Rational Concept Analysis

Lucas Carr

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University Name

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Acknowledgements

There is a dog

Abstract

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Introduction

Part I

Foundations

Chapter 1

Mathematical Preliminaries

This chapter is intended to serve as a reference point for the more fundemental concepts that are used throughout this dissertation. In particular, this dissertation contains a signficant amount of discussion on *Formal Concept Analysis* and *Preference Relations*; which, in turn, are based on order theory, which is where we begin. The second half of this chapter provides an introduction to basic notions in logic, given in the setting of propositional logic.

1.1 Order and Lattice Theory

Definition 1. A **binary relation** R over two sets X and Y is a set of pairs (x, y) with $x \in X$ and $y \in Y$; and so $R \subseteq X \times Y$. In many cases we express this pair using infix notation, and we write xRy.

Binary relations are not particularly interesting until they satisfy certain properties. We now discuss certain binary relations which occur frequently enough to deserve a distinct name.

Definition 2. A **partial-order** is a binary relation $\preceq \subseteq X \times X$ that satisfies the following properties:

$$\text{(Reflexivity)} \qquad x \preceq x \tag{1.1}$$

$$\text{(Antisymmetry)} \qquad x \preceq y \text{ and } y \preceq x \text{ implies } x = y \tag{1.2}$$

$$\text{(Transitivity)} \qquad x \preceq y \text{ and } y \preceq z \text{ implies } x \preceq z \tag{1.3}$$

for all $x, y, z \in X$.

We write $x \npreceq y$ to indicate that $x \preceq y$ does not hold, and $x \prec y$ for the case where $x \preceq y$ and $x \neq y$. When $x \npreceq y$ and $y \npreceq x$—i.e., that x and y are incomparable—we write $x \| y$ [1]. From a partial-order we can quite easily induce the notion of a *strict partial-order*.

Definition 3. A **strict partial-order** is a binary relation $\prec \subseteq X \times X$ that satisfies:

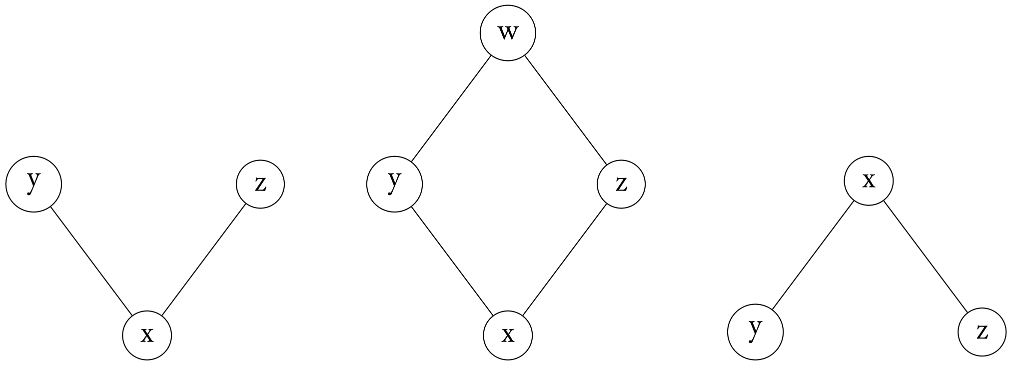
$$\text{(Irreflexivity)} \qquad x \not\prec x \tag{1.4}$$

$$\text{(Asymmetry)} \qquad x \prec y \text{ implies } y \not\prec x \tag{1.5}$$

$$\text{(Transitivity)} \qquad x \prec y \text{ and } y \prec z \text{ implies } x \prec z \tag{1.6}$$

for all $x, y, z \in X$.

An ordered set is a pair $(X, \preceq)$ with X being a set and $\preceq$ being an ordering on X. If $\preceq$ is a partial-ordering, we might then refer to X as a *poset*.



(a) $\preceq_a$ (b) $\preceq_b$ (c) $\preceq_a$

Figure 1.1: Three partial-orders over a set P

Chapter 2

Formal Concept Analaysis

2.1 Basic Notions

Chapter 3

Non-Monotonic Reasoning

Hello there my dog A wfaew

Part II

Rational Concept Analysis

Chapter 4

Defeasible Reasoning in Formal Concept Analysis

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Rational Concepts

Bibliography

[1] B. A. Davey and H. A. Priestley, *Introduction to Lattices and Order*, 2nd ed. Cambridge University Press, 2002.

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