TYPED-UP (IMPORTANT) NOTES 9/30

LUCAS CHEN

1. Cauchy Sequences Proof via inf = sup

Definition: A Cauchy sequence is a sequence $\{x_n\}, x_n \in \mathbb{R}, n \in \mathbb{N}$ s.t. $\forall \epsilon$, $\exists a \in \mathbb{N}$ s.t. $|x_m - x_n| \leq \epsilon$, $\forall m, n \geq a$

Then, given any sequence $\{y_n\}$ we define $\limsup_{n \to \infty} x_n = \inf\{\sup\{x_m : m \geq n\} n \in \mathbb{N}\}$

We define $\liminf$ similarly with the $\inf$, $\sup$ reversed.

If $\liminf_{n \to \infty} x_n = \limsup_{n \to \infty} x_n$ then the lim exists and the sequence is convergent, as established by the conditions below:

We prove that Cauchy sequences satisfy $\liminf = \limsup$.

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