Rational Concept Analysis

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1 INTRODUCTION

Formal Concept Analysis (FCA) provides a lattice theoretic framework for mathematically reasoning about *formal concepts* and their hierarchies [5, 6]. The matter of *concepts* has largely been of philosophical concern: the view of concepts as a dualism between *intension* and *extension* has its foundations in Aristotle's *Organon* and, much later on, in the *Logic of Port-Royal* [3, 16]. Here, the *extension* of a concept as "those things" that one would reference as instances of the concept. Dually, the *intension* describes the meaning, or sense, of a concept.

FCA—which adopts this view of concepts—introduces a *formal context* as a data structure from which concepts can be derived. This is a triple consisting of a finite set of objects G, attributes M, and an incidence relation $I \subseteq G \times M$, which indicates when a particular object has a respective attribute [5, 6]. A *formal concept* is a pair comprised of an extension and intension. That is, a set of objects and of attributes, respectively. When the set of all concepts belonging to a context is ordered by the *sub/super-concept* relation, a lattice is formed called the *concept lattice*.

Another important topic in FCA is the discovery of *implications* that pertain to, or are *respected* by, a context [5, 16]. Implications are used to express correspondence between (sets of) attributes. The semantics of these implications resemble Tarskian notions of logical consequence, and are accordingly monotonic. Implications hold only if they hold in every object intent (i.e., *valuation*).

The property of monotonicity in the logic underlying FCA means that it is ill-suited to reason about concepts with *exceptional members*, or *defeasible* implications. This work argues that developing the expressivity to do so would be useful for the kinds of problems FCA is used to tackle. Despite this, there have been few attempts to introduce non-monotonic reasoning of any kind to FCA. We attempt this through introducing preferential reasoning, and more specifically the KLM framework to the attribute logic of FCA, developing notions of a non-monotonic conditional, and concepts which behave in accordance to the defined preferential view.

The rest of this proposal is structured as follows: Section 2 is devoted to providing a brief introduction to both FCA (Section 2.1) and KLM-style non-monotonic reasoning (Section 2.2). Section 3 then presents arguments in favour of introducing non-monotonicity into FCA. In turn, Section 4 clarifies the previous section, explaining the current view of how this might be achieved, as well as identifying particularly challenging obstacles. We discuss related work(s) in Section 5, and then provide administrative details concerning this project in Section 6 and Section 7.

2 BACKGROUND

The proceeding subsections provide an introduction to aspects of FCA and the KLM-framework for non-monotonic reasoning, which are particularly relevant to the scope of this research.

2.1 Formal Concept Analysis

The starting point of FCA is the *formal context*. This is a triple consisting of a finite set of objects, a finite set of attributes, and a relation which describes when an object g 'has' an attribute m.

Definition 2.1. A *formal context* is a triple, $\mathbb{K} = (G, M, I)$, where G is a finite set of objects, M is a finite set of attributes, and $I \subseteq G \times M$ is an incidence relation such that for an object g and attribute m, $(g, m) \in I$ is interpreted as *g having m.*

For contexts of reasonable size, a cross-table is frequently used as a visual aid. Each row represents an object, and each column an attribute. A '×' is used to indicate where an object has an attribute [5, 6]. See Table 1 for an example. For any subset of objects (*resp.* attributes) we use a derivation operator to describe the attributes (*resp.* objects) common to all members of that subset. These derivation operators form a anti-tone Galois connection between the power sets $\mathcal{P}(G)$ and $\mathcal{P}(M)$ [5].

Definition 2.2. In a context, $\mathbb{K} = (G, M, I)$, we define two *derivation operators*, both denoted by $(\cdot)'$, as follows:

$$A \subseteq G: \quad A' := \{m \in M \mid \forall g \in A : (g, m) \in I\}$$
$$B \subseteq M: \quad B' := \{g \in G \mid \forall m \in B : (g, m) \in I\}$$

In case of a singleton set, $\{g\}$, the braces are omitted. If g is an object, then g' describes the *object intent*; if g is an attribute, g' is the *attribute extent*. A derivation operator can be applied to the result of an earlier derivation operator, this is then referred to as a *double-prime operator*, and expressed as $(\cdot)''$. Any set derived from a double-prime operator is closed, and so each derivation operator describes a closure system on G and M, respectively [5, 6, 16]. As such, the double-prime operator $(\cdot)''$ satisfies:

$$\begin{array}{ll} \text{Monotonicity:} & \text{if } A_1 \subseteq A_2 \text{ then } A_1'' \subseteq A_2'' \\ \text{Idempotency:} & A'' = (A'')'' \\ \text{Extensivity:} & A \subseteq A'' \end{array}$$

Definition 2.3. A *formal concept* of a context, $\mathbb{K} = (G, M, I)$, is a pair, (A, B), with $A \subseteq G$ and $B \subseteq M$ and where $A' = B$ and $B' = A$. Then A is the *concept extent* and B is the *concept intent*. The set of all concepts of a context is given by $\mathfrak{B}(G, M, I)$.

There is obvious redundancy in this definition of concepts: if (A, B) is a formal concept, it could be expressed equivalently as (A, A') or (B', B). Moreover, for an arbitrary set A of objects (*resp.* attributes), A' defines a concept intent (*resp.* extent), and A'' defines a concept extent (*resp.* intent) [6].

Definition 2.4. Let (G, M, I) be a formal context, then for every object, $g \in G$, we define a *object concept* as

$$\gamma g := (g'', g')$$

and for each attribute, $m \in M$, we define the *attribute concept* as

$$\mu m := (m', m'')$$

The idea of sub and super-concepts gives rise to a natural partial order on $\mathfrak{B}(G, M, I)$. Specifically, if (A_1, B_1) and (A_2, B_2) are concepts in $\mathfrak{B}(G, M, I)$, then (A_1, B_1) is a *sub-concept* of (A_2, B_2) iff $A_1 \subseteq A_2$ (equivalently, $B_2 \subseteq B_1$). Respectively, (A_2, B_2) would be a *super-concept* of (A_1, B_1) and we say that $(A_1, B_1) \leq (A_2, B_2)$ [5]. Then, $\mathfrak{B}(G, M, I)$ and the relation $\leq$ form a complete lattice called the *concept lattice*. For an example, refer to Figure 1.

Previously, it was mentioned that contexts of a reasonable size have a tidy representation in the form of a cross-table. In case of larger contexts, the concept lattice can instead be inferred from *attribute implications* [5, 6].

Definition 2.5. Let M be an arbitrary attribute-set. An *implication* over M takes the form $B_1 \longrightarrow B_2$ where $B_1, B_2 \subseteq M$. Another set $C \subseteq M$, *respects* the implication iff $B_1 \not\subseteq C$ or $B_2 \subseteq C$. Then, $C \vDash B_1 \longrightarrow B_2$. C *respects a set* of implications $\mathcal{L}$ if it respects every implication in $\mathcal{L}$.

Respecting an implication can be generalised to the notion of *validity* w.r.t a formal context.

Definition 2.6. For a formal context $\mathbb{K} = (G, M, I)$, and an implication $B_1 \longrightarrow B_2$ over M, the implication is *valid* in the formal context ($\mathbb{K} \models B_1 \longrightarrow B_2$) iff for every object $g \in G$, it is the case that the object intent g' respects the implication. Then, the following are equivalent:

$$\mathbb{K} \models B_1 \longrightarrow B_2$$
$$B_1' \subseteq B_2'$$
$$B_2 \subseteq B_1''$$

If $\mathcal{L}$ is the set of all implications valid in a formal context, $\mathbb{K} = (G, M, I)$, then the concept intents can be given by $\{B \subseteq M \mid B \models \mathcal{L}\}$.

2.2 KLM-Style Non-monotonic Reasoning

In [17, 18], Shoham developed a semantic framework for non-monotonicity, which he called *preferential reasoning*. The style of reasoning is based on the idea that an ordering can be imposed on the valuations of a knowledge base, expressing that some valuations are preferred to others. One might, for example, argue that a world in which pigs fly is worthy of less consideration than one where they do not.

In the classical setting, $\alpha \vDash \beta$ means that all models of α are also models of β. A corollary of this is that $\alpha \wedge \gamma \vDash \beta$, which implies monotonicity. Under preferential reasoning, $\alpha \vDash_{\sqsubset} \beta$ does not enforce the preferred models of $\alpha \wedge \gamma$ to be models of β, thus constituting a non-monotonic consequence [17].

Independently, Kraus, Lehmann, and Magidor [11] introduced systems of non-monotonic reasoning as consequence relations, these have since been unified under the *KLM framework*. The central contribution of [11] is *system P*: a preferential consequence relation who's semantics are a variation of Shoham's preferential reasoning. A preferential consequence relation satisfies certain properties, called the *KLM postulates*.[1] Of particular interest is *cautious monotonicity*, which stipulates that new knowledge—which could already be inferred—cannot invalidate previous conclusions. Put plainly, only genuinely new information should cause retraction of existing information [9, 11].

Later, Lehmann and Magidor [13] introduced the stronger *rational consequence relation*, which satisfies all the properties of P and an additional property: *rational monotonicity*. Rational monotonicity says that new information which is consistent with existing knowledge should not lead to the retraction of prior inferences. *Ranked interpretations* lead to a semantic characterisation of a rational consequence relation.

Definition 2.7. A *ranked interpretation* is a function $\mathsf{R} : \mathcal{U} \mapsto \mathcal{N} \cup \infty$ s.t. for every $n \in \mathcal{N}$ there exists a $u \in \mathcal{U}$ such that $\mathsf{R}(u) = n$. Then, there exists $u' \in \mathcal{U}$ s.t. $\mathsf{R}(u') = n'$ with $0 \leq n' < n$

Following the intuition of preferential reasoning, for two valuations, v, v', v is regarded as being representative of a more typical (i.e., preferable, less exceptional) state of affairs if $\mathsf{R}(v) < \mathsf{R}(v')$ [13]. A ranked interpretation, R, induces a rational consequence relation $\mid\sim_{\mathsf{R}}$. For $\alpha \mid\sim_{\mathsf{R}} \beta$, we define $[\![\alpha]\!]^{\mathsf{R}} := \{u \in \mathcal{U} \mid u \Vdash \alpha\}$. Then, the conditional, $\alpha \mid\sim_{\mathsf{R}} \beta$, holds iff for all minimal valuations $v \in [\![\alpha]\!]^{\mathsf{R}}$, $v \Vdash \beta$.

What remains is to describe a process of non-monotonic entailment which is consistent with the pattern of reasoning given by rational consequence relations. Rational closure was introduced in [13] as a definition for non-monotonic entailment. We borrow the following description from Giordano et al. [7].

Definition 2.8. Let $\mathcal{K}$ be a knowledge base, and α a propositional formula. α Is *exceptional* in $\mathcal{K}$ iff $\mathcal{K} \models \top \mid\sim \neg\alpha$. A conditional formula $\alpha \mid\sim \beta$ is *exceptional* in $\mathcal{K}$ iff the antecedent is exceptional. The set of exceptional formulae in $\mathcal{K}$ is denoted as $E(\mathcal{K})$.

For a set of defeasible formulae, $\mathcal{K}$, we construct a sequence of subsets $C_0, \ldots, C_n$ where $C_0 = \mathcal{K}$, and when $i > 0$ the set $C_i = E(C_{i-1})$. In essence, each subsequent set holds the formulae exceptional in the previous set. Since $\mathcal{K}$ is finite, either there exists an n where $C_n = \emptyset$ or for all $n' > n$ $C_n' = Cn$ [7]. A formula α is given rank i, $(R(\alpha) = i)$, where i is the smallest number for which α is not exceptional in C_i.

The rational closure $\overline{\mathcal{K}}$ of $\mathcal{K}$ is then the set of formulae $\alpha \mid\sim \beta$ such that $R(\alpha) < R(\alpha \wedge \neg\beta)$ or where $R(\alpha)$ does not exist.

[1]Further discussion of these postulates can be found in [11]

3 MOTIVATION FOR RESEARCH AREA

In practice, FCA has been implemented as a framework for information retrieval, program analysis, ontology engineering [8, 14, 15]. Until very recently [2, 4] there have been no attempts to lift the expressivity of FCA's attribute logic to a non-monotonic counterpart. To justify non-monotonicity as a desirable property in FCA, toy example is provided.

When introducing younger children to the vertebrates it is often expressed that a defining feature of mammals is that they give birth to live young, and that reptiles lay eggs instead. Mammals and reptiles can be formalised as concepts $\mathtt{m}$ and $\mathtt{r}$:

$$\mathtt{m} \coloneqq \big(\{\texttt{platypus,...,horse}\},\ \{\texttt{warm-blooded,...,live young}\}\big)$$

$$\mathtt{r} \coloneqq \big(\{\texttt{snake,...,J-chameleon}\},\ \{\texttt{cold-blooded,...,lay eggs}\}\big)$$

There is, however, a problem with this construction. Namely, Jackson chameleons are reptiles that give birth to live young, and platypodes are mammals that lay eggs, and so they should not belong to these concepts. The classical FCA response to this issue might be that there should be two sub-concepts, $\mathtt{m}_1$ and $\mathtt{m}_2$, of $\mathtt{m}$. Then, $\mathtt{m}_1$ may specify those mammals which give birth to live young, and $\mathtt{m}_2$ those that lay eggs.

This argument forces the abandon the kind of reasoning that appears so central to our intuition. We do not think of reptiles as being divided between those that give birth to live young and those that do not; moreover, there are likely several properties which cause similar rifts in conception. A more natural way of reasoning about this matter would be to think of Jackson chameleons as *exceptional* reptiles. *Exceptionality* obviously suggests a counter notion of *typicality*. Then, we might express that typical reptiles lay eggs, while exceptional ones may give birth to live young.

Another important concern is the use implications of a formal context to discover correspondence between attribute(s). As a reminder of Definition 2.6, classical FCA implications are only valid in a formal context when they are respected by every object. Consequently, a context modelling birds and their attributes might find that, although most objects have wings and fly, there are exceptions (e.g., penguins, ostriches) which prevent $\texttt{wings} \rightarrow \texttt{fly}$ from being valid; and so we remain unable to express the relationship between these attributes. Once again, the classical framework does not handle exceptions well.

A method to describe implications which only partially hold in a formal context is desirable. *Association rules* are an existing approach, and association rule mining in FCA has been discussed [6, 12]. Association rules, however, use *confidence* and *support* as a means of discovering relationships. These relationships then do not correspond to some formal pattern of reasoning—e.g., rational consequence relations—but are rather a flavour of 'majority rules'.

4 RESEARCH QUESTIONS & OBJECTIVES

In light of Section 3's argument for introducing non-monotonic reasoning to FCA, we now develop a more thorough framing of the problem. Ultimately, the aim of this work is to develop a holistic notion of KLM-style non-monotonic reasoning in FCA. That is, a semantics for rational consequence that gives rise to both defeasible implications and "rational concepts". The following details the pertinent questions, and how we intend to approach them.

The starting point is to begin developing a syntax and semantics for preferential reasoning in an FCA setting. More specifically, the aim is to develop semantics for implications which give rise to a rational consequence relation. The current approach finds an analogue between preference over worlds (from [11, 18]) and a preference over the objects in a formal context. Progressing from this idea to a semantics is quite intuitive. This brings about immediate questions regarding the *Or* and *Rational Monotonicity* postulates: typically, the attribute logic of FCA does not have the expressivity for negation or disjunction, which are central to these postulates. To this end, it needs to be investigated how this expressivity can be introduced to FCA, and what their ramifications may be.

To make this explicit, the aim is to enable a non-monotonic implication $A \rightsquigarrow B$ between attribute(s). This implication would be valid in a formal context if it is respected by the most typical (i.e., preferred, best) object intents which are supersets of A. It would then be of interest to investigate how we might naturally induce an ordering over objects. An idea to be explored further is to supplement a formal context with a defeasible knowledge base, $\mathcal{K}$, which acts as an authoritative view over the domain. The statements in $\mathcal{K}$ are then ranked, which in turn provides a way to assign each object a rank.

Recalling the discussion in Section 1, one of the uses for FCA was mining (classical) implications as a method of discovering relationships between attribute(s). We would aim to introduce an parallel notion for mining defeasible implications from a formal context. This suggests developing a notion of defeasible entailment from a formal context; specifically, rational closure in FCA. The intuition for what this means is quite clear: given a ranking of objects, can we find a closed set of defeasible implications which are valid in the context. While the rational closure already has a clear definition, part of this work would involve translation from the setting of propositional logic to the attribute logic of FCA. Alongside this task, we intend to evaluate complexity of the algorithm in its new setting.

Echoing the final points of Section 2.1, in certain cases it can be beneficial to use the set of (classical) implications of a formal context to derive the set of concept intents. We are interested in exploring this in relation to the rational closure of a formal context, and whether it leads to, or if we can otherwise create, a notion of a *rational concept*. Rational concepts are intended to convey the sense of a "typical" version of some concept. We return to the example of mammals from Section 3: the rational concept for mammal might describe the attributes we (rationally) associate mammals with, such as giving birth to live young.

It is perhaps a good idea to clarify that the intention is not to create concepts the necessarily align with human psychology or intuition (see [1] for this kind of approach). Rather, the concepts should coincide with the typicality relation encoded by the ranking on objects and subsequent rational closure.

The existence of rational concepts raises another point of interest: if a similar ordering of sub/super concept(s) can be induced over rational concepts—ideally, this should be the case—then investigating the resulting structure would be of great interest.

In FCA, the set of all implications valid in a formal context contains many redundant implications (i.e., if $A \to C$ then via monotonicity $A \cup B \to C$). This has led to the development of algorithms to find the *canonical basis* of a formal context. That is, the smallest set of implications from which all other implications follow [6]. Although not central to the scope of this work, and perhaps not a question we will attempt to solve, attempting to find an analogous basis for defeasible implications would be an endeavour which may provide some utility to the larger KLM community.

5 RELATED WORK

Outside of [4] we are not aware of any direct attempts to introduce non-monotonic reasoning of any kind to FCA. In [4], the authors attempt to introduce KLM-style defeasible reasoning on the sub/super concept relation in FCA. Specifically, where in classical FCA $C_1 \vdash C_2$ indicates that "all the objects in concept C_1 are in concept C_2", their aim is to introduce a non-monotonic operation $C_1 \mid\sim C_2$ meaning "the *typical* objects in C_1 are in C_2". They restrict their attention to cumulative reasoning, and thus avoid issues around disjunction and negation.

In addition, there is some work which runs parallel to broad ideas around "typical concepts". [10] introduces *Rough Concept Analysis*, a merging of rough set theory and FCA that uses equivalence classes on objects to define upper and lower-bound approximations of concepts. [19] investigates an expansion which enables "rough concepts" to be defined not only by objects.

6 ETHICAL, PROFESSIONAL, AND LEGAL ISSUES

This work is strictly of theoretical nature, which has no identifiable potential to bring about risks to populations, environments, or others. No personal data will be collected or stored. Copyrighted material will not be used, and all other sources will be referenced appropriately.

7 PROJECT PLAN

7.1 Risks, Timeline, & Deliverables

For the risks, deliverables, and Gantt chart, please refer to Section 8.1, Section 8.2, and Section 8.3 in the Appendix.

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8 APPENDIX

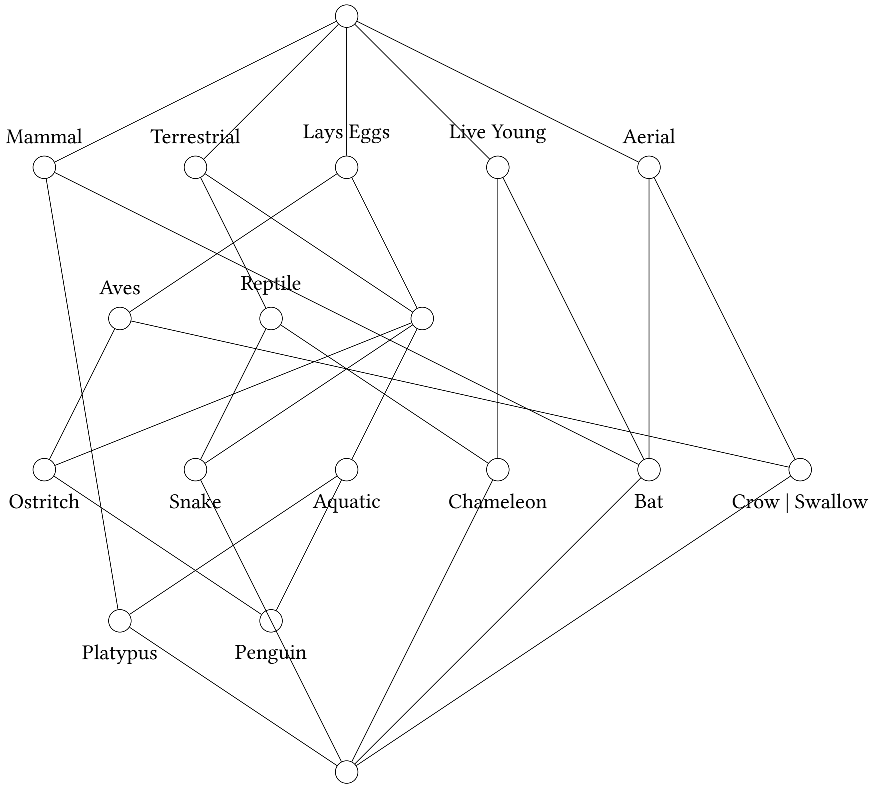

Figure 1: Concept lattice for the formal context in Table 1.

	Aves	Mammal	Reptile	Aerial	Aquatic	Terrestrial	Live Young	Eggs
Bat		×		×			×	
Crow	×			×				×
Ostrich	×					×		×
Penguin	×				×	×		×
Platypus		×			×	×		×
Snake			×			×		×
Swallow	×			×				×
J. Chameleon			×			×	×	

Table 1: A formal context describing animals (objects) and some of their features (attributes)

8.1 Risks

Risk Description	Probability	Impact	Mitigation	Monitor	Manage
Not being able to achieve the research goals I set out to, (i.e. showing it cannot be done, or failing otherwise)	0.4	0.5	Ensure realistic goals, and that each phase of research is scrutinised to avoid realising error much later on	Keep track of progress	In case of showing this method does not work, recognition that positive results are not necessarily required. Otherwise, consult with supervisors on how the scope may be redefined.
Novel contributions of my work are published elsewhere beforehand	0.2	0.3	Ensure that I try publish results as soon as it is ready. Pay attention to upcoming conferences and submission dates.	Make sure I keep up to date with FCA publications to observe research trends.	Since this work is a MSc, there is no requirement for novelty. Ensure that I mention these results alongside my own.
Problem is too difficult/I get stuck on something.	0.4	0.2	Ensure frequent discussion with supervisors and lab-mates, asking for help when things are not clear. In addition, refer back to foundational texts to make sure my understanding of the broader field(s) is good.	A sign of not understanding things fully would be frequently making errors; monitor work for signs of this.	If I reach a point where I am not understanding the work, go back to the last point where I feel a solid grasp of the concepts, and move forward slowly.
Suffering from fatigue, or burnout.	0.4	0.6	Ensure I develop a healthy work routine, which allows me to do things outside of work, while easing fears that I'm not doing enough.	Generally check-up on myself.	If I do begin to feel like this, speak to supervisors about taking a short break, and then return to work later on.
Loss of work/material due to technical issues.	0.1	0.8	Ensure all digital work is backed up to secure location, written work should be copied and digitised.	-	If this does happen, make attempts to recover lost files. Otherwise, reconstruct lost work.
Writing/documentation challenges	0.4	0.3	Start writing early. Break thesis into manageable sections. Maintain good research notes from day one.	Ensure I do regular progress checks. Keep track of references and sources properly.	Seek writing support if needed. Consider writing workshops or peer review groups.

8.2 Deliverables

Task	Start	End	Comments
Proposal	05.10.24	28.10.24	Submit draft one week before deadline
Background Thesis Chapters	01.11.24	31.01.25	Submit rough outlines, and semi-incremental updates for review before deadline.
Novel Results Write-Up	01.02.25	31.04.25	While this process occurs, have meetings/feedback sessions. results
Thesis Consolidation	01.05.25	31.06.25	This should be a more-or-less finished document.
Final Review and Intention to Submit	01.07.25	31.07.25	Final adjustments and corrections

8.3 Gantt Chart

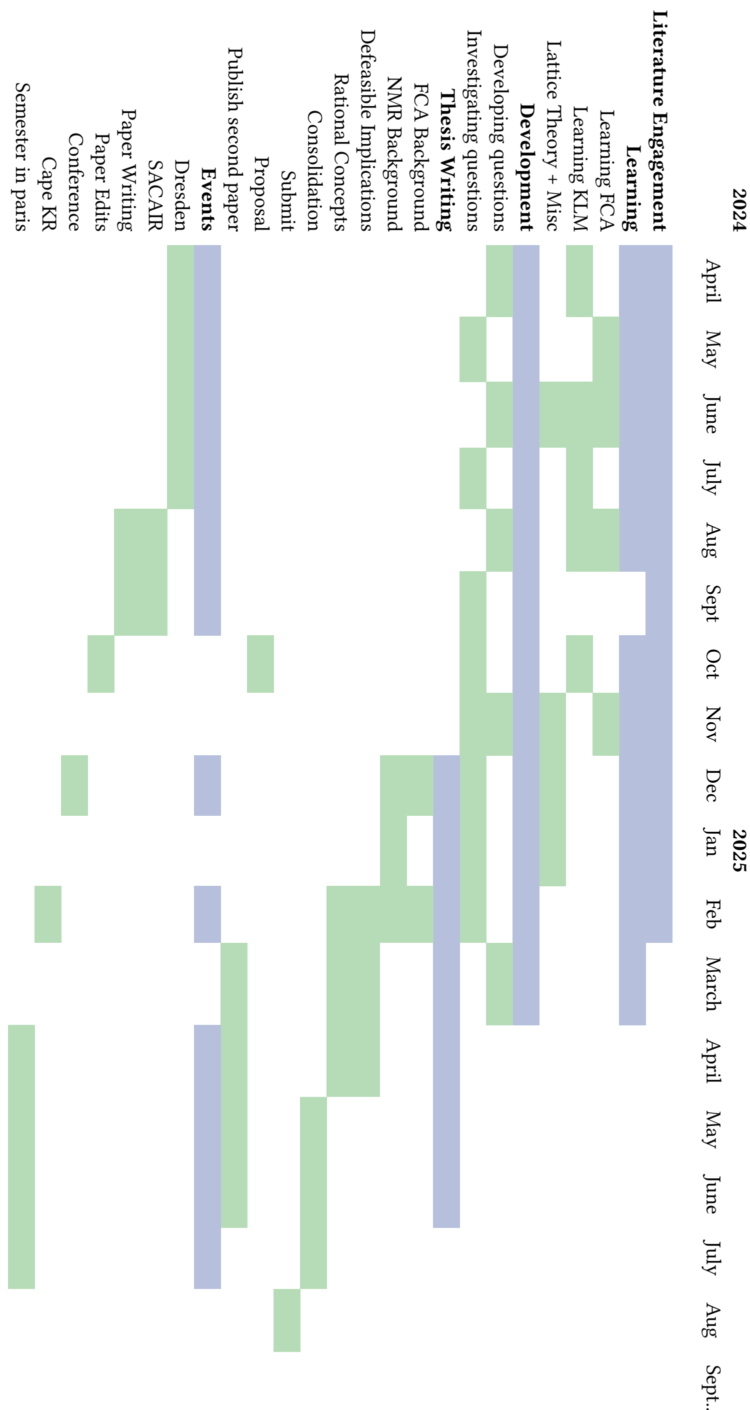

Table 2: Blue indicates main task, while green indicates sub-task