BUSINESS APPLICATIONS OF HYPOTHESIS TESTING AND CONFIDENCE INTERVAL ESTIMATION

[redacted]

<u>NORMAL DISTRIBUTION</u>: SYMMETRIC CONTINUOUS DISTRIBUTION WITH TWO PARAM. (THE MEAN AND STD)

THUS, STANDARD NORMAL DISTRIBUTION HAS MEAN = 0 AND STD = 1

<u>T-DISTRIBUTION</u>: VERY SIMILAR TO NORMAL DIST. BUT WITH DEGREES OF FREEDOM (DF).

DF $\rightarrow \infty$ => T-DISTR. === NORMAL DISTR.

<u>DEF.</u>: DEGREES OF FREEDOM IS THE NUMBER OF INDEPENDENT PIECES OF INFO. THAT WENT INTO CALCULATING THE ESTIMATE. BY DEFINITION, DEGREES OF FREEDOM EQUALS TO N-1 (**N IS** THE SIZE OF **POP.**)

<u>CONFIDENCE INTERVAL</u>: DEFINES AN INTERVAL WITH A PROBABILITY THAT CERTAIN SHARE WILL BE WITHIN THIS INTERVAL. <u>E.G.</u>:

95% CONFIDENCE INTERVAL FOR VOTE SHARE OF CANDIDATE A IS [55.7%, 64.3%] WHERE 4.3% IS THE MARGIN OF ERROR.

CONFIDENCE INTERVALS ARE USUALLY USED FOR

- POPULATION PROPORTION (PROPORTION OF VOTES FOR A CERTAIN CANDIDATE)
- POPULATION MEAN (AVERAGE SALARY OF BUSINESS STUDENTS)

THE Z STATISTIC AND THE T STATISTIC

$$Z\text{-STATISTIC} = \frac{\bar{x} - \mu}{\sigma / \sqrt{N}} \sim \text{NORMAL}(0, 1)$$

$$T\text{-STATISTIC} = \frac{\bar{x} - \mu}{S / \sqrt{N}} \sim t_{n-1} = T\text{-DISTRIBUTION}$$

ONE SHOULD USE THE Z-STATISTIC WHEN THE σ IS KNOWN, AND THE T-STATISTIC OTHERWISE, SINCE S (THE SAMPLE STANDARD DEVIATION, USUALLY IS THE INFORMATION KNOWN).

BOTH STATISTICS ARE USED TO BUILD CONFIDENCE INTERVALS.

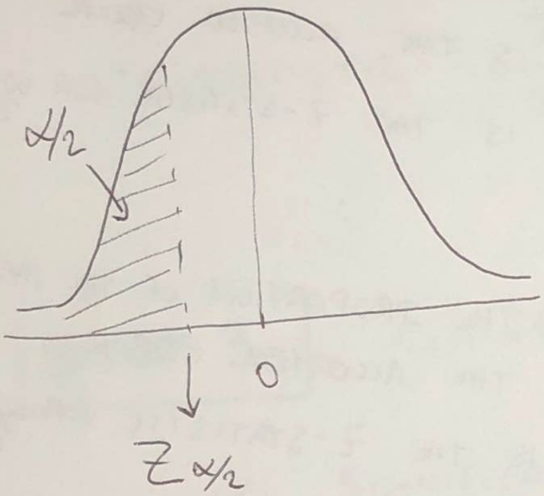
★ THE PROBABILITY FOR OUTSIDE THE CONFIDENCE INTERVAL IS REFERRED AS α, SO, ONE WANTS TO BUILD A $(1-\alpha)$ CONFIDENCE INTERVAL FOR THE POP. MEAN.

THE FORMULA IS GIVEN BY:

$$\underbrace{\bar{x} - \left| Z_{\alpha/2} \right| \cdot \frac{\sigma}{\sqrt{N}}}_{\text{LOWER LIMIT}} < \mu < \underbrace{\bar{x} + \left| Z_{\alpha/2} \right| \cdot \frac{\sigma}{\sqrt{N}}}_{\text{UPPER LIMIT}}$$

MARGIN OF ERROR: $Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{N}}$

SO, A CONFIDENCE INTERVAL OF 80% EQUALS TO $\alpha = 20\%$



WHAT IS GIVEN:

- $\bar{x}$ (SAMPLE MEAN)
- σ (POP. SD)
- N (SIZE OF POP.)
- α (PROB. OF OUTSIDE OF C.I.)

FOR A C.I. OF 100% $\Rightarrow$ $(-\infty < \mu < \infty)$

IN CASE THE SD FOR THE POPULATION IS UNKNOWN, USE THE t-STATISTIC ($t_{\alpha/2}$) INSTEAD OF THE Z-STATISTIC.

THUS,

$$\bar{x} - \left| t_{\alpha/2} \right| \cdot \frac{s}{\sqrt{N}} < \mu < \bar{x} + \left| t_{\alpha/2} \right| \cdot \frac{s}{\sqrt{N}}$$

FOR A $(1-\alpha)$ CONFIDENCE INTERVAL FOR THE POPULATION MEAN.

CONFIDENCE INTERVAL FOR THE POP. PROPORTION

IS GIVEN BY THE FOLLOWING FORMULA:

$$\hat{p} - |Z_{\alpha/2}| \sqrt{\frac{\hat{p}(1-\hat{p})}{N}} < p < \hat{p} + |Z_{\alpha/2}| \sqrt{\frac{\hat{p}(1-\hat{p})}{N}}$$

WHERE $\hat{p}$ IS THE SAMPLE PROPORTION.

SAMPLE SIZE CALCULATION

- FOR POPULATION MEAN

$$n = \frac{Z_{\alpha/2}^2 \cdot \sigma^2}{e^2}$$

WHERE σ IS THE SD OF THE POP.
e IS THE ACCEPTED ERROR
$Z_{\alpha/2}$ IS THE Z-STATISTIC FOR $\alpha/2$

- FOR POPULATION PROPORTION

$$n = \frac{Z_{\alpha/2}^2 \cdot p(1-p)}{e^2}$$

WHERE p IS THE PROPORTION OF THE POP.
e IS THE ACCEPTED ERROR
$Z_{\alpha/2}$ IS THE Z-STATISTIC FOR $\alpha/2$

* THE p EQUALS TO 50% MAXIMIZES THE ESTIMATION

HYPOTHESIS TESTING (T-TEST)

THE IDEA/GOAL IS TO TEST TWO HYPOTHESIS/PROPOSALS FROM A STATISTICAL POINT OF VIEW.

BELOW THERE IS AN EXAMPLE WITH THE FOUR STEPS TO PERFORM THE TEST.

SUPPOSE YOU HAVE THE FOLLOWING:

$n = 10$, $\bar{x} = 199$ mL, $s = 0.8$ mL

STEP 1 FORMULATE THE HYPOTHESIS

NULL HYPOTHESIS $H_0: \mu = 200$

ALTERNATE HYPOTHESIS $H_A: \mu \neq 200$

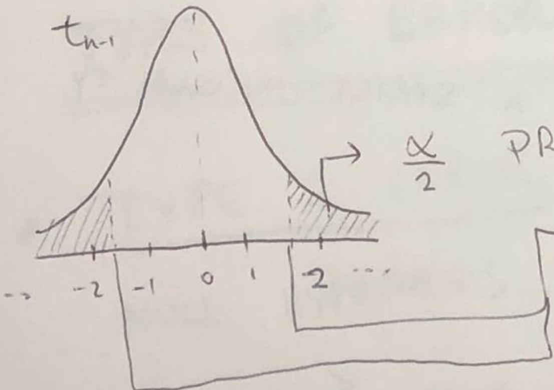
} TWO TAILED TEST (SINCE IT CAN BE TOO MUCH OR TOO LITTLE)

STEP 2 CALCULATE THE T-STATISTIC

$$\text{T-STATISTIC} = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{199 - 200}{0.8/\sqrt{10}} = -3.9528$$

STEP 3 CUTOFF VALUES FOR THE T-STATISTIC

α: SIGNIFICANCE LEVEL ($(1-\alpha)$ IS ~~USUALLY~~ THE CONFIDENCE INTERVAL, SO USUALLY 0.05 OR 0.01)



$\rightarrow \frac{\alpha}{2}$ PROBABILITY

$\rightarrow$ THOSE ARE GOING TO BE THE CUTOFFS, LET'S SAY ± 1.5

STEP 4 CHECK WHETHER T-STATISTIC FALLS IN THE REJECTION REGION

THUS, IN THIS EXAMPLE WE WOULD REJECT THE NULL HYPOTHESIS.

★ FOR A SINGLE TAIL HYPOTHESIS TEST, THE DIFFERENCE WILL RELY ON THE FOLLOWING STEPS:

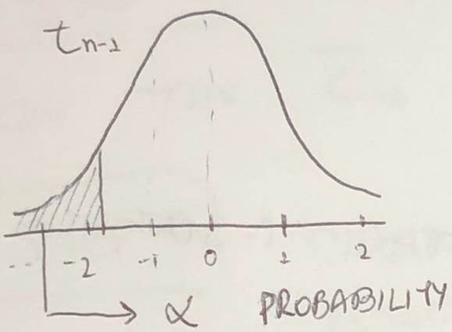
EXAMPLE: THE CLAIM IS THAT μ WILL BE GREATER OR EQUAL TO 200

STEP 1

$$H_0 : \mu \geq 200$$

$$H_A : \mu < 200$$

STEP 3 THE CUTOFF IS DEFINED ONLY FOR THE LEFT SIDE OF THE DISTRIBUTION:



THE VALUE OF THE CUTOFF PER SE IS GIVEN BY THE STATISTIC $t_{\alpha}^{(n-1)}$ AND THE VALUE WILL BE POSITIVE OR NEGATIVE DEPENDING ON THE HYPOTHESIS.

- IMPORTANT OBSERVATION:

NULL HYPOTHESIS SHOULD NOT HAVE A STRICT INEQUALITY

SO, IT CAN ONLY HAVE:

$=$	$\geq$	$\leq$
TWO TAILED HYPOTHESIS TEST	ONE TAIL TEST, REJECTION REGION ON LEFT SIDE	ONE TAIL TEST, REJECTION REGION ON RIGHT SIDE

HYPOTHESIS TESTING FOR POPULATION PROPORTION

STEP 1 $H_0 : p \geq 0.70$

$H_A : p < 0.70$

STEP 2 CALCULATE THE Z-STATISTIC

$$\text{Z-STATISTIC} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

, WHERE $\hat{p}$ IS THE SAMPLE PROPORTION AND p IS THE PROPORTION WE WANT TO TEST

STEP 3 THE CUTOFFS ARE CALCULATED BASED ON THE Z_α, WHERE α ~~[illegible]~~ IS THE PROBABILITY

STEP 4 CHECK WHETHER Z-STATISTIC FALLS IN THE REJECTION REGION.

TYPES OF ERRORS

- TYPE 1 ERROR OCCURS WHEN WE REJECT THE NULL HYPOTHESIS WHEN ITS TRUE.
- TYPE 2 ERROR OCCURS WHEN WE FAIL TO REJECT THE NULL HYPOTHESIS WHEN ITS FALSE.

- PROBABILITY OF TYPE 1 ERROR IS SET BY OUR CHOICE OF α.
- PROBABILITY OF TYPE 2 ERROR CAN BE REDUCED BY TAKING A LARGER SAMPLE SIZE.

DIFFERENCE IN MEANS TEST

EXAMPLE: DIFFERENCE BETWEEN THE POPULATION MEAN HEIGHT OF MEN AND WOMEN IS 12.5 cm (CLAIM)

STEP 1

$$H_0: \mu_{men} - \mu_{women} = 12.5$$

$$H_1: \mu_{men} - \mu_{women} \neq 12.5$$

- DIFFERENCE IN MEANS TEST
- TWO TAILED TEST

STEP 2

- NEED TO ASSUME EITHER EQUAL VARIANCE OR UNEQUAL VARIANCE ACROSS THE TWO POPULATIO
- SUBJECT JUDGEMENT
- IN MOST CASES, EQUAL OR UNEQUAL DO NOT CHANGE THE CONCLUSION.

"EQUAL VARIANCE"

$$\frac{\bar{x}_1 - \bar{x}_2 - \mu}{\sqrt{\left(\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}\right)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \text{T-STAT}$$

"UNEQUAL VARIANCE"
$$\frac{\bar{x}_1 - \bar{x}_2 - \mu}{\sqrt{\left(\frac{s_1^2 + s_2^2}{n_1 + n_2}\right)}} = \text{T-STAT}$$

$\boxed{\text{STEP 3}}$ CALCULATE THE T-CUTOFF (TWO TAIL TEST)

$\boxed{\text{STEP 4}}$ CHECK IF T-STATISTIC FALLS IN THE REJECTION REGION.

$\boxed{\text{THIS IS CALLED THE T-TEST}}$

ON THIS SPECIFIC CASE ITS THE "T-TEST: TWO SAMPLE ASSUMING EQUAL VARIANCES"

THERE IS ALSO THE PAIRED T-TEST USED FOR CASES LIKE SCORES OF EXAMS FOR INDIVIDUALS BEFORE AND AFTER A CERTAIN EXPERIMENT.

$\boxed{\text{STEP 1}}$ $H_0: \mu_{AFTER} - \mu_{BEFORE} \geq 10$

$H_A: \mu_{AFTER} - \mu_{BEFORE} < 10$

FINALLY, THERE ARE CASES WHERE ONE WANTS TO KNOW WHETHER TWO AVERAGES ARE THE SAME, EXAMPLE BELOW.

$\boxed{\text{STEP 1}}$ $H_0: \mu_{GROUP\,1} - \mu_{GROUP\,2} = 0$

$H_A: \mu_{GROUP\,1} - \mu_{GROUP\,2} \neq 0$

ADDITIONAL INFORMATION:

- ALL TESTS PERFORMED IN THIS COURSE USING THE T-DISTRIBUTION IS OF THE TYPE "T-TEST". ALSO, T-DISTRIBUTION CAN ALSO BE CALLED "STUDENT'S T-DISTRIBUTION".

- FOR NON-PARAMETRIC DISTRIBUTION, i.e., WHEN THERE IS NO PARAMETERS DEFINING THE DISTRIBUTION, THE TEST TO BE USED SHOULD BE THE WILCOXON TEST.

- IN CASE THERE ARE MORE THAN TWO HYPOTHESIS (A, B, C, D, ...), YOU SHOULD USE ANOVA TEST