Everything I know about [Formal Concept Analysis]

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1 Introduction

1.1 Lattices

A lattice $\mathcal{C}$ is a poset s.t.for any pair $(a, b) \in \mathcal{C}$, the supremum $a \wedge b$, and infimum $a \vee b$ exist. We extend this to a complete lattice, which has the requirement that for any subset $\mathcal{D} \subseteq \mathcal{C}$ the supremum $\bigvee \mathcal{D}$ and infimum $\bigwedge \mathcal{D}$ exist.

1.2 Formal Contexts

A Formal Context is a triple $\langle G, M, I \rangle$ where G refers to a set of objects, M to a set of properties, and I an incidence relation over $G \times M$.

We have derivation operators A' and B'; for A', where $A \subseteq G$, the derivation operator tells us which properties belong to the objects in A, the dual holds for properties and their objects. **Formally**,

Definition 1.1

$$A' := \{m \in M \,|\, \forall g \in A, gIm\}$$
$$B' := \{g \in G \,|\, \forall m \in B, gIm\}$$

We also have closure operators, A'', which works intuitively by applying the derivation operator on A (B), which yields a set of properties. Then applying it again on A' (B'), which yields back a set of objects (properties).

Proposition 1.2 *For subsets $A, B \subseteq G$ (defined dually for properties $C, D \subseteq M$), we have*

a. $A \subseteq B \implies B' \subseteq A'$

b. $A \subseteq A''$

c. $A' = A'''$

For more natural discussion, a describes the behaviour that if we have two sets of objects A and B, where $A \subseteq B$; then it follows that objects in A will have at *least* all the properties of objects in B.

1.3 Formal Concepts

Presume we are working with a formal context $\langle G, M, I \rangle$.

Definition 1.3 (A, B) *is a **formal concept** of our formal context iff $A \subseteq G$, $B \subseteq M$, $A' = B$, and $B' = A$*

A is called the **extent**, and B is called the **intent**. We can refer to the set of all formal concepts of a formal context as a $\mathcal{B}(G, M, I)$.