## **CPLEX**

## **Introduction**

#### **Check this first:**

TI OG	Solvers	
ILOG CONCERT	CPLEX	mathematical programming (LP, QP, MILP, MIQP)
COTTOLLIVI	CP	constraint programming

- ➤ A common way to define the model (CONCERT environment)
- > All classes are tied to the environment
- > Components (variables/constraints/objective) are added to the model
- ➤ Model is extracetd by CPLEX

#### Base classes to solve models:

	Steps	Classes
	environment (main class)	IloEnv
	model	IloModel
CONCERT	variables	IloNumVar, IloNumColumn, IloNumVarArray, IloConversion
	constraints	IloRange, IloExpr, IloRangeArray
	objective	IloObjective
CPLEX	solve	IloCplex

- > All classes (except IloCplex) are listed in the Concert section of the reference guide (see link of online documentation at the bottom of this page).
- > IloCplex class also has an "advanced" section with methods that give users an opportunity to interfere with the normal progress of branch&cut (=> callbacks)

## **IMPORTANT**:

- > By default, CPLEX (as Gurobi, Matlab and others do) will use **ALL** CPUs and cores of the machine on which the program runs.
- > To limit the number if threads used by CPLEX, we ask you to use this instruction to fix the limit to 1:

## $cplex.setParam(IloCplex::Param::Threads,\ 1);$

#### **Interfaces:**

Interactive	load a model from disk and solve it while fixing parameters manually
Library	build and solve a model with classes and functions in various languages (C, C++, Java, Python, .NET, MATLAB)
OPL	special language from ILOG to interact with Excel among other things

#### **Compilation**:

	Headers	Preprocessor directives (-D)
C	#include <ilcplex cplex.h=""></ilcplex>	
C++	#include <ilcplex ilocplex.h=""></ilcplex>	IL_STD ==> STL access NDEBUG ==> assert (checking disabled)
Java	import ilog.concert.*; import ilog.cplex.*;	

	Libraries
C	libcplex.a libpthread.a libm.a
C++	libilocplex.a libconcert.a libcplex.a libpthread.a libm.a
Java	cplex.jar

## N.B.: In C++, the order of libraries listed in bold is IMPORTANT!!!

## <u>Macro (C++)</u>: **ILOSTLBEGIN** ( ===> using namespace std; )

- Must be used before any call to CPLEX classes.
- ➤ In the future, could be extended to include ther features.

# **CPLEX C++ (Concert):**

## Overview:

• Classes, definitions and base types:

IloExtractable	base class for model building classes
IloAlgorithm	base class for model solving classes
IloNum,	alias for double,
IloInt	int,
IloBool	bool (coded as int)
ILOFLOAT	types for numeric,
ILOINT	integer,
ILOBOOL	binary variables
IloInfinity	infinity (bounds of variables/constraints)

#### Environment and model

IloEnv	create a common environment (build + solve model)
IloModel	create LP, QP, MILP, MIQP model (with variables, constraints, etc.)

## • Objective:

IloObjective	define objective function
IloMinimize, IloMaximize	sets optimization direction (min / max)

## Variables:

IloNumColumn	populate variables adding coefficients one by one
IloNumVar	declare numeric (defaut),
IloIntVar	integer,
IloBoolVar	binary variables
IloConversion	convert numeric variables in other types

#### • Constraints:

IloExpr	build an expression adding coefficients one by one
IloRange	declare constraints
IloSum	return the sum of variables
IloScalProd	return the scalar product between an array of variables and an array of coefficients
IloIfThen	create "Big M" or logic constraints

## • Arrays + macro:

IloArray	Class template to create multi-dimension extensible arrays
IloExtractableArray	Extensible array of IloExtractable
IloNumArray IloIntArray IloBoolArray	Extensible arrays of base types (IloNum, IloInt, IloBool)
IloNumVarArray IloIntVarArray IloBoolVarArray	Extensibles arrays of variables (IloNumVar, IloIntVar, IloBoolVar)
IloRangeArray	Extensibles arrays of constraints
IloAdd	Macro to define objects and add them to the model in one shot.

## • Solve:

IloCplex
----------

## Base classes:

• IloExtractable: base class for model building classes

end()	destroy the element (replace the destrcutor)	
getEnv()	retrieve the environment to which the element is tied	
removeFromAll()	remove element from all other related objects	
setName(const char*)	give name to the element	
getObject()	retrieve "object" assigned to the element	
setObject(IloAny)	assign object to element	

• IloAlgorithm: base class for model solving classes

clear()	reinitialize solver
end()	destroy solver (replace destructor)
extract(const IloModel)	load model in the solver
setError(ostream&) setWarning(ostream&) setOut(ostream&)	redirect error, warning and output channels for solver

## **Environment and model:**

• **IloEnv** : create a common environment (build + solve model)

end()	destroy environment (replace destructor) as well as ALL objects tied to this environment
<pre>setError(ostream&amp;) setWarning(ostream&amp;) setOut(ostream&amp;)</pre>	redirect error, warning and output channels for environment
getMemoryUsage()	heap usage (in bytes)
${\bf getTotal Memory Usage}()$	allocated heap (in bytes)

ex.: **IloEnv** env;

• **IloModel**: create LP, QP, MILP, MIQP model (with variables, constraints, etc.)

<pre>add(const IloExtractableArray &amp;X) add(const IloExtractable &amp;X)</pre>	add element or array X to model
<pre>remove(const IloExtractableArray &amp;X) remove(const IloExtractable &amp;X)</pre>	remove element or array X from model
inherited methods	see IloExtractable

**==> you should add to model**: variables, constraints, objective

ex.: IloModel model(env);
 model.add(obj);

#### Objectif:

• **IloObjective**: define objective function

setConstant(IloNum)	sets constant term of objective
setLinearCoef(variable, value) setLinearCoefs(variables, values)	modify one or multiple objective coefficients
setExpr(expression)	sets expression of objective
setSense(IloObjective::Sense)	sets sense (min/max) of the objective
inherited methods	see IloExtractable

ex.: <u>ilobendersatsp.cpp</u>

IloObjective obj(env);
obj.setSense(IloObjective::Minimize);
obj.setExpr(exp);

N.B.: IloObjective::Sense = IloObjective::Minimize or IloObjective::Maximize

• **IloMinimize**, **IloMaximize**: sets optimization direction (min / max)

 $ex.: \quad \textbf{\it IloObjective} \ obj = \textbf{\it IloMinimize} (env);$ 

#### Variables:

• **IloNumColumn**: populate variables adding coefficients one by one

clear()	reinitialize object
operator +=()	add coefficients to the variable
end()	destroy object

# N.B.: USE end() METHOD ONCE VARIABLE HAS BEEN DEFINED IN ORDER TO PREVENT MEMORY LEAKS

• IloNumVar, IloIntVar, IloBoolVar : declare numeric, integer or binary variables by default

setLB(value) setUB(value) setBounds(value, value)	modify variable bounds (lower, upper or both)
inherited methods	see IloExtractable

• **IloConversion**: convert numeric variables in other types

end()	end the conversion (GET BACK TO INITIAL TYPE)
inherited methods	see IloExtractable

#### N.B.: We cannot add 2 conversions back to back to the same variables!!!

```
ex.: IloConversion conv(env, varX, ILOINT);
model.add(conv);
model.remove(conv);
conv.end();
```

#### **Constraints:**

• **IloExpr**: build an expression (constraint)

setConstant(IloNum)	initialize the constant term of the expression
setLinearCoef(variable, value) setLinearCoefs(variables, values)	change some coefficients of the expression
operator += (variable/expression/value)	add an element to the expression (with positive sign)
operator -= (variable/expression/value)	add an element to the expression (with negative sign)
inherited methods	see IloExtractable

# N.B.: USE end() METHOD ONCE VARIABLE HAS BEEN DEFINED IN ORDER TO PREVENT MEMORY LEAKS

• **IloRange**: declare constraints

setLB(value) setUB(value) setBounds(value, value)	change bounds of constraint (lower, upper or both)
setLinearCoef(variable, value) setLinearCoefs(variables, values)	change some coefficients of constraint
setExpr(expression)	Set a new expression to the constraint (LHS)
inherited methods	see IloExtractable

## ex.: facility.cpp

• **IloSum**(*variables*): return the sum of the variables in the array

```
ex.: facility.cpp
```

IloArray<IloNumVarArray> supply(env, nbClients);

```
for(i = 0; i < nbClients; i++)

supply[i] = \textbf{IloNumVarArray}(env, nbLocations, 0, 1, ILOINT);

for(i = 0; i < nbClients; i++)

model.add(\textbf{IloSum}(supply[i]) == 1); // \Sigma (j) supply[i][j] == 1
```

**N.B.**: IloSum can be applied only on the <u>LAST</u> dimension of the matrix. *Otherwise, we have to use* **IloExpr** *to build the constraint.* 

• **IloScalProd**(*variables*, *values*): return the scalar product of an array of variables and an array of values

ex.: facility.cpp

```
IloNumArray fixedCost(env);

IloNumVarArray open(env, nbLocations, 0, 1, ILOINT);

IloArray<IloNumVarArray> supply(env, nbClients);

IloExpr obj = IloScalProd(fixedCost, open); // \Sigma(j) fixedCost[j] * open[j]

for(i = 0; i < nbClients; i++)

obj += IloScalProd(cost[i], supply[i]); // \Sigma(i,j) supply[i][j] * cost[i][j]
```

• **IloIfThen**(condition If, condition Then) : create « Big-M » or logic constraints

```
ex.: x \le M * y = =>  IloIfThen(env, y == 0, x == 0);
```

ex.: <u>foodmanufact.cpp</u>

If products p1 or p2 are used (more than 20), then product p3 will be used also:

 $model.add(\textbf{IloIfThen}(env, (use[p1] >= 20) \mid | (use[p2] >= 20), use[p3] >= 20));$ 

#### Arrays + macro:

• **IloAdd**: macro which defines objects and add them to the model at the same time

```
ex.: <u>ilodiet.cpp</u>
```

• IloArray: class template for multi-dimensions extensible array

add(X)	add X to the array
operator[](int i)	get access to the i-th element of the array
clear()	clear the array => size = 0
getSize()	returns the size of the array
end()	destroy the array and all its elements (replaces destructor)

## ex.: facility.cpp

```
typedef IloArray<IloNumArray> FloatMatrix; // 2D matrix of numbers typedef IloArray<IloNumVarArray> NumVarMatrix; // 2D matrix of variables
```

• **IloExtractableArray** : class for using extensible arrays to define one or multidimensional arrays for variables/constraints

end()	destroy array and all its elements (replaces destructor)
removeFromAll()	remove elements of the array from all other objects where they were referenced
setNames(const char*)	give name to the elements of the array

• IloNumArray, IloIntArray, IloBoolArray : extensible arrays for numbers

contains(value)	look for a specific value in the array
inherited methods	see IloExtractableArray et IloArray

• IloNumVarArray, IloIntVarArray, IloBoolVarArray : declare numeric, integer or binary arrays of variables

add(variable) add(variables)	add variables to the array
setBounds(values, values)	change bounds of variables in the array
inherited methods	see IloExtractableArray et IloArray

ex.: <u>facility.cpp</u>

```
typedef IloArray<IloNumVarArray> NumVarMatrix;
// 2D matrix of variables
```

```
NumVarMatrix\ supply(env,\ nbClients); for(i = 0;\ i < nbClients;\ i++) supply[i] = \textbf{IloNumVarArray}(env,\ nbLocations,\ 0,\ 1,\ ILOINT);
```

ex.: <u>cutstock.cpp</u>

IloNumArray newPatt(env, nWdth); IloNumVarArray Use(env, nWdth, 0.0, IloInfinity, ILOINT); patSolver.getValues(newPatt, Use);

• IloRangeArray: declare extensible arrays of constraints

<pre>add(constraint) add(constraints)</pre>	add constraints to the array
setBounds(values, values)	Change bounds of constraints in the array
inherited methods	see IloExtractableArray et IloArray

```
ex.: IloRangeArray tab(env);
tab.add(IloRange(env, 0.0, 100.0));
IloRangeArray range (env, nutrMin, nutrMax); ===> ilodiet.cpp
```

## Solver class:

• **IloCplex** : solving a model with CPLEX

LP	solve()	solve the model (LP relaxation followed by MIP if needed)
	getCplexStatus()	returns optimization status
	exportModel(char*)	Export model to a file
+	getObjValue()	returns objective function value
MIP	getValue(variable/expression) getValues(values, variables)	retrieve values of variables at the end of otpimization
	setParam(parametre, value)	change parameter values
	tuneParam()	automatic search for better parameter values
	<pre>getDual(constraint), getDuals(values, variables/expressions)</pre>	retrieve dual values of constraints
LP	getReducedCost(variable) getReducedCosts(values, variables)	retrieve reduced costs of variables
	getSlack(constraint), getSlacks(values, constraints)	retrieve slack value associated to constraints
	getBestObjValue()	retrieve best known bound of all the remaining open nodes
	addLazyConstraint(constraint) addLazyConstraints(constraints)	add LAZY constraints in a cut pool (integer solutions)
	addUserCut(constraint), addUserCuts(constraints)	add USER constraints in a cut pool (any non-integer node)
MIP	solveFixed()	solve MIP while fixing variables to LP solution (get access to duals, slacks,)
	addMIPStart(variables, values)	define starting point to solve MIP
	setPriority(variable, value) setPriorities(variables, values)	Set priorities on variables for MIP branching
	use(IloCplex::Callback)	Usage of callbacks to change the progress of MIP B&C
	inherited methods	see IloAlgorithm

## <u>Useful parameters</u>:

IloCplex::Param::	(prefix)	
Threads	Limit on number of threads used (default: ALL CPUs) Please use: 1	
Advance	keep the current basis or not	
TimeLimit	max time for solving (in seconds)	
ClockType	to specify wall clock or CPU time	
Preprocessing::Presolve	apply presolve or not	
Preprocessing::Symmetry	Break model symmetry (MIP)	
Preprocessing::BoundStrength	Ry to fix variables (MIP)	
Simplex::Tolerances::Optimality	tolerance on solution optimality	
Simplex::Tolerances::Feasibility	tolerance on feasibility of variables	
Simplex::Display	display simplexe optimization	
Emphasis::MIP	MIP search emphasis (FEASIBLE vs OPTIMAL)	
Emphasis::Numerical	numerical precision	
Emphasis::Memory	keep memory usage as low as possible	
RootAlgorithm	algorithm to solve root node	
NodeAlgorithm	algorithm to solve other nodes	
MIP::Display	display of B&C	
MIP::Strategy::File	create a file to store nodes	
WorkMem	limit on memory used before storing nodes	
MIP::Strategy::VariableSelect	select next variable (MIP branching)	
MIP::Strategy::NodeSelect	select next node (MIP branching)	
MIP::Strategy::Branch	decide which branch should be taken first	
MIP::Tolerances::LowerCutoff	cut branches where solution value < threshold	
MIP::Tolerances::UpperCutoff	cut branches where solution value > threshold	
MIP::Tolerances::MIPGap	limit on gap (%)	
MIP::Tolerances::Integrality	tolerance on integrality of variables	
MIP::Limits::Solutions	limit on number of integer solutions found	
MIP::Limits::Nodes	limit on number of evaluated nodes	
MIP::Limits::TreeMemory	limit on branching tree memory	

## Basic callbacks:

==> to give users a way to change the progress of branch&cut.

## • Callbacks types:

informational	retrieve information on current optimization without changing anything on the solving process	
diagnostic	check the progress of optimization and eventually terminate it	
control control the search process during B&C procedure and interfere directly in the solving process		

## • Control callbacks:

UserCutCallback	add cuts to non-integer nodes	
LazyConstraintCallback	add cuts to integer nodes	
NodeCallback	choose the next node the B&C will be solving	
SolveCallback	change the way the node is solved	
HeuristicCallback	implement heuristic to give better incumbents	
BranchCallback	choose the next nodes to be created	
IncumbentCallback	check incumbent solutions and reject « false » integer solutions	

• Macros to define callbacks (LAZY/USER):

#### ILOLAZYCONSTRAINTCALLBACKO(name)

**ILOLAZYCONSTRAINTCALLBACK1**(name, type1, x1)

**ILOLAZYCONSTRAINTCALLBACK2**(name, type1, x1, type2, x2)

...

#### ILOLAZYCONSTRAINTCALLBACK7

(name, type1, x1, type2, x2, type3, x3, type4, x4, type5, x5, type6, x6, type7, x7)

#### ILOUSERCUTCALLBACKO(name)

**ILOUSERCUTCALLBACK1**(name, type1, x1)

**ILOUSERCUTCALLBACK2**(name, type1, x1, type2, x2)

. . .

#### ILOUSERCUTCALLBACK7

(name, type1, x1, type2, x2, type3, x3, type4, x4, type5, x5, type6, x6, type7, x7)

# ex.: ILOUSERCUTCALLBACK3(CtCallback, IloExprArray, lhs, IloNumArray, rhs, IloNum, eps)

-> define a function of type UserCutCallback with name CtCallback receiving 3 parameters (lhs, rhs, eps) of types (IloExprArray, IloNumArray, IloNum)

#### ex.: cplex.use(CutCallback(env, lhs, rhs, 1e-05);

-> ask the solver to use the UserCutCallback named *CutCallback* during optimization with parameters: lhs, rhs, 1e-05

LAZY	add(constraint)	add global cut to the model	
+	addLocal(constraint)	add local cut to the model	
USER	inherited methods	see next table	
	abortCutLoop()	get out of the cutting phase and go back to branching	
USER	isAfterCutLoop()	returns IloTrue if callback called one last time after the last cut has been generated (IloFalse otherwise)	

• inherited methods (common to LazyConstraintCallback and UserCutCallback )

getLB(variable) getLBs(values, variables) getUB(variable) getUBs(values, variables)	retrieve lower/upper bounds from variables	
getNodeId()	retrieve unique ID from current node	
getObjValue()	retrieve objective value of current node	
getSlack(constraint) getSlacks(constraints)	retrieve slack values of constraints at current node	
getValue(variable/expression) getValues(values, variables)	retrieve values of variables (Expression) at current node	
getBestObjValue()	retrieve best bound of the remaining nodes	
getIncumbentObjValue()	retrieve best incumbent found so far	
getIncumbentValue (variable/expression) getIncumbentValues (values, variables)	retrieve values of variables (expressions) from the best integer solution found so far	
getNnodes()	retrieve the number of evaluated nodes	
getNremainingNodes()	retrieve the number of remaining nodes	
hasIncumbent()	returns IloTrue if an integer solution has been found (IloFalse otherwise)	
getModel()	retrieve the current model	
abort()	terminate optimization	
getEnv()	retrieve current environment	

<u>N.B.</u>: Inside the user-defined function (macro), the only way to retrieve values of variables/constraints at current node is to give all needed arrays as parameters to the callback like this:

```
ILOUSERCUTCALLBACK5(Benders User Callback, IloArray < IloIntVarArray >, x,
                                  IloCplex, workerCplex, IloNumVarArray, v,
                                  IloNumVarArray, u, IloObjective, workerObj)
    // Skip the separation if not at the end of the cut loop
    if (!isAfterCutLoop() )
                               return;
    IloInt i:
    IloEnv \ masterEnv = getEnv();
    IloInt\ numNodes = x.getSize();
    // Get the current x solution
    IloArray<IloNumArray> xSol(masterEnv, numNodes);
    for (i = 0; i < numNodes; ++i) {
      xSol[i] = IloNumArray(masterEnv);
      getValues(xSol[i], x[i]);
    // Benders' cut separation
    IloExpr cutLhs(masterEnv); // expression de la coupe
    IloNum cutRhs;
    IloBool\ sepStat = separate(x, xSol, workerCplex, v, u, workerObj, cutLhs, cutRhs);
    if (sepStat) {
      add(cutLhs >= cutRhs).end();
      // cree la coupe, l'ajoute au modele et detruit l'objet temporaire cree
    // Free memory
    cutLhs.end();
    for (i = 0; i < numNodes; ++i)
      xSol[i].end();
    xSol.end();
    return;
  }
```

#### Generic callbacks: (version 12.8 and +)

- More flexibility than the basic callbacks :
  - in the same generic function, it is possible to:
    - get informations about solving status
    - give new heuristic solutions to the solver
    - reject « integer » solutions (with or without lazy constraints)
    - get current relaxed solution
    - add user cuts
    - abort optimization

However, some fucntions usable in some basic callbacks (*SolveCallback*, *NodeCallback*, *BranchCallback*) cannot be called from within generic callbacks, and furthermore, both kinds of callback could not be used at the same time.

• Callback calling contexts: (of type IloCplex::Callback::Context::Id::)

ThreadUp	Thread activation	
ThreadDown	Thread deactivation	
LocalProgress	Local progress (thread based)	
GlobalProgress	Global progress (whole solving)	
Candidate Integer or unbounded solution		
Relaxation	Relaxed solution	

• How to use a generic callback : *ex.* : *lilobendersatsp2.cpp* :

```
CPXLONG\ context mask = IloCplex::Callback::Context::Id::Candidate \\ |\ IloCplex::Callback::Context::Id::ThreadUp \\ |\ IloCplex::Callback::Context::Id::ThreadDown; \\ masterCplex.use(\&cb,\ context mask);
```

In this example, the generic callback (defined here by variable cb) will be called if:

- · an integer or unbounded solution is found
- a thread is activated or deactivated

ex.: iload mip ex 8.cpp, iload mip ex 9.cpp, iloben ders at sp 2.cpp

## • Methods:

	getID()	get the callback calling context
	<pre>inThreadUp() inThreadDown() inLocalProgress() inGlobalProgress() inCandidate() inRelaxation()</pre>	Verify if callback has been called from a particular context
	postHeuristicSolution (variables, valeurs, objective, stratégie)	Give a new integer solution to CPLEX
I	isCandidatePoint() isCandidateRay()	Test if callback called for integer (point) or unbounded (ray) solution
N T	getCandidatePoint(variables, valeurs) getCandidatePoint(variable)	Get values of current <b>integer</b> solution variables
E	getCandidateValue(expression)	Get current value of expression
G E	getCandidateObjective()	Get current objective value
R	rejectCandidate(contraintes) rejectCandidate(contrainte = 0)	Reject current integer solution by adding (lazy) constraints – if NULL, then CPLEX automatically add cut
R E	getRelaxationPoint(variables, valeurs) getRelaxationPoint(variable)	Get values of current <b>relaxed</b> solution variables
L A	getRelaxationValue(expression)	Get current value of expression
X	${\bf getRelaxationObjective}()$	Get current objective value
E D	addUserCut(contrainte, cutManagementFlag, localFlag)	Add a local or global user cut
	getIncumbent(variable) getIncumbent(variables, valeurs)	Get variables values from current incumbent
	getIncumbentValue(expression)	Get value of expression from incumbent
	getIncumbentObjective()	Get incumbent objective value

• How to define a generic callback:

```
In order to define a generic callback, you have to inherit from
IloCplex::Callback::Function . The new class should also redefine :
void invoke(Context const &context) :
ex.: iloadmipex8.cpp:
class FacilityCallback: public IloCplex::Callback::Function {
 private:
   /* Empty constructor is forbidden. */
   FacilityCallback ()
   /* Copy constructor is forbidden. */
   FacilityCallback(const FacilityCallback &tocopy);
                                 /// Destructor
   virtual ~FacilityCallback();
   void separateDisagregatedCuts (const IloCplex::Callback::Context &context);
   void lazyCapacity (const IloCplex::Callback::Context &context);
   IloNumVarArray opened;
   NumVarMatrix supply;
   IloRangeArray cuts;
 public:
   /* Constructor with data */
   FacilityCallback(const IloNumVarArray &_opened,
                    const NumVarMatrix & supply):
     opened(_opened), supply(_supply), cuts(opened.getEnv())
   {}
  virtual void invoke (const IloCplex::Callback::Context &context)
  {
      if ( context.inRelaxation() ) {
            separateDisagregatedCuts(context);
      }
      if (context.inCandidate())
             lazyCapacity (context);
  }
```