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CDTec
centro de desenvolvimento tecnológico

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Computer Graphics
Today: Geometric Transformation

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Motivation

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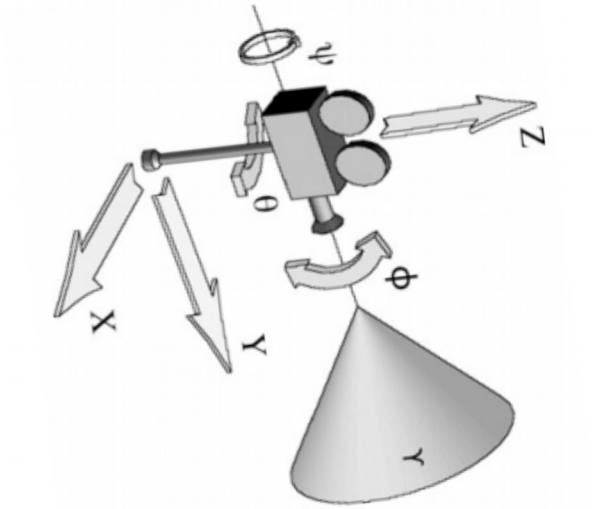
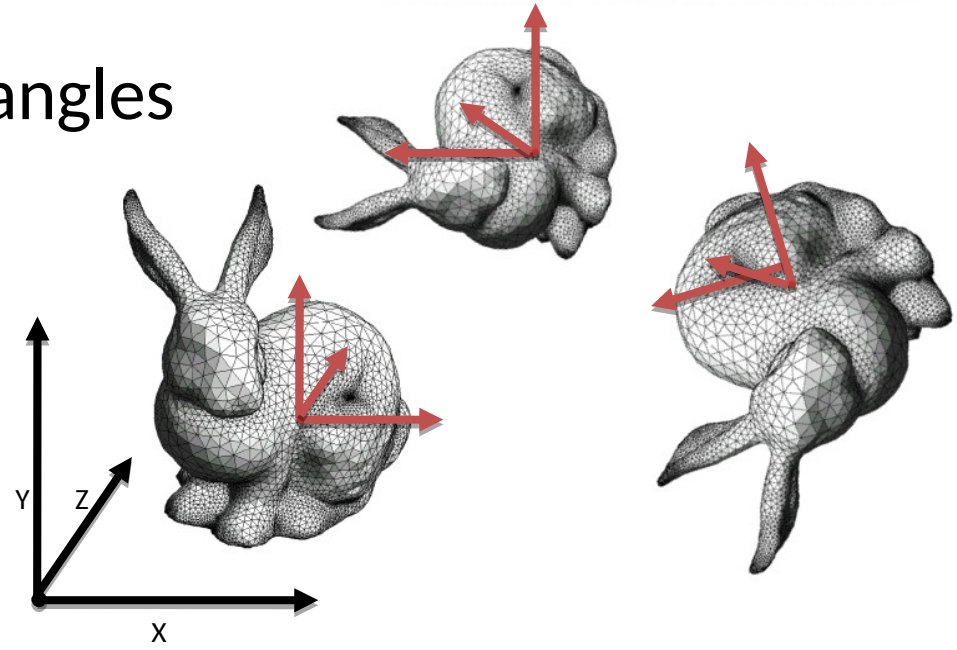
- Different coordinate systems
- Modeling
  - Place an object
  - Size
  - View it from different angles
- Animation





Each basis has a purpose

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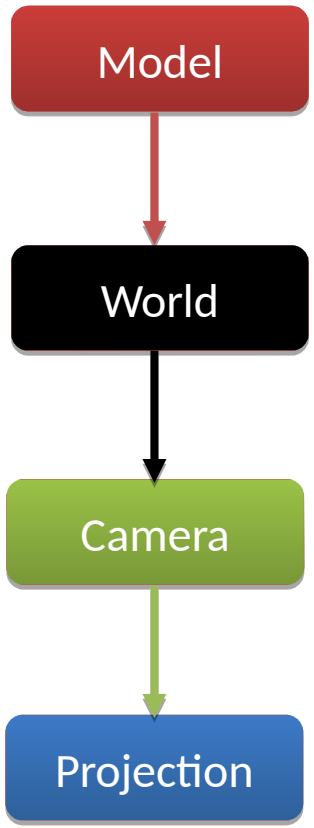

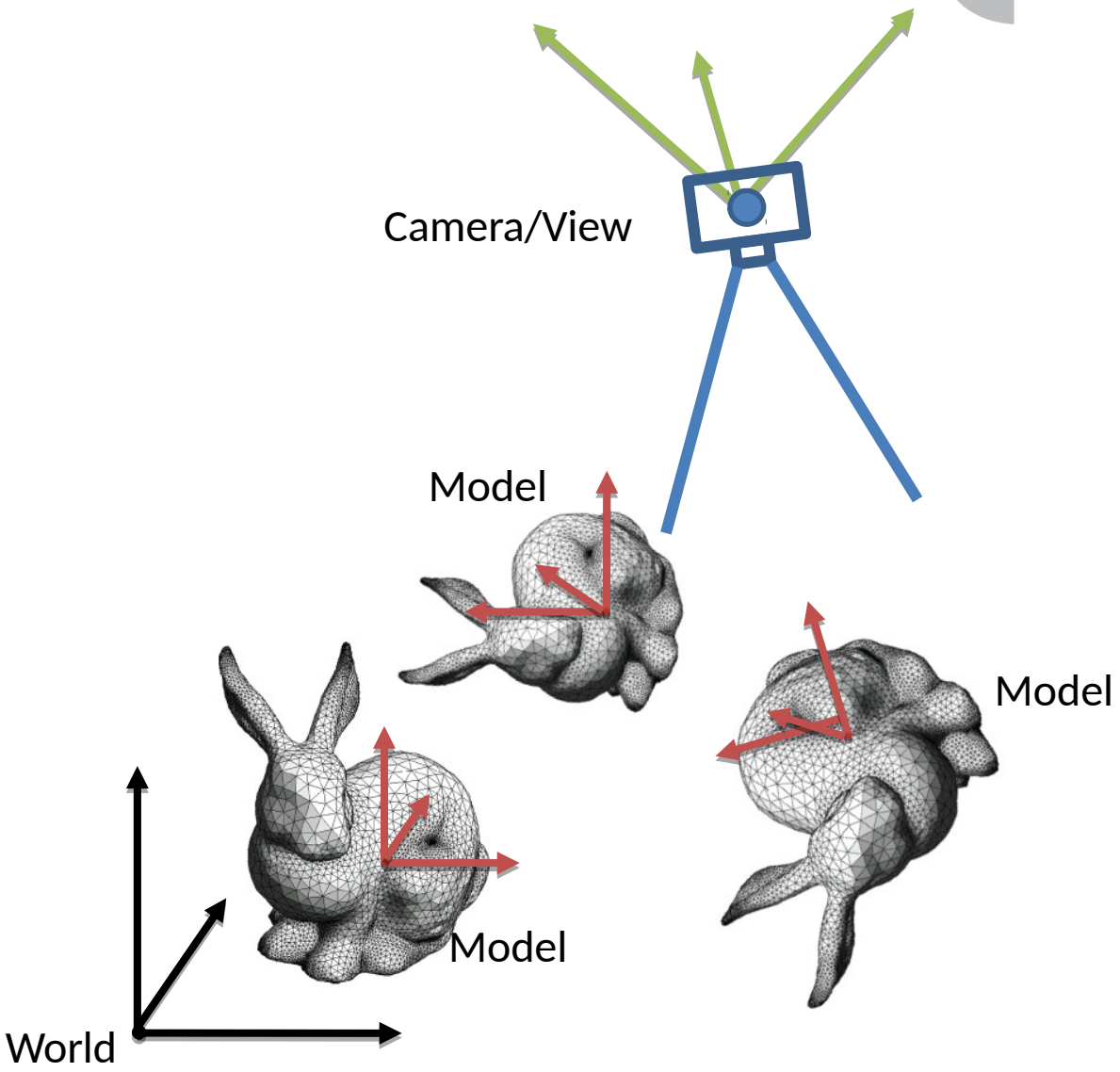

Transformations

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- Transformations 2D and 3D
  - Translation
  - Rotation
  - Scale
  - Shear
- Transformation classes
- Combination of transformations

Concept

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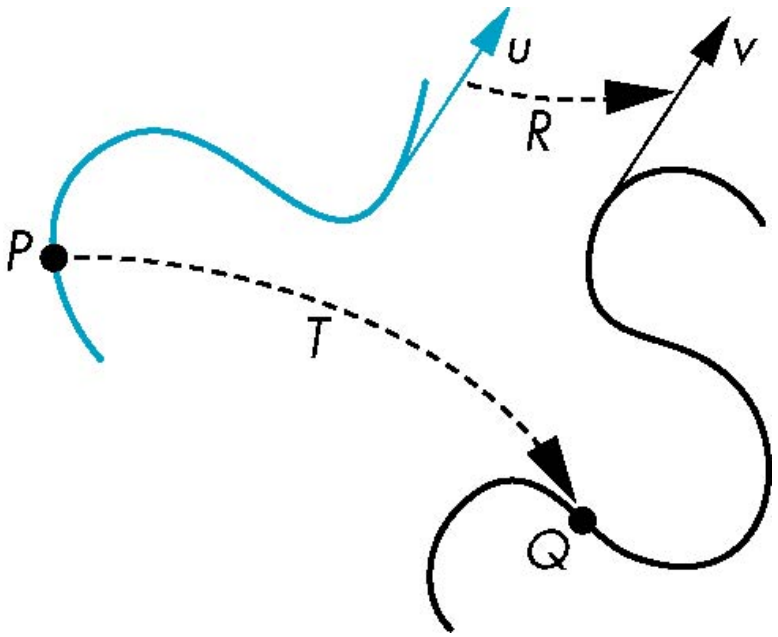

v=T(u)

A transformation **maps** points to other points and/or vectors to other vectors

Pipeline Implementation

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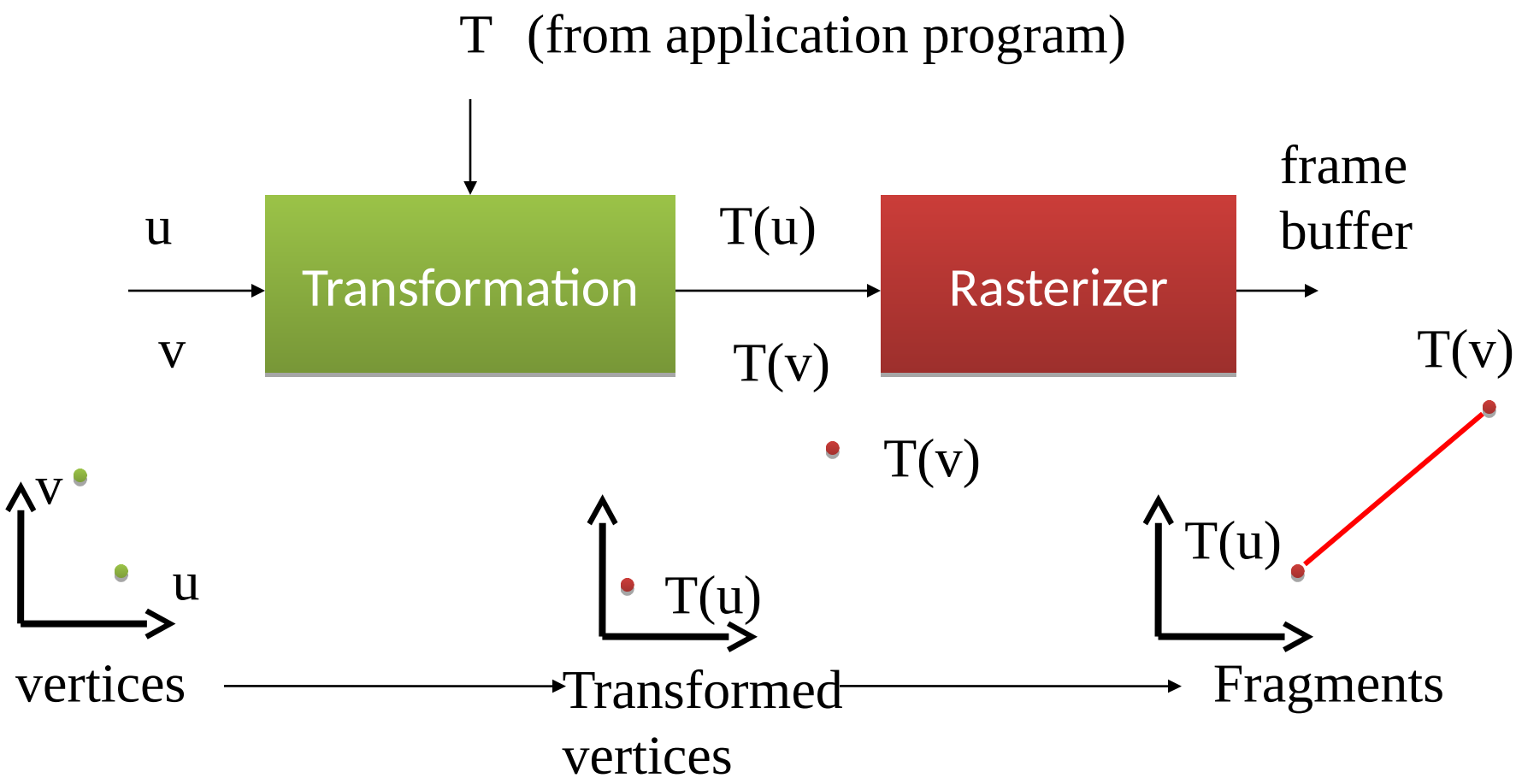

Current GPU **Graphics** Pipeline

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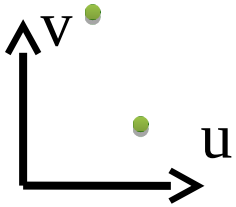
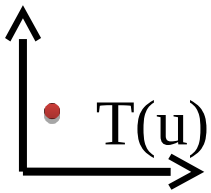
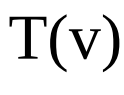
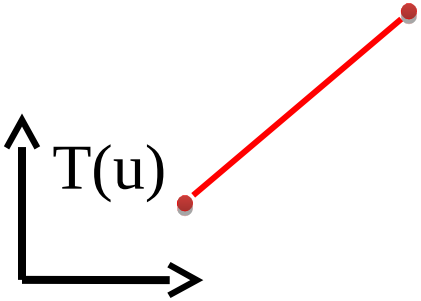


vertices

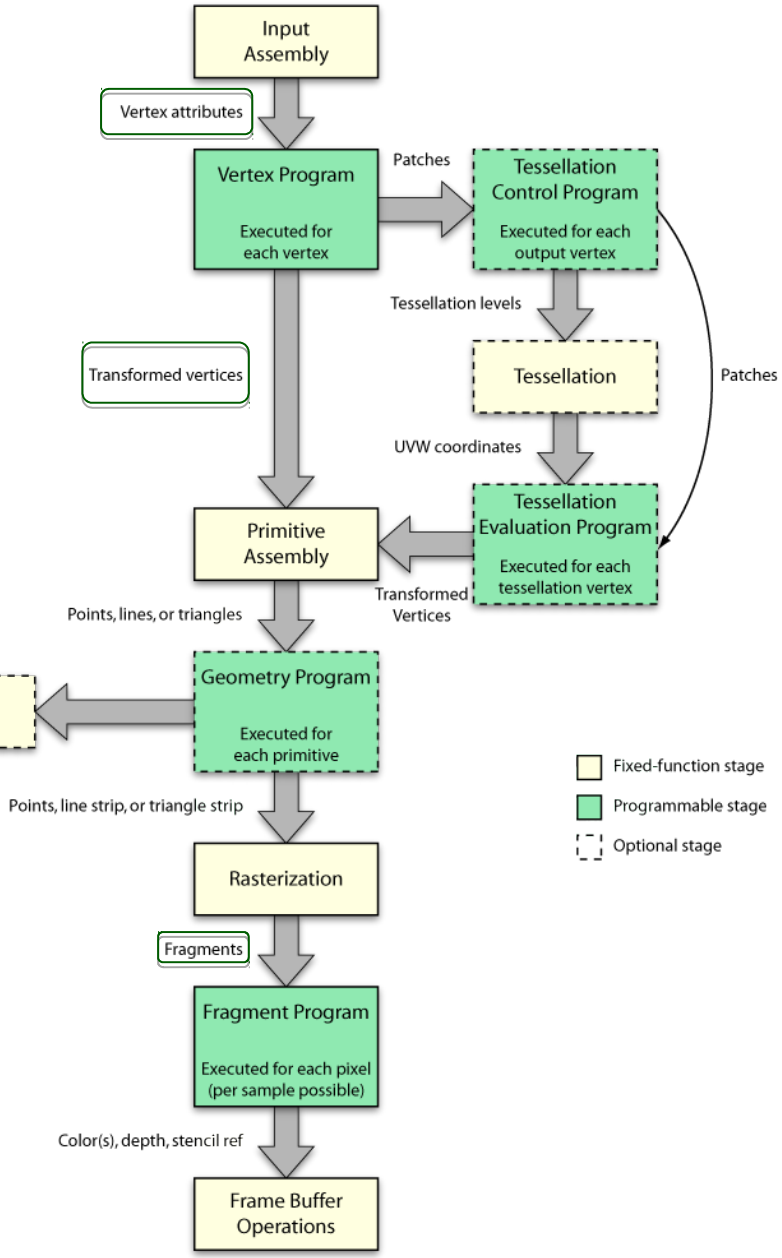


Transformed vertices





Fragments



Translation: Vertice

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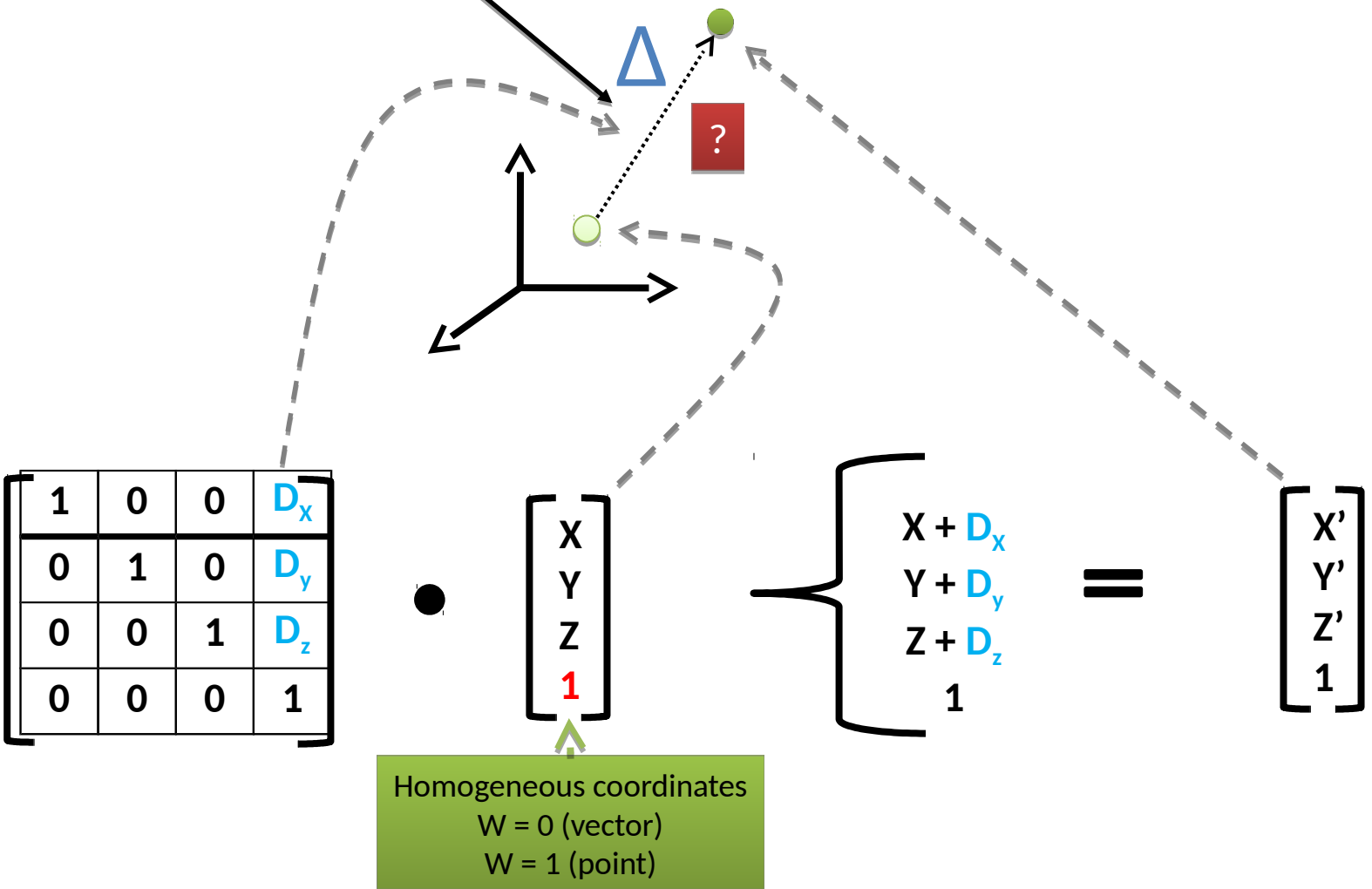
How we compute this vector given two points?



Homogeneous coordinates

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Vector + Vector = Vector

$$\begin{bmatrix} U_x \\ U_y \\ U_z \\ 0 \end{bmatrix} + \begin{bmatrix} V_x \\ V_y \\ V_z \\ 0 \end{bmatrix} = \begin{bmatrix} U_x + V_x \\ U_y + V_y \\ U_z + V_z \\ 0 \end{bmatrix}$$

Point + Vector = Point

$$\begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix} + \begin{bmatrix} V_x \\ V_y \\ V_z \\ 0 \end{bmatrix} = \begin{bmatrix} P_x + V_x \\ P_y + V_y \\ P_z + V_z \\ 1 \end{bmatrix}$$

Point - Point = Vector

$$\begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix} - \begin{bmatrix} R_x \\ R_y \\ R_z \\ 1 \end{bmatrix} = \begin{bmatrix} P_x - R_x \\ P_y - R_y \\ P_z - R_z \\ 0 \end{bmatrix}$$

Translation: Mesh

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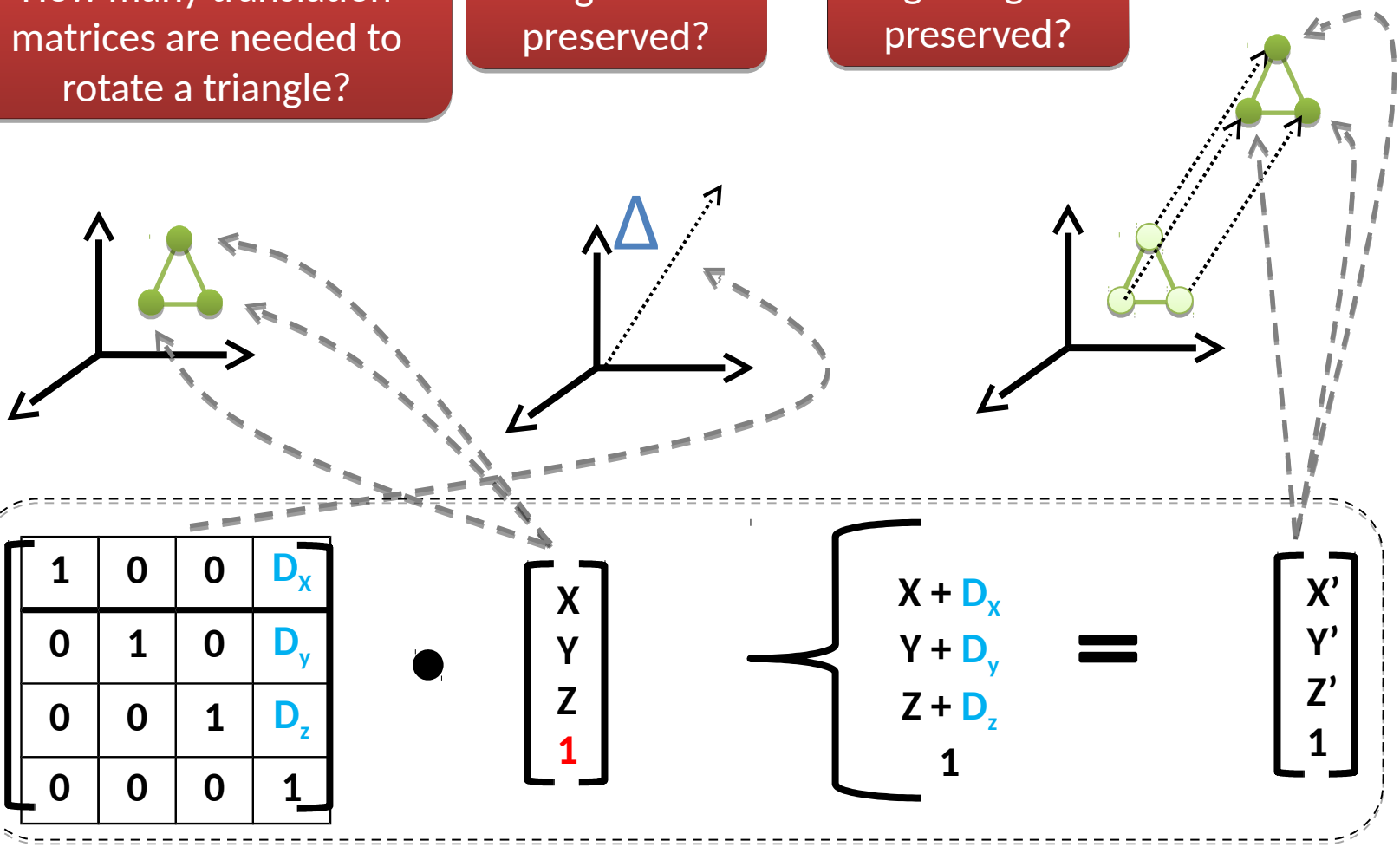
How many translation matrices are needed to rotate a triangle?

Angles are preserved?

Edge length is preserved?



Per vertice

One matrix per model

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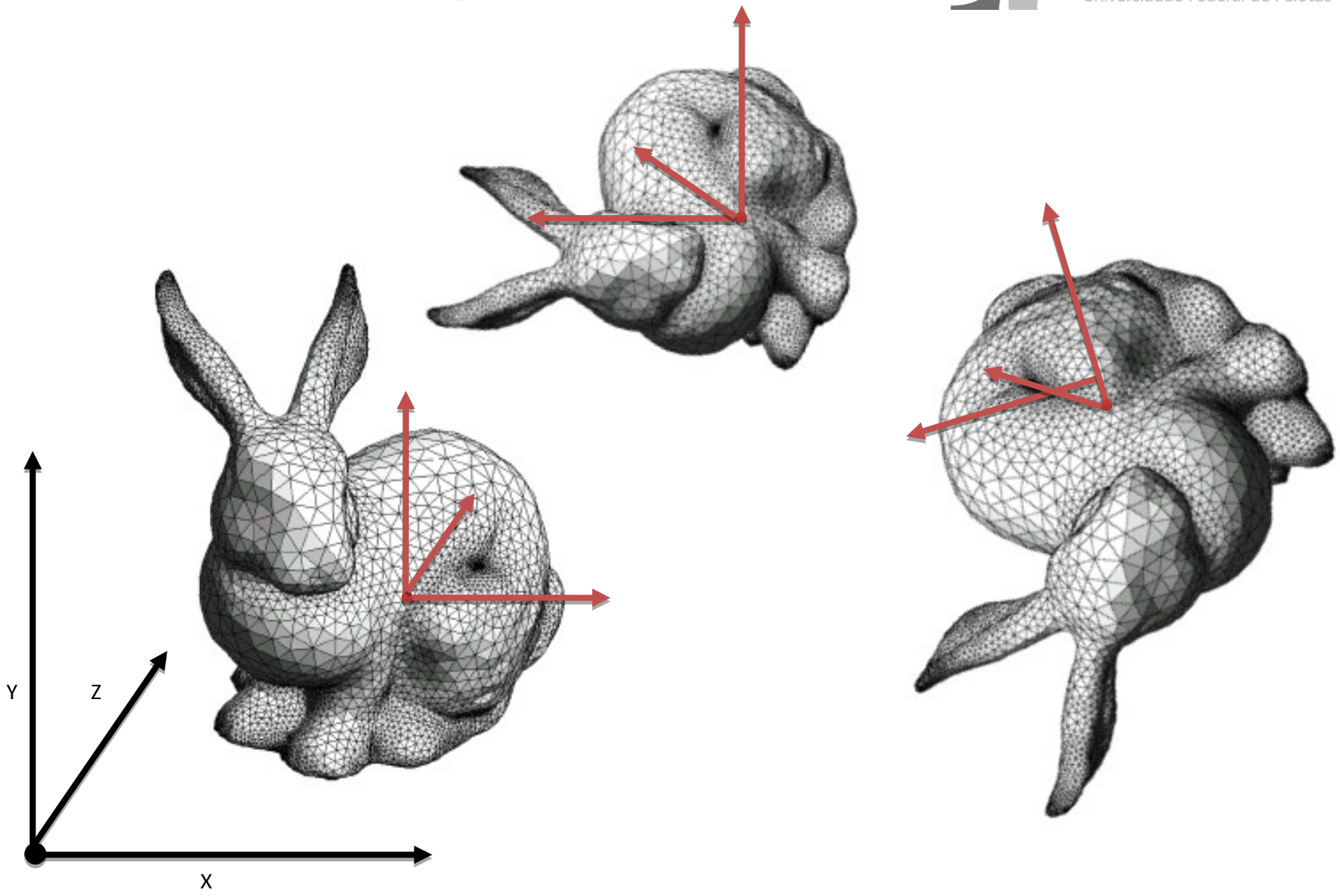
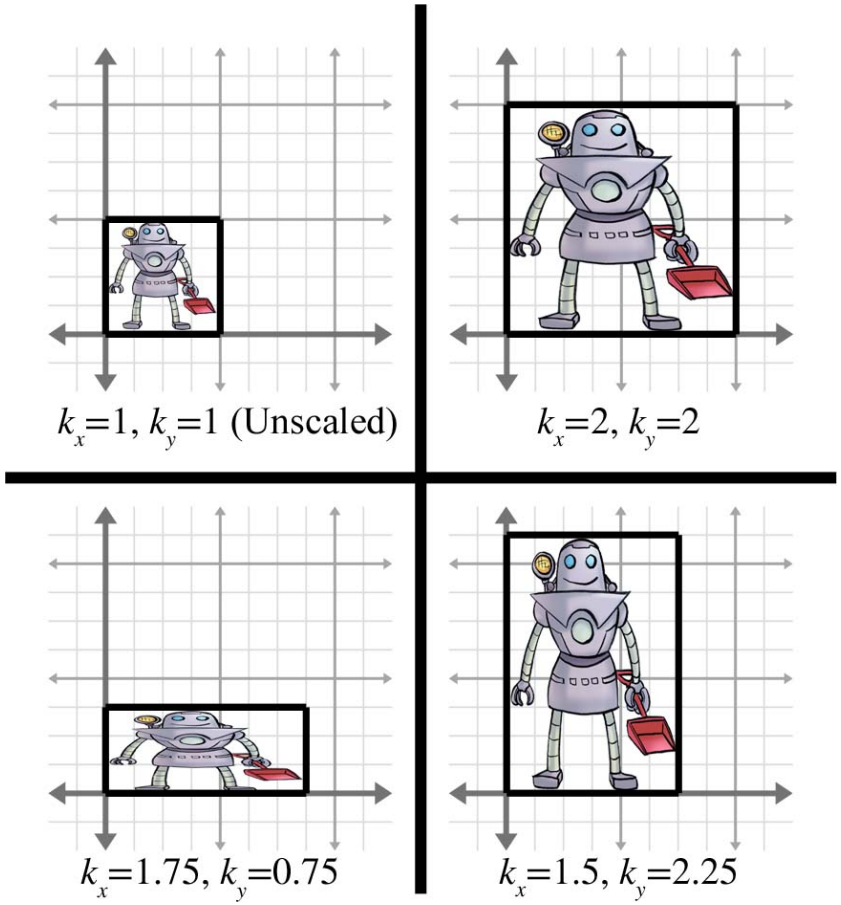


Scale



Scale

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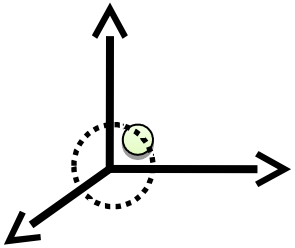
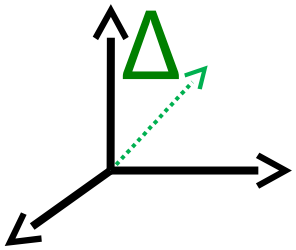
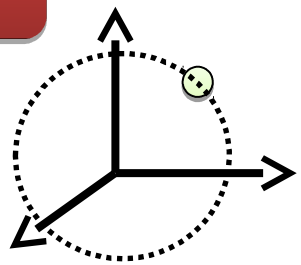
Edge length is preserved?

Angle is preserved?

Always?







$$\begin{bmatrix} D_x & 0 & 0 & 0 \\ 0 & D_y & 0 & 0 \\ 0 & 0 & D_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \begin{cases} X \cdot D_x \\ Y \cdot D_y \\ Z \cdot D_z \\ 1 \end{cases} = \begin{bmatrix} X' \\ Y' \\ Z' \\ 1 \end{bmatrix}$$

Rotation

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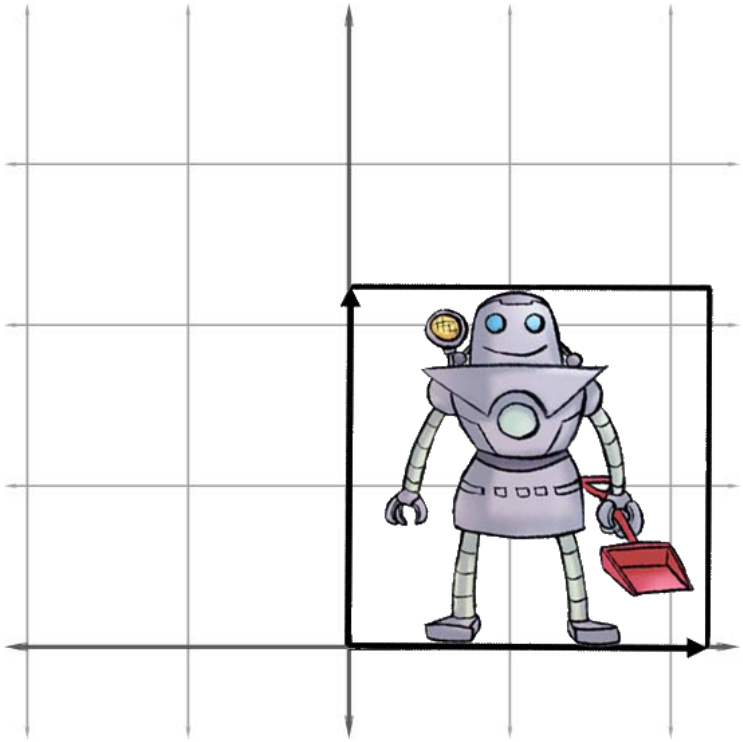
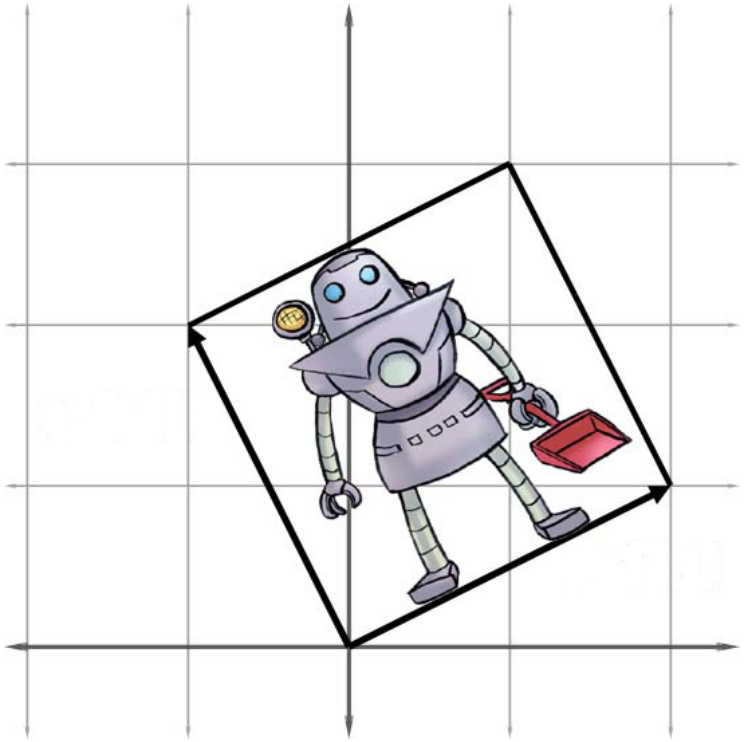
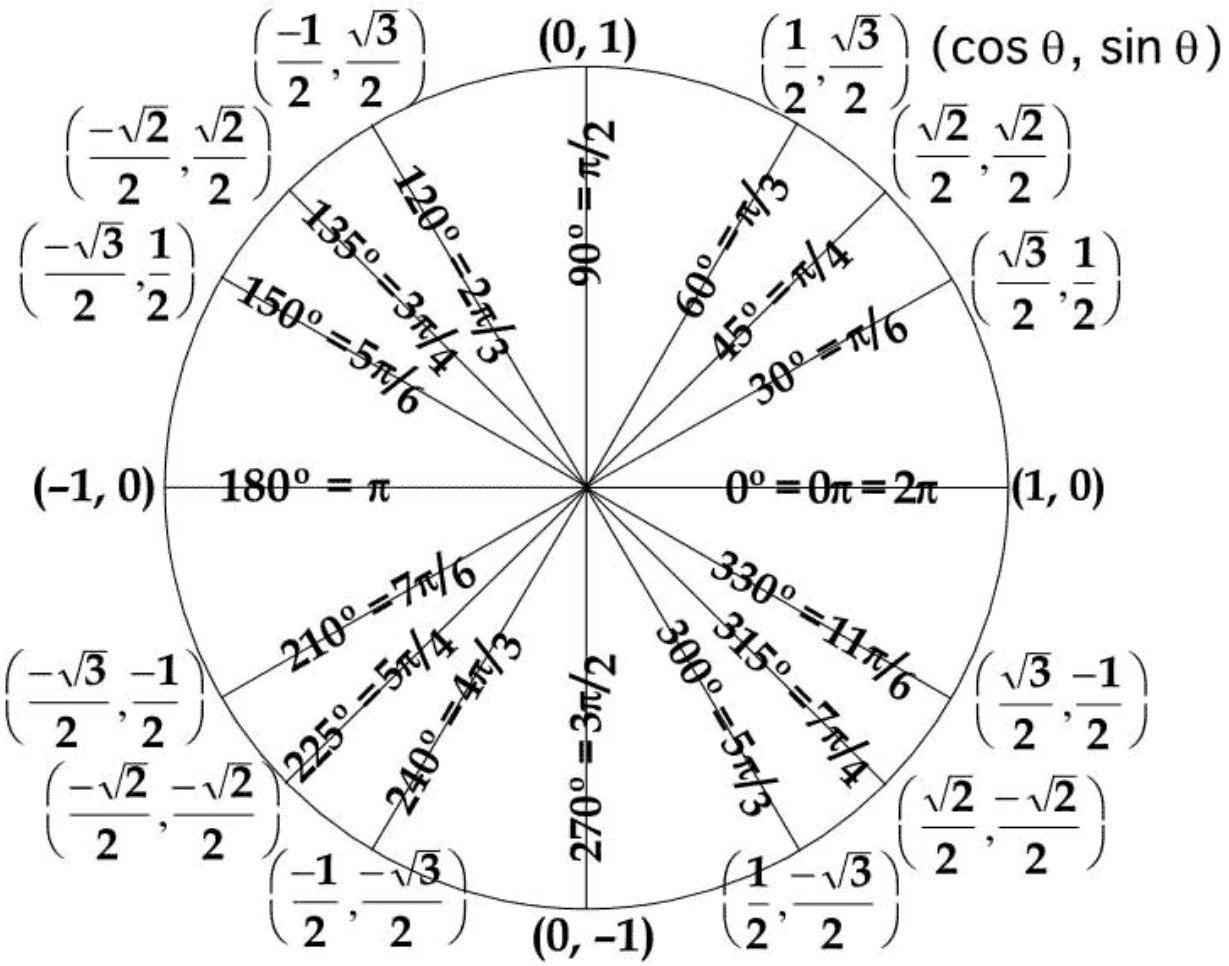


Before



After



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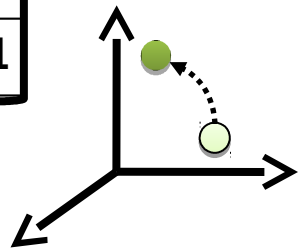
Rotation around which axis?

X

0	Sα	Cα	0
0	0	0	1

Y

Cα	0	Sα	0
0	1	0	0
-Sα	0	Cα	0
0	0	0	1



Z

Cα	-Sα	0	0
Sα	Cα	0	0
0	0	1	0
0	0	0	1

$\bullet$

X
Y
Z
1

X · Cα + Y · -Sα
X · Sα + Y · Cα
Z
1

$=$

X'
Y'
Z'
1

Inverses

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- **Translation**: $\mathbf{T}^{-1}(d_x, d_y, d_z) = \mathbf{T}(-d_x, -d_y, -d_z)$
- **Rotation**: $\mathbf{R}^{-1}(\theta) = \mathbf{R}(-\theta)$
  - Holds for any rotation matrix
  - Note that since $\cos(-\theta) = \cos(\theta)$ and $\sin(-\theta) = -\sin(\theta)$

  $\mathbf{R}^{-1}(\theta) = \mathbf{R}^{T}(\theta)$
- **Scaling**: $\mathbf{S}^{-1}(s_x, s_y, s_z) = \mathbf{S}(1/s_x, 1/s_y, 1/s_z)$

Are these operation invertible?

Yes, except scale = 0

Transformations

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- **Affine**: preserves ratio of areas
  - Translation
  - Rotation
  - Non-uniform or uniform scaling
  - Shearing

- **Conformal**: preserves angles
  - Translation
  - Rotation
  - Uniform scaling

- **Isometric**: preserves length
  - Translation
  - Rotation

Concatenation

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- We can form arbitrary **affine transformation** matrices by multiplying together rotation, translation, and scaling matrices
- Because the same transformation is applied to many vertices, the cost of forming a matrix $\mathbf{M}=\mathbf{ABCD}$ is **not significant compared** to the cost of computing $\mathbf{Mp}$ for many vertices $\mathbf{p}$

Concatenation

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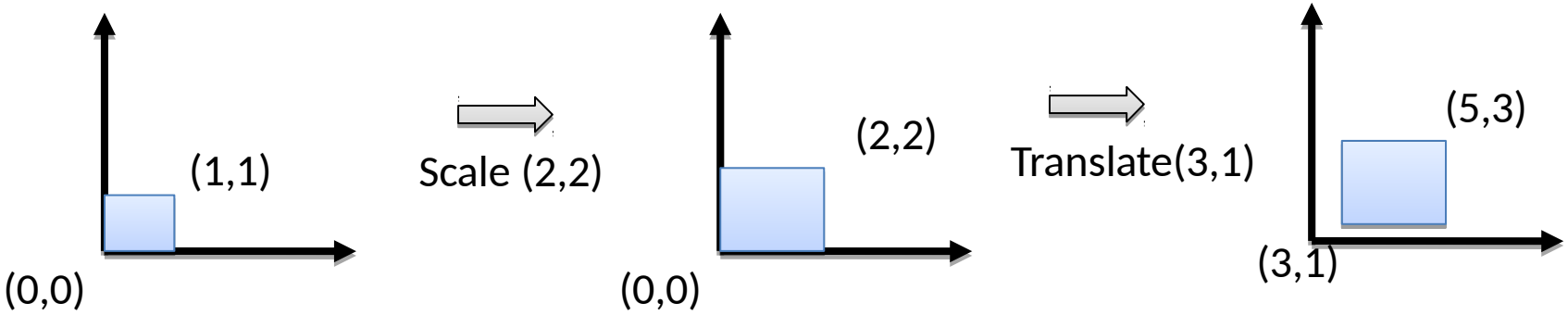

1	0	3
0	1	1
0	0	1

Translate(3,1)

2	0	0
0	2	0
0	0	1

Scale (2,2)

=

2	0	3
0	2	1
0	0	1

TS

Concatenation

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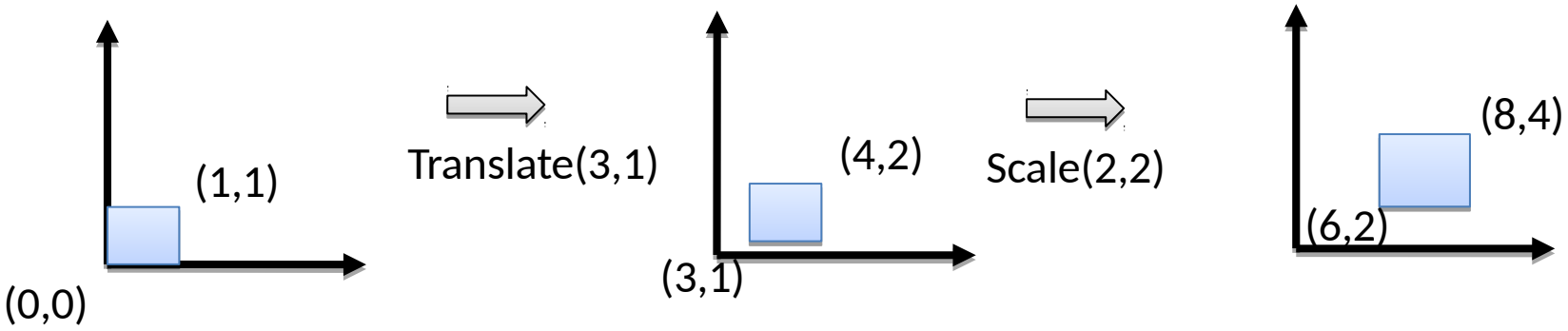

2	0	0
0	2	0
0	0	1

Scale (2,2)

1	0	3
0	1	1
0	0	1

Translate(3,1)

=

2	0	6
0	2	2
0	0	1

ST

Which one is the correct?
It's application dependent

Concatenation

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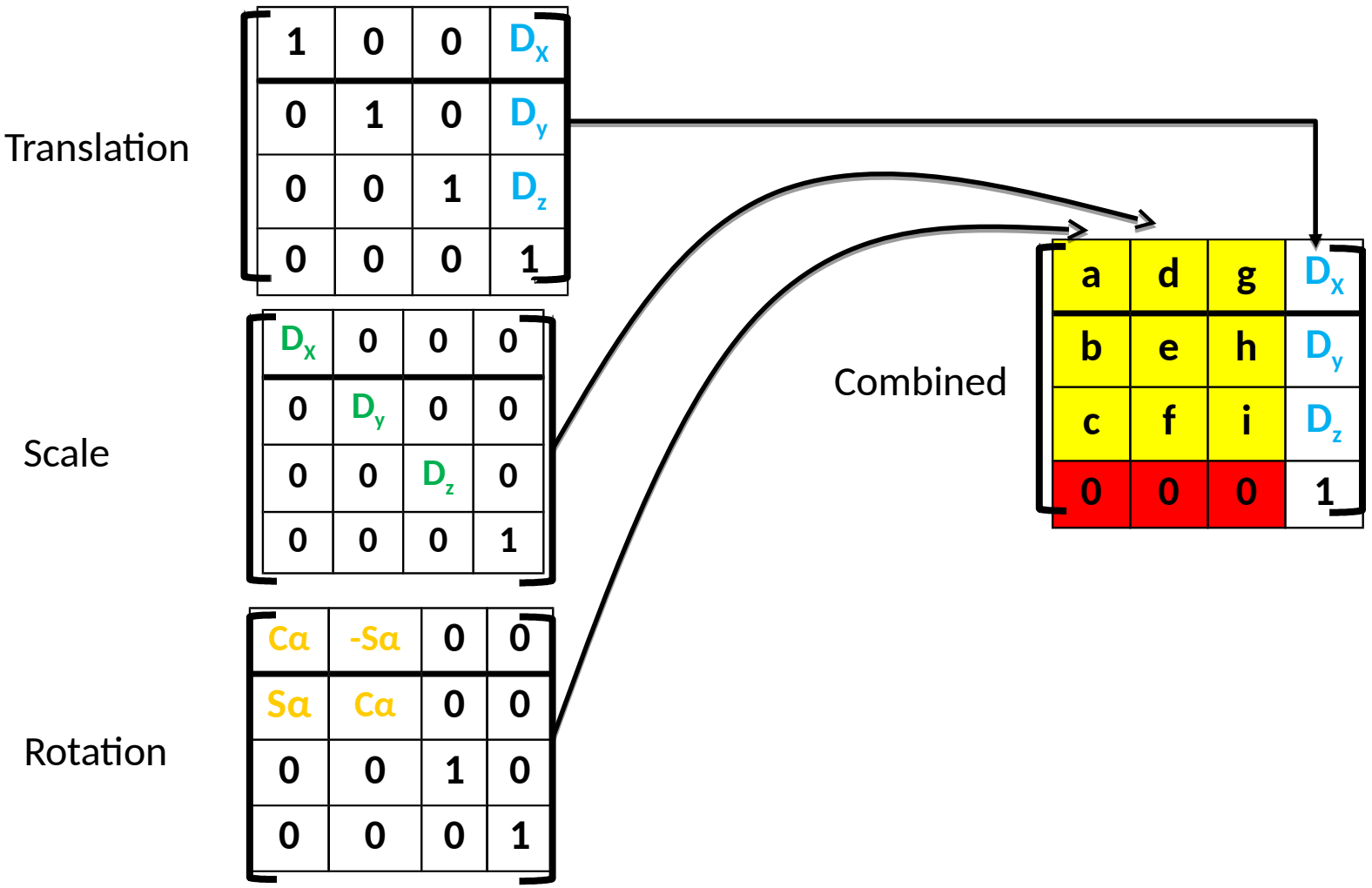

Normal transformation

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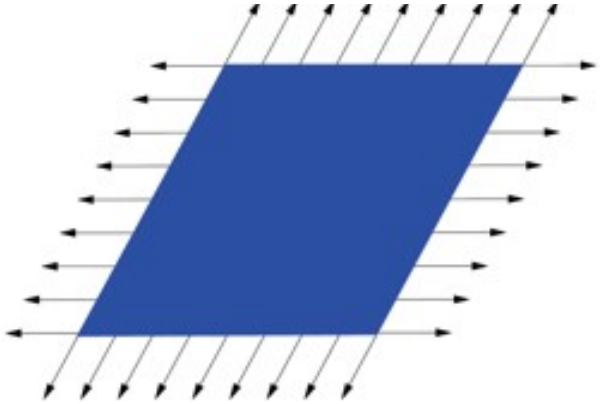
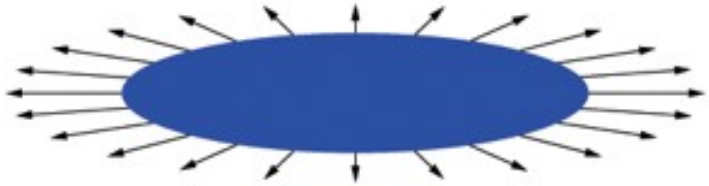
Shear

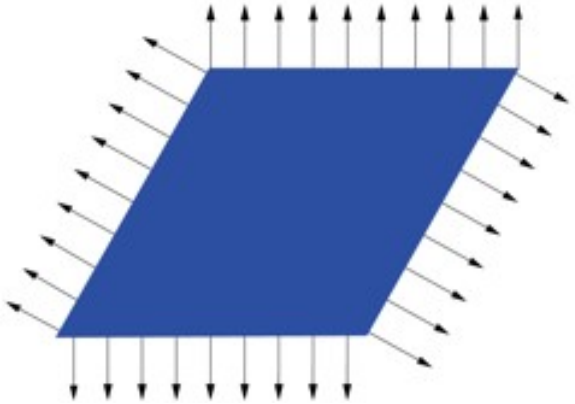
Scale

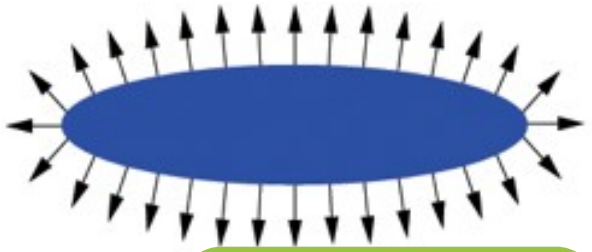
Incorrect Normal Transformation





Correct Normal Transformation





Bonus: 0,5
Explain how normals are transformed

Only one student, first to appear in the forum. Complete lesson!

OpenGL

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https://learnopengl.com/Getting-started/Transformations

Books

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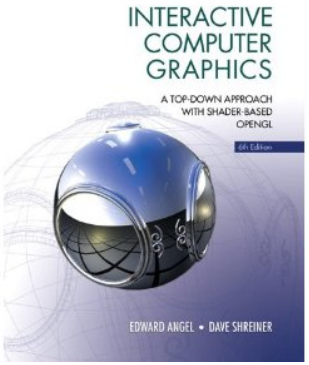
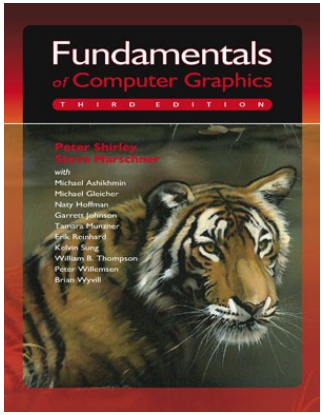


Chapter 3



Chapter 6

Chapter 3 and 4

Read at least one!

Links

http://www.cs.princeton.edu/~gewang/projects/darth/stuff/quat_faq.html
http://www.realtimerendering.com/#xforms
http://www.geometrictools.com/
http://mathworld.wolfram.com/
http://www.gamedev.net
http://solarianprogrammer.com/2013/05/22/opengl-101-matrices-projection-view-model/