Performance evaluation of a single core

This project was developed for the CPD Curricular Unit (Parallel and Distributed Computation) in the year 2021/22 by the following students:

José Pedro Peixoto Ferreira	up201904515@edu.fe.up.pt
Lucas Calvet Santos	up201904517@edu.fe.up.pt
Sérgio Rodrigues da Gama	up201906690@edu.fe.up.pt

Problem Description and Algorithms Explanation

In this project, we analyse the performance of a single core program, taking into account the impact of accessing large amounts of data. This analysis is made by running different algorithms for solving the same problem, in two completely different languages, in this case C++ and Python. The problem to be solved is the multiplication of two square matrices, which are matrices of the same width and height.

Meanwhile, the algorithms chosen to solve this problem were the simple (naive) matrix multiplication, line matrix multiplication and block matrix multiplication, where the initial hypothesis was that they should be incrementally more efficient in terms of cache memory usage, and therefore have a better performance.

Naive Matrix Multiplication

This algorithm is the simpler approach to solve the matrix multiplication problem. It is composed by three loops, where the two inner loops traverse all the matrix elements, line by line in the first matrix and column by column in the second one. The outermost loop just fills the resultant matrix with the results.

Line Matrix Multiplication

At a first glance, this algorithm may seem very similar to the previous one. However, the major difference relies in the way that it traverses the matrix. The naive algorithm had to get the column of the second matrix, which means that in each iteration it obtains the whole matrix line, when it is only going to need a single element. As it is imaginable, this is very memory (cache) expensive, as it in each iteration `[(n-1)/n × 100] %` of the data loaded to memory will be useless (being n the number of elements loaded into cache). For example, if all 10 elements from a line of a matrix of size 10 by 10 are loaded to cache, 90% of the data loaded won't be used for the actual calculations.

Block Matrix Multiplication

Finally, in this last algorithm, we partition the matrices in blocks of a constant size. This way, the elements to be multiplied are matrices themselves instead of numbers like in previous algorithms. The multiplication of said matrices are made individually.

Performance Metrics

In order to better analyse the performance of a single core we came up with a group of relevant metrics:

- **Time** - The time an instance of the program takes to complete the multiplication. This is the most significant way to infer an algorithm's efficiency, as we always crave to obtain the solution in the shortest time possible.
- **L1 Data Cache Misses** - Number of times a request to retrieve data from the level 1 cache was not met.
- **L2 Data Cache Accesses** - Number of requests made to retrieve data from the level 2 cache.
- **L2 Data Cache Misses** - Number of times a request to retrieve data from the level 2 cache was not met.
- **L2 Miss Rate** - The ratio between the data cache misses and accesses for memory of level 2.
- **L3 Data Cache Accesses** - Number of requests made to retrieve data from the level 3 cache.
- **Waiting Cycles** - Cycles stalled waiting for memory write.
- **Total Cycles** - Total of executed cycles.
- **Waiting Cycles Rate** - Ratio between waiting and total cycles.

- **GFlops** - The number of 10^9 floating-point operations per second, an important metric for CPU manufacturers.

Results and Analysis

The data we collected is consistent overall, and makes sense in most instances. However, in some cases the L2 Miss Rate is measured above 1, which is by definition impossible. It's the result of the division of the L2 Data Cache Accesses by the L2 Data Cache Misses, and we know the Accesses are equal to the sum of the Misses and the Hits, so in the most extreme scenario where there are no Hits, the rate is 1, but should never surpass that number. A possible explanation for this could be the occurrence of overflows in the Accesses metric.

C++ Naive Multiplication

Matrix Size	Time (seconds)	PAPI_L1_DCM	PAPI_L2_DCA	PAPI_L2_DCM	L2 Miss Rate	PAPI_L3_DCA	PAPI_MEM_WCY	PAPI_TOT
600	0.188	244598334	219353364	39009750	0.177839762	39009750	97498	8.64E+08
1000	1.172	1220936881	1101117640	304101155	0.276174992	304101155	333545	5.45E+09
1400	3.501	3513170912	3187244834	1561519018	0.489927539	1561519018	722983	1.61E+10
1800	17.592	9089664951	8457800627	3918204925	0.463265227	3918204925	1277852	8.11E+10
2200	38.296	17626548453	16431515657	20221528599	1.230655103	20221528599	1909794	1.77E+11
2600	68.68	30896545029	28797112516	49453183847	1.717296615	49453183847	2734223	3.17E+11
3000	118.634	50303752649	46733943387	94045934297	2.012368901	94045934297	3701625	5.43E+11

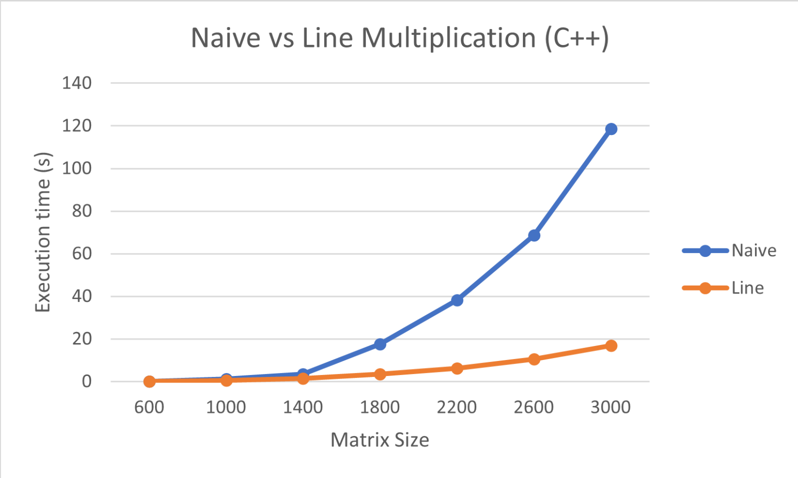
We observe that as the matrix size increases, the Cache Miss Rate also increases, while both the Waiting Cycles Rate and the GFlops decrease. The GFlops decrease as there are more operations to be executed. The drop in the Waiting Cycles Rate could be happening because there are more cycles overall.

C++ Line Multiplication

Matrix Size	Time (seconds)	PAPI_L1_DCM	PAPI_L2_DCA	PAPI_L2_DCM	L2 Miss Rate	PAPI_L3_DCA	PAPI_MEM_WCY	PAPI_TOT
600	0.101	27101946	1433075	57514888	40.13389948	57514888	181471	4.6E+08
1000	0.472	125893758	8756611	262349196	29.9601291	262349196	827186	2.2E+09
1400	1.545	346566909	23440976	704825887	30.0681118	704825887	1791618	7.18E+09
1800	3.391	746026658	46605620	1434727637	30.78443409	1434727637	2405016	1.56E+10
2200	6.255	2075149799	180010949	2515396048	13.97357251	2515396048	3747273	2.89E+10
2600	10.544	4413465124	423133562	4138796506	9.781300463	4138796506	5309851	4.84E+10
3000	16.806	6781743520	626914918	6232935497	9.942235091	6232935497	7292675	7.69E+10
4096	41.092	17543305277	4926884303	15954163982	3.23818523	15954163982	12500610	1.92E+11
6144	137.636	59078982990	9443373849	52758417311	5.586818668	52758417311	29882283	6.43E+11
8192	335.283	1.40382E+11	59305983948	1.30568E+11	2.201603369	1.30568E+11	56325393	1.57E+12
10240	643.39	2.73296E+11	54953617395	2.45559E+11	4.468477781	2.45559E+11	104352448	3E+12

Just by looking at the time, we can see that this algorithm is more efficient than the Naive one. For instance, for a matrix size of 3000 by 3000, the Line Algorithm is about 7 times faster than the simpler one. In terms of the rates we observe similar results in the GFlops and Waiting Cycles Rate.

The following is a comparison between the times of the two mentioned algorithms:

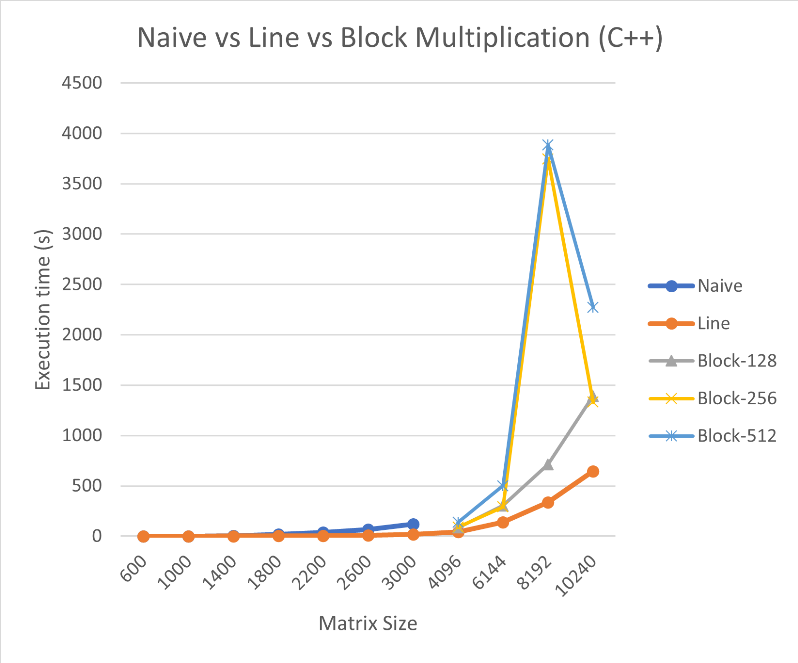


C++ Block Multiplication

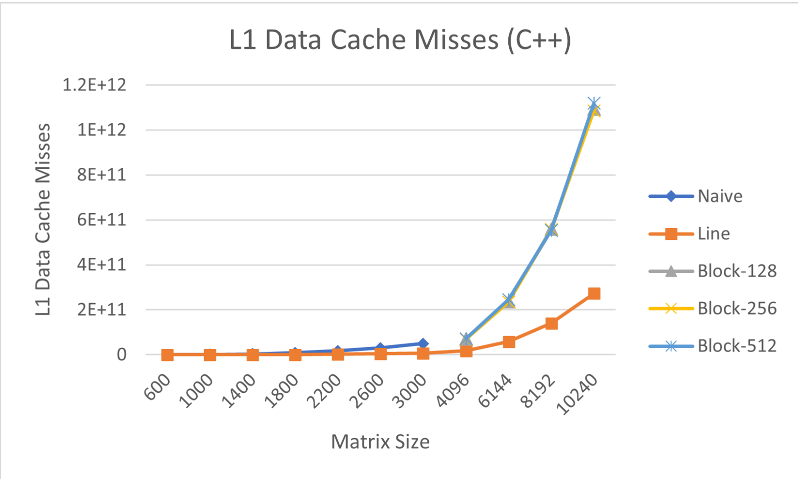
Block Size	Matrix Size	Time (seconds)	PAPI_L1_DCM	PAPI_L2_DCA	PAPI_L2_DCM	L2 Miss Rate	PAPI_L3_DCA	PAPI_MEM_WCY	F
128	4096	85.86	69997820793	69924220733	1.25258E+11	1.791337597	1.25258E+11	18408091	4
128	6144	303.127	2.36257E+11	2.3603E+11	3.37344E+11	1.429237692	3.37344E+11	43965926	1
128	8192	712.229	5.59987E+11	5.59262E+11	1.03808E+12	1.856167086	1.03808E+12	80153733	3
128	10240	1391.825	1.09376E+12	1.09272E+12	1.55481E+12	1.422880809	1.55481E+12	128357361	6
256	4096	89.122	69696703580	69644128307	1.2712E+11	1.825279101	1.2712E+11	18438593	4
256	6144	291.88	2.34691E+11	2.34539E+11	4.0898E+11	1.74375777	4.0898E+11	43792065	1
256	8192	3747.446	5.57276E+11	5.56783E+11	5.8426E+11	1.049349333	5.8426E+11	80342772	1
256	10240	1331.728	1.08521E+12	1.08451E+12	1.90543E+12	1.756943453	1.90543E+12	128466433	6
512	4096	137.103	71716184386	71637546430	1.17852E+11	1.645121031	1.17852E+11	18444372	6
512	6144	501.668	2.45511E+11	2.45177E+11	3.72529E+11	1.519429761	3.72529E+11	43963544	2
512	8192	3884.154	5.53155E+11	5.5287E+11	5.76147E+11	1.042101362	5.76147E+11	80162499	1
512	10240	2273.866	1.11853E+12	1.11713E+12	1.69747E+12	1.519490162	1.69747E+12	128643592	1

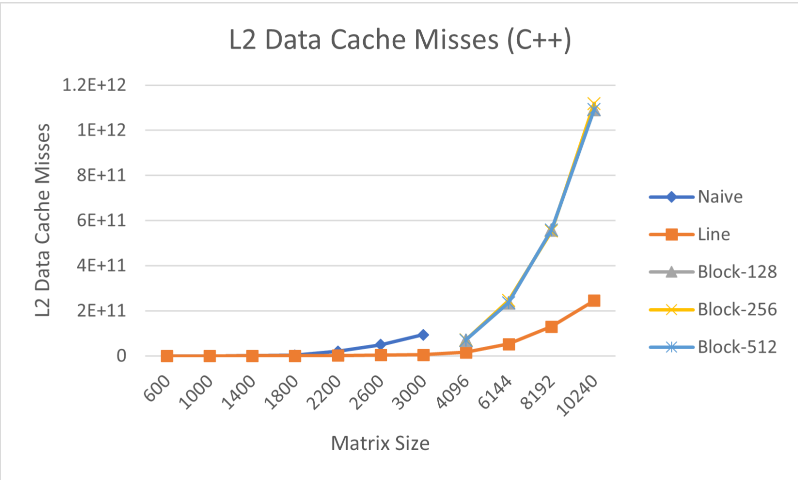
The Block Multiplication Algorithm seems to take more or less twice as long to reach the solution as the Line Multiplication Algorithm for a given matrix size. It's, however, less time-consuming than the Naive approach. This time, metrics like the GFlops, Waiting Cycles Rate and Cache Miss Rate stay nearly constant. The block size with the best performance appears to be 256, followed by 128. This could be because the increase from 128 to 256 blocks is beneficial, but increasing to 512, results in blocks that are too big and therefore the application of the Naive Algorithm slows the process down. The advantage that dividing in blocks brings to the table, is under the presupposition that the blocks are small enough that applying a less effective strategy, like the first one, doesn't affect the overall performance.

The following is a comparison between the times of the three algorithms:

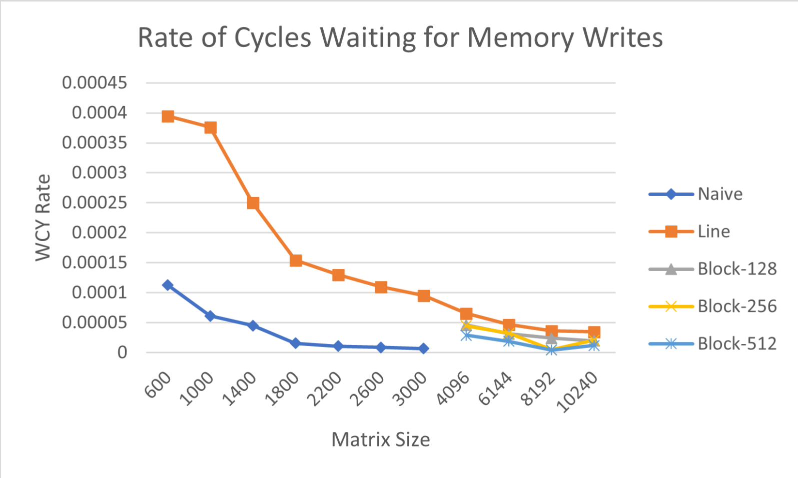


The following graphs show the comparison between Level 1 and Level 2 Data Cache Misses between the different algorithms:





The following graph shows the comparison between the Rate of Waiting Cycles from the different C++ algorithms:



Python Naive Multiplication

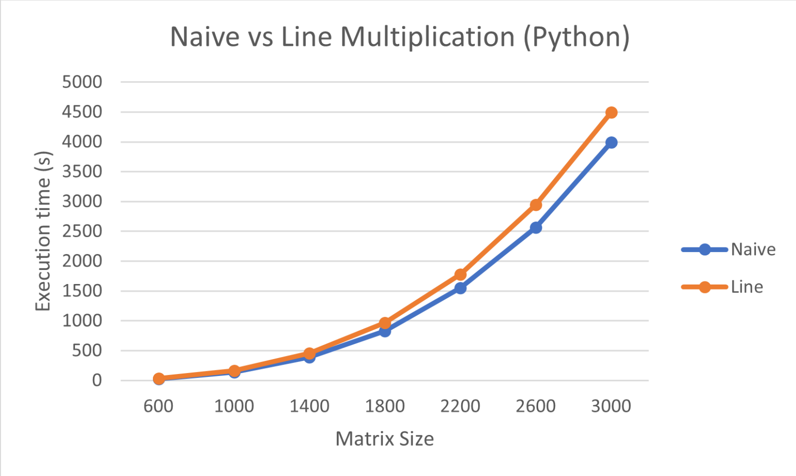
Matrix Size	Time (seconds)
600	27.511
1000	138.652
1400	389.192
1800	826.688
2200	1554.341
2600	2564.582
3000	3991.616

We can easily see that Python is much, much slower than C++ regarding these operations. For a matrix of 3000 by 3000, Python takes 250 times more seconds than C++.

Python Line Multiplication

Matrix Size	Time (seconds)
600	35.179
1000	166.376
1400	452.353
1800	965.341
2200	1778.496
2600	2944.469
3000	4497.765

Unlike the Line Multiplication Algorithm in C++, the very same algorithm in Python doesn't have a better performance than the simpler, Naive approach. In fact, it was slower than the first algorithm in every iteration we tested. And consequently the gap of efficiency between Python and C++ got even bigger, with the ratio rising to 264 at the already mentioned size of 3000 by 3000.



Conclusions

This project was an interesting way to kick-off this Curricular Unit, as it helped us understand the importance of well written single-thread, sequential algorithms to the overall performance and efficiency of a program. Only after we perfect the art of coding for a single-core, can we aim to produce good multi-thread, parallel programs.

As we expected, in C++, the first algorithm (naive multiplication) presented the worst performance results, being 7 times slower than the next one (line multiplication), with a matrix of size 3000. The last algorithm was worse than the second one, because the strategy that it is adopted can only be beneficial when using multiple threads. On the opposite side, it is better than the naive algorithm, as it doesn't load as much useless data as the first one.

However, in Python, we were surprised to see that the overall performance was worse than in C++ and even the second algorithm performed slower than the naive approach.