 - x_1$ and the absolute acceleration $\ddot{y}_1$.

Fig. 8 shows the time histories of the standard deviations of the SDOF displacement and the control force for the two cases, one in Fig. 5 with $g_0/2m_0\omega_0 = 0.03$ for velocity feedback and the other in Fig. 6 with $\mu_2 = -0.005$ for complete feedback. These two cases were chosen so that they gave almost the same standard deviation of the SDOF displacement $x_0$. The control force required using complete feedback is only about one-eighth of that needed using velocity feedback. The advantage of using the optimal tuned mass damper together with an actuator generating control forces based on the proposed complete feedback is demonstrated obviously in this case.

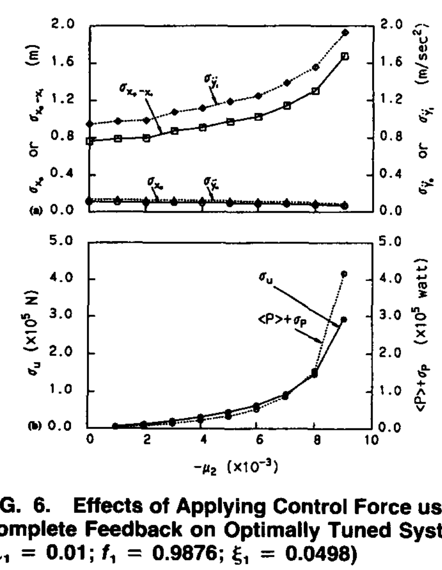

FIG. 6. Effects of Applying Control Force using Complete Feedback on Optimally Tuned System ($\mu_1 = 0.01$; $f_1 = 0.9876$; $\xi_1 = 0.0498$)

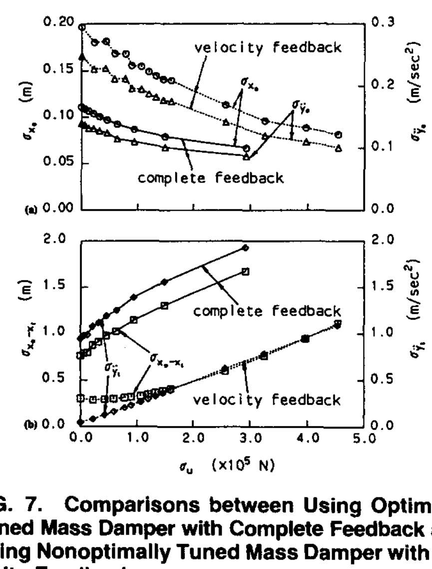


FIG. 7. Comparisons between Using Optimally Tuned Mass Damper with Complete Feedback and Using Nonoptimally Tuned Mass Damper with Velocity Feedback



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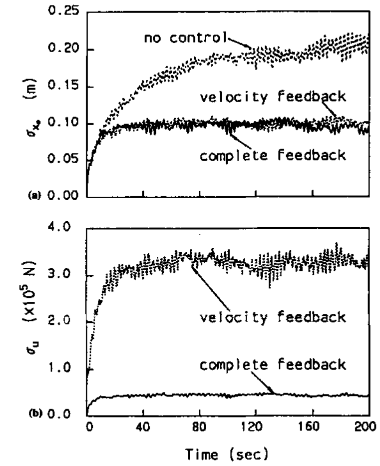
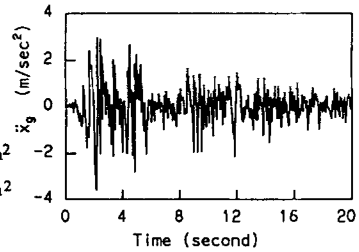
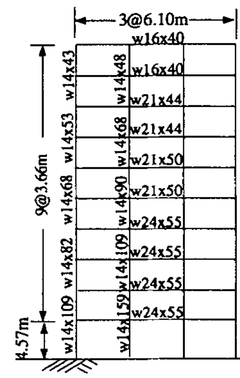



FIG. 8. Time Histories of Deviations of SDOF Displacement and Control Force for Two Cases, One in Fig. 5 with $g_0/2m_0\omega_0$ = 0.03 for Velocity Feedback and One in Fig. 6 with $\mu_2 = -0.005$ for Complete Feedback

Tributary width : 9.14 m
Load : Roof : 4,790 N/m²
Floor : 5,748 N/m²




(b) The NS component of the ground acceleration record of the 1940 El-Centro earthquake




| Mode | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Natural Frequency (Hz) | 0.40 | 1.10 | 1.87 | 2.71 | 3.63 |
| Modal Mass (kN- s²/m) | 406 | 370 | 398 | 573 | 775 |
| Modal Stiffness (MN/m) | 2.60 | 17.7 | 54.7 | 166 | 403 |
| Modal Damping Ratio (%) | 5 | 5 | 5 | 5 | 5 |
| Participation Factor | 1.38 | 0.58 | 0.32 | 0.19 | 0.12 |

(a) A 10-story 3-bay steel building frame (c) Dynamic properties of the building frame

FIG. 9. Modeling of 10-Story Three-Bay Steel Building Frame under NS Component of Ground Acceleration Record of 1940 El Centro Earthquake

## 10-Story Three-Bay Building Frame

A 10-story, three-bay moment-resistant steel building frame used by Naiem (1989) was adopted to demonstrate the effects of passive and active tuned mass dampers on the vibration control of a structure modeled as a multiple-degrees-of-freedom system. The building frame was modeled using a two-node, six-degree-of-freedom planar beam-column element. One element was used to model each of the 70 members of the frame (30 beam and 40 column members). The design loads on the frame were assumed as follows: roof load = 4,790 N/m² and floor load = 5,748 N/m², all with tributary width 9.14 m. The total mass of this building frame was calculated as 963.5 kN-s²/m. The damping ratios were assumed as 5% for each mode. The dynamic properties of the building frame, such as the natural frequency, modal mass, modal stiffness, and participation factor are calculated and listed in Fig. 9. The first 20 sec of the north-south (NS) component of the ground acceleration record of the 1940 El Centro earthquake were used as the input excitation. The time increment for the Newmark integration procedure was assumed to be 0.01 sec. It was noted that under the assumed building properties and the ground excitation, the displacement response due to the first mode constitutes approximately 90% of the total displacement response. Thus the first mode was selected for the designs of TMD and ATMD system.

In the following analysis, the time-average quantities, such as average displacement $J_d$, average absolute acceleration $J_{\ddot{D}}$, and average control force $J_u$ were used for discussion. These values were calculated based on the following equation:

$$J_y = \frac{1}{t_f}\int_0^{t_f} Y'Y\,dt \tag{52}$$

*Tuned Mass Damper*

It was assumed that a tuned mass damper with the stiffness of the damper tuned to the optimal value ($k_1 = k_{1_{opt}}$) was installed on top of the building frame. Three values of mass ratio for the auxiliary mass, which amounted to 1%, 2%, and 3% of the total mass of the building frame, respectively, were assumed for the study.

Fig. 10 shows the average displacement $J_d$ of the building frame for the damping ratio $\xi_1$ ranging between 0 and 20%. It is noted that significant decreases in $J_d$ are seen as $\xi_1$ increases from 0 to 10% for all three values of mass ratio. For $\xi_1$ greater than 10%, however, slight increases of $J_d$ are seen. The solid dots in the figure represent the results of the tuned mass damper with optimal damping ratios calculated using (12). These dots seem to fall in the neighborhoods of the minimum $J_d$ values for all three cases studied. This observation



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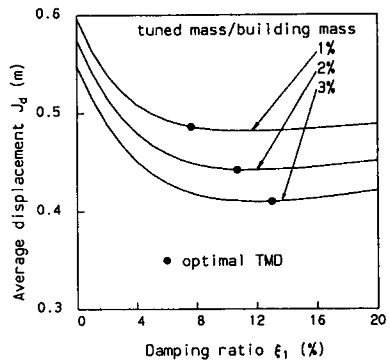


**FIG. 10. Average Displacement of $J_d$ of Building Frame Controlled by Tuned Mass Damper with Various Values of Damping Ratio $\xi_1$**

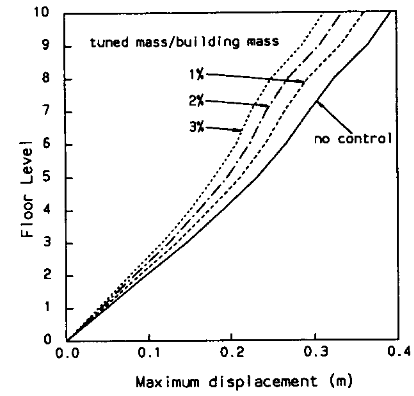


**FIG. 11. Maximum Floor Displacement Profiles for Building Frame Controlled by Tuned Mass Damper with Optimal Properties**

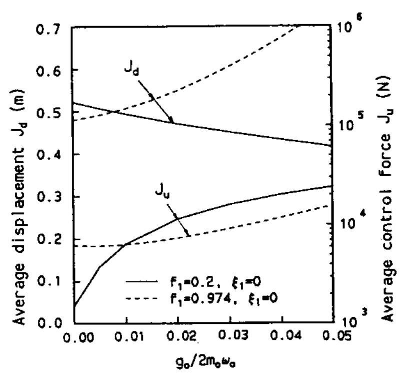


**FIG. 12. Average Displacement $J_d$ and Average Control Force $J_u$ of Building Frame Controlled by Active Tuned Mass Damper with Nonoptimal Properties Using Velocity Feedback**

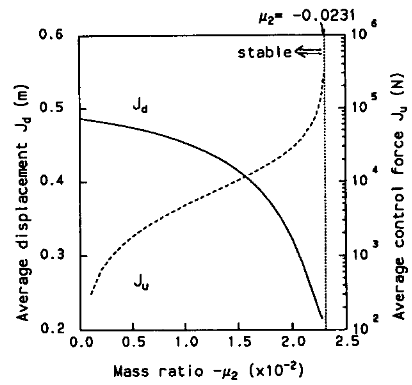


**FIG. 13. Average Displacement $J_d$ and Average Control Force $J_u$ as Functions of Equivalent Mass Ratio $\mu_2$ for Building Frame Controlled by Active Tuned Mass Damper Using Complete Feedback**

confirms that the set of optimal properties derived in (11) and (12) can minimize the response of the building frame.

The maximum floor displacements for the building frame controlled by the tuned mass damper with optimal properties were plotted in Fig. 11. The displacement reduction becomes more prominent as the mass ratio increases. Generally speaking, the reduction does not seem to be significant, because only about 20% of the displacements are suppressed using a tuned mass damper weighted 3% of the total weight of the building. In order to reduce the seemingly heavy mass required for the TMD system, adding active control systems appears to be an alternative.

*Active Tuned Mass Damper Using Velocity Feedback*

It was assumed that an actuator was placed between the building and the TMD and the control force was calculated based on the velocity feedback only. The mass of the damper was assumed to be 1% of the total mass of the building. The optimal frequency ratio $f_{1_{opt}}$ and damping ratio $\xi_{1_{opt}}$ were calculated to be 0.974 and 0.0765, respectively. Two sets of properties were considered for the mass damper, one with $f_1 = 0.2$ and $\xi_1 = 0$, the other with $f_1 = f_{1_{opt}}$ and $\xi_1 = 0$. Also, one of the velocity feedback gain coefficients, $g_1$, was assumed to be constant with a value $0.4m_1\omega_1$, while the other coefficient, $g_0$, varied from 0 to $0.1m_0\omega_0$.

Fig. 12 shows the results of the average displacement $J_d$ and the average control force $J_u$ using these two sets of properties. It is seen that for the TMD with $f_1 = 0.2$ and $\xi_1 = 0$, $J_d$ decreases and $J_u$ increases as $g_0/2m_0\omega_0$ moves from 0 to 0.05. In the mean time, both $J_d$ and $J_u$ increase for the TMD with optimal stiffness ratio.

*Active Tuned Mass Damper Using Complete Feedback*

It was assumed that the control force generated by the actuator placed between the building and the TMD was calculated using complete feedback, and the mass ratio of the TMD was assumed to be 1% with the stiffness and damping constants tuned to their optimal values.

Fig. 13 shows the results of the average displacement $J_d$ and the average control force $J_u$



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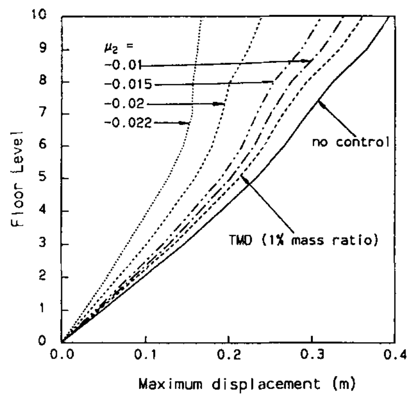


**FIG. 14. Maximum Floor Displacement Profiles for Building Frame Controlled by Active Tuned Mass Damper Using Complete Feedback**

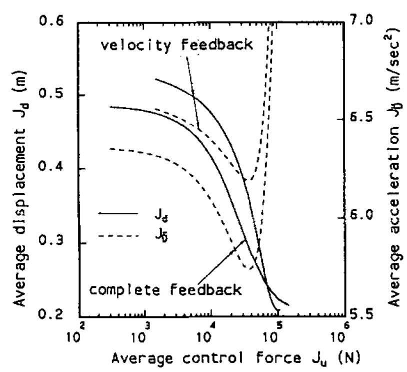


**FIG. 15. Comparisons of Control Efficiency between Complete Feedback and Velocity Feedback**

plotted against the equivalent mass ratio $\mu_2$. It is seen that $J_d$ decreases and $J_u$ increases as $\mu_2$ moves from 0 to $-0.023$. This observation suggests that the ATMD with optimal properties using complete feedback is effective in reducing the displacement of the building frame. It is noted that, in this case, the complete feedback diverges when $\mu_2$ is smaller than $-0.0231$ [(51)]. The divergence is due to the fact that the diagonal stiffness coefficient corresponding to the ATMD system becomes negative.

Fig. 14 shows the maximum floor displacements of the building frame controlled by the ATMD using complete feedback with $\mu_2$ equal to $-0.01$, $-0.015$, $-0.02$, and $-0.022$, respectively. The results for the frame without control and controlled by a TMD were also plotted in the figure for comparison. The maximum-floor-displacement profile decreases steadily as $\mu_2$ moves from $-0.01$ to $-0.022$. The largest control effect is seen for $\mu_2 = -0.022$, which gives approximately 60% reduction in displacement at the top of the building.

*Comparisons*

Fig. 15 shows the comparison of the results between using complete feedback on a TMD with optimal properties and using velocity feedback on a TMD with nonoptimal properties. The mass ratios for both TMDs were assumed as 1% of the total mass of the building frame. The properties of the nonoptimal TMD were assumed as $f_1 = 0.2$ and $\xi_1 = 0$. This set of nonoptimal properties were similar to those of an active mass driver system used by Kobori et al. (1991a) to suppress the response of a 10-story office building subjected to earthquake and typhoon. The results of the average displacement $J_d$ and the average absolute acceleration $J_{\ddot{D}}$ were plotted against the average control force $J_u$. For both cases, as $J_u$ increases, $J_d$ decreases and $J_{\ddot{D}}$ decreases at first and increases after $J_u$ exceeds $3 \times 10^4$ N. Between these two cases, it is observed that for the same amount of $J_u$, the average displacement and absolute acceleration of the building frame controlled by ATMD with optimal properties using complete feedback are smaller than those controlled by ATMD with nonoptimal properties using velocity feedback for the range of $J_u$ up to $7 \times 10^4$ N. This observation suggests that, for the cases studied and the parameters assumed, it is more efficient to control the building frame by ATMD with optimal properties using complete feedback.

## CONCLUDING REMARKS

A closed-loop complete-feedback control algorithm was proposed for the control of a structure modeled as a single-degree-of-freedom (SDOF) system by using an active tuned mass damper (ATMD). The SDOF system was assumed to be under Gaussian white noise stationary ground excitation. The control force was calculated from the acceleration, velocity, and displacement feedbacks of the SDOF system and the auxiliary mass. The passive properties and the gain coefficients of the actuator were derived by minimizing the displacement variance of the SDOF system. The stability of the proposed algorithm was also discussed using the Routh-Hurwitz criterion.

Monte Carlo simulations were performed to verify and evaluate the performance of the ATMD design on the examples of a SDOF system and a 10-story three-bay building frame. The results show that control efficiency of the ATMD based on the velocity feedback depends on the passive-control-device properties assumed. Such a velocity feedback based ATMD can not decrease the structural response when the control-device properties are optimal. The responses of the SDOF system could be reduced either by using velocity feedback on an ATMD with nonoptimal parameters or by using complete feedback on an ATMD with optimal parameters.



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The results also show that for the same level of reduction in the structural displacement, the control force required is smaller using complete feedback.

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## APPENDIX II. NOTATIONS

*The following symbols are used in this paper:*

$c_0$ = damping constant of single-degree-of-freedom system;
$c_1$ = damping constant of tuned mass damper;
$c_2$ = equivalent damping constant of active tuned mass damper;
$f_1$ = frequency ratio defined as $\omega_1/\omega_0$;
$g_0$ = velocity feedback gain associated with $\dot{x}_0$;
$g_1$ = velocity feedback gain associated with $\dot{x}_1$;
$J_D$ = average absolute acceleration of building frame;
$J_d$ = average displacement of building frame;
$J_u$ = average control force;
$k_0$ = stiffness constant of single-degree-of-freedom system;
$k_1$ = stiffness constant of tuned mass damper;
$k_2$ = equivalent stiffness constant of active tuned mass damper;
$m_0$ = mass constant of single-degree-of-freedom system;
$m_1$ = mass constant of tuned mass damper;
$m_2$ = equivalent mass constant of active tuned mass damper;
$P$ = power of control force;
$u$ = control force;
$x_0$ = relative displacement of single-degree-of-freedom system;
$x_1$ = relative displacement of tuned mass damper;
$x_0 - x_1$ = stroke, relative displacement between single-degree-of-freedom system and a tuned mass damper;
$\ddot{y}_0$ = absolute acceleration of single-degree-of-freedom system;
$\ddot{y}_1$ = absolute acceleration of tuned mass damper;
$\beta_0$ = modal participation factor;
$\mu_1$ = mass ratio defined as $m_1/m_0$;



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$\mu_2$ = equivalent mass ratio defined as $m_2/m_0$;
$\xi_0$ = damping ratio of a single-degree-of-freedom system;
$\xi_1$ = damping ratio of tuned mass damper;
$\sigma$ = standard deviation;
$\omega_0$ = frequency of single-degree-of-freedom system; and
$\omega_1$ = frequency of tuned mass damper.

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